Togai InfraLogic, Inc.

THE WORLD'S SOURCE FOR FUZZY LOGIC SOLUTIONS

# Fuzzy Logic Primer

## A Brief Introduction to Fuzzy Logic


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Written By: Peter Pacini and Andrew Thorson
Togai InfraLogic, Inc.
5 Vanderbilt
Irvine, CA 92718
Phone: (714) 975-8522
Fax: (714) 975-8524
Compuserve: 70571,467
email: info@til.com

**What is Fuzzy Logic?**

Fuzzy Logic, invented in 1965 by Dr. Lotfi Zadeh, is a branch of mathematics that allows a computer to model the real world the same way that people do. Unlike computers, people are not always precise. People think and reason using linguistic terms such as "hot" and "fast", rather than in precise numerical terms such as "100 degrees" and "70 miles per hour." In addition, people can make "shades of gray" decisions, rather than absolute "black and white" or "yes/no" decisions. Fuzzy Logic provides a computer with the capability to make the same kinds of classifications and decisions that people do. The most common use of Fuzzy Logic is in fuzzy expert systems.

**How Does Fuzzy Logic Work?**

The way Fuzzy Logic works is through the use of *fuzzy sets*, which are different from traditional sets. A set is simply a collection of objects. Traditional sets impose rigid membership requirements upon the objects within the set. An object is either completely in the set, or it is not in the set at all. Another way of saying this is an object is a member of a set to degree 1 (completely in the set) or 0 (not in the set at all). For example, the set of "TALL" men could be defined to be all men 6 feet tall or taller (see figure 1). We would say a man 6 feet tall is TALL, but a man 5 feet 11 inches is not TALL. But isn't this unrealistic? Most people would classify a man 5 feet 11 inches tall as somewhat TALL, however the traditional set classifies a man as either completely TALL, or not TALL at all—there is no middle ground.

In contrast, fuzzy sets have more flexible membership requirements that allow for partial membership in a set. The degree to which an object is a member of a fuzzy set can be any value between 0 and 1, rather than strictly 0 or 1 as in a traditional set. With a fuzzy set, there is a gradual transition from membership to nonmembership. For example, a man 6 feet tall is a member of the fuzzy set TALL to degree .5 (see figure 1). A man 5 feet 6 inches tall is TALL to degree .25, and a man 6 feet 6 inches tall is TALL to degree .75.

