# Adaptive Multi-Stage Distance Join Processing *

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**Abstract**

A spatial distance join is a relatively new type of operation introduced for spatial and multimedia database applications. Additional requirements for ranking and stopping cardinality are often combined with the spatial distance join in on-line query processing or internet search environments. These requirements pose new challenges as well as opportunities for more efficient processing of spatial distance join queries. In this paper, we first present an efficient $k$-distance join algorithm that uses spatial indexes such as R-trees. Bi-directional node expansion and plane-sweeping techniques are used for fast pruning of distant pairs, and the plane-sweeping is further optimized by novel strategies for selecting a sweeping axis and direction. Furthermore, we propose adaptive multi-stage algorithms for $k$-distance join and incremental distance join operations. Our performance study shows that the proposed adaptive multi-stage algorithms outperform previous work by up to an order of magnitude for both $k$-distance join and incremental distance join queries, under various operational conditions.

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# 1 Introduction

A spatial distance join operation was recently introduced to spatial databases to associate one or more sets of spatial data by distances between them [13]. A distance is usually defined in terms of spatial attributes, but it can be defined in many different ways according to various application specific requirements. In multimedia and image database applications, for example, other metrics such as a *similarity distance function* can be used to measure a distance between two objects in a feature space.

In on-line decision support and internet search environments, it is quite common to pose a query that finds the best $k$ matches or reports the results incrementally in the decreasing order of well-matchedness. This type of operations allow users to interact with database systems more effectively and focus on the "best" answers. Since users can say "It is enough already" at any time after obtaining the best answers [8], the waste of system resources can be reduced and thereby delivering the results to users more quickly.

This ranking requirement is often combined with a spatial distance join query, and the ranking requirement provides a new opportunity of optimization for spatial distance join processing [9, 10]. For example, consider a query that retrieves the top $k$ pairs (*i.e.*, the nearest pairs) of hotels and restaurants:

```
SELECT h.name, r.name
FROM Hotel h, Restaurant r
ORDER BY distance(h.location, r.location)
STOP AFTER k;
```

For a relatively small stopping cardinality $k$, the processing time can be reduced significantly by sorting only a fraction of intermediate results enough to produce the $k$ nearest pairs, instead of sorting an entire set of intermediate results (*i.e.*, a Cartesian product of hotels and restaurants).

A spatial distance join query with a stopping cardinality can be formulated as follows:

$$\sigma_{dist(r,s)<\mathcal{D}_{max}}(R \bowtie S)$$

where $dist(r, s)$ is a distance between two spatial objects $r \in R$ and $s \in S$, and $\mathcal{D}_{max}$ is a cutoff distance that is determined by a stopping cardinality $k$ and the spatial attribute values of two data sets $R$ and $S$. It may then be argued that a spatial distance join query can be processed by a spatial join operation [1, 6, 7, 15, 16, 19] followed by a sort operation. Specifically, if a $\mathcal{D}_{max}$ value can be predicted precisely for a given stopping cardinality $k$, we can use a spatial join algorithm with a `within` predicate instead of an `intersect` predicate to find the $k$ nearest pairs of objects. Then, a sort operation will be performed only on the $k$ pairs of objects.

In practice, however, it is almost impossible to estimate an accurate $\mathcal{D}_{max}$ value for a given stopping cardinality $k$, and, to the best of our knowledge, no method for estimating such a cutoff value has been reported in the literature. If the $\mathcal{D}_{max}$ value is overestimated, then the results from a spatial join operation may contain too many candidate pairs, which may cause a long delay in a subsequent stage to sort all the candidate pairs. On the other hand, if the $\mathcal{D}_{max}$ value is underestimated, a spatial join operation may not return a sufficient number of object pairs. Then, the spatial join operation should be repeated with a new estimate of $\mathcal{D}_{max}$, until $k$ or more pairs are returned. This may cause a significant amount of waste in processing time and resources.

There is another reason that makes it impractical to apply a spatial join algorithm to spatial distance join queries. A spatial join query is typically processed in two steps, *filter* and *refinement*, as proposed in [18]. In a filter step, MBR approximations are used to find pairs of potentially intersected spatial objects. Then, in a refinement step, it is guaranteed that all the qualified (*i.e.*, actually intersected) pairs can be produced from the results returned from the filter step.

In contrast, it is completely unreasonable to process a spatial distance join query in two separate filter and refinement steps, because of the fact that a filtering process is based on MBR approximations. A set of object pairs sorted by distances measured by MBR approximations does not reflect a true order based on actual representations. This is because, for any two pairs of spatial objects $\langle r_1, s_1\rangle$ and $\langle r_2, s_2\rangle$, the fact that $dist(MBR(r_1), MBR(s_1)) < dist(MBR(r_2), MBR(s_2))$ does not necessarily imply that $dist(r_1, s_1) < dist(r_2, s_2)$. Consequently, any processing done in the filter step will be of no use for finding the $k$ nearest object pairs.

In this paper, we propose new strategies for efficiently processing spatial distance join queries combined with ranking requirements. The main contributions of the proposed solutions are:

- New efficient methods are proposed to process distance join queries using spatial index structures such as R-trees. *Bi-directional node expansion* and *optimized plane-sweep* techniques are used for fast pruning of distant pairs, and the plane-sweep is further optimized by novel strategies for selecting a sweeping axis and direction.
- Adaptive multi-stage algorithms are proposed to process distance join queries in a way that the $k$ nearest pairs are returned *incrementally*. When a stopping cardinality is not known a priori (*e.g.*, in on-line query processing environments or a complex query containing a distance join as a sub-query whose results need to be pipelined to the next stage of the complex query), the adaptive multi-stage algorithms can produce pairs of objects in a stepwise manner.
- We provide a systematic approach for *estimating the maximum distance* for a distance join query with a stopping cardinality. This estimated distance allows the adaptive multi-stage algorithms to avoid a *slow start* problem, which may cause a substantial delay in the query processing. This approach for estimating the maximum distance also allows the size of memory to be parameterized into a queue management scheme, so that data movement between memory and disk can be minimized.

The proposed algorithms achieve up to an order of magnitude performance improvement over previous work for both $k$-distance join and incremental distance join queries, under various operational conditions.

The rest of this paper is organized as follows. Section 2 surveys the background and related work on processing spatial distance join queries. Major limitations of previous work are also discussed in the section. In Section 3, we present a new improved algorithm based on bi-directional expansion and optimized plane-sweep techniques for $k$-distance join queries. In Section 4, adaptive multi-stage algorithms are presented for $k$-distance join and incremental distance join queries. A queue management scheme parameterized by memory capacity is also presented. Section 5 presents the results of experimental evaluation of the proposed solutions. Finally, Section 6 summarizes the contributions of this paper and gives an outlook to future work.

## 2 Background and Previous Work

A spatial index structure R-tree and its variants [3, 5, 11] have been widely used to efficiently access multidimensional data – either spatial or point. Like other tree-structured index structures, an R-tree index partitions a multidimensional space by grouping objects in a hierarchical manner. A subspace occupied by a tree node is always contained in the subspace of its parent node. This hierarchy of spatial containment between R-tree nodes is readily used by spatial distance join algorithms as well as spatial join algorithms.

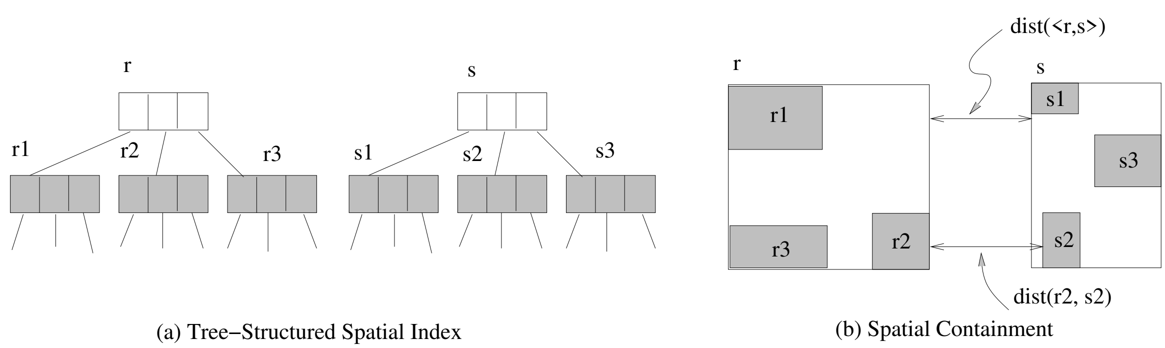


Figure 1: Hierarchy of Spatial Containment of R-Tree Nodes

Suppose $r$ and $s$ are non-leaf nodes of two R-tree indexes $R$ and $S$, respectively, as in Figure 1. Then, the minimum distance between $r$ and $s$ is always less than or equal to the minimum distance between one of the child nodes of $r$ and one of the child nodes of $s$. Likewise, the maximum distance between $r$ and $s$ is always greater than or equal to the maximum distance between one of the child nodes of $r$ and one of the child nodes of $s$. This observation leads to the following lemma.

**Lemma 1** *For two R-tree indexes $R$ and $S$, if neither $r \in R$ nor $s \in S$ is a root node, then*

$$\begin{aligned} dist(r,s) &\geq dist(parent(r), parent(s)), \\ dist(r,s) &\geq dist(r, parent(s)), \\ dist(r,s) &\geq dist(parent(r), s). \end{aligned} \tag{1}$$

*where $dist(r,s)$ is the minimum distance between the MBR representations of $r$ and $s$.*

*Proof.* From the observation above. □

Lemma 1 allows us to limit the search space, while R-tree indexes are traversed in a top-down manner to process a spatial distance join query. For example, if a pair of non-leaf nodes $\langle r, s\rangle$ turn out to be too far from each other (or their distance is over a certain threshold), then it is not necessary to traverse further down the tree indexes below the nodes $r$ and $s$. Thus, this lemma provides the key leverage to processing distance join queries efficiently using R-tree indexes.

