THEORY OF LOCAL TIMES

Hitoshi Kitada
Department of Mathematical Sciences, University of Tokyo
Komaba, Meguro, Tokyo 153, Japan

(accepted on Nov. 30, 1993, for publication in "IL NUOVO CIMENTO," Sect. B)

A model of a stationary universe is proposed. In this framework, time is defined as a local and quantum mechanical notion in the sense that it is defined for each local and quantum mechanical system consisting of finite number of particles. The total universe consisting of infinite number of particles has no time associated. It is a stationary bound state of the total Hamiltonian of infinite degrees of freedom. The quantum mechanics and the theory of general relativity are consistently united in this context if one uses this notion of local and quantum mechanical time. As one of the consequences, the Einstein-Podolsky-Rosen paradox is resolved. The Hubble's red shift is explained as a consequence of general relativity which is consistent with the quantum mechanics. This does not require us to argue on the beginning nor the end of the universe. (Conjectures (1), (4) in Section 10.) The universe just exists without time.

TABLE OF CONTENTS

INTRODUCTION
1. WHAT IS TIME ?
2. GLOBAL AXIOM
3. LOCAL AXIOMS
4. LOCAL TIMES
5. RELATIVITY
6. A PARADOX OF CYCLOTRON
7. EPR PARADOX
8. OBSERVATION
9. CONCLUDING DISCUSSIONS
10. SOME CONJECTURES
References

INTRODUCTION

As stated in the abstract, the main theme of the paper is to present one possible consistent unification of the quantum mechanics and general relativity. This is stated intentionally with anticipating the naiverefutation that the Euclidean geometry which quantum mechanics follows and the nonflat Riemannian geometry which the relativity follows can never be united consistently.

Our trick of the consistent unification of these two theories is to adopt a ten dimensional vector bundle $X \times R^6$(the reason R^6 is adopted instead of R^4 will be touched below) as the total physics space, where the base space X and the fibre R^6 are mutually orthogonal. The quantum mechanics is seton the Euclidean space R^6 and the relativity theory on the curved Riemannian space X. Each point $(t, x) \in X$ is correlated to the *center* of mass of the local system consisting of finite number of (quantum mechanical) particles, and these centers of mass are considered as the *classical* particles. These classical particles are regarded as moving following the general relativity in the Riemannian manifold X on the one hand, and the particles inside the local systems are regarded as moving following thequantum mechanics on the other hand.

In this sense each point (t, x) of the base Riemann space X of the vector bundle $X \times R^6$ corresponds to the local system consisting of finite number of particles which follow the quantum mechanics in each fibre R^6.

Because the fibre R^6 where quantum mechanics holds and the base space X where the relativity holds are mutually orthogonal in the total space$X \times R^6$, it can be shown that the principles of quantum mechanics and the principles ofgeneral relativity are united consistently in this formulation, with the aid of the notion of quantum mechanical local time introduced after Section 1 below.

The reader may ask: There are many sub local systems in a local system$(H, \mathcal{H})$, and the centers of mass of these sub local systems follow the classical relativity. But inside the local system $(H, \mathcal{H})$, the quantum mechanics holds. Then what mechanics do these sub local systems follow? The answer is

> These sub local systems follow the classical relativity *as far as* the observer observes them in accordance with *the observer's* own time, but if the *time* of the system $(H, \mathcal{H})$ itself is adopted as the reference time, then the local system $(H, \mathcal{H})$ follows the quantum mechanics.

Now here enters the notion of local time, which is the main ingredient of our consistent unification of the quantum and relativity theories.

Our starting point is the recognition that the time t is given by the ratio $|x|/|v|$ of the position x and the motion (velocity) v.This standpoint is fully explained in Section 1. WHAT IS TIME? This formulation is justified by the result (Theorem 1) of the many particle scattering theory. Because of this definition of time, the Euclidean space of quantum mechanics becomes R^6 of (x, v) instead of R^4 of (t, x), and the usual four dimensional structure R^4 is recovered as an approximation through the uncertainty principle stated in Section 4 after Definition 3.

Like this, the quantum mechanics controls the mechanics inside each local system, and the relative motion of the centers of mass of these local systems follows the general relativity. In actual observations or experiments, the observer's time must be transformed into the local time ofthe observed local system. The rules of this transformation are given by the covariance principle and the equivalence principle of the general relativity. An actual example of this kind of explanation of the relativistic quantum phenomena is given in Section 8. OBSERVATION.

As consequences, the EPR paradox is explained without contradiction, and the Hubble's red shift is explained even though our model is constructed on the basis of a stationary universe. As another result, the dark matter is unnecessary in our theory to explain the stability of galaxies, clusters of galaxies, and so on.

1. WHAT IS TIME ?

This question seems to have a history as long as that of the mankind itself has. In physics literature the time was defined by Newton as a kind of absolute notion in the seventeenth-century. In the first decade of the twentieth-century, a reflection on the notion of time was given by Einstein in his theory of special relativity. Later Einstein gave a more profound reflection on space-time in his theory of general relativity. Even though these reflections required us a reconstruction of our notion of space-time on a very deep level, time together with space is still considered to give a reference frame according to which one measures the physical quantities like positions, motions, velocities, and so on: Matters determines the space-time structure of the universe according to the theory of general relativity. Nonetheless the space-time remains as a frame according to which the position of each matter is determined.

What we propose in this paper is a different point of view on the notion of time based on the following simple but overlooked observation: the time does not appear until it is measured by some equipments, *clocks*. We need tools to know the time. This situation is different from the perception of the positions and motions, which are recognized directly by our senses. Even when we use some tools like a rule to measure the length of a thing, what we actually do is to *see* which scales of the rule coincide with the both sides of the thing. The fundamental recognition done here is the perception of this coincidence, which cannot be done without our senses. The positions and motions are recognized through our senses in this sense. On the contrary, if one reflects the measurement procedure of time by clocks, one notice that he or she compares several motions or movements of matters, and takes the quotient of position and velocity. In fact the usual (analog) clock measures the time by the motion of its hands. We look at the hands, and recognize that one second passes if the second hand 'moves' one 'scale'. We do not measure the time directly by our senses, but we know the time by perceiving the positions and motions of the hands of clocks. In this sense the time is neither a quantity nor a frame given a priori. What exist first are the positions and movements of the matters relative to our own position. The perception of the positions and motions indicates an introduction of the common parameter in each system of matters consisting of *finite* number of particles. This parameter is called *time* and it is a local notion by nature. This recognition is our starting point.

In the twentieth-century several theories of universe have been investigated. Most of these are related with the theory of (general) relativity. Manytrials have been made to "quantize" the theory of relativity. Butit seems that none of them can be said to have succeeded. (See Ashtekar's Introduction of Ashtekar and Stachel [**?**] and Streater's paper inBrown-Harré [**?**] for the history of many trials including the recent ones.See also Glimm-Jaffe [**?**] for the point of view that the quantum field theoryis successful to a *certain* extent.) One category of such theories is the stationary theories of universe. Another category consists of non-stationary ones. The typical one of the latter category is the so-called "Big Bang" theory. Tracing in this direction, Hawking introduced imaginary time for instance.

In this article we introduce one stationary model of universe. This enables us to define the notion of (local) times. According to our theory, there exists the total universe which has no time associated. It is a stationary bound state of the total Hamiltonian with infinite degrees of freedom. Our theory is quantized from the beginning in this context. Relative coordinates and momenta are postulated to exist according to our reflection of the measuring procedure of time stated above.

From these postulates we define *time* as a local (or 'glocal') and quantum mechanical quantity. Our theory may also be called a quantization of time in this sense. We further show that our theory is consistent with the theory of general relativity. According to the so-called EPR paradox (see Redhead [**?**], for instance), the quantum mechanics and the theory of relativity cannot be consistently united, because the former denies the local causality which is a consequence of the latter, as far as we suppose that the quantum mechanics is complete in the sense prescribed below in the argumentof EPR paradox in Section 7. In this sense the unification ofthese two theories is impossible on the same level or on the same 'plane'. In our unification of these theories, we 'orthogonalize', so to speak,these two theories, or set these theories on mutually 'orthogonal planes', with the introduction of local times. We do not try to quantize the theory of relativity itself, as it has been tried in many theories of quantum fields. We just leave the theory of general relativity as aclassical theory. What we actually do is to show that our theory of local quantum mechanical times is consistent with the classical relativity.

Our theory is basically the non-relativistic quantum mechanics 'orthogonalized' to the general relativity. The actual way of this orthogonalization is shown in Section 5. We take the standpoint that**the nature follows the quantum mechanics intrinsically, and the relativistic effects only appear associated with the observation**.Namely, the gravitation is nothing but an outlook; it is no intrinsicnature of universe according to our theory. Note that this standpoint is different from the standpoint of quantum gravity being discussed recently, e.g., in Ashtekar and Stachel[**?**], where it is implicitly supposed that the quantum andgravitational aspects of universe are possible to be discussed on thesame 'plane' or on the same level. Our theory differs from this traditional standpoint.

