Power law in the performance of the human memory and a simulation with a neural network model[1]

Tatsuhiro Nakajima, Nobuko Fuchikami
Department of Physics, Tokyo Metropolitan University
1-1 Minami-Osawa, Hachioji, Tokyo 192-03, Japan

Hideyuki Câteau[2]
Department of Physics, University of Tokyo
Bunkyo-ku, Tokyo 113, Japan

Hiroshi Nunokawa[2]
National Laboratory for High Energy Physics (KEK)
Tsukuba-shi, Ibaraki 305, Japan

1 Introduction

Since the birth of the neural network model as a model of the brain, a lot of features similar to brain's behaviors were found in this artificial model[1]. Despite of such intensive studies, we have not yet come to a satisfactory understanding of the complexity of the brain.

In this paper, we suggest a new approach for understanding the human brain. We took psychological data of the human memory as quantitative characterization of the performance of the living brain. Specifically, we show in this paper that both the neural network model and the human brain are subject to the same power law when they memorize something. This quantitative correspondences will deepen our understanding of the brain.

In the next section we examine the power law in the human memory. In section 3 we examine the power law in the neural network model. The last section is devoted to some discussions.

2 Power law in the human memory

Now imagine the situation that you are memorizing some objects. You surely experience that as the number of objects increases, your pace of memory declines. This phenomena is believed to be caused by the interference among different objects to be memorized.

M.Foucault expressed this fact quantitatively as follows[2]:

$$t(M) = cM^D$$

with $D = 2$, where M denotes the number of objects to memorize, t is the learning time,

[1]This paper will appear in proceedings of ISKIT'92. The report number of this paper is TMUP-HEL-9203, TU-611, KEK-TH-334 or KEK preprint 92-40.

[2]Fellow of Japan Society for the Promition of Science

which is necessary to complete memory and c is a constant. Various exponents D were proposed by several researchers, but most of their estimations agreed with $1 < D < 2$.

Since these psychological experiments stated above are very old (they were taken in the early times of this century), we tested some people in our university to confirm the power law by ourselves. The way of our experiment is as follows. We let the subject memorize a sequence of random numbers and check how many numbers he/she has memorized every 30 seconds.

The results is shown in t versus M plot with log axis (Fig.1). We carried out the χ^2-test to check the validity of the power law. The result of the test shows that the power law is supported with relatively high significance level. For almost a half of our subjects its values are more than 80 percent. Despite the fact that our experiment is not under a strict control like the experiment which psychologist does, it is fairly a nice fit. Thus we think it is reasonable to accept the power law to be true from these facts.

3 Power law in the neural network model

In this section we discuss the problem whether the neural network model has the power law observed in the performance of the human brain. We perform the simulation using a back propagation network (BPN) with a single hidden layer[1].

We make our BPN learn M objects which we call patterns. Then we record the number of iterations spent in convergence, which we call learning time t. Our purpose here is to examine the relation between the number of patterns and the learning time.

As is well known, however, the learning time t is sharply dependent on the initial value on the connection weights. Displayed in Fig.2 is a histogram of the typical distribution of the learning time. They show the following features: (1) the existence of minimal time, (2) beyond the minimum its curve sharply peaks.

In this situation, we face a problem which we shall call the learning time.

To avoid this problem we propose a model of the following memory system. Let us suppose that the large system composed of n BPNs and one central neuron. Each BPN is connected to the central neuron and has a different initial condition of its connection weights. Suppose that these BPNs start their learning at the same time and finish it sequentially. At the completion of learning, we suppose, they send signals to the central neuron. Conversely, the central neuron receives those signals. If the bias of central neuron is set to an appropriate value, it sparks when the signals pile up enough to go over the bias. Hence we can naturally define the learning time as the moment when the central neuron sparks.

Here we set the bias of central neuron so that it sparks when the signals from 20 percent of total sub-networks are received. There is no importance to specify this value, since we can show that any other definitions such as 30 percent lead to essentially the same result. Hence we take the setting above in this paper.

Taking the average value of the distribution as a definition of the learning time is another simpler definition. However, the average is calculated using the data which come from faster paced systems to slower ones. It seems physiologically unnatural to take account of much longer time after the true learning time. Thus we adopt the former definition conceptually. But it is again shown that the latter definition leads to essentially the same plot. Because of the technical reason we present here the plots based on the latter definition in Fig.3.

Those graphs show that the plots are fitted with the line in high accuracy. That is, the power law relation surely holds in this model. In our simulation the exponents D are distributed around the value of 2.

4 Discussions

As we have discussed in the previous section, the power law is observed in our model. We believe that it is valuable to obtain such a result from the BPN, one of the most standard learning algorithms. Moreover, the fact that our model has the same feature to the performance of the human brain might be a hint to the secret of the system of human memory.

Finally we point out two interesting topics.

By identifying the power laws in human beings and the BP, we can determine the correspondence between the time in the human case and the BP case. Then we find that one iteration corresponds to $0.01 \sim 0.1$ second (note that it does not mean CPU-time). This means that $10 \sim 100$ learning processes are run in one second. From the physiological restraint of the neuron a brain can execute the task at most 100 steps in a second. If the one learning iteration corresponds to 1 step of program in the brain of human being, our estimation is consistent with the physiological fact[3].

The second is on the form of curve in bilog plot graph. We notice that there is a certain vibrating mode around the line in the curve,which is not merely a statistical fluctuation. This may be a key to understand the detailed structure of the learning dynamics of the model.

Acknowledgement

We wish to express my thanks to Dr. Chris Love for his helpful suggestions. This research was supported in part by *A Fund for Special Research Project at Tokyo Metropolitan University.*

References

[1] Rumelhart, D.E., McClelland, J.L., and the PDP research Group (1986), *Parallel Distributed Processing,* The MIT Press;
Rumelhart, D.E., Hinton, G.E. and Williams, R.J.(1986), *Nature 323,* 533;
Amari, S.(1967) *IEEE Trans. EC, 16,3,* 299;
Widrow, B. and Hoff, M.E.(1960), *Institute of Radio Engineers, Western Electronic Show and Convention, Convention Record Part 4*, 94.

[2] Foucault, M.(1913), *Anée Phychol. 19,* 218.

[3] Feldman, J.A.(1985), *Cognitive Science 9,* 1.