## 2.1 Incremental Distance Join and $k$-Distance Joins

During top-down traversals of R-tree indexes, it is desirable to store examined node pairs in a priority queue, where the node pairs are kept in an increasing order of distances. We call it a *main queue* as opposed to a *distance queue* we will describe later. The main queue initially contains a pair of the root nodes of two R-tree indexes. Each time a pair of non-object nodes are retrieved from the main queue, the child nodes of one node are paired up with the child nodes of the other to generate a new set of node pairs, which are then inserted into the main queue. This process that we call *node expansion* is repeated until the main queue becomes empty or until stopped by an interactive user. If an element retrieved from the main queue is a pair of two objects, the pair is returned immediately to the user as a query result. This is how a spatial distance join query is processed *incrementally.* Figure 2 depicts a typical framework of processing an incremental distance join (**IDJ**) query using R-tree indexes.

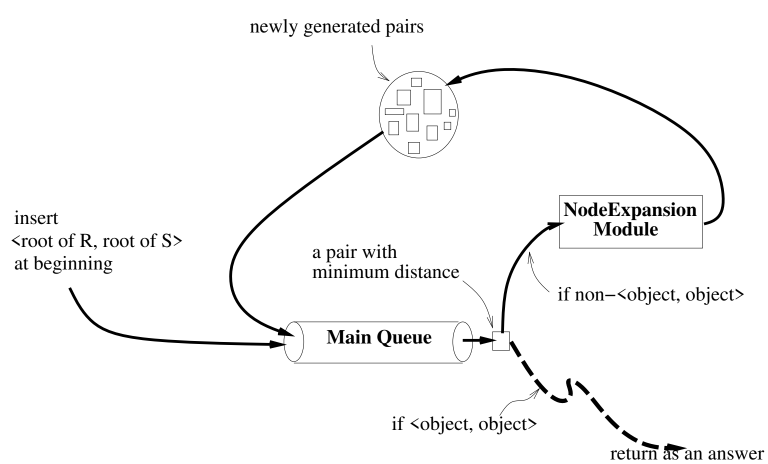


Figure 2: Framework of Incremental Distance Join (**IDJ**) Processing

A distance join query is often given with a stopping cardinality $k$ as in the "stop after" clause of the sample query in Section 1. Since it is known a priori how many object pairs need to be produced for a distance join query, this knowledge can be exploited to improve the performance of the query processing. Suppose a maximum of $k$ nearest pairs of objects are to be retrieved by a query. One plausible approach is to maintain $k$ candidate pairs of objects during the entire course of query processing. As they are the $k$ nearest object pairs known at each stage of query processing, any pair of nodes (and any pair of their child nodes) whose distance is longer than *all* of the $k$ candidate pairs cannot be qualified as a query result. Thus, we can use another priority queue to store the $k$ minimum distances, and use the queue to avoid having to insert unqualified pairs into the main queue during the node expansions. We call the priority queue a *distance queue*. Figure 3 depicts a typical framework of processing a $k$-distance join (**KDJ**) query using R-tree indexes and both main and distance queues.

Both main and distance queues can be implemented by heap structures. A main queue is normally implemented as a min-heap, because the query results are produced in an increasing order of distances. In contrast, a distance queue should be implemented as a max-heap, as the cutoff distance is determined by the maximum value among the $k$ distances stored in the distance queue at each stage of query processing. Pruning node pairs by the distance queue was shown to be very efficient from our experiments, especially when $k$ was rather small.

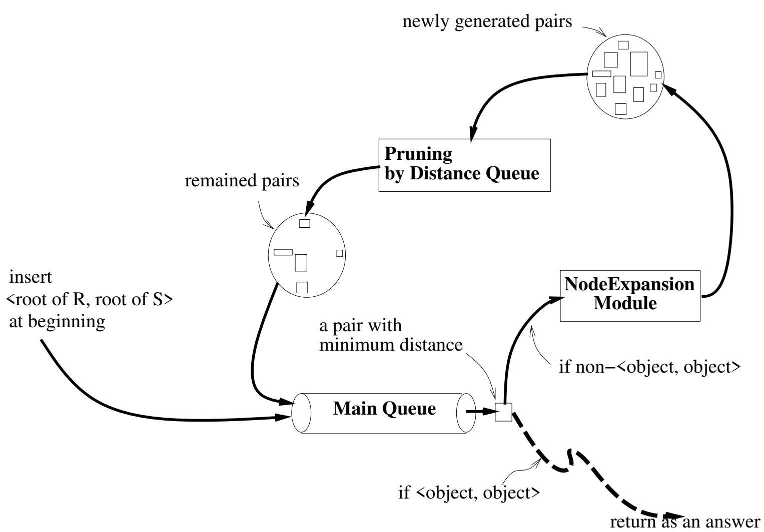


Figure 3: Framework of $k$-Distance Join (**KDJ**) Processing

## 2.2 Previous Work

The distance join algorithms proposed in [13] are based on uni-directional node expansions. When a pair of nodes $\langle r, s\rangle$ are retrieved from a main queue, either node $r$ is paired up with the child nodes of $s$, or node $s$ is paired up with the child nodes of $r$. None of the pairs are generated from a child node of $r$ and a child node of $s$. The advantage of the uni-directional expansion is that the number of pairs generated at each expansion step is limited to the fanout of an R-tree index being traversed, and an explosion of the main queue can be avoided. As is acknowledged by the authors of the algorithms, however, the main disadvantage of this approach is that the uni-directional expansion may lead to each node being accessed from disk more than necessary. And also, the uni-directional expansion requires pairing up node $r$ exhaustively with all the child nodes of node $s$ or vice versa.

For a spatial distance join query with a relatively small stopping cardinality $k$, the use of a distance queue is an effective means to prevent distant pairs from entering a main queue. For a large $k$ value, however, the distance queue may not work well as an effective pruning tool, because the cutoff value stored in the distance queue may remain too high for a long duration. This may in turn lead to a long delay particularly in the early stage of query processing. For these reasons, the previous algorithms suffer from poor performance for a $k$-distance join query with a large $k$ and an incremental distance join query, for which $k$ is unknown in advance.

Moreover, there is an important issue that was not fully addressed in [13]. A hybrid memory/disk technique was proposed as a queue management scheme, which partitions a queue based on the distance range. This technique keeps a partition in the shortest distance range in memory, while the rest of partitions are stored on disk. However, no mechanism was provided to determine a boundary distance value between the partition in memory and the rest, which may have a crucial impact on the performance of queue management.

Several closely related studies for nearest neighbor queries have been reported in the literature. Among those are nearest neighbor search algorithms based on Voronoi cells [2, 4] and branch and bound techniques [21, 22], a nearest neighbor search algorithm for ranking requirement [12], and multi-step $k$-nearest neighbor search algorithms [14, 23].

# 3 Optimized Plane-Sweep for Fast Pruning

In this section, we propose a new distance join algorithm $\mathcal{B}$**-KDJ** (Bidirectional expanding K-Distance Join) using a *bi-directional* node expansion, in an attempt to avoid redundant accesses to R-tree nodes. As is pointed out in Section 2, distance join algorithms based on an uni-directional expansion require accessing an R-tree node more than those based on bi-directional expansions. Under the bidirectional node expansion, for a pair $\langle r, s\rangle$, each of the child nodes of $r$ is paired up with each of the child nodes of $s$. This is essentially a Cartesian product, which may generate more redundant pairs than the uni-directional expansion does. Nonetheless, we will show $\mathcal{B}$**-KDJ** algorithm can effectively avoid generating redundant pairs by a plane sweeping technique [20] and novel strategies for choosing an axis and a direction for sweeping. The $\mathcal{B}$**-KDJ** algorithm is described in Algorithm 1.

## 3.1 Bidirectional Pair Expansion

**Algorithm 1:** $\mathcal{B}$**-KDJ**: $K$-Distance Join Algorithm with Bi-directional Expansion and Plane Sweep

set $\mathcal{A}nswerSet \leftarrow$ an empty set;
set $\mathcal{Q}_M$, $\mathcal{Q}_D \leftarrow$ empty main and distance queues;
insert a pair $\langle R.root, S.root\rangle$ into the main queue $\mathcal{Q}_M$;
**while** $|\mathcal{A}nswerSet| < k$ *and* $\mathcal{Q}_M \neq \emptyset$ **do**
  set c $\leftarrow$ dequeue($\mathcal{Q}_M$);
  **if** c *is an* $\langle object, object\rangle$ **then** $\mathcal{A}nswerSet \leftarrow \{c\} \cup \mathcal{A}nswerSet$;
  **else** *PlaneSweep*(c);
**end**

**procedure** PlaneSweep($\langle l, r\rangle$)
set L $\leftarrow$ sort_axis({child nodes of $l$}); // Sort the child nodes of $l$ by axis values.
set R $\leftarrow$ sort_axis({child nodes of $r$}); // Sort the child nodes of $r$ by axis values.
**while** $L \neq \emptyset$ *and* $R \neq \emptyset$ **do**
  $n \leftarrow$ a node with the min axis value $\in L \cup R$; // $n$ becomes an anchor.
  **if** $n \in L$ **then**
    $L \leftarrow L - \{n\}$; *SweepPruning*$(n, R)$;
  **else**
    $R \leftarrow R - \{n\}$; *SweepPruning*$(n, L)$;
  **end**
**end**

**procedure** SweepPruning($n, List$)
**for** *each node* $m \in List$ *in an increasing order of axis value* **do**
  **if** *axis_distance*$(n, m) > q\mathcal{D}_{max}$ **then** return; // No more candidates.
  **if** *real_distance*$(n, m) \leq q\mathcal{D}_{max}$ **then**
    insert $\langle n, m\rangle$ into $\mathcal{Q}_M$;
    **if** $\langle n, m\rangle$ *is an* $\langle object, object\rangle$ **then** insert *real_distance*$(n, m)$ into $\mathcal{Q}_D$; // $q\mathcal{D}_{max}$ modified.
  **end**
**end**

Like the distance join algorithms proposed in [13], $\mathcal{B}$**-KDJ** algorithm uses $q\mathcal{D}_{max}$ from the distance queue $\mathcal{Q}_D$ as a cutoff value to examine node pairs. If a pair of nodes $\langle r, s\rangle$ removed from the main queue are a pair of objects, then the object pair is returned as a query result. Otherwise, the pair is expanded by the *PlaneSweep* procedure for further processing.