The point of our theory of orthogonalization of the quantum mechanics and the relativity theory is summarized as follows:

> The *observer's* coordinate system (t^0, x^0) of R^4 is *independent of* the coordinate system (t^1, x^1) of the *observed* system, according to our definition of local times t^1, t^0 (Section 4). Thus the observed system can follow the quantum mechanics*with respect to* the coordinate system (t^1, x^1), *as well as* it can follow the classical relativity *with respect to* the coordinate system (t^0, x^0). In this sense, any local system can follow the classical relativity *when observed*, and at the same time, it can follow the quantum mechanics *inwardly* or *intrinsically.*

This is our key of the consistent unification of quantum mechanics andgeneral relativity.

Accordingly any local system behaves in classical mechanical way as well as in quantum mechanical way, depending on the choice of coordinate systems.For the relation between these two looks of the behavior of local systems,we make the following fundamental assumption:

> **The observer can see only the motions of centers of mass of local systems, and these motions are observed following the classical relativity. The quantum effects inside the local systems are unobservable directly, but can be deduced from these classical observations of the sub local systems.**

The way of deduction of the internal quantum mechanics from the classical observations which we propose is discussed in Section 8 as a procedure of observation.

The unification of quantum and relativity theories in the above form means that we abandon the *relativistic* quantum field theory.A positive reason which supports this abandonment isthat there seems to be only a trivial model in the axiomatic quantum field theory in actual four dimensional space-time (see Streater's paper in [**?**]). Owing to this abandonment of quantum field theory, we are free from such problems as renormalizations, divergence problems, and so on, which constitute the fundamental difficulties of quantum field theories. The classical fields are not quantized *relativistically* in our theory. They are left as classical notions. The quantum fields are auxiliary tools, which will be introduced to treat the creation and annihilation phenomena (see conjecture (2) below). The quantum fields are confined to the explanation of the nonrelativistic quantum phenomena.

Summarizing, the most important result of the paper is that there exists at least one consistent theory which unites the *non*relativistic quantum mechanics and the general relativity. (As an example of other trials in this direction, we refer to Prugovečki [**?**], where geometro-stochastic approach is adopted for the problem of the unification of quantum theory and general relativity.) Even in our approach,we can give an explanation of the so-called *relativistic* quantum phenomena. These phenomena will be explained as the consequences of the observation activities: the actual experimental data are different from their *true* quantum mechanical values because of the relativistic effects of observations, and the observer has to calculate the true values from his experimental data through some relativistic considerations. This point will be discussed in Section 8 concerning the observation. The necessity of considering of high energy particle physics is not strong in our context, since the universe does *not* begin via something like the Big Bang. However, some of the high energy phenomena related with the relativistic quantum mechanics can be explained in our context. Hopefully, a full explanation of these phenomena would be given in the future.

2. GLOBAL AXIOM

As a foundation of the theory of times, we adopt a model of stationary universe. Differently from the usual stationary theories where time of the universe is assumed to exist a priori, our universe ϕ, which is an element of a Hilbert space $\mathcal{U}$, has no time associated. It is assumed as a stationary bound state of a Hamiltonian H of infinite degrees of freedom.In this sense our universe ϕ is quantized from the beginning:

Axiom 1. There exist a (separable) Hilbert space of (possible) universes

$$\mathcal{U} = \{\phi\} = \bigoplus_{n=0}^{\infty}\left(\bigoplus_{\ell=0}^{\infty}\mathcal{H}^n\right) \qquad (\mathcal{H}^n = \underbrace{\mathcal{H}\otimes\cdots\otimes\mathcal{H}}_{n \text{ factors}}) \tag{1}$$

with $\mathcal{H}$ being a separable Hilbert space, a set of selfadjoint operators (observables) $\mathcal{O} = \{A\}$ of the form $A\phi = (A_{n\ell}\phi_{n\ell})$ for $\phi = (\phi_{n\ell}) \in \mathcal{U}$, and a selfadjoint operator $H \in \mathcal{O}$ in $\mathcal{U}$ such that for some $\phi \in \mathcal{U} - \{0\}$ and $\lambda \in R^1$

$$H\phi = \lambda\phi \tag{2}$$

in the following sense: There exists an infinite matrix $(\lambda_{n\ell})$ of real numbers such that $H_{n\ell}\phi_{n\ell} = \lambda_{n\ell}\phi_{n\ell}$ for each $n \geq 1$, $\ell \geq 0$ and $\lambda_{n\ell_n} \to \lambda$ as $n \to \infty$ along any ℓ_n such that $F_n^{\ell_n} \subset F_{n+1}^{\ell_{n+1}}$. Here F_n is a finite subset of $\mathbf{N} = \{1, 2, \cdots\}$ with $\sharp(F_n) =$ (the number of elements in F_n) $= n$ and $\{F_n^\ell\}_{\ell=0}^\infty$ is the totality of such F_n.

H is an infinite matrix $(H_{n\ell})$ of selfadjoint operators $H_{n\ell}$ in $\mathcal{H}^n$. Axiom 1 asserts that this matrix converges in the sense of (2).

We should repeat our standpoint here. Our purpose in this paper is to construct at least one consistent theory which unites the quantum mechanics and the relativity. As a mathematical problem, it is of course possible to consider the propagation e^{-iTH} along some global time T in the grand universe $\mathcal{U}$. However, this does not seem to lead to any consistent unification of quantum mechanics and relativity. Axiom 1 is adopted so that it gives a starting point of our consistent unification.

3. LOCAL AXIOMS

What we can perceive in the world are the (local) positions and movements of other matters relative to our own position. Comparing various movements, we determine time as a ratio of the positions and the movements (velocities).So there exist first the local and relative positionsand velocities or momenta. This fact is formulated as in the following axiom.

Axiom 2. Let $n \geq 1$ and F_{n+1} be a finite subset of$\mathbf{N} = \{1, 2, \cdots\}$ with $\sharp(F_{n+1}) = n+1$. Then for any $j \in F_{n+1}$, there exist selfadjoint operators $X_j = (X_{j1}, X_{j2}, X_{j3})$, $P_j = (P_{j1}, P_{j2}, P_{j3})$ in $\mathcal{H}^n$ and constants $m_j > 0$ such that

$$[X_{j\ell}, X_{km}] = 0, \qquad [P_{j\ell}, P_{km}] = 0, \qquad [X_{j\ell}, P_{km}] = i\delta_{jk}\delta_{\ell m}, \tag{3}$$

$$\sum_{j \in F_{n+1}} m_j X_j = 0, \qquad \sum_{j \in F_{n+1}} P_j = 0. \tag{4}$$

By the Stone-von Neumann theorem Axiom 2 also specifies the space dimension (see Abraham-Marsden [?], p.452). We identify $\mathcal{H}^n$with $L^2(R^{3n})$ in the following.

What we want to mean by $H_{n\ell}$ $(n, \ell \geq 0)$ in Axiom 1 is the $N = n+1$ body Hamiltonian in the usual quantum mechanics. For the local Hamiltonian $H_{n\ell}$ we thus make the following postulate.

Axiom 3. Let $n \geq 0$ and F_N $(N = n+1)$ be a finite subset of $\mathbf{N} = \{1, 2, \cdots\}$ with $\sharp(F_N) = N$.Let $\{F_N^\ell\}_{\ell=0}^\infty$ be the totality of such F_N. Then the Hamiltonians$H_{n\ell}$ $(\ell \geq 0)$ are of the form

$$H_{n\ell} = H_{n\ell 0} + V_{n\ell}, \qquad V_{n\ell} = \sum_{\alpha=(i,j), 1 \leq i < j < \infty, i,j \in F_N^\ell} V_\alpha(x_\alpha) \tag{5}$$

on $C_0^\infty(R^{3n})$, where $x_\alpha = x_i - x_j$ with x_i being the position vector of the i-th particle, and $V_\alpha(x_\alpha)$ is a real-valued measurable function of $x_\alpha \in R^3$ which is $H_{n\ell 0}$-bounded with $H_{n\ell 0}$-bound of $V_{n\ell}$ less than 1. $H_{n\ell 0} = H_{(N-1)\ell 0}$ is the free Hamiltonian of the$N$-particle system. The concrete form is expressed as in [?], (1.4), if one use clustered Jacobi coordinates.

This axiom implies that $H_{n\ell} = H_{(N-1)\ell}$ is uniquely extended to a selfadjoint operator in $\mathcal{H}^n = \mathcal{H}^{N-1} = L^2(R^{3(N-1)})$ by theKato-Rellich theorem.

4. LOCAL TIMES

For the N-body Hamiltonian $H_{N-1} = H_{n\ell}$ $(N = n+1)$ the following Theorem 1 is known [?] to hold under suitable assumptions on the pair potentials (Assumption 1 in [?]) and on the decay property of subsystem eigenvectors (Assumption 2 in [?]).

