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Drawing On The Sphere And Its Paper Model

I have been impressed with M.C. Escher's ornaments and so decided to transfer some of them onto the surface of the sphere. I began with an IBM PC for this purpose, and gradually this work turned into a program, which gave the opportunity to draw a picture on the screen of the PC and then this picture was replicated all over the surface of the sphere according to one of the famous symmetries -- tetrahedral, octahedral or dodecahedral.

Everyone has played with the children's toy -- a kaleidoscope in which a few colored stones reflect in the edges of the triangular mirror. These reflections create an attractive ornament on the plane. The surface of the sphere also has triangles with the same property -- reflections through a triangle's edges cover the whole sphere. Those spherical triangles are referred to as Möbius triangles. Thus, in order to create a program of drawing ornaments on the sphere it was necessary to input a picture on a Möbius triangle and then to create the image of the whole sphere covered by these pictures.

The computer screen is a plane, not a sphere--what planar area can serve as a Möbius triangle? It proved that a projection of a Möbius triangle onto the cylinder circumscribed around the sphere can fit very well after unrolling the cylinder. Cylinders, round or elliptic, are excellent surfaces for modeling because they can be unrolled and their points can be stored in the computer memory as two-dimensional arrays. Thus, we get the opportunity to draw a picture in the planar model of a Möbius triangle in the computer screen.

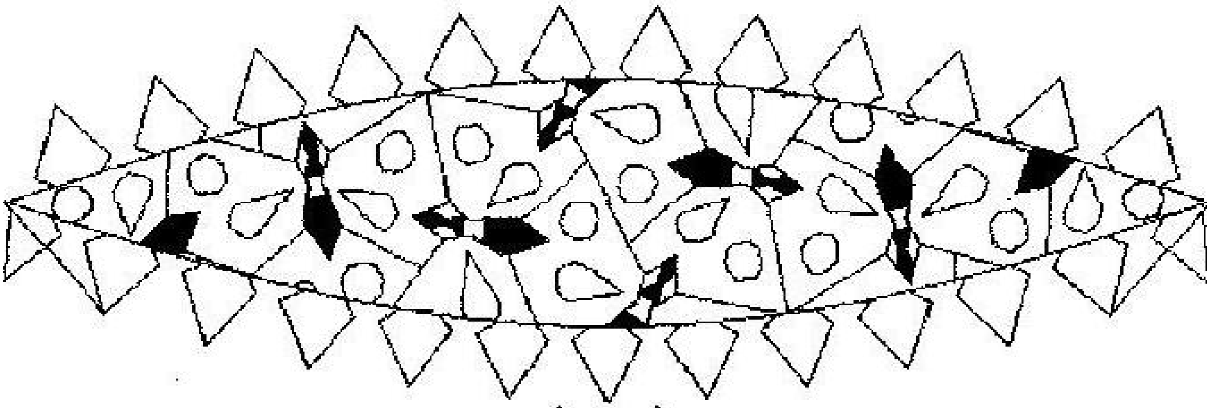
Then the whole sphere must be drawn in the screen. Again the cylinder surface appears useful. Let's consider dodecahedral symmetry. The whole spherical surface can be divided into ten lunar digons, stretching from South to North like sections of an orange. Each digon consists of exactly twelve Möbius triangles and there are only two different digons, so the whole image of the sphere can be constructed from these two basic digons. Again let's make two projections of these digons onto the cylinder circumscribed around the sphere. Now the whole image of the sphere can be created by comparatively simple conversions of these two planar digons into five spherical digons seen on the screen.