Assume that a sweeping axis (*i.e.*, $x$ or $y$ dimensional axis) and a sweeping direction (*i.e.*, forward or backward) are determined, as we will describe in Sections 3.2 and 3.3. Then, the child nodes of $r$ and $s$

are sorted by $x$ or $y$ coordinates of one of the corners of their MBRs in an increasing or decreasing order, depending on the choice of sweeping axis and sweeping direction. Each node encountered during a plane sweep is selected as an *anchor*, and it is paired up with child nodes in the other group. For example, in Figure 4, a child node $r_1$ of $r$ is selected as an anchor, and the child nodes $s_1, s_2, s_3$ and $s_4$ of $s$ are examined for pairing, as they are within $q\mathcal{D}_{max}$ distance from $r_1$ along the sweeping axis (lines 11-14 and line 16).

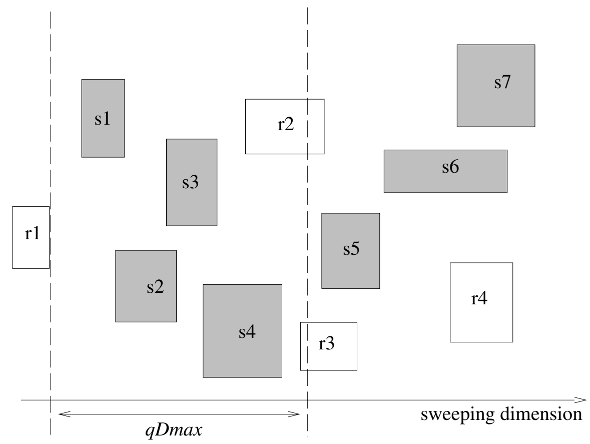


Figure 4: Bidirectional Node Expansion with Plane Sweeping

Since an axis distance between any pair $\langle r, s\rangle$ is always smaller than or equal to their real distance (*i.e.*, $axis\_distance(r,s) \leq real\_distance(r,s)$), real distances are computed only for nodes whose axis distances from the anchor are within the current $q\mathcal{D}_{max}$ value (line 17). Given that a real distance is more expensive to compute than an axis distance, it may yield non-trivial performance gain. Then, each pair whose real distance is within $q\mathcal{D}_{max}$ is inserted into the main queue $\mathcal{Q}_M$ (line 18). If it is a pair of objects, then update the current $q\mathcal{D}_{max}$ value by inserting the real distance of the object pair into the distance queue $\mathcal{Q}_D$ (line 19). [1]

For a relatively small $q\mathcal{D}_{max}$ value and two sets of evenly distributed spatial objects, the number of pairs for which $\mathcal{B}$**-KDJ** algorithm computes real distances and performs queue management operations is expected to be $\mathcal{O}(|r|+|s|)$ roughly. This justifies the additional cost of sorting child nodes for plane-sweeping, because the overall cost of $\mathcal{B}$**-KDJ** algorithm would otherwise be $\mathcal{O}(|r| \times |s|)$ by Cartesian products.

## 3.2 Sweeping Axis

We can improve $\mathcal{B}$**-KDJ** algorithm one step further by deciding the sweeping axis and direction on an individual pair basis. Intuitively, if child nodes (or data objects) are spread more widely along one dimension (say, $x$) than the other dimensions, then the bi-directional node expansion is likely to generate a smaller number of node pairs to compute the real distances for by plane-sweeping along the dimension $x$. This is because, when the nodes are more widely spread along a sweeping axis, the chance that a pair of nodes are within a $q\mathcal{D}_{max}$ distance along the sweeping axis is lower. For a pair of parent nodes shown in Figure 5, as an example, it would be better to choose $y$-axis as a sweeping axis, as the child nodes are more widely spread along the $y$-dimension. On the other hand, if $x$-axis is chosen as a sweeping axis, no pair of the child nodes will be pruned by $x$-axis distance comparison with $q\mathcal{D}_{max}$, because the $x$-axis distance between any pair of the child nodes is shorter than the $q\mathcal{D}_{max}$ value.

[1] There are alternatives as to what pairs are to be inserted into a distance queue: (1) any pairs encountered during node expansions, or (2) pairs of objects only. If a pair of non-object R-tree nodes is inserted into a distance queue, its maximum distance should be inserted as well [13]. Since the maximum distance tends to be larger than those of pairs of objects, most of non-object pairs are inserted into a distance queue only to be removed from the distance queue without reducing $q\mathcal{D}_{max}$ value. Thus, we decide to follow the second

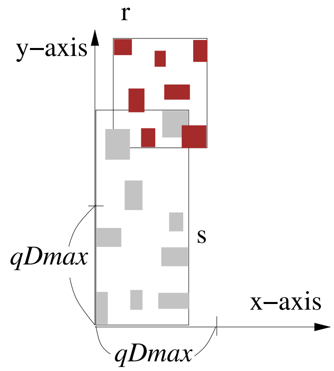


Figure 5: Effect of Right Selection of the Sweeping Axis

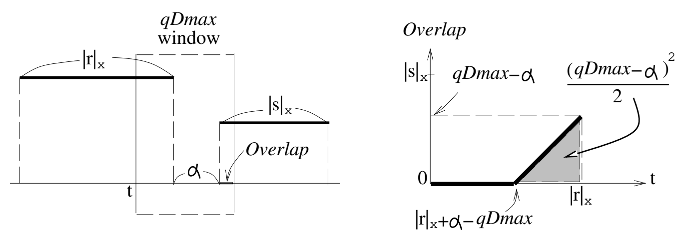


Figure 6: Sweeping Index

Formally, we define a new metric **sweeping index** as follows, and we use the metric to determine which axis a plane-sweep will be performed on. For a given pair $\langle r, s\rangle$ of R-tree nodes and a given $q\mathcal{D}_{max}$value, we can compute a sweeping index for each dimension. Conceptually, a sweeping index is an estimated number of node pairs that we need to compute the real distances for.[2]

$$\text{Sweeping Index}_x = \int_0^{|r|_x} Overlap(q\mathcal{D}_{max}, r, t)dt + \int_0^{|s|_x} Overlap(q\mathcal{D}_{max}, s, t)dt \tag{2}$$

In the first integral term of the equation above, $|r|_x$ is the side length of node $r$ along the dimension $x$. The function $Overlap(q\mathcal{D}_{max}, r, t)$ is a portion of the side length of $s$ along the dimension $x$, overlapped with a window of length $q\mathcal{D}_{max}$ whose left end point is located at a point $t$ within $|r|_x$ (*i.e.*, $0 \le t \le |r|_x$). (See the left diagram in Figure 6.) Thus, $Overlap(q\mathcal{D}_{max}, r, t)$ represents an estimated number of $s$'s child nodes intersected with a window $[t, t + q\mathcal{D}_{max}]$. The value of the function varies as the window moves along the dimension $x$ from $[0,q\mathcal{D}_{max}]$ to $[|r|_x, |r|_x + q\mathcal{D}_{max}]$. Therefore, the first integral term represents an estimated number of $s$'s child nodes encountered during the plane-sweeps performed for all the child nodes of $r$. The second integral term is symmetric with the first integral, and an identical description can be offered by exchanging $r$ and $s$.

A smaller sweeping index indicates that the bi-directional expansion needs to compute real distances for a smaller number of nodes pairs. For the reason, $\mathcal{B}$-**KDJ** algorithm chooses a dimension with the minimum sweeping index as a sweeping axis.

One thing we may be concerned about is the cost of computing a sweeping index for each dimension. The sweeping index may appear expensive to compute, as it includes two integral terms. For given $q\mathcal{D}_{max}$, $|r|_x$ and $|s|_x$ values, however, the sweeping index is reduced to a formula that involves only a few simple arithmetic operations. Suppose nodes $r$ and $s$ are not intersected along a dimension $x$, the minimum $x$-axis

option.

[2] An actual number of node pairs for which we need to compute the real distances would be computed by counting the number of $s$'s child nodes within $q\mathcal{D}_{max}$ axis distance from each child node of $r$, counting the number of $r$'s child nodes within $q\mathcal{D}_{max}$ axis distance from each child node of $s$, and then adding all the counts and dividing the count sum by two. However, this process will be very expensive.

distance between them is $\alpha$, and node $r$ appears before node $s$ in the plane-sweep direction along $x$-axis. (Again, see the left diagram in Figure 6.) Then, the second integral term of Equation (2) become zero, because all the child nodes of $r$ have already been swept when the first child node of $s$ is encountered. The first integral term varies depending on the conditions among $q\mathcal{D}_{max}$, $|r|_x$ and $|s|_x$ values and the proximity (*i.e.*, $\alpha$) of nodes $r$ and $s$ along a chosen dimension. Table 1 summaries the formulae of the sweeping index for non-overlapping nodes $r$ and $s$. The right diagram in Figure 6 illustrates how we can compute the first integral term and obtain a simple expression when $|s|_x + \alpha \le q\mathcal{D}_{max} \le |r|_x + \alpha$ is satisfied.