We here follow the notation and conventions in [?], [?] for the N-body quantum systems. In particular $H_b = H_{(N-1)b} = H_{N-1} - I_b = H_{n\ell}^b + T_{n\ell b} = H^b + T_b$ is the truncated Hamiltonian for the cluster decomposition $1 \le |b| \le N$, and P_b^M is the M-dimensionalpartial projection of the eigenprojection $P_b = P_{H^b}$ associated withthe subsystem H^b, i.e., P_b is the orthogonal projection in $\mathcal{H}^b = L^2(R^{3(N-|b|)})$ onto the eigenspace of H^b. q_b is the velocity conjugate to the intercluster coordinates x_b. We define for a k-dimensional multi-index $M = (M_1, \cdots, M_k)$, $M_j \ge 1$,

$$\hat{P}_k^M = \left(I - \sum_{|b|=k} P_b^{M_k}\right) \cdots \left(I - \sum_{|d|=2} P_d^{M_2}\right)(I - P^{M_1}), \qquad k = 1, \cdots, N-1, \tag{6}$$

where $P^{M_1} = P_a^{M_1}$ with $|a| = 1$, and fora $|b|$-dimensional multi-index $M_b = (M_1, \cdots, M_{|b|-1}, M_{|b|}) = (\hat{M}_b, M_{|b|})$

$$\tilde{P}_b^{M_b} = P_b^{M_{|b|}} \hat{P}_{|b|-1}^{\hat{M}_b}, \qquad 2 \le |b| \le N. \tag{7}$$

It is clear that

$$\sum_{2 \le |b| \le N} \tilde{P}_b^{M_b} = I - P^{M_1}, \tag{8}$$

provided that the component M_k of M_b depends only on the number$k$ but not on b. In the following we use such M_b's only. Under these circumstances, the following is known to hold.

Theorem 1([?]). Let Assumptions 1 and 2 in [?] be satisfied. Let $f \in \mathcal{H}^{N-1}$. Then there are a sequence $t_m \to \pm\infty$ (as $m \to \pm\infty$) and a sequence M_b^m of multi-indices whose components all tend to∞ as $m \to \pm\infty$ such that for all cluster decompositions b, $2 \le |b| \le N$, $\psi \in C_0^\infty(R^1)$, and $\varphi \in C_0^\infty(R^{3(|b|-1)})$

$$\left\| \frac{|x^b|^2}{t_m^2} \tilde{P}_b^{M_b^m} e^{-it_m H_{N-1}} f \right\| \to 0, \tag{9}$$

$$\|\{\psi(H_{N-1}) - \psi(H_b)\} \tilde{P}_b^{M_b^m} e^{-it_m H_{N-1}} f\| \to 0, \tag{10}$$

$$\|\{\varphi(x_b/t_m) - \varphi(q_b)\} \tilde{P}_b^{M_b^m} e^{-it_m H_{N-1}} f\| \to 0 \tag{11}$$

as $m \to \pm\infty$.

Definition 1. Let $\phi = (\phi_{n\ell})$ with$\phi_{n\ell} = \phi_{n\ell}(x_1, \cdots, x_n) \in L^2(R^{3n})$be the universe in Axiom 1. We define $\mathcal{H}_{n\ell}$ as the sub-Hilbert space of $\mathcal{H}^n$ generated by the functions $\phi_{mk}(x^{(\ell)}, y)$ of $x^{(\ell)} \in R^{3n}$ with regarding $y \in R^{3(m-n)}$ as a parameter, where $m \ge n$, $F_{n+1}^\ell \subset F_{m+1}^k$, and $x^{(\ell)}$ is the (relative) coordinates of $n+1$ particles in F_{n+1}^ℓ. $\mathcal{H}_{n\ell}$ is called a *local universe* of ϕ. $\mathcal{H}_{n\ell}$ is said to be non-trivial if $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \ne \{0\}$.

The total universe ϕ is a single element in $\mathcal{U}$. The local universe $\mathcal{H}_{n\ell}$ may be richer. This isbecause we consider the subsystems of the universe consisting of a finite number of particles. These subsystems receive the influence from the other particles of infinite numberoutside the subsystems, and may vary to constitute a non-trivial subspace$\mathcal{H}_{n\ell}$.

Definition 2. The restriction of H to $\mathcal{H}_{n\ell}$ is also denoted by the same notation $H_{n\ell}$ as the (n,ℓ)-th component of H. We call the pair $(H_{n\ell}, \mathcal{H}_{n\ell})$ a local system. The unitary group $e^{-itH_{n\ell}}$ $(t \in R^1)$ on $\mathcal{H}_{n\ell}$ is called the *proper clock* of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, if $\mathcal{H}_{n\ell}$ is non-trivial: $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \neq \{0\}$. (Note that the clock is defined only for $N = n+1 \geq 2$, since $H_{0\ell} = 0$.) The universe ϕ is called *rich* if $\mathcal{H}_{n\ell}$ equals $\mathcal{H}^n = L^2(R^{3n})$ for all $n \geq 1$, $\ell \geq 0$. For a rich universe ϕ, $H_{n\ell}$ equals the (n,ℓ)-th component of H.

The formula (11) indicates that t_m is asymptotically equal to $\pm|x_b|/|q_b|$ as $m \to \pm\infty$, independently of the choice of cluster decompositions b. This is precisely the actual procedure of measuring the time t_m in the mechanics. The implication of this theorem is therefore interpreted as follows: If one 'measures' the time of a state $f \in (I - P_{H_{(N-1)\ell}})\mathcal{H}_{(N-1)\ell} - \{0\}$ in the local system $(H_{(N-1)\ell}, \mathcal{H}_{(N-1)\ell})$ by the associated proper clock $e^{-itH_{(N-1)\ell}}f$, namely if one measures the quotient $\pm|x_b|/|q_b|$ of the scattered particles which are regarded as moving almost in a steady velocity, then that time is asymptotically equal to the parameter t_m in the exponent of $e^{-it_mH_{(N-1)\ell}}f$ as $m \to \pm\infty$. In this sense t_m is interpreted as the quantum mechanical proper time of the local system $(H_{n\ell}, \mathcal{H}_{n\ell}) = (H_{(N-1)\ell}, \mathcal{H}_{(N-1)\ell})$, if $(I - P_{H_{(N-1)\ell}})\mathcal{H}_{(N-1)\ell} \neq \{0\}$.

Definition 3. The parameter t in the exponent of the proper clock $e^{-itH_{n\ell}} = e^{-itH_{(N-1)\ell}}$ of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ is called the (quantum mechanical) *proper time*or *local time* of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, if $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \neq \{0\}$.This time t is denoted by $t_{(H_{n\ell},\mathcal{H}_{n\ell})}$ indicating the local system under consideration.

This definition is the one reverse to the usual definition of the motion or dynamics of the N-body quantum systems, where the time t is given *a priori* and the motion of the particles is defined by $e^{-itH_{(N-1)\ell}}f$ for a given initial state f of the system.

We notice here that there are two possible 'directions' or'orientations' of time t; the one where t increases to $+\infty$, and the other where t decreases to $-\infty$. So far discussed, one can choose an arbitrary orientation from them depending on each local system. However the Axiom 4 (the general principle of relativity) which will be introduced below determines the orientation of time to be common to *all* local systems (see Hawking-Ellis [?], p.181).

Time is thus defined only for the local systems$(H_{n\ell}, \mathcal{H}_{n\ell})$ and is determined by the associated proper clock $e^{-itH_{n\ell}}$. Therefore there are infinitely many times $t = t_{(H_{n\ell},\mathcal{H}_{n\ell})}$ each of which is proper to the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$. In this sense time is a local notion. There is no time for the total universeϕ in Axiom 1, which is a (stationary) bound state for the totalHamiltonian H.

This local time is an approximate one in double senses: First t_m is only *asymptotically* equal to $\pm|x_b|/|q_b|$ as $m \to \pm\infty$. This fact explains the so-called principle of uncertainty in our context. In the usual explanation, the position x_b and the velocity q_b or the momentum p_b cannot be determined in equal accuracy.According to our theory, this is rephrased as follows: The time t cannot be determined accurately, even if x_b and q_b could be determined precisely.It is only determined in some *mean* sense as in (11). (See Enss [?].Also see Dereziński [?], Sect.5, for more precise inequalities which holdfor x_b, q_b and the local time t under some decompositions of the phase space other than (8).) Second the local Hamiltonian $H_{n\ell}$ is not the total Hamiltonian H. Or rather, the time arises from this approximation of H by $H_{n\ell}$. This approximation may make $\mathcal{H}_{n\ell}$ non-trivial, and the clock $e^{-itH_{n\ell}}$ can be defined as in Definition 2 owing to $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \neq \{0\}$. On the contrary the total universe ϕ has no associated clock and time, since $(I - P_H)\phi = 0$.