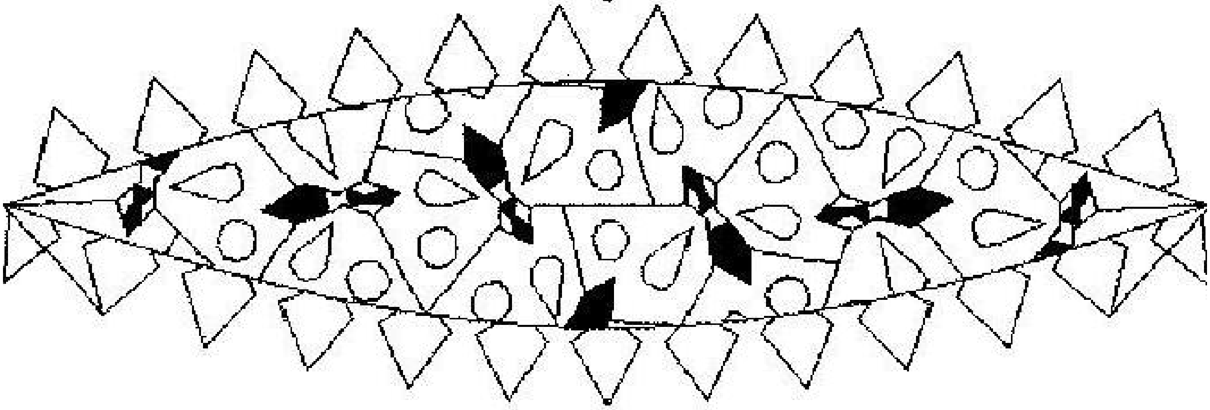
Though such an image of the sphere may seem rough, it proved to be good enough for the eye to see the sphere in this image. This method is very fast and it gives the opportunity to rotate the sphere in the computer screen after an accumulation of several dozens of images at different angles. Nowadays when the speed of computers has been increased substantially, there is a chance to simulate the real spherical kaleidoscope in which an ornament is changing on the rotating sphere like the changing picture in an ordinary planar kaleidoscope.

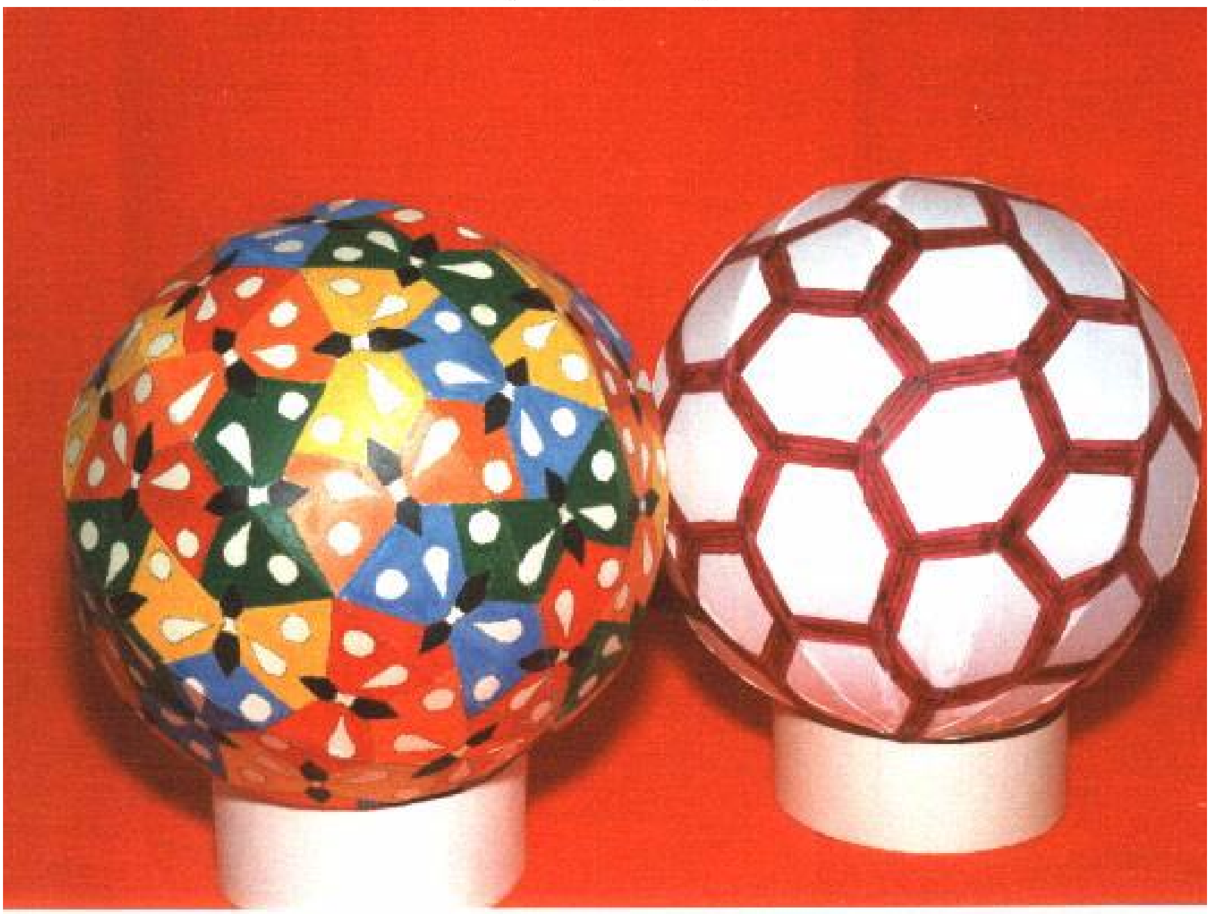
If it is possible to draw the sphere in the screen, why don't we output patterns of the sphere to the printer for gluing a paper model of it? We have already created the planar digons-- ten such digons can be printed intact and glued. They will fit each other and create a model of the sphere--unfortunately our eye will see a slight deviation of this model from the real sphere, probably because the model circumscribes the sphere. A good model can be achieved if we create a projection of the spherical digons onto the elliptic cylinder which goes through the edges of the spherical digon. In this case the model inscribes in the sphere, however the calculation of projections becomes much more complicated.