If nodes $r$ and $s$ are intersected, both the integral terms of Equation 2 become non-zero. By a similar reasoning, each integral term is also transformed into a formula with only a few simple arithmetic operations. Considering that each R-tree node may contain hundreds of child nodes, it will be a trivial cost to compute a sweeping index for each dimension, while the performance gain by the sweeping axis selection is expected to be significant. This is empirically corroborated by our experiments in Section 5.

| Condition | The first integral term of Equation 2 |
|---|---|
| $q\mathcal{D}_{max} \le \alpha$ | 0 |
| $\alpha < q\mathcal{D}_{max} \le \|r\|_x + \alpha$ | $\begin{cases} \frac{(q\mathcal{D}_{max}-\alpha)^2}{2} & \text{if } q\mathcal{D}_{max} \ge \|s\|_x + \alpha, \\ \frac{(q\mathcal{D}_{max}-\alpha)^2}{2} - \frac{(\|s\|_x)^2}{2} & \text{otherwise.} \end{cases}$ |
| $q\mathcal{D}_{max} \ge \|r\|_x + \alpha$ | $\begin{cases} \frac{(\|r\|_x)^2}{2} - \frac{(q\mathcal{D}_{max}-\|r\|_x-\alpha)^2}{2} & \text{if } \|r\|_x \le \|s\|_x, \\ \frac{(\|r\|_x)^2}{2} - \frac{(q\mathcal{D}_{max}-\|r\|_x-\alpha)^2}{2} - \frac{(\|r\|_x-\|s\|_x)^2}{2} & \text{if } (q\mathcal{D}_{max} - \|r\|_x - \alpha) \le \|s\|_x < \|r\|_x, \\ \|r\|_x \times \|s\|_x & \text{if } \|s\|_x < (q\mathcal{D}_{max} - \|r\|_x - \alpha). \end{cases}$ |

Table 1: Sweeping index for non-overlapping $r$ and $s$ ($\alpha$ is the minimum distance between $\langle r, s \rangle$)

### 3.3 Sweeping Direction

Once a sweeping axis is determined, a sweeping direction can be chosen to be either a *forward* sweep or a *backward* sweep. For a pair of nodes $r$ and $s$, we can define the forward and backward sweeps as follows.

- A forward plane-sweep scans the child nodes of $r$ and $s$ in an increasing order of coordinates along the chosen sweeping axis.
- A backward plane-sweep scans the child nodes of $r$ and $s$ in a decreasing order of coordinates along the chosen sweeping axis.

Consider nodes $r$ and $s$ projected on a sweeping axis. The projected images generate three consecutive closed intervals on the sweeping axis, unless the projected images are completely overlapped. For example, if nodes $r$ and $s$ are intersected as in Figure 7(a), an interval in the left is projected from $r$, one in the middle from both $r$ and $s$, and one in the right from $s$. The interval in the middle may be projected from none of $r$ and $s$, if $r$ and $s$ are separate as in Figure 7(b). Both the intervals in the left and right may be projected from the same node, if one node is contained in the other as in Figure 7(c).

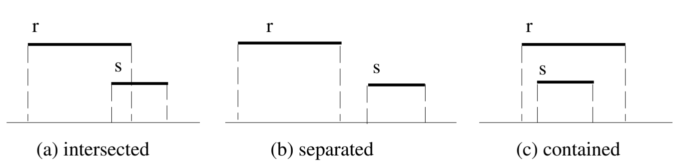


Figure 7: Three intervals projected from two nodes $r$ and $s$

However, it does not matter which node an interval is projected from, because the a sweeping direction is determined solely on the relative length of the intervals in the left and right. A sweeping direction is determined by comparing the length of the left and right intervals: *if the left projected interval is shorter than the right one, then a forward direction is chosen. Otherwise, a backward direction is chosen.* By this strategy of choosing a sweeping direction, a pair of nodes closer to each other are likely to be examined earlier than those farther to each other. This in turn allows a pair of closer nodes are inserted into the main queue (and the distance queue as well if they are an object pair), and helps reduce the $q\mathcal{D}_{max}$ value more rapidly.

In summary, the sweeping axis selection improves the bi-directional node expansion step by pruning more child node pairs whose axis distances are larger than the $q\mathcal{D}_{max}$, while, the sweeping direction selection does by reducing the $q\mathcal{D}_{max}$ value more rapidly.

# 4 Adaptive Multi-Stage $k$-Distance Join

In $\mathcal{B}$-**KDJ** algorithm, $q\mathcal{D}_{max}$ value is initially set to an infinity and becomes smaller as the algorithm proceeds. The adaptation of the $q\mathcal{D}_{max}$ value has a crucial impact on the performance of $\mathcal{B}$-**KDJ** algorithm, as $q\mathcal{D}_{max}$ is used as a cutoff to prevent pairs of distant nodes from entering the main queue. If the $q\mathcal{D}_{max}$ value approaches to the real $\mathcal{D}_{max}$ value slowly, the early stage of $\mathcal{B}$-**KDJ** algorithm will be delayed considerably for handling too many pairs of distant nodes. Consequently, at the end of the algorithm processing, the main queue may end up with a large number of distant pairs whose insertions to the main queue were not necessary. The performance effect of *slow start* is more pronounced for a larger $k$, as the main queue and distance queue tend to grow large for a large $k$, and thereby increasing the $q\mathcal{D}_{max}$ value. From our experiments with $k$ as high as 100,000, we observed that more than 90 percent of execution time of $k$-distance join algorithms was spent to produce the first one percent (*i.e.*, 1,000 pairs) of final query results.

In this section, we propose new adaptive multi-stage distance join algorithms $\mathcal{AM}$-**KDJ** and $\mathcal{AM}$-**IDJ** that mitigate the slow start problem by *aggressive pruning* and *compensation*.

## 4.1 Adaptive Multi-Stage $k$-Distance Join

The slow start problem is essentially caused by a pruning strategy using $q\mathcal{D}_{max}$, whose value is dynamically updated as tree indexes are traversed and therefore not under direct control of the distance join algorithms. To circumvent this problem, we introduce a new pruning measure $e\mathcal{D}_{max}$, which is an estimated $\mathcal{D}_{max}$ value for a given $k$. The $e\mathcal{D}_{max}$ value is set to an initial estimation at the beginning and adaptively corrected during the algorithm processing. We will discuss techniques for initial estimation and adaptive correction in Section 4.3.

$\mathcal{AM}$-**KDJ** algorithm is similar to $\mathcal{B}$-**KDJ** algorithm in that both the algorithms use a bi-directional node expansion. However, unlike the single-stage $\mathcal{B}$-**KDJ** algorithm, where only $q\mathcal{D}_{max}$ is used for pruning, both $q\mathcal{D}_{max}$ and $e\mathcal{D}_{max}$ are used as cutoff values for pruning distant pairs in $\mathcal{AM}$-**KDJ** algorithm. Specifically, in the *aggressive pruning* stage (described in Algorithm 2),

- $e\mathcal{D}_{max}$ is used for pruning based on *axis distances* for aggressive pruning and thereby limiting the size of main and distance queues (line 22),
- $q\mathcal{D}_{max}$ is used for further pruning on *real distances* for nodes whose axis distances are within $e\mathcal{D}_{max}$, in the same way as $\mathcal{B}$-**KDJ**.

With a properly estimated $e\mathcal{D}_{max}$ value, $\mathcal{AM}$-**KDJ** algorithm can prune a large number of distant pairs in the first stage and avoid a significant portion of delay due to the slow start. However, if $\mathcal{AM}$-**KDJ** algorithm becomes too aggressive by choosing an underestimated $e\mathcal{D}_{max}$ value, even close enough pairs may be discarded incorrectly. To avoid any false dismissals, we introduce another queue called *compensation queue* ($\mathcal{Q}_C$). The compensation queue stores every node pair retrieved from the main queue (line 11), if it is not a pair of objects or all the child nodes of the pair are examined by plane sweeping. It should also be noted that $q\mathcal{D}_{max}$ *but not* $e\mathcal{D}_{max}$ is used for nodes whose axis distances are within $e\mathcal{D}_{max}$. If $e\mathcal{D}_{max}$ values are used instead, the compensation stage will become very costly in order to keep track of an exhaustive set of pruned pairs and recover qualified pairs from them. Using $q\mathcal{D}_{max}$ values also makes the performance of $\mathcal{AM}$-**KDJ** fairly insensitive to estimated $e\mathcal{D}_{max}$ values.

**Algorithm 2:** $\mathcal{AM}$**-KDJ**: Adaptive Multi-Stage K-Distance Join Algorithm (Aggressive Pruning)

1: set $AnswerSet \leftarrow$ an empty set;
2: set $\mathcal{Q}_M$, $\mathcal{Q}_D$, $\mathcal{Q}_C \leftarrow$ empty main, distance and restart queues ;
3: set $e\mathcal{D}_{max} \leftarrow$ an initial estimated $\mathcal{D}_{max}$;
4: insert a pair $\langle R.root, S.root \rangle$ to the main queue $\mathcal{Q}_M$;
5: **while** $|AnswerSet| < k$ *and* $\mathcal{Q}_M \neq \emptyset$ **do**
6:     set c $\leftarrow$ dequeue($\mathcal{Q}_M$);
7:     **if** c *is an* $\langle object, object \rangle$ **then** $AnswerSet \leftarrow \{c\} \cup AnswerSet$;
    **else**
8:         **if** $q\mathcal{D}_{max} \leq e\mathcal{D}_{max}$ **then** $e\mathcal{D}_{max} \leftarrow q\mathcal{D}_{max}$; // overestimated $e\mathcal{D}_{max}$
9:         **if** c.$distance < e\mathcal{D}_{max}$ **then**
            reinsert c back into $\mathcal{Q}_M$;
            break; // Terminate the Aggressive Pruning stage.
        **end**
10:         AggressivePlaneSweep(c);
11:         enqueue($\mathcal{Q}_C$, c);
    **end**
**end**
12: **if** $|AnswerSet| < k$ **then** execute Algorithm 3;