Our theory of local times further implies in particular that local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ cannot be decomposed into pieces and are *mutually independent*: It is true that for a subset $F_{N'}^{\ell'} \subset F_N^{\ell}$ with $N' < N$, $H_{(N'-1)\ell'}$ is a subsystem Hamiltonian of $H_{(N-1)\ell}$. However the correspondent times $t_{N'\ell'}$ and $t_{N\ell}$ are measured mutually independently as in Theorem 1. Namely the clocks $e^{-itH_{(N'-1)\ell'}}$ and $e^{-itH_{(N-1)\ell}}$ are different. More precisely speaking, the base Hilbert spaces $\mathcal{H}_{(N'-1)\ell'}$ and $\mathcal{H}_{(N-1)\ell}$ have different representations $L^2(R^{3(N'-1)}; N', \ell')$ and $L^2(R^{3(N-1)}; N, \ell)$ in general even on the common configuration space $R^{3(N'-1)}_{x(\ell')}$. Thus the corresponding $|x'_b|/|q'_b|$ and $|x_b|/|q_b|$ are not correlated in general. The same is true for arbitrary two different local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$.

The present theory differs from most of the existing theories of universe in this point. They start from the existence of particles or material points obtained by dividing matters into pieces infinitesimally. In our theory the local systems are regarded as the generic points oras the classical particles and cannot be divided further, because any divisions vary the associated space-time correspondingly. In this sense each local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ is a 'glocal' existence: It is neither a local thing nor a global one.

We have defined the (local) time $t = t_{(H_{n\ell}, \mathcal{H}_{n\ell})}$ for each local system $(H_{n\ell}, \mathcal{H}_{n\ell})$. This time t satisfies Theorem 1-(11). If one regards the time t as a *given* quantity, this fact is interpreted as follows: In each local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, the physics follows the quantum mechanics, i.e. follows the Schrödinger equation.

Our definition of times is consistent with the theory of (general) relativity of Einstein. Our (quantum mechanical) proper time of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ can be regarded as the quantum mechanical correspondent to the classical proper time in the theory of relativity.Within the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ the velocity q_b can be arbitrarily large. This is because one uses the propertime of the system $(H_{n\ell}, \mathcal{H}_{n\ell})$. If one measures the velocity of other systems from one's own system using the associated proper time, then those other systems move in accordance with the relativity theory.

5. RELATIVITY

For the relative motions of the *centers* of mass of local systems, we postulate the principle of (general) relativity and the principle of equivalence as in Einstein [?].

What should be stated first on our introduction of relativity is that **only** the relative classical motions of the **centers** of mass of local systems are **observable** in our theory. The **internal** quantum mechanical motion within each local system is independent of **observation**, at least at the present stage of our theory till Section 8. In this sense, the internal quantum mechanical motion within a local system is **unobservable**. We postulate Axiom 6 in Section 8,which gives a principle of the **deduction** of the **internal**quantum mechanical motion within each local system from classicalobservations of its **sub** local systems, through certain relativistic considerations.

Following Definition 3 and Axiom 2-(4), we define the local space-time $(t, x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ with $x = (x_1, x_2, x_3) \in R^3$ of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ such that the center of mass of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ is at the *origin* of the space coordinates $x = (x_1, x_2, x_3) \in R^3$. In this sense time $t = t_{(H_{n\ell},\mathcal{H}_{n\ell})}$ is interpreted as the relativistic proper time associated with the center of mass of the system $(H_{n\ell}, \mathcal{H}_{n\ell})$. Once the local space-time $(t, x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ has been defined, one can *observe*the movements of other systems from *this* space-time coordinates. The space-time $(t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$ of other system $(H_{mk}, \mathcal{H}_{mk})$ is defined *independently of* that of one's systems$(H_{n\ell}, \mathcal{H}_{n\ell})$. There is no quantum mechanical correlation between two local coordinate systems $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ and $(t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$ unless one or an observer unites those two systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$ to a single system for his particular purpose of observation.(We remark that the combined or united system existsa priori by Definition 2,independently of the observer's concern.) In this case, the coordinates of the resulting combined system $(H_{pj}, \mathcal{H}_{pj})$ is again independent of those of the subsystems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$.Thus the quantum mechanics which governs the inside of each local system puts no restriction on the relative motions of the centers of mass of local systems, as will be seen in Theorem 2 below. Therefore there is no reason to exclude the classical mechanics in describing the relative motions of the *centers* of mass of other systems *observed* in one's coordinate system $(t, x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$.

Axiom 4. Those laws of physics which controlthe *relative* motions of the *centers* of mass of the*observed* local systems are expressed by the classical equations which are covariant under the change of *observer's* coordinate systems of R^4: $(t, x) = (t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$ to $(t, x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ for any pairs (m, k), (n, ℓ).

It is included in this axiom that one can observe the positions of other systems (i.e., their centers of mass) in his coordinate system (t, x). The relative velocities of the observed systems are then defined as quotients of the relative positions of those systems and the(local and quantum mechanical) time t of one's own or the observer's system. These are our definitions of the measurement procedure of *classical* quantities, which accord with the ordinary (implicit) agreement among physicists where the time is given a priori. (This is the point where we reversed the order of the time and the velocity in our definitions of quantum mechanical times.)

Axiom 5. The coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ associatedwith the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ is the local Lorentz system of coordinates. Namely the gravitational potentials

$g_{\mu\nu}$ for the *center* of mass of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, *observed* in this coordinates $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$, are equal to $\eta_{\mu\nu}$. Here $\eta_{\mu\nu} = 0$ $(\mu \neq \nu)$, $= 1$ $(\mu = \nu = 1, 2, 3)$, and $= -1$ $(\mu = \nu = 0)$.

For the field equation which determines the metric $g_{\mu\nu}$, we refer to Hawking-Ellis [?], p.74 or Friedman [?], p.180.

Axiom 5 implies that for the coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ associated with the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, the principle of constancy of light velocity hold in the following special sense: The light radiated from another system $(H_{mk}, \mathcal{H}_{mk})$ moving in a steady velocity relative to one's own system $(H_{n\ell}, \mathcal{H}_{n\ell})$ propagates through the *flat* region where $g_{\mu\nu} = \eta_{\mu\nu}$, in a constant speedindependent of the velocity of the system $(H_{mk}, \mathcal{H}_{mk})$ relative to one's own system $(H_{n\ell}, \mathcal{H}_{n\ell})$.

Theorem 2. Axioms 4 and 5 are consistent with Axioms 1–3.

Proof. This may be clear by the mutual independency of local systems and the associated times stated at the end of the previous section: We can assume any equations on relative motions, which follow arbitrarily given transformation rules under the change of the coordinates systems between any two local systems, and give arbitrary numbers to $g_{\mu\nu}$, as far as these are consistent with "physics". Whichever equations may one adopt to govern the relative motions of centers of mass of local systems, these equations are consistent with the quantum mechanics that controls the inside of the local systems.

For the sake of clarity, however, we repeat the argument in the previous section. The local coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ is determined only within each local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, through the quantum mechanical *internal* motions of the system. This coordinate system is independent of local coordinate system $(t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$ of any other local system $(H_{mk}, \mathcal{H}_{mk})$. This is due to the mutualindependency of the L^2 representations of the base Hilbert spaces $\mathcal{H}_{n\ell}$ and $\mathcal{H}_{mk}$.

The relativity axioms, Axioms 4 and 5, are concerned merely with the *centers* of mass of local systems $(H_{mk}, \mathcal{H}_{mk})$, *observed* by an observer system $(H_{n\ell}, \mathcal{H}_{n\ell})$ with coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$. This *observer's* coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ is *independent of* the coordinate system $(t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$ of the *observed* system $(H_{mk}, \mathcal{H}_{mk})$, as stated in the previous paragraph. Because of this independency, the system $(H_{mk}, \mathcal{H}_{mk})$ can follow the quantum mechanics (Axioms 1–3) *inside* the system *with respect to* its own coordinate system $(t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$, *as well as* its *center* of mass can follow the general relativity (Axioms 4) or any other given postulates *with respect to* the observer's coordinate system$(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$. This is the case, even if the coordinate system$(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$of the observer coincides with the coordinate system $(t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$of the observed system itself, because the motion of the *center* of mass and the internal *relative* motion of a local system aremutually independent. Therefore the local Lorentz postulate (Axiom 5)of the *center* of mass of the system $(H_{n\ell}, \mathcal{H}_{n\ell})$ also does not contradictthe Euclidean postulates in Sections 2–4 of the *internal*space-time of that system.