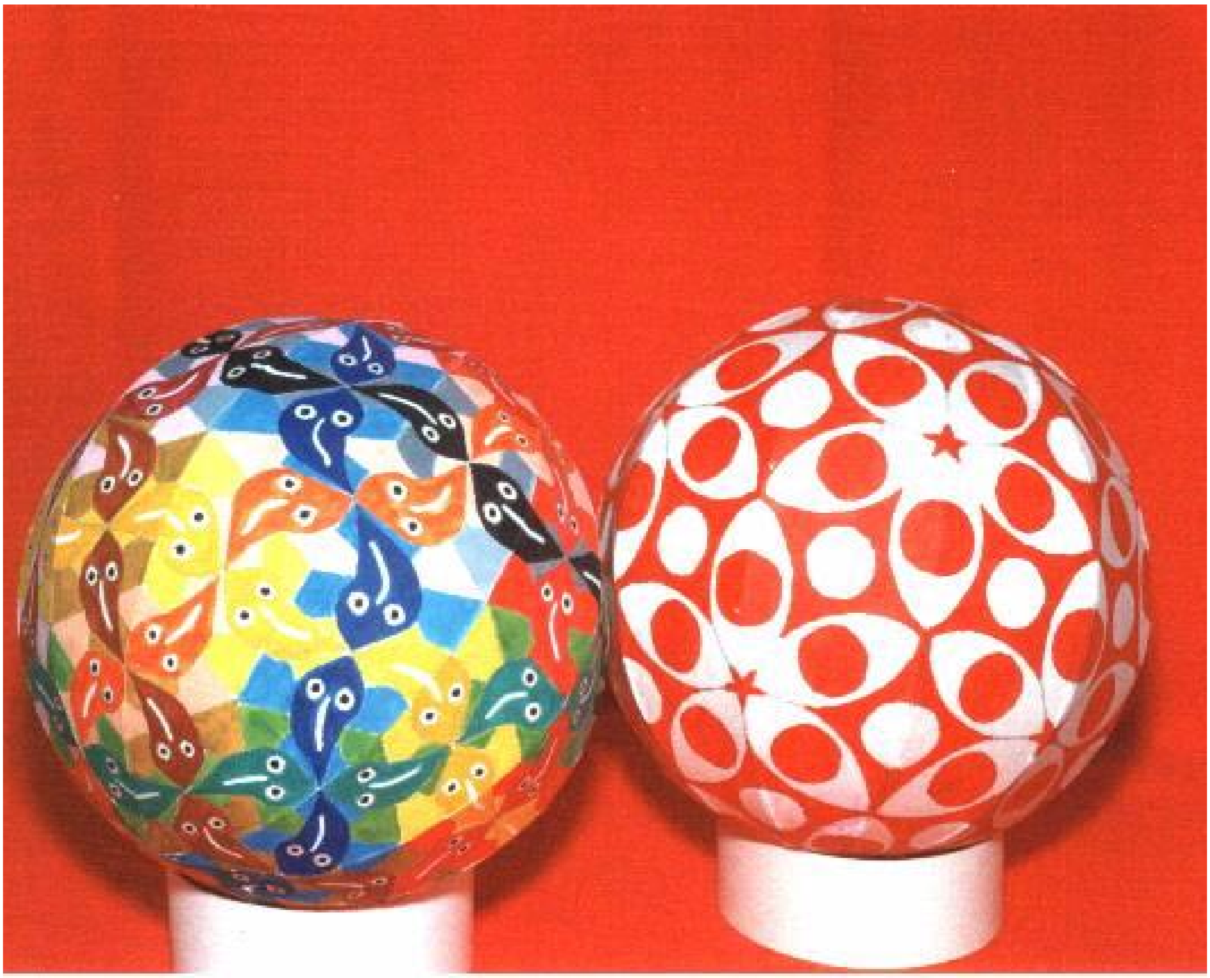
In order to draw regular geometric shapes, some sort of coordinates of the drawing pen are output to the screen. During input of a picture into a Möbius triangle, the program calculates and shows distances from the pen to the vertices and to the edges of the Möbius triangle. Using these coordinates I succeeded in transferring onto the sphere some of M.C. Escher's ornaments and a row of famous planar ornaments.