**procedure** AggressivePlaneSweep($\langle l, r \rangle$)
13: set L $\leftarrow$ sort_axis({child nodes of $l$}); // Sort the child nodes of $l$ by axis values.
14: set R $\leftarrow$ sort_axis({child nodes of $r$}); // Sort the child nodes of $r$ by axis values.
15: **while** $L \neq \emptyset$ *and* $R \neq \emptyset$ **do**
16:     $n \leftarrow$ a node with the min axis value $\in L \cup R$; // $n$ becomes an anchor.
17:     **if** $n \in L$ **then**
18:         $L \leftarrow L - \{n\}$; $AggressiveSweepPruning(n, R)$;
19:         $n.compensate \leftarrow$ a node in $R$ with the min axis value and not yet paired with $n$;
    **else**
20:         $R \leftarrow R - \{n\}$; $AggressiveSweepPruning(n, L)$;
21:         $n.compensate \leftarrow$ a node in $L$ with the min axis value and not yet paired with $n$;
    **end**
**end**

**procedure** AggressiveSweepPruning($n, List$)
Same as the SweepPruning procedure in Algorithm 1 except line 16 replaced with the following:
22: **if** $axis\_distance(n, m) > e\mathcal{D}_{max}$ **then** return;

For example, in Figure 8 (drawn from Figure 4), an anchor node $r_1$ is paired up with nodes $s_1$ and $s_2$ but not with $s_3$ and $s_4$ in the aggressive pruning stage, because only $s_1$ and $s_2$ are within $e\mathcal{D}_{max}$ from the anchor node $r_1$. Thus, $\mathcal{AM}$**-KDJ** algorithm inserts only two pairs ($\langle r_1, s_1 \rangle$, $\langle r_1, s_2 \rangle$) into a main queue, instead of all four pairs ($\langle r_1, s_1 \rangle$, $\langle r_1, s_2 \rangle$, $\langle r_1, s_3 \rangle$, $\langle r_1, s_4 \rangle$) that would be enqueued by $\mathcal{B}$**-KDJ** algorithm. Then, the pair $\langle r, s \rangle$ currently being expanded is inserted into a compensation queue.

The aggressive pruning stage ends when one of the following conditions is satisfied: (1) the main queue becomes empty (line 5), (2) $k$ or more query results have been returned (line 5), or (3) the distance of a node pair retrieved from the main queue becomes smaller than $e\mathcal{D}_{max}$ (line 9). When the condition (2) is satisfied, obviously it is not necessary to execute the compensation stage of the $\mathcal{AM}$**-KDJ** algorithm. (An overestimated $e\mathcal{D}_{max}$ can also be detected by comparing with $q\mathcal{D}_{max}$ value (line 8). In this case, instead of terminating the first stage, $\mathcal{AM}$**-KDJ** behaves exactly the same as $\mathcal{B}$**-KDJ** algorithm by using $q\mathcal{D}_{max}$ alone as a cutoff value.) When the condition (3) is satisfied, $e\mathcal{D}_{max}$ must have been underestimated and the compensation stage (described in Algorithm 3) begins its processing by inserting all the pairs stored in the compensation queue to the main queue.

In the compensation stage, the pairs in the main queue are processed in a similar way as $\mathcal{B}$**-KDJ** algorithm, but there are two notable differences from $\mathcal{B}$**-KDJ** algorithm. First, the child nodes are not sorted again because they have already been sorted in the first stage. Second, for the pairs already expanded once in the first stage, only child pairs not examined in the first stage are processed by plane sweeping.

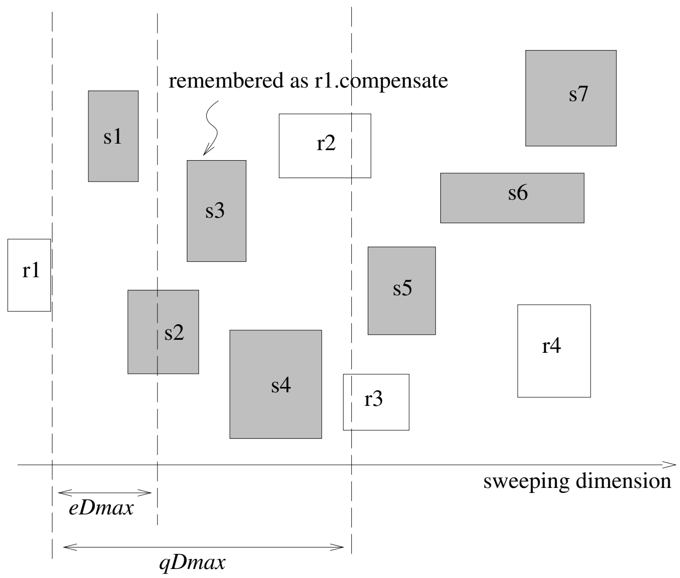


Figure 8: Aggressive pruning with $q\mathcal{D}_{max}$ and $e\mathcal{D}_{max}$

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**Algorithm 3: $\mathcal{AM}$-KDJ**: Adaptive Multi-Stage K-Distance Join Algorithm (Compensation Stage)

**23:** insert all elements in $\mathcal{Q}_C$ into $\mathcal{Q}_M$;<br>
**24:** **while** $|\mathcal{A}nswerSet| < k$ *and* $\mathcal{Q}_M \neq \emptyset$ **do**<br>
**25:**     set c $\leftarrow$ dequeue($\mathcal{Q}_M$);<br>
**26:**     **if** c *is an* $\langle object, object \rangle$ **then** $\mathcal{A}nswerSet \leftarrow \{c\} \cup \mathcal{A}nswerSet$;<br>
**27:**     **else** CompensatePlaneSweep(c);<br>
**end**

**procedure** CompensatePlaneSweep($\langle l, r \rangle$)<br>
**28:** L $\leftarrow$ { child nodes of $l$ sorted in Stage One}; // $\{L[1], L[2], \ldots, L[|L|]\}$<br>
**29:** R $\leftarrow$ { child nodes of $r$ sorted in Stage One}; // $\{R[1], R[2], \ldots, R[|R|]\}$<br>
**30:** **while** $L \neq \emptyset$ *and* $R \neq \emptyset$ **do**<br>
**31:**     $n \leftarrow$ a node with the min axis value $\in L \cup R$; // $n$ becomes an anchor.<br>
**32:**     **if** $n \in L$ **then**<br>
**33:**         $L \leftarrow L - \{n\}$; $R' \leftarrow$ {node list in $R$ not paired with $n$ in the Stage One }; // { $R[n.compensate], R[n.compensate + 1], \ldots, R[|R|]$}<br>
**34:**         $SweepPruning(n, R')$;<br>
    **else**<br>
**35:**         $R \leftarrow R - \{n\}$; $L' \leftarrow$ {node list in $L$ not paired with $n$ in the Stage One }; // { $L[n.compensate], L[n.compensate + 1], \ldots, L[|L|]$}<br>
**36:**         $SweepPruning(n, L')$;<br>
    **end**<br>
**end**

---

This is feasible by bookkeeping done in the first stage (lines 19 and 21). For these reasons, the cost of the compensating stage is not considerable compared with the cost of restarting the algorithm. In summary, $\mathcal{AM}$-**KDJ** algorithm uses $e\mathcal{D}_{max}$ to avoid the slow start problem in the aggressive pruning stage and speeds up the query processing.

## 4.2 Adaptive Multi-stage Incremental Distance Join

Consider on-line query processing and internet database search environments, where users interact with database systems in a way the number of required matches can be determined interactively or changed at any point of query processing. Consider also a complex query that pipelines the results from a spatial distance join to a filter stage. Under these circumstances, the number of pairs ($k$) that should be returned from a distance join is not known a priori, and hence a $k$-distance join algorithm proposed in [13] and $\mathcal{B}$-**KDJ** algorithm presented in Section 3 cannot be used directly.

An important advantage of $\mathcal{AM}$-**KDJ** algorithm proposed in the previous section is that $\mathcal{AM}$-**KDJ** algorithm can be extended to an incremental algorithm (we call $\mathcal{AM}$-**IDJ**) to support the interactive applications described above. The main difference between $\mathcal{AM}$-**KDJ** and $\mathcal{AM}$-**IDJ** algorithms is that $\mathcal{AM}$-**IDJ** does not maintain a distance queue. This is because it is not feasible to keep an unknown number of distances in a distance queue, due to the lack of a priori knowledge about $k$, Thus, $\mathcal{AM}$-**IDJ** algorithm uses $e\mathcal{D}_{max}$ alone as a cutoff value for pruning distant pairs, because $q\mathcal{D}_{max}$ would be drawn only from a distance queue.

Without $q\mathcal{D}_{max}$, $\mathcal{AM}$-**IDJ** works as a stepwise incremental algorithm. First, $\mathcal{AM}$-**IDJ** starts by determining an initial value $k_1$ and estimating an initial $e\mathcal{D}_{max1}$ for $k_1$. Then, it performs the same way as the first stage of $\mathcal{AM}$-**KDJ** algorithm without $q\mathcal{D}_{max}$. However, the first stage may terminates before producing enough object pairs (*i.e.*, less than $k_1$), because $\mathcal{AM}$-**IDJ** does not use $q\mathcal{D}_{max}$ as a cutoff value. If that happens, $\mathcal{AM}$-**IDJ** algorithm estimates $e\mathcal{D}_{max2}$ value for $k_2$ ($k_2 > k_1$) and initiates a compensation stage.

Even when a sufficient number of object pairs have been returned from the first stage, users may request more answers. Then, $\mathcal{AM}$-**IDJ** initiates a compensation stage by determining $k_2$ and estimating a new $e\mathcal{D}_{max2}$ accordingly. As shown in Figure 9 (drawn from Figure 4), the compensation stage can initiate another compensation stage at the end of its processing, by choosing $k_3$ and $e\mathcal{D}_{max3}$. This process continues until users stop requesting more answers. In this way, $\mathcal{AM}$-**IDJ** algorithm can be used to produce query results incrementally without limiting the maximum number of pairs in advance.