In this sense, Axioms 4 and 5 are chosen so thatthe relativity theory holds between the *observed*motions of *centers* of mass of local systems, and *have nothing to do with* the *internal* motion of each local system, which obeys Axioms 1–3. Thus Axioms 4 and 5 are consistent with Axioms 1–3. □

Remark. As stated in INTRODUCTION, theory of local times can be reformulated or redescribed as a kind of vector bundle theory with ten dimensionaltotal space $X \times R^6$ and four dimensional base space X. R^6 corresponds to the Euclidean space of quantum mechanics inside thelocal systems. X is the Riemannian manifold with metric $g_{\mu\nu}$, the gravitational potentials.X corresponds to the classical space-time observed by a fixed observer,which gives the observer's reference frame or coordinates system for measurements of the classical particles = the centers ofmass of other local systems. Theorem 1 due to Enss is interpreted as a contraction procedure of the fibre R^6 to four dimensional space-time R^4 in an approximate context of the uncertainty principle described in Section 4 after Definition 3.Owing to the orthogonality of X and R^6 in the total space$X \times R^6$, the quantum and relativity theories hold in R^6 and in X respectively without mutual contradiction.

This is the mathematical explanation of the consistency of Axioms 1–5, namely the proof of Theorem 2.

This proof describes the way of the 'orthogonalization' of the quantum mechanics and the relativity, as announced in Section 1.

Under Axiom 5, the local time $t_{(H_{n\ell},\mathcal{H}_{n\ell})}$ of the system $(H_{n\ell},\mathcal{H}_{n\ell})$ coincides with the relativistic proper time of the center of mass of the system, because, at the center of mass, the space coordinates $x_{(H_{n\ell},\mathcal{H}_{n\ell})} = 0$.

The relation of the *internal* motion of a localsystem with the motion of *centers* of mass of its *sub*systems will be discussed in a later Section 8 on observation.

Summing up, we have obtained the following physical picture of the universe: The universe ϕ is a stationary bound state of the total Hamiltonian H with infinite degrees of freedom. Times$t = t_{(H_{n\ell},\mathcal{H}_{n\ell})}$ appear only for local systems $(H_{n\ell},\mathcal{H}_{n\ell})$. According to this time t, the physics laws within the local system $(H_{n\ell},\mathcal{H}_{n\ell})$ obey the quantum mechanics, and the physics laws outside the system which govern the relative motions between the centers of mass of local systems obey the classical theory of relativity. These two sorts of view are consistent, because the quantum mechanics inside the local systems puts no restriction on the motions of the centers of mass of local systems owing to the mutual independency of local systems.

6. A PARADOX OF CYCLOTRON

As an illustration of the unification of the quantum mechanics and the relativity in our context, we consider the experiment by a cyclotron. To clarify the point of our argument, we consider an ideal situation.

We suppose that some electrons for example are accelerated by a cyclotron, and can have velocities very near the velocity of light. By this experiment the observer can 'see' many phenomena: the electrons are accelerated to have the velocities near the velocity of light hence some relativistic phenomena occur, and some of the electrons may hita nucleus and they together produce or change into several elementary particles. From these observations, the observer 'knows' the masses, velocities, energies, and so on, of these particles by some reasoning or analysis of the experimental data. In this experiment, the quantum mechanical and relativistic effects look like appearing at the same 'time'. This might be taken a contradiction: As we shall see in the next section, the quantum mechanics yields the nonlocality, which contradicts the local causality deduced from the relativity.

The answer to this problem is as follows: The local causality is only concerned with the *observed* relative motions between the centers of mass of local systems, e.g., between the system $(H_{n\ell}, \mathcal{H}_{n\ell})$ of the electrons under the acceleration, and the system $(H_{mk}, \mathcal{H}_{mk})$ of the stationary nucleus. In this case the *observed* motion between the two systems naturally follows the relativity theory, and the causality holds, according to our theory. When the observer 'watches' the collision of the accelerated electrons and the stationary nucleus, it looks at the inside of the combined local system $(H_{pj}, \mathcal{H}_{pj})$ of the two systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$. Since what can be observed is only the relativeclassical motions of the centers of mass of sub local systems of $(H_{pj}, \mathcal{H}_{pj})$, the observer cannot see the inside of the system $(H_{pj}, \mathcal{H}_{pj})$.However, as we shall discuss in Section 8, one can deduce the internal quantum mechanical motion within $(H_{pj}, \mathcal{H}_{pj})$ from these observations ofsub local systems. This deduction leads one to conclude that the physics*within* the system $(H_{pj}, \mathcal{H}_{pj})$ follows the quantum mechanics, and he sees the quantum mechanical phenomena of collision in $(H_{pj}, \mathcal{H}_{pj})$.

These two kinds of 'observation' are mutually consistent in the sense described in the proof of Theorem 2. The *observed* motion *between* the centers of mass of the two systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$ can follow the classical relativity and the causality, without any contradiction with the nonlocality of quantum mechanics *within* the combined system $(H_{pj}, \mathcal{H}_{pj})$. This consistency which follows from the mutual independencyof the coordinate systems of the observer and the observed local systems is our solution of the unification of the quantum mechanics and the relativity. We shall analyze the problem in more detail in the following sections.

7. EPR PARADOX

Einstein-Podolsky-Rosen [?] arguedthat the quantum mechanics (QM) combined with the locality principle (Redhead [?], p.75):

(L) Elements of reality pertaining to one system cannot be affected by measurements performed 'at a distance' on another system

implies the incompleteness of the quantum mechanics. Here completeness means that

([?]):

(C) Every element of the physical reality must have a counterpart in the physical theory.

This argument is rephrased as follows:

$$\text{QM} \Rightarrow \neg(\text{L}) \text{ or } \neg(\text{C}), \tag{12}$$

which is called *Einstein Dilemma* in Redhead [?].Here QM is the theory of quantum mechanics, and '$\neg$' denotes the negation. We refer to Jammer [?], Selleri [?],Schommers [?] for further references.

Contrary to Einstein-Podolsky-Rosen [?],we adopt the standpoint that the quantum mechanics is completein the sense of (C). Thus the quantum mechanics yields that the nonlocality holds within each local system. (There are arguments that the negation of the Bell inequality implies the nonlocality. Thus in these arguments, the quantum mechanics automatically yields the nonlocality. See d'Espagnat's paper in Schommers [?]. However, there are refutations to this type of arguments. See Redhead [?], Chap.4 for instance.) In this sense, quantum mechanics contradicts the local causality in the relativity theory (see the arguments in Redhead [?], p.75, also see Hawking-Ellis [?], Chap.3). However this situation is not a contradiction in our context of local times. The reason is that, in our theory, the classical theory of relativity is concerned *only* with the relative motions *between* the *centers* of mass of local systems, *observed* in the*observer's* coordinate system (t, x). Thus the local causality is required *only* to the *observed* relative motions of the*centers* of mass of local systems, *not* to the physics *within* the local systems, which is *unobservable* directly according to our fundamental assumption made in Section 1. This allows us to admit the *unobservable* nonlocality *within* each local system, where the quantum mechanics holds consistently with the nonlocality (by (12)).

For the illustration, let us take the well-known example of two photons polarized mutually orthogonally. (See Redhead [?], Chap.3 and Sect.4.5.)Let $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$ be the local systems of each of these two photons, and let $(H_{pj}, \mathcal{H}_{pj})$ be the combined system of the two photons. According to our theory, within this local system $(H_{pj}, \mathcal{H}_{pj})$ of two photons, the interactions propagate with *infinite* speed by the quantum mechanics valid inside the system,as far as the two photons are considered to constitute one local system $(H_{pj}, \mathcal{H}_{pj})$, and are 'observed' from their creation to the measurements of their polarization traced along the proper local time $t_{(H_{pj},\mathcal{H}_{pj})}$. In this sense the nonlocality is no contradiction within the combined local system of the two photons. Only when the two photons are considered as two different local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$ and are *observed* in the observer'scoordinate system, the classical mechanicalspeed of interactions appears and the situation looks like a contradiction. However, since $(H_{pj}, \mathcal{H}_{pj})$, $(H_{n\ell}, \mathcal{H}_{n\ell})$, and $(H_{mk}, \mathcal{H}_{mk})$ are mutually independent quantum mechanically in their respective local times, this situation is not a contradiction: The postulate that the relative motions between the centers of mass of$(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$ observed in the observer's coordinates is governed by classical relativity, hence the local causality holds between the two systems, *does not contradict*, by the mutual independency of the coordinates of these two systems, the fact that the combined system $(H_{pj}, \mathcal{H}_{pj})$ follows the quantum mechanicsin its own local time $t_{(H_{pj}, \mathcal{H}_{pj})}$, according to which the nonlocality holds.

We remark that, in this solution of EPR paradox, no "hidden variables" are used.

8. OBSERVATION

So far we have postulated five Axioms, which describe the fundamental properties of the universe, and we have seen that these axioms are mutually consistent. We turn to the considerations of the observation process by an observer's system.