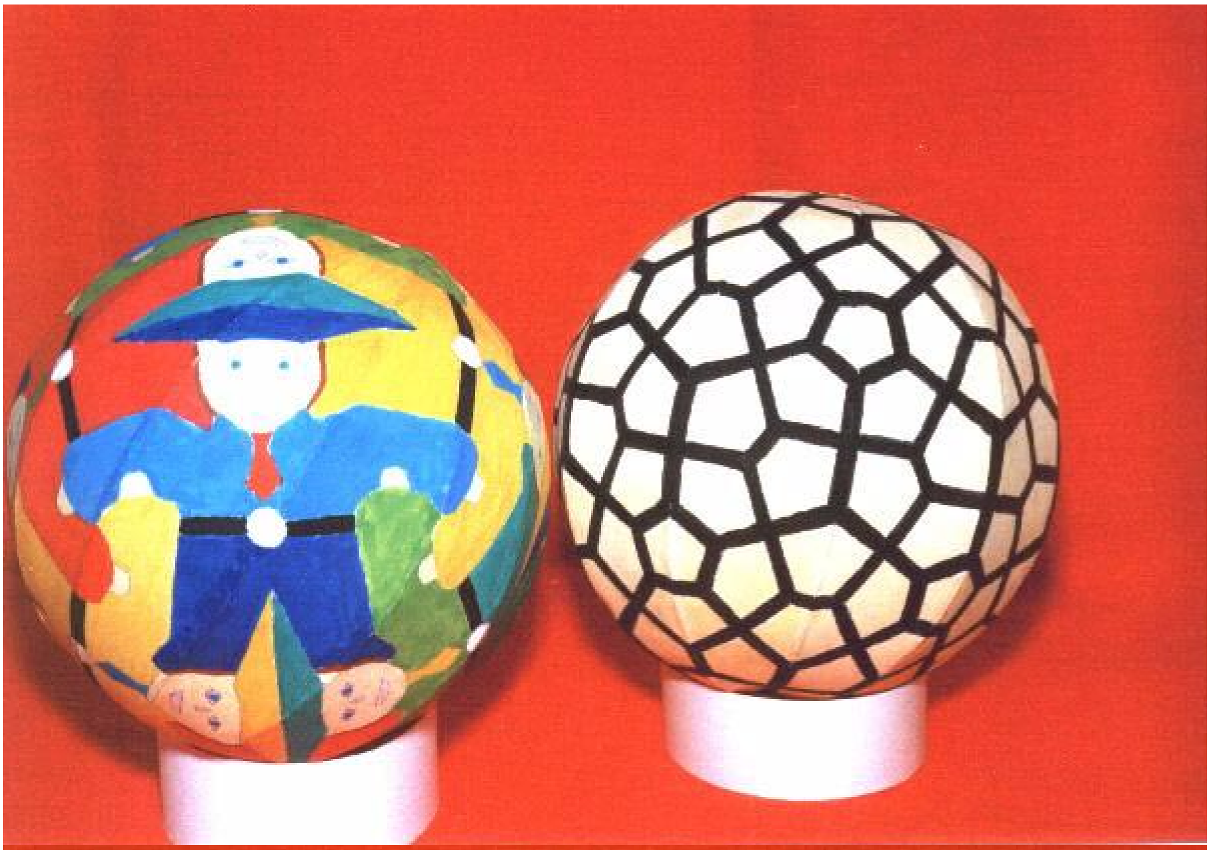
Illustrations of a drawing on digons follows. In the first picture you can see a pair of patterns: two basic digons for gluing--five of these pairs create the sphere covered by butterflies. The other six pictures show the models of the sphere, covered by the famous ornaments: butterflies, fish, interlacing rings, men, reptiles and tadpoles. The ornaments also have color symmetry.