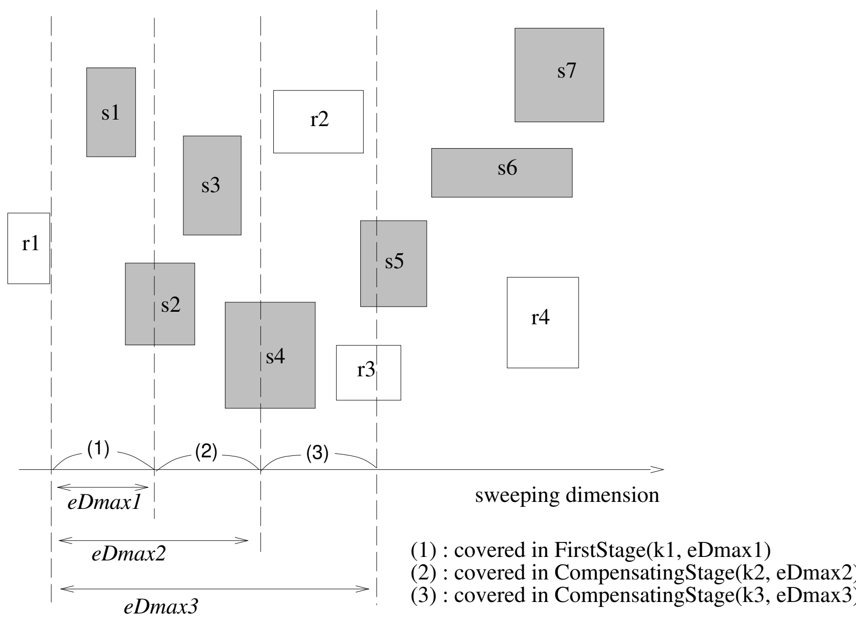



Figure 9: Adaptive Multi-Stage Incremental Distance Join

### 4.3 Estimating the Maximum Distance ($e\mathcal{D}_{max}$)

Both $\mathcal{AM}$-**KDJ** and $\mathcal{AM}$-**IDJ** algorithms process a distance join query based on an estimated cutoff value $e\mathcal{D}_{max}$. Thus, there should be a way to obtain an initial estimate and correct the estimate adaptively as the algorithms proceed. Assuming data sets are uniformly distributed, we provide mechanisms to choose an initial estimate of $e\mathcal{D}_{max}$, and to adaptively correct it.

If the distribution of a data set is skewed, then a larger number of close pairs can be found in a smaller dense region of the data space. We expect that the formulae given in this section tend to overestimate $e\mathcal{D}_{max}$ value for non-uniformly distributed data sets, especially when a stopping cardinality $k$ is far smaller than the number of all pairs of objects (*i.e.*, $k \ll |R| \times |S|$). This was corroborated by our experiments as described in Section 5.4.

#### 4.3.1 Initial estimation

Let $|R|$ and $|S|$ be the number of data objects in sets $R$ and $S$, respectively. Then, the number of data objects in $S$ within a distance $d$ from a data object in $R$ is approximated by $|S| \times \frac{\pi \times d^2}{area(R \cap S)}$. Therefore, the total number of object pairs ($k$) within a distance $d$ is given by

$$k = |R| \times |S| \times \frac{\pi \times d^2}{area(R \cap S)}.$$

For a given $k$ value as the number of requested query results, an initial estimation of $e\mathcal{D}_{max}$ can be obtained from the above equation as follows.

$$e\mathcal{D}_{max} = \sqrt{k \times \rho} \qquad (\text{where } \rho = \frac{area(R \cap S)}{\pi \times |R| \times |S|}). \tag{3}$$

#### 4.3.2 Adaptive Correction of Estimated Distance $e\mathcal{D}_{max}$

The performance of $\mathcal{AM}$-**KDJ** and $\mathcal{AM}$-**IDJ** algorithms can be further improved by adaptively adjusting the value of $e\mathcal{D}_{max}$ at runtime. Adaptive correction of $e\mathcal{D}_{max}$ can be done at any point of query processing by estimating a new $e\mathcal{D}_{max}$ from the number of object pairs $k_0$ ($k_0 < k$) obtained up to the point and the real distance of the $k_0$-th object pair, $\mathcal{D}_{max(k_0)}$. Specifically, the new estimate $e\mathcal{D}_{max}'$ can be computed from Equation (3) as

$$e\mathcal{D}_{max}' = \sqrt{\mathcal{D}_{max(k_0)}^{2} + (k - k_0)\rho} \tag{4}$$

by arithmetic correction, or as

$$e\mathcal{D}_{max}' = \mathcal{D}_{max(k_0)} \times \sqrt{k/k_0} \tag{5}$$

by geometric correction if $\mathcal{D}_{max}(k_0) \neq 0$. In practice, we propose computing $e\mathcal{D}_{max}'$ in both ways, and then choose the minimum if the query processing needs to be err on the aggressive side. Otherwise, the maximum is chosen as $e\mathcal{D}_{max}'$.

Note that the new estimate $e\mathcal{D}_{max}'$ can sometimes grow beyond the previous estimate. If this happens, some pairs whose distances are larger than the previous estimate but smaller than the new estimate could have already been pruned and will never be examined in the current processing stage under the new estimate. Thus, to guarantee the correctness of the distance join, the algorithm should initiate a compensation stage, as soon as a pair whose distance is smaller than the smallest $e\mathcal{D}_{max}$ is dequeued from the main queue.

### 4.4 Queue Management

Efficient queue management is one of the key components of the distance join algorithms proposed in this paper. Each of the $\mathcal{B}$-**KDJ**, $\mathcal{AM}$-**KDJ**, and $\mathcal{AM}$-**IDJ** algorithms relies on the use of one or more priority queues for query processing. In particular, the main queue ($\mathcal{Q}_M$) is heavily used by all of the proposed algorithms, and its performance impact is significant. In the worst case, the main queue can grow as large as the product of *all* objects of two R-tree indexes. That is, the size of $\mathcal{Q}_M$ is in $\mathcal{O}(|R_{obj}| \times |S_{obj}|)$, where $|R_{obj}|$ and $|S_{obj}|$ are the number of all objects in $R$ and $S$, respectively. Thus, it is not always feasible to store the main queue in memory.

It was reported in [13] that a simple memory-based implementation might slow down query processing severely, due to excessive virtual memory thrashing. A hybrid memory/disk scheme [13] and a technique based on range partitioning [9] have been proposed to improve queue management and to avoid wasted sorting I/O operations. We adopt a similar scheme for queue management, which partitions a queue by range based on distances of pairs. A partition in the shortest distance range is kept in memory as a heap structure, while the rest of partitions are stored on disk as merely unsorted piles.

When the in-memory heap becomes full, it is *split* into two parts, and then one in the longer distance range is moved to disk as a new segment. When the in-memory heap becomes empty, a disk-resident segment in the shortest distance range or a part of the segment is *swapped in* to memory to fill up the in-memory heap. Each of the split and swap-in operations requires $\mathcal{O}(n \log n)$ computational cost for a heap of $n$ elements as well as I/O cost for reading and writing a segment. Thus, it is important to minimize the required number of those operations, which largely depends on the partition boundary values between the in-memory heap and the first disk-resident segment, and between those consecutive segments. However, as it is impossible to predict an exact $\mathcal{D}_{max}$ value for a given $k$, so is it difficult to determine optimal distance values as segment boundaries.

To address this issue, we use Equation (3) to determine the boundary distance values. Suppose $n$ is the number of elements that can be stored in an in-memory heap. Then, the boundary value between the in-memory heap and the first disk-resident segment is given by $\sqrt{n \times \rho}$, and the boundary value between the first and second segments is given by $\sqrt{(2 \times n) \times \rho}$, and so on.

In addition to a main queue, multi-stage algorithms $\mathcal{AM}$-**KDJ** and $\mathcal{AM}$-**IDJ** use a compensation queue ($\mathcal{Q}_C$) in the compensation stage. Unlike the main queue, a compensation queue does not store any pair of objects. In other words, a compensation queue can store pairs of non-object R-tree nodes only. Thus, the size of $\mathcal{Q}_C$ is in $\mathcal{O}(|R_{node}| \times |S_{node}|)$, where $|R_{node}|$ and $|S_{node}|$ are the number of nodes (both internal and leaf nodes) in $R$ and $S$, respectively. This is a significantly lower upper-bound than a main queue has. We also observed from our experiments that compensation queues were several orders of magnitude smaller than main queues. As for a distance queue used by $\mathcal{B}$-**KDJ** and $\mathcal{AM}$-**KDJ** algorithms, its size is always bounded by a given $k$ value. For these reasons, under most circumstances, we assume either a compensation queue and a distance queue fits in memory. If any of these queues outgrows memory, the same partitioning technique used for a main queue will be applied.

# 5 Performance Evaluation

In this section, we evaluate the proposed algorithms empirically and compare with previous work. In particular, the proposed $\mathcal{B}$-**KDJ**, $\mathcal{AM}$-**KDJ** and $\mathcal{AM}$-**IDJ** algorithms were compared with Hjaltason and Samet's $k$-distance and incremental distance join algorithms (hereinafter denoted as *HS*-**KDJ** and *HS*-**IDJ**, respectively) for $k$-distance join (**KDJ**) and incremental distance join (**IDJ**) queries. We also include the performance of an R-tree based spatial join algorithm [7] combined with a sort operation (denoted as *SJ*-**SORT**) in most of the experiments. For each distance join query, a spatial join operation was performed with a real $\mathcal{D}_{max}$ value to generate the $k$ nearest pairs. Then, a sort operation was performed to return the query results in an increasing order of distances. Note that we made a favorable assumption for *SJ*-**SORT** that a real $\mathcal{D}_{max}$ value was known a priori.