When the observer sees the universe, the potentials $V_\alpha(x_\alpha)$ of the *observed* systems effective *between* these local systems do not give any *quantum mechanical* affection but should be interpreted as the classical potentials: A local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ is closed when it is considered within its own proper coordinate system $(t_{(H_{n\ell}, \mathcal{H}_{n\ell})},\ x_{(H_{n\ell}, \mathcal{H}_{n\ell})})$, since the time $t_{(H_{n\ell}, \mathcal{H}_{n\ell})}$ is determined only within the system by the associated clock $e^{-itH_{n\ell}}$. Other local systems $(H_{mk}, \mathcal{H}_{mk})$ can influence the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ in the quantum mechanical way only when these are considered by an observer as a single combined system of $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$. Only at this stage the time is common to these two systems,and the quantum mechanical correlation between these systems through pair potentials $V_\alpha(x_\alpha)$ can be discussed on the same scales of space-time. The principle of uncertaintyin the sense stated before within the united system of $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$ holds only at this stage. *Between* the two independent systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$, the physics looks following the classical mechanics for the observer, and the pair potentials $V_\alpha(x_\alpha)$ effective between these two systems operate as the classical ones as in electromagnetism. One system $(H_{n\ell}, \mathcal{H}_{n\ell})$ of these systems playsa role of the observer system, and another $(H_{mk}, \mathcal{H}_{mk})$ is an observed system. In this case the coordinate system is $(x^\lambda)_{\lambda=0}^3 = (t, x) = (t_{(H_{n\ell}, \mathcal{H}_{n\ell})}, x_{(H_{n\ell}, \mathcal{H}_{n\ell})})$.By Axioms 4 and 5 the motion of the center of mass of the observed system relative to the observer system, when we neglect the effects otherthan the gravitational force, is thus expressed as in the theory ofgeneral relativity by the equation of motion:

$$\frac{d^2x^\lambda}{d\tau^2} + \sum_{\mu,\nu=0}^{3} \Gamma^\lambda{}_{\mu\nu} \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0, \tag{13}$$

where $\tau = t_{mk} = t_{(H_{mk}, \mathcal{H}_{mk})}$is the proper time for the center of mass of the system $(H_{mk}, \mathcal{H}_{mk})$ and $\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2}\sum_{\alpha=0}^3 g^{\lambda\alpha}\left(\frac{\partial g_{\alpha\mu}}{\partial x^\nu} + \frac{\partial g_{\nu\alpha}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\alpha}\right)$ is the Christoffel's symbol.

We have to note that the above process of 'observations' is concerned only with the classical mechanical phenomena between the centers of mass of local systems. As we have stated, the *observation* is restricted only to the classical relative motions of centers of mass of local systems. However, the observer's system can also 'see' the *unobservable* quantum mechanical effects inside the observed system $(H_{mk}, \mathcal{H}_{mk})$. Reversely to the classical mechanical observations, the observer sees that the physics *within* the observed system is described by the Hamiltonian H_{mk} in a quantum mechanical way. The observer can 'see' the clock and motions in the observed system through the classical observations using, e.g., light as in the astronomical observation of electromagnetic radiations from stars.The observer deduces from these classical observations, with some relativistic corrections of the observed values as will be discussed below, that the quantum mechanical laws hold within the system $(H_{mk}, \mathcal{H}_{mk})$, *as far as* the time of the system is defined as the quantum mechanical proper one of the system $(H_{mk}, \mathcal{H}_{mk})$ as in Definition 3.

The results obtained in the usual relativistic quantum theories could be explained in our context as follows: The quantum phenomena occurring in a local system follow the non-relativistic quantum mechanics, but the observed values of quantum mechanical quantities should be corrected according to the classical relativity so that the corrected values equal the values predicted by the (non-relativistic) quantum mechanics. The relativistic effects arise only related with the *observation* of the classical quantities of the local system. The so-called *propagation* of fields or forces is interpreted in our theory also as the classical phenomena which appear through the observation process. What the observer measures concerning the observed system $(H_{mk}, \mathcal{H}_{mk})$ is the classical physics values of the subsystems or the portions of $(H_{mk}, \mathcal{H}_{mk})$, e.g., the positions, momenta, etc., of the subsystem's centers of mass, measured according to the observer's local time. From these classical quantities, the observer deduces through the classical relativistic corrections that the quantum mechanics is working in the observed system $(H_{mk}, \mathcal{H}_{mk})$.

If we see this procedure in reverse order, as in the second paragraph of Section 4 from the last, beginning with the quantum mechanics, i.e. with the Schrödinger propagator $e^{-it_{mk}H_{mk}}$, given the system $(H_{mk}, \mathcal{H}_{mk})$ with coordinates $(t_{mk}, x_{mk}) = (t_{(H_{mk}, \mathcal{H}_{mk})}, x_{(H_{mk}, \mathcal{H}_{mk})})$, it is described more precisely as follows: The quantum mechanical velocities of the particles in the system $(H_{mk}, \mathcal{H}_{mk})$ are given by the quotients $q_b = x_b/t_{mk}$, asymptotically as $t_{mk} \to \infty$, of the position vectors x_b of the particles and the local time t_{mk}. This is the case, since we have assumed that the coordinates (t_{mk}, x_{mk}), in particular, the local time t_{mk}, are given through the propagator or clock $e^{-it_{mk}H_{mk}}$ and Axiom 2. Our fundamental assumption here on the observation is as follows:

Axiom 6. The momenta $p_j = m_j x_j/t_{mk}$ of the particles j with mass m_j in the observed local system $(H_{mk}, \mathcal{H}_{mk})$ with coordinate system (t_{mk}, x_{mk}), given as above, are observed, by the observer system $(H_{n\ell}, \mathcal{H}_{n\ell})$ with coordinate system $(t_{n\ell}, x_{n\ell})$, as $p'_j = m_j x'_j/t_{n\ell}$, where x'_j is obtained from x_j by the relativistic transformation of coordinates: (t_{mk}, x_{mk}) to $(t_{n\ell}, x_{n\ell})$ as in Axiom 4. The same is true for the observation of the energies of the particles: the energies of the particles in the observed local system are observed by the observer as the ones transformed in accordance with the relativity.

Namely it is assumed that the quantum mechanical momenta $p_j = m_j x_j/t_{mk}$ of the particles within the system $(H_{mk}, \mathcal{H}_{mk})$ are observed in actual experiments by the observer system $(H_{n\ell}, \mathcal{H}_{n\ell})$ with coordinate system $(t_{n\ell}, x_{n\ell})$, as the *classical* quantities

$p'_j = m_j x'_j / t_{n\ell}$ whose values are calculated or predicted by correcting the quantum mechanical values p_j with taking the relativistic effects of observation into account. A similar assumption is made for the energies of the particles.

In this sense, Axiom 6 should be called a principle of deduction of the experimentally observed values from the nonrelativistic quantum mechanics through certain relativistic corrections, rather than be called a deduction rule of the internal quantum mechanical motionsfrom the classical observations of the sub local systems. We adopted the latter expressionin Sections 1 and 5 for the sake of simplicity of expressions.

We should note that Axiom 6 is consistent withour Axioms 1–5 in the following sense: Axiom 6 is concerned only with the quantum mechanics *within* the local system $(H_{mk}, \mathcal{H}_{mk})$so that it gives the rules to transform the quantum mechanical values,e.g., p_j, of the system $(H_{mk}, \mathcal{H}_{mk})$ to the values, e.g.,p'_j, *observed* experimentally by the observer. It is therefore not related with *any* physics laws of the particles*within* the system $(H_{mk}, \mathcal{H}_{mk})$, unless the transformed values(e.g., p'_j) are compared with the actual experimental values. In this sense, Axiom 6 is concerned only with *how the naturelooks to the observer*. Together with Axioms 1–5, it gives the prediction of the physical values observed in actual experiments, and is checked solely through the experimental data.