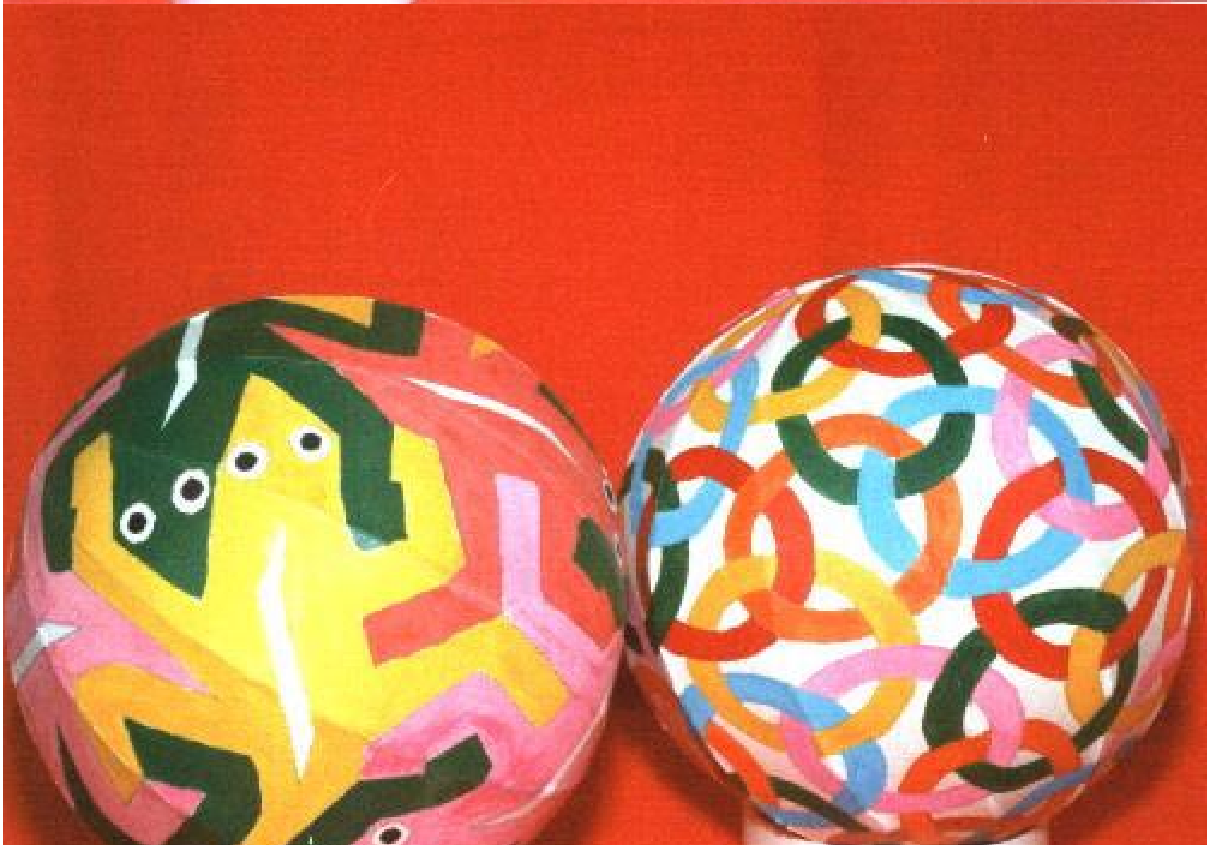












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