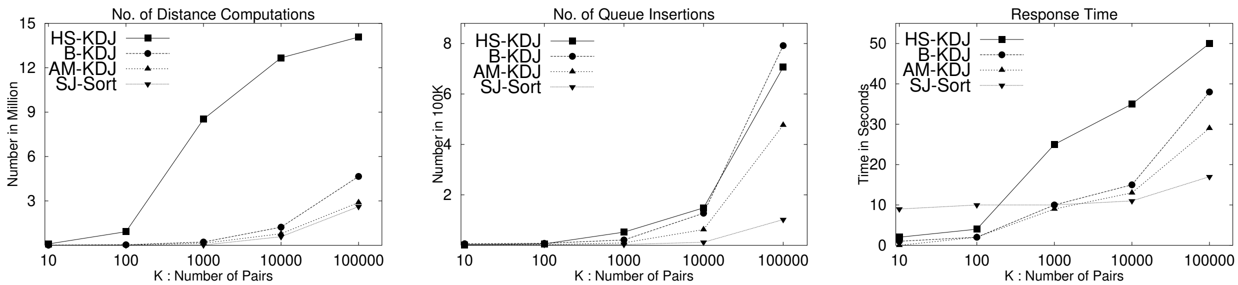
## 5.1 Experimental Settings

Experiments were performed on an Intel Pentium II workstation with 200 MHz clock rate. This workstation has 128 MBytes of memory and 4 GBytes of disk storage with Ultra-wide SCSI interface, and runs on Linux kernel version 2.0.34.

**Data sets** To evaluate distance join algorithms, we used real-world data sets in TIGER/Line97 from the U.S. Bureau of Census [17]. The particular data sets we used were 64,952 streets and 191,289 hydrographic objects from the Arizona state. Throughout all the experiments, the same page size of 4 KBytes was used for disk I/O and R*-tree [3] nodes.

**Metrics** We measured the performance of various algorithms based on the following metrics to compare the algorithms in different aspects such as computational cost and I/O cost.

1. *number of distance computations*: The cost of computing distances between pairs of nodes (or objects) constitutes a significant portion of the computational cost of a distance join operation. Thus, the total number of distance computations required by a distance join algorithm provides a direct indication of its computational performance.




(a) Distance Computations (b) Queue Insertions (c) Response Time

Figure 10: Performance $k$-Distance Joins

2. *number of queue insertions*: The task of managing a main queue is largely I/O intensive as well as CPU intensive. Thus, the total number of insertions to a main queue required by a distance join algorithm provides a reasonable indication of its I/O performance, because insertions are much more frequent than deletions.

3. *response time*: Actual query response times were measured for overall performance of distance join algorithms.

## 5.2 Evaluation of $k$-Distance Joins

In this set of experiments, we varied a stopping cardinality $k$ from 10 to 100,000 to compare the performance of *HS*-**KDJ**, $\mathcal{B}$-**KDJ** and $\mathcal{AM}$-**KDJ** algorithms. The size of in-memory portion of a main queue was fixed to 100,000 elements. For $\mathcal{AM}$-**KDJ** algorithm, we used Equation (3) to estimate $e\mathcal{D}_{max}$ values, and we observed a tendency for $e\mathcal{D}_{max}$ values to be overestimated with respect to real $\mathcal{D}_{max}$ values. For example, for $k = 100,000$, $e\mathcal{D}_{max}$ was about 1.9 times larger than a real $\mathcal{D}_{max}$.

Figure 10(a) shows that both $\mathcal{B}$-**KDJ** and $\mathcal{AM}$-**KDJ** reduced the number of distance computations significantly. The numbers of distance computations required by the algorithms were smaller than those required by *HS*-**KDJ** algorithm by up to two orders of magnitude. $\mathcal{AM}$-**KDJ** was almost identical to *SJ*-**SORT** by this metric. This demonstrates that the optimized plane-sweep method was very effective in pruning distant pairs generated by bi-directional expansions. On the other hand, *HS*-**KDJ** algorithm examines all possible pairs exhaustively in uni-directional expansions.

In Figure 10(b), *HS*-**KDJ** and $\mathcal{B}$-**KDJ** were comparable in queue insertions. $\mathcal{B}$-**KDJ** was slightly better than *HS*-**KDJ** for small $k$ values, and vice versa for large $k$ values. $\mathcal{AM}$-**KDJ** was always better than both *HS*-**KDJ** and $\mathcal{B}$-**KDJ**. This result confirms our conjecture that the optimized plane-sweep method can prevent an explosion of a main queue that would be caused by bi-directional node expansions without the optimized plane-sweep.

As Figure 10(c) shows, $\mathcal{B}$-**KDJ** and $\mathcal{AM}$-**KDJ** outperformed *HS*-**KDJ** in response time by a factor of 1.7 ($k$ = 100,000) up to 3 ($k$ = 1,000 or 10,000). For large $k$ values ($k > 10,000$), the response time of $\mathcal{AM}$-**KDJ** was within about 70 percent of that of *SJ*-**SORT**. Note that *SJ*-**SORT** was worse than all three $k$-distance join algorithms in response time for small $k$ values ($k \leq 1000$). This was because there were about 1,000 pairs of intersected objects in the Arizona data sets, and all the intersected object pairs were returned as a distance join query results, no matter what distance cutoff was provided for the *SJ*-**SORT** processing.

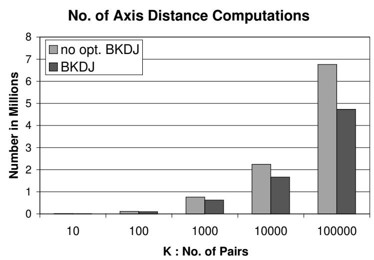
Table 2 shows that the proposed $\mathcal{B}$-**KDJ** and $\mathcal{AM}$-**KDJ** algorithms based on bi-directional node expansions require a far smaller number of R-tree node accesses than *HS*-**KDJ** algorithm, which is based on uni-directional node expansions.

## 5.3 Impact of Optimized Plane-Sweep

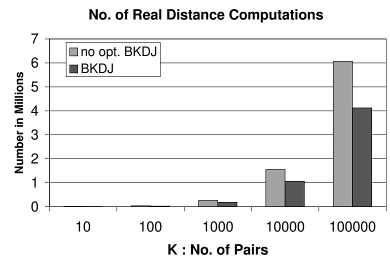
To further analyze the performance impacts of the optimized plane-sweep method proposed in Section 3, we measured the performance of $\mathcal{B}$-**KDJ** with the optimization turned off. Specifically, a sweeping axis

| **KDJ** Algorithms | Stopping cardinality $k$ | | | | |
|---|---|---|---|---|---|
| | 10 | 100 | 1,000 | 10,000 | 100,000 |
| *HS*-**KDJ** | 456 | 4,345 | 37,450 | 56,016 | 62,432 |
| $\mathcal{B}$-**KDJ** | 42 | 456 | 3,444 | 4,120 | 4,244 |
| $\mathcal{AM}$-**KDJ** | 36 | 442 | 3,308 | 4,120 | 4,244 |
| *SJ*-**SORT** | 4,106 | 4,106 | 4,106 | 4,120 | 4,244 |

Table 2: No. of R-Tree Node Accesses for $k$-Distance Joins




(a) Axis Distance Computations




(b) Real Distance Computations

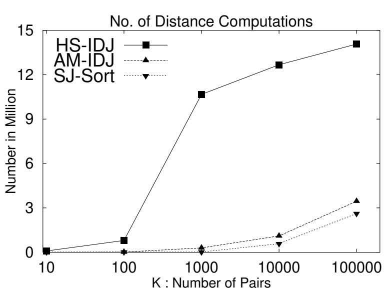
Figure 11: Improvements by Optimized Plane Sweep

and direction were fixed to $x$-axis and forward direction, for $\mathcal{B}$-**KDJ** with the optimization turned off. As Figure 11 shows, the optimized plane-sweep alone reduced the number of required axis and real distance computations by about 30 to 42 percent.

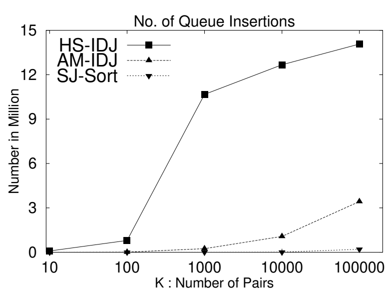
## 5.4 Evaluation of Incremental Distance Joins

As in the previous section, we varied a stopping cardinality $k$ from 10 to 100,000 to compare the performance of incremental distance join algorithms *HS*-**IDJ** and $\mathcal{AM}$-**IDJ**. Unlike the previous experiments, the size of in-memory portion of a main queue was set to $k + 100,000$ elements instead of 100,000 elements, so that the incremental distance join algorithms were evaluated under the same memory constraints as the $k$-distance join algorithms, which used $k$ additional elements for a distance queue.

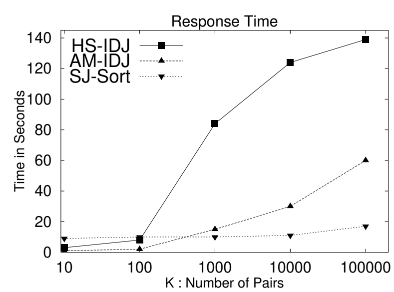
As Figures 12(a) and 12(b) show, 75 to 98 percent of distance computations and queue insertions performed by *HS*-**IDJ** algorithm were eliminated by $\mathcal{AM}$-**IDJ** algorithm. The significant improvement in these two metrics in turn led to improvement in response time by a factor of four to six in Figure 12(c). Like $\mathcal{AM}$-**KDJ** algorithm, Equation (3) was used to estimate $e\mathcal{D}_{max}$ values for $\mathcal{AM}$-**IDJ** algorithm.




(a) Distance Computations




(b) Queue Insertions




(c) Response Time

Figure 12: Performance of Incremental Distance Joins

## 5.5 Impact of Memory Size

In this set of experiments, we examined the performance impact of memory constraint on queue management. We measured the response time of *HS*-**KDJ**, $\mathcal{B}$-**KDJ** and $\mathcal{AM}$-**KDJ** algorithms for a fixed stopping cardinality $k = 100,000$. The size of in-memory portion of a main queue was varied from 5,000 to 500,000. As Figure 13 shows, the response time of all three algorithms improved as the size of available memory increased. Moreover, the proposed $\mathcal{B}$-**KDJ** and $\mathcal{AM}$-**KDJ** algorithms showed consistently better performance than *HS*-**KDJ** all over the examined range of memory size.