In this sense, Axiom 6 together with Axioms 1–5 gives our prediction for the relativistic observations of the quantum mechanical local systems. For the illustration, let us take an example of calculation of the differential cross section $\frac{d\sigma}{d\Omega}$ for the scattering phenomenon of an electron by a Coulomb potential $-\frac{Ze^2}{r}$, where $r = |x|$ and x is the position vector of the electron relative to the scatterer.We assume that the scatterer has a very large mass comparedto the electron and that $\frac{Z}{137}$ is small. Then as usual the quantum mechanics gives, in a Born approximation,

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 e^4}{16 E^2 \sin^4(\theta/2)}, \tag{14}$$

where θ is the scattering angle, and E is the total energy of the system consisting of the electron and the scatterer. Thus, since the electron is far away from the scatterer after the scattering, we may assume that the energy E is equal to the kinetic energy of the electron and the scatterer. We assume that the observer is stationary relativeto this local system of the electron and the scatterer. This means that the observer is approximately stationary relative to the scatterer, since we assumed the mass of the scatterer much larger than that of the electron. From the observer, which is stationary relative to the scatterer, the energy E is equal to the kinetic energy of the electron and is observed as a classical quantity by Axiom 6. Therefore its actual value observedequals $p^0 - p^0_0$ by the special relativity. Here p^0_0 is the rest energyof the electron, and p^0 is the relativistic energy of the electron given as follows. Let v be the absolute value of the coordinate velocity of the electron: $v = |dx/dt|$, where t is the observer's local time. Then by the theory of special relativity, p^0 is expressed as follows:

$$p^0 = \frac{1}{\sqrt{1-v^2}}\, p_0^0. \tag{15}$$

Here we adopted a unit system such that the speed of light $c = 1(> v \geq 0)$. Thus p_0^0 equals the rest mass m_0 of the electron: $p_0^0 = m_0$. We can then compute

$$E = p^0 - p_0^0 = \frac{1-\sqrt{1-v^2}}{\sqrt{1-v^2}}\, p_0^0 \approx \frac{m_0 v^2/2}{\sqrt{1-v^2}} \approx \frac{m_0 v^2}{2}(1+v^2/2) \approx \frac{m_0 v^2}{2}. \tag{16}$$

Thus taking the relativistic effects of the observation into account,we have the differential cross section given by

$$\frac{d\sigma}{d\Omega} \approx \frac{Z^2 e^4}{4m_0^2 v^4 \sin^4(\theta/2)}(1-v^2). \tag{17}$$

This coincides with the usual relativistic prediction of the Klein-Gordon equation obtained by a Born approximation, if the observer is assumed to be stationary relative to the scatterer.

The effect of the spin of the electron can also be included by introducing the spin-orbit interactions (see Mott-Massey [?], Chap.X). In this treatment, the wave function of the electron is regarded as a two-dimensional vector-valued function as usual.The result corresponding to (14) is the same as the one obtained through Dirac theory as far as the spin correction is concerned (Mott-Massey [?], Chap.X, Sect.3):

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 e^4}{16E^2 \sin^4(\theta/2)}\left(1 - \frac{2E}{m_0}\sin^2(\theta/2)\right). \tag{18}$$

Inserting the relativistic correction (16), we get

$$\frac{d\sigma}{d\Omega} \approx \frac{Z^2 e^4}{4m_0^2 v^4 \sin^4(\theta/2)}(1-v^2\sin^2(\theta/2))(1-v^2). \tag{19}$$

This is exactly the relativistic Dirac prediction in a Born approximation.We can proceed to finer Born approximations as well, as in Mott-Massey [?], Chap.IX, Sect.4.5, and can recover the relativistic prediction of Dirac equation.

In sum, the quantum mechanics has the intrinsic nature, and the relativity is concerned with how the nature looks to the observer.The observed relativistic quantum phenomena are explained as the consequences of the relativistic effects of the observation of thenon-relativistic quantum systems.

9. CONCLUDING DISCUSSIONS

The times are defined only for local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$. The total universe ϕ has no time associated. The local times arise through the affections from other particles outside the local systems. (Definitions 1–3.) The uncertainty principle holds only within these local systems as the uncertainty of the local times. The quantum mechanics is confined within each local system in this sense. The quantum mechanical phenomenabetween two local systems appear only when they are combined asa single local system. In the local system, the interaction and forces propagatewith infinite velocity or in other words they are *unobservable*.

Each local system can be the observer of other systems. In this situation the local systems are mutually independent in the sense thatthe associated quantum mechanical local times are not correlatedin general. Therefore there are no reasons to exclude the classicalmechanics in describing the *observable* relative behavior of the observed systems with respect to the observer. Thus the gravitational potentials can be introduced in accordance with the theory of general relativity. These potentials determine the global space-time structure around the observer system. Inside the observer system the space-time is Euclidean. The observer itself cannot detect the gravitational correlation or the space-time structure inside its own system,and the interactions and forces cannot be detected inside itself. On the contrary, between the local systems, the observer can detect only the classical mechanical effects. Namely the gravitational forces appear and the quantum mechanical potentials $V_\alpha(x_\alpha)$ effective outside the local systems operate as the classical ones between the local systems. Nevertheless, through the media (e.g., light in classical sense) which connect the observer and the observed systems and obey the classical physics, the observer sees,through some relativistic corrections of the observed classical values,that the physics laws inside the other local systems follow the quantum mechanics.

These facts are all the consequences of the introduction of *local times* which are proper to each local system. The time is neithera given thing nor a common one to the total universe. On the contrarythere can be defined no global time. More strongly the total universe is a(stationary) bound state of the total Hamiltonian H of infinite degreesof freedom. The times arise only when the observers restrict their attention to its subsystems as approximations of the total Hamiltonian H. The universe itself is correlated within it as a bound state of H. The observer always separates a subsystem from it, so to speak, artificially, and the (steady) motion and time appear. Inside the subsystem this local time explains the quantum effects,and outside the subsystem it explains the gravitation and the classical mechanics. The relativistic quantum phenomena are explained as the relativistic effects of the observation of the non-relativistic quantum systems. All these physical phenomena occur by this artificial separation of the universe. The universe itself does not 'change': It is a stationary bound state.

10. SOME CONJECTURES

As conclusions, we state some conjectures which would likely hold in our theory.

(1) The universe ϕ would be confined in a local region of the infinite dimensional configuration space R^∞ in *some* sense as an eigenvector of the total Hamiltonian H: This is an analogy from the corresponding fact for the finite dimensional Hamiltonian $H_{n\ell}$. The eigenvectors for this Hamiltonian are local in the sense that they are in $L^2(R^{3n}_{x^{(\ell)}})$. Or more strongly it is known that they decay exponentially with respect to $|x^{(\ell)}|$ if the associated eigenvalues are not the thresholds of $H_{n\ell}$. This conjecture meansthat the universe is a 'closed' or 'finite' one in quantum mechanical sense. Therefore this conjecture will eliminate the arguments on the so-called 'dark matter', which is supposed to exist in the usual 'Big Bang' theory to make the universe almost stable, i.e., to make it have the density very close to the 'critical density', therefore to let the universe have matters enough for stars and planets to exist. A similarexplanation will work for explaining the stability of such local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ as galaxies, clusters of galaxies, etc., if one regards their actual quantum mechanical states $\phi_{n\ell} \in \mathcal{H}_{n\ell}$ as (approximate) bound states of thelocal Hamiltonian $H_{n\ell}$, namely as 'resonances' of $H_{n\ell}$ in some sense (cf. Jensen-Kato [**?**]). Then the quantum mechanical effect would explain their stability or 'locality' as in the explanation of the locality of the total universe without introducing the dark matter.

(2) It would be possible to accommodate bosons, fermions, photons (i.e.,the quantum theory of radiation), and so on in our theory by using sub-Fock spaces $\mathcal{F} = \bigoplus_{n=0}^{\infty} \mathcal{H}^n$ of $\mathcal{U} = \bigoplus_{n=0}^{\infty} (\sum_{\ell=0}^{\infty} \mathcal{H}^n)$.As usual the spin of the particles can be introduced by taking the vector-valued representations $L^2(R^{3n}; \mathbf{C}^k)$ $(k \geq 1)$of $\mathcal{H}^n$ in Axiom 2. ($\mathbf{C}$ is the set of complex numbers.)

For the photonswithout interactions with matter, the Hamiltonians $H_{n\ell}$in the sense of Axiom 3 shouldbe taken as a sum of those for harmonic oscillators. Since the correspondent total Hamiltonian $H_p = \bigoplus_{n=0}^{\infty} H_{n\ell}$ has a complete system of eigenvectors, one has $(I - P_{H_p})\mathcal{F} = \{0\}$. Thus it has no associated proper clock.In this sense the light as a *wave* propagates with infinite speedwithin the system$(H_p, \mathcal{F})$.It should be remarked that this statement has a different meaning than that the light, as an electromagnetic interaction, propagateswith infinite speed. As such a field, light interacts with other matters through vector potentials A(x), which are quantum mechanical electromagnetic field interacting instantaneously with matters. On the contrary, as a *wave*, light propagates with the constant speed c within general local systems. The reason is that the Hamiltonian of a general local system contains terms comprising the interactions between photons and other particles. In consequence, that Hamiltonian does not have a complete system of eigenvectors, and the local time of the system is defined by Definition 3. Then with some additional arguments to Chap.III of von Neumann[**?**], one can show that the speed of the wavefront of light within that local system is the constant c.