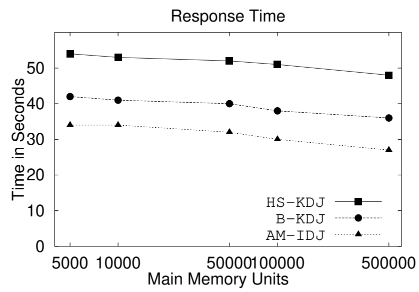



Figure 13: Performance Impact of Memory Size

## 5.6 Impact of $e\mathcal{D}_{max}$ Estimation on $\mathcal{AM}$-KDJ Performance

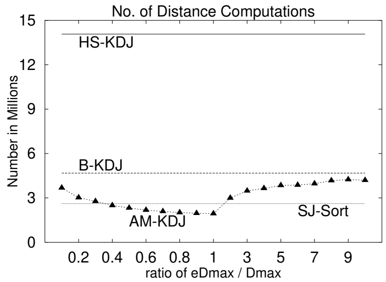
We designed a set of experiments to characterize the performance of $\mathcal{AM}$-**KDJ** algorithm with respect to the accuracy of estimated $e\mathcal{D}_{max}$ values. Instead of using Equation (3) to estimate $e\mathcal{D}_{max}$, we varied the $e\mathcal{D}_{max}$ value from $0.1 \times \mathcal{D}_{max}$ to $10 \times \mathcal{D}_{max}$. Recall that $\mathcal{D}_{max}$ is a real distance between the $k$-th nearest pair of objects. Again, we fixed a stopping cardinality $k$ to 100,000, and the size of in-memory portion of a main queue was fixed to 100,000 elements.

When $e\mathcal{D}_{max}$ is overestimated ($e\mathcal{D}_{max} > \mathcal{D}_{max}$), the compensation stage of $\mathcal{AM}$-**KDJ** algorithm is not necessary, because all the $k$ nearest pairs will be produced in the first (aggressive pruning) stage. Even when $e\mathcal{D}_{max}$ is overestimated, $\mathcal{AM}$-**KDJ** guarantees that $e\mathcal{D}_{max}$ is always smaller than or equal to $q\mathcal{D}_{max}$ (obtained from a distance queue) throughout the first stage. Thus, $\mathcal{AM}$-**KDJ** always requires no more distance computation and queue insertion operations than $\mathcal{B}$-**KDJ** algorithm does.

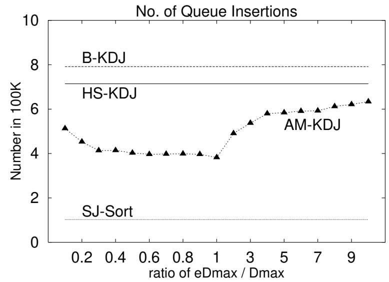
On the other hand, if $e\mathcal{D}_{max}$ is underestimated ($e\mathcal{D}_{max} < \mathcal{D}_{max}$), the node pairs in the compensation queue will be revisited in the compensation stage. Thus, the cost of tree traversals and queue management will increase, but it will be bounded by twice the cost of $\mathcal{B}$-**KDJ** algorithm. As discussed in Section 4.1, for a pair already expanded once in the first stage, only child pairs not examined in the first stage are paired up in the compensation stage and thereby wasting no time for redundant work. The value of $q\mathcal{D}_{max}$ is likely to become quite close to a real $\mathcal{D}_{max}$ value in the compensation stage. So, $\mathcal{AM}$-**KDJ** algorithm usually prunes distant pairs much more efficiently in the compensation stage than $\mathcal{B}$-**KDJ** algorithm would do in a single stage. Therefore, $\mathcal{AM}$-**KDJ** outperforms the $k$-distance join algorithms *HS*-**KDJ** and $\mathcal{B}$-**KDJ**, despite the additional cost of compensation stage.

Figure 14 shows that as $e\mathcal{D}_{max}$ approaches to a real $\mathcal{D}_{max}$ value, the performance of $\mathcal{AM}$-**KDJ** improves consistently in all three metrics. When $e\mathcal{D}_{max}$ increases far beyond the real $\mathcal{D}_{max}$ value, the performance of $\mathcal{AM}$-**KDJ** converges to that of $\mathcal{B}$-**KDJ** algorithm. Importantly, however, $\mathcal{AM}$-**KDJ** always outperformed $\mathcal{B}$-**KDJ**, not to mention *HS*-**KDJ**, with $e\mathcal{D}_{max}$ in a wide spectrum of estimated value range.

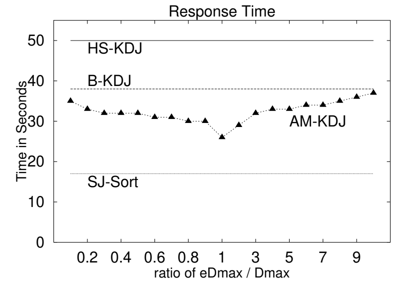
We have not measured the cost of compensation queue management. A compensation queue contains pairs of non-object R-tree nodes. During the first (aggressive pruning) stage of $\mathcal{AM}$-**KDJ** algorithm, The number of pruned pairs is far larger than the number of non-object pairs inserted into a compensation queue. In most of our experiments, the size of a compensation queue was *less than 0.5 percent* of the size of a main queue. Thus, the additional cost required for the compensation queue was almost negligible. This is one of the reasons why $\mathcal{AM}$-**KDJ** algorithm always outperformed $\mathcal{B}$-**KDJ**, which does not need a compensation queue.




(a) Distance Computations




(b) Queue Insertions




(c) Response Time

Figure 14: Performance Impact of $e\mathcal{D}_{max}$

## 5.7 Stepwise Incremental Execution of $\mathcal{AM}$-IDJ

Incremental distance join algorithms do not require a preset stopping cardinality $k$. Thus, in this set of experiments, we simulated a situation where users repeatedly requested a set of 10,000 nearest pairs at a time until a total of 100,000 nearest pairs were generated. Incremental algorithms *HS*-**IDJ** and $\mathcal{AM}$-**IDJ** each were executed once in a single experiment run, until a total of 100,000 nearest pairs were generated. The size of in-memory portion of a main queue was fixed to 10,000 elements both for *HS*-**IDJ** and $\mathcal{AM}$-**IDJ**.

On the other hand, since *SJ*-**SORT** is not an incremental algorithm, we restarted its processing each time $i \times 10,000$ nearest pairs were generated for $i\,(1 \leq i \leq 9)$. Thus, the performance measurements of *SJ*-**SORT** presented in Figure 15 are cumulative. For example, the response time of *SJ*-**SORT** for $k = 20,000$ includes the times spent on executing *SJ*-**SORT** twice, once for $k = 10,000$ and another for $k = 20,000$. For each run of *SJ*-**SORT**, we used a real $\mathcal{D}_{max}$ value for each of different stopping cardinalities.

In Figure 15, we measured the response time of $\mathcal{AM}$-**IDJ** algorithm in two different ways: (1) with $e\mathcal{D}_{max}$ values estimated by Equation (3), and (2) with real $\mathcal{D}_{max}$ values. When real $e\mathcal{D}_{max}$ values were provided, $\mathcal{AM}$-**IDJ** initiated a compensation stage each time another set of 10,000 pairs of object were requested by users. When estimated $e\mathcal{D}_{max}$ values were provided, $\mathcal{AM}$-**IDJ** needed compensation processing only after generating 40,000 pairs and 70,000 pairs, due to overestimated $e\mathcal{D}_{max}$ values. $\mathcal{AM}$-**IDJ** showed a fairly consistent performance over varying $e\mathcal{D}_{max}$ estimates, as $\mathcal{AM}$-**KDJ** did in Section 5.6. For all the $k$ values, $\mathcal{AM}$-**IDJ** with estimated $e\mathcal{D}_{max}$ improved the response time by a factor of two to four, when compared with *HS*-**IDJ**.

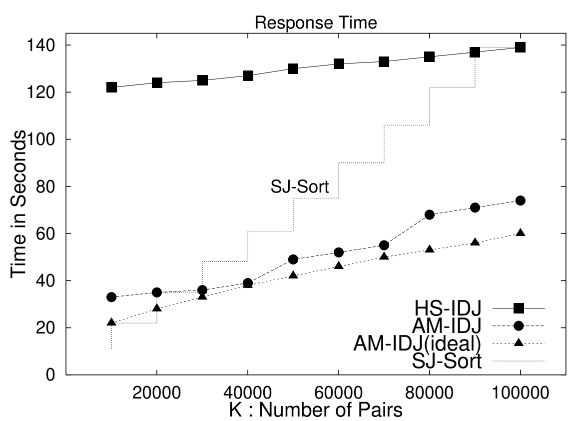



Figure 15: Step-Wise Incremental Execution

# 6 Conclusions

We have developed new distance join algorithms for spatial databases. The proposed algorithms provide significant performance improvement over previous work. The plane-sweep technique optimized by novel strategies for selecting a sweeping axis and direction minimizes the computational overhead incurred by bi-directional node expansions. We have shown that this optimized plane-sweep technique alone improves processing of a $k$-distance join query considerably.

The adaptive multi-stage algorithms employ aggressive pruning and compensation methods to further optimize the distance join processing. These algorithms address a slow start problem by using estimated maximum distances as cutoff values for pruning distant pairs. Assuming data objects are uniformly distributed, we have developed strategies to choose an initial estimate and to correct the estimate adaptively during the query processing. Our experimental study shows that the proposed algorithms outperformed previous work significantly and consistently over a wide spectrum of estimated maximum distances. In particular, for a relatively small stopping cardinality, the proposed algorithms achieved up to an order of magnitude improvement over previous work.

When the stopping cardinality of a distance join query is unknown (as in on-line query processing environments or a complex query that contains a distance join as a sub-query), the adaptive multi-stage algorithms process the query in a stepwise manner so that the query results can be returned incrementally.

We plan to develop new strategies for estimating the maximum distances and managing queues for non-uniform data sets.

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