In the line of this treatment of the interactions between electromagnetic field and matters, it would be possible to explain the Lamb shift. It is probable that the explanation could be given even without appealing the path integral techniques described, e.g., in Feynman-Hibbs [**?**]. For this possibility, we refer to the argument around the footnote 149) of [**?**], Chap.III, section 6, remarking that the argument there and the one in [**?**], Chap.9, are essentially the same. As there in [**?**], let H be the total Hamiltonian of the system consisting of matter and radiation field (i.e., photons), let I be the interaction between matter and photons, andlet a sequence of complex numbers $a_{kM_1M_2\cdots}$belong to the space of sequences $a_{kM_1M_2\cdots}$ with normalization condition that $\sum_{k,M_1,M_2,\cdots}|a_{kM_1M_2\cdots}|^2 = 1$ and let its component$a_{kM_1M_2\cdots}$express the state where matter is in the state k and the number of photons in state $j$is M_j. In other words, $a_{kM_1M_2\cdots}$denotes the eigenvector of $H - I$ with eigenvalue $W_k + \sum_{j=1}^{\infty} h\rho_j \cdot M_j$, where W_k is the eigenenergy of the matter state k, ρ_j is the eigenfrequency of photons in state j, and h is the Planck constant. Set $b_{kM_1M_2\cdots}(t) = \exp\{-itH\}\exp\{it(H-I)\}a_{kM_1M_2\cdots}$. Consider the transition process where the matter part of the total system begins and ends in the same state $\bar{k}$. Integrating the differential equation

$$\begin{aligned}\frac{1}{i}\frac{d}{dt}b(t) &= \exp\{-itH\}(-I)\exp\{it(H-I)\}b(t)\\ b(0) &= a_{\bar{k}\bar{M}_1\bar{M}_2\cdots}\end{aligned}$$

in some approximation, and estimating the solution, as in [?], Chap.III, section 6, we see that

$$\sum_{\forall M_j}|b_{\bar{k}M_1M_2\cdots}(t)|^2 = 1 - \sum_{k\neq\bar{k},\ \forall M_j}|b_{kM_1M_2\cdots}(t)|^2$$

with the second term on the right hand side $\geq$ *const.*$|t|$ (*const.* > 0) in norm for small $|t|$. This, in turn by the definition above of $b_{kM_1M_2\cdots}(t)$, yields that there exists some energy shift $\delta E \neq 0$ to the total energy of the systemduring the transition from the state $\bar{k}$ to the same state $\bar{k}$ of the matter part of the system. In this sense, it seems that the Lamb shift was already predicted at least implicitly around 1930. We expect that this kind of argument could be refined in rigorous sense to give the explanation of Lamb shift with no divergence problem, in our context of abandonment of relativistic quantum field theory.

These conjectures stated here do not require us to introduce the notion of relativistic quantum field, because the system should be considered as anon-relativistic one.(See also Dirac [?], Chap.X other than [?], Chap.III.) The relativistic quantum phenomena could be explained as the relativistic effects of observations as discussed in the section on observation.

(3) We adopt the standpoint that the quantum mechanics is symmetric with respect to the reversal of (local) time: $t \to -t$. In this sense we stand upon the continuous "Schrödinger-like" picture concerning the measurement of the local systems by the observer's local system: As far as the local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ consisting of finite number of particles are concerned, no bound state $\psi_{n\ell}$ of $H_{n\ell}$ in $\mathcal{H}_{n\ell}$ with eigenvalue μ can be detectedif one observes the system in accord with the local time $t = t_{(H_{n\ell},\mathcal{H}_{n\ell})}$, because the state $\psi_{n\ell}$ does not change in its amplitude: $|e^{-itH_{n\ell}}\psi_{n\ell}(x)|^2 = |e^{-it\mu}\psi_{n\ell}(x)|^2 = |\psi_{n\ell}(x)|^2$. Therefore it emits no light and informations outside. Namely only the scattering state which liesin the continuous spectral subspace $\mathcal{H}^c_{n\ell}(\subset \mathcal{H}_{n\ell})$ of the local Hamiltonian $H_{n\ell}$ can be detected. We need not consider herethe so-called reduction of the wave packets. The observer measures only, e.g., the light emitted from the scattering state in the observed system $(H_{n\ell}, \mathcal{H}_{n\ell})$. In this sense the measurement does not include the irreversible procedure in

our theory, and the (local) time is reversible. (We refer to von Neumann [?], Chap.III, VI for the arguments which support our standpoint including the explanation of emission of light in the line of conjecture (2).)

(4) By the consistency of Axioms 4–5 with Axioms 1–3, the theory of(general) relativity implies the Hubble's red shift. Namely the universe looks like itis expanding (or contracting) for any observer's local system. At the sametime, Axioms 1–3 and Definitions 1–3 imply that the universe has no proper time associated and is stationary. This situation is no contradiction, because Axioms 1–5 are consistent. The expansion appears only when theobserver's local system observes the universe in accordance with the classical mechanics. In other words the universe looks like expanding only when the observer puts its concerns on the centers of mass of the stars, galaxies, etc., regarding them as other local systems. Only in this context it is required to argue on the beginning and the end of the universe. But an appearance of the universe as this comes from the view point merely based on the classical mechanics. When the observer looks the inside of those local systems, he sees that the quantum mechanics is working inside them as we have seen in Section 8. Then he reasons from these observations and consideration that the universe itself is a quantum mechanical one, and concludes that the consistent introduction of the quantum mechanics into such a situation would remove the problem of the beginning and the end of the universe. His conclusion would be that the universe just 'exists' without time.

References

[1] R. Abraham, J.E. Marsden, *Foundations of Mechanics*, The Benjamin/Cummings Publishing Company, 2nd edn., London-Amsterdam-Don Mills, Ontario-Sydney-Tokyo, 1978.

[2] A. Ashtekar, J. Stachel (eds.), *Conceptual Problems of Quantum Gravity*, Birkhäuser, Boston-Basel-Berlin, 1991.

[3] H.R. Brown, R. Harré (eds.), *Philosophical Foundations of Quantum Field Theory*, Clarendon Press, Oxford, 1990.

[4] J. Dereziński, *Asymptotic completeness of long-range N-body quantum systems*, preprint (Centre de Mathématiques, Ecole Polytechnique, U.R.A.169 du C.N.R.S.), 1992.

[5] P.A.M. Dirac, *The Principles of Quantum Mechanics*, Clarendon Press, 4th edn., Oxford, 1958.

[6] V. Enss, *Introduction to asymptotic observables for multiparticle quantum scattering*, in "Schrödinger Operators, Aarhus 1985," ed. E. Balslev, Lect. Note in Math. **1218**, Springer-Verlag, 1986, pp.61-92.

[7] A. Einstein, *Die Grundlage der allgemeinen Relativitätstheorie*, Ann. der Phys. Ser. 4, **49**, 1916, 769-822.

[8] A. Einstein, B. Podolsky, N. Rosen, *Can quantum-mechanical description of physical reality be considered complete* ?, Phys. Rev. **47**, 1935, 777-780.

[9] R.P. Feynman, A.R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill Book Company, New York-St. Louis-San Francisco-Toronto-London-Sydney, 1965.

[10] M. Friedman, *Foundations of Space-Time Theories, Relativistic Physics and Philosophy of Science*, Princeton University Press, Princeton, 1983.

[11] J. Glimm, A. Jaffe *Quantum Physics, A Functional Integration Point of View*, Springer-Verlag, 2nd edn.,New York-Berlin-Heidelberg-London-Paris-Tokyo, 1987.

[12] S.W. Hawking, G.F.R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge University Press, Cambridge-New York-Port Chester-Melbourne-Sydney, 1973.

[13] M. Jammer, *The Philosophy of Quantum Mechanics, The interpretations of Quantum Mechanics in Historical Perspective*, John Wiley & Sons, Inc., New York, 1974.

[14] A. Jensen, T. Kato, *Spectral properties of Schrödinger operators and time-decay of the wave functions*, Duke Math. J.**46**, 1979, 583-611.

[15] H. Kitada, *Asymptotic completeness of N-body wave operators I. Short-range quantum systems*, Rev. Math. Phys. **3**, 1991, 101-124.

[16] H. Kitada, *Asymptotic completeness of N-body wave operators II. A new proof for the short-range case and the asymptotic clustering for long-range systems*, in "Functional Analysis and Related Topics, 1991," ed. H. Komatsu, Lect. Note in Math. **1540**, Springer-Verlag, 1993, pp.149-189.

[17] N.F. Mott, H.S.W. Massey, *The Theory of Atomic Collisions*, Clarendon Press, 3rd edn., Oxford, 1965.

[18] E. Prugovečki, *Quantum Geometry, A Framework for Quantum General Relativity*, Kluwer Academic Publishers, Dordrecht-Boston-London, 1992.

[19] M. Redhead, *Incompleteness, Nonlocality, and Realism, A Prolegomenon to the Philosophy of Quantum Mechanics*, Clarendon Press, Oxford, 1990.

[20] F. Selleri, *Die Debatte um die Quantentheorie*, Frieder. Vieweg & Sohn Verlagsgesellschaft mbH, Braunschweig, 1983.

[21] W. Schommers (ed.), *Quantum Theory and Pictures of Reality, Foundations, Interpretations, and New Aspects*, (with contributions by B. d'Espagnat, P. Eberhard, W. Schommers, F. Selleri), Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong, 1989.

[22] J. von Neumann, *Die Mathematische Grundlagen der Quantenmechanik*, Springer-Verlag, Berlin, 1932.