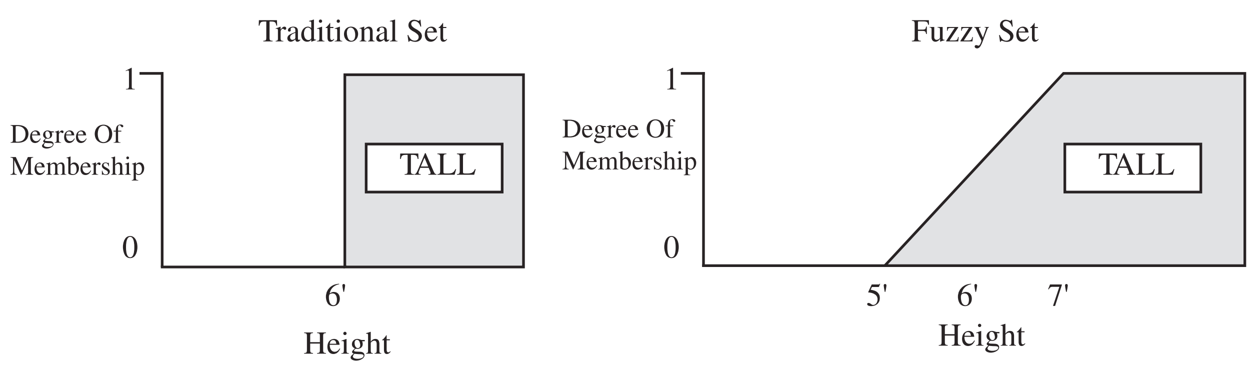


*Figure 1 - Traditional and fuzzy sets for TALL*

Figure 2 below shows the traditional and fuzzy versions of the sets SHORT and TALL. Notice the abrupt change of membership in the traditional SHORT and TALL sets at a height of 6 feet. Compare this with the fuzzy SHORT and TALL sets, which allow a gradual change from SHORT to TALL for men between 5 and 7 feet tall. Is a man 5 feet 11 inches tall SHORT or TALL? It's ambiguous. People would say he is somewhat SHORT, and they would say he is somewhat TALL. Traditional sets do not model this ambiguity well at all. However, fuzzy sets, which allow partial membership, provide a way for a computer to deal with this ambiguity by classifying the man as partially SHORT and partially TALL at the same time.

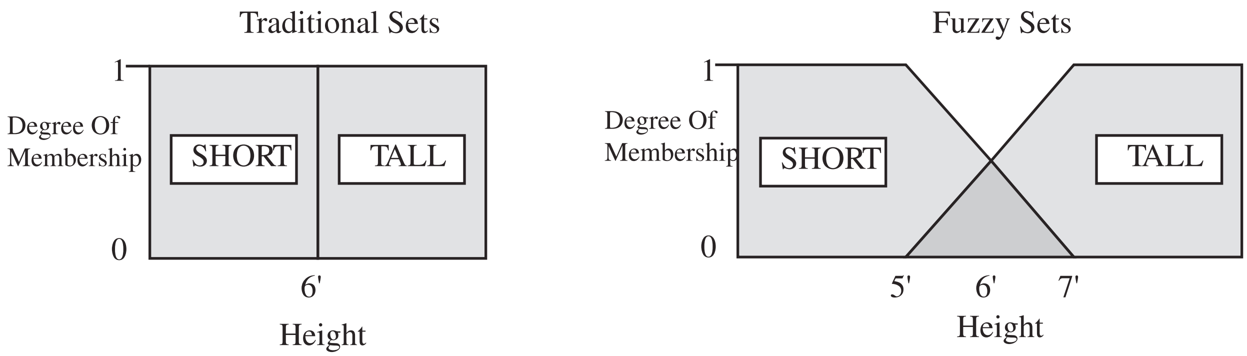


*Figure 2 - Traditional and fuzzy sets for SHORT and TALL*

**Fuzzy Logic Versus Probability**

People often confuse Fuzzy Logic and probability because both degrees of membership in fuzzy sets and probabilities are numbers between 0 and 1. However, they are not the same thing, and it is important not to confuse the two. The difference between probability and Fuzzy Logic is that probability measures the likelihood that a future event will occur, whereas Fuzzy Logic measures the ambiguity of events that have already occurred. For example, consider the shape in Figure 3.

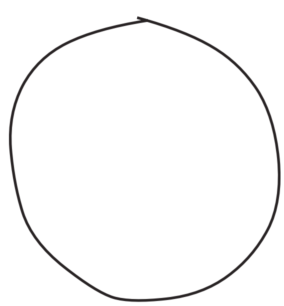

*Figure 3 - Fuzzy Circle*

Would you describe this shape as "probably a circle," or "circular?" The circle has already been drawn, so it is a past event, and there is no probability associated with it. The shape is not a perfect circle, but it resembles a circle, so it is fair to say it is "circular." In other words, the shape is a member to some degree in the fuzzy set of shapes that are circular.

**Membership Functions**

Each fuzzy set has a corresponding membership function that returns the degree of membership of a precise numerical value in the set. If we define a variable, *Height*, then the notation we use to obtain the degree of membership of *Height* in the set TALL is:

*Height* IS TALL.

For example, if *Height* has a value of 6 feet, then *Height* IS TALL would return a degree of membership of .5. The clause "*Height* IS TALL" is also called an antecedent.

**Fuzzy Operators**

Fuzzy operators allow you to combine antecedents into premises. The fuzzy logic operators are AND, OR and NOT. Given two antecedents *a* and *b*, the operators are defined as follows:

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| *a* AND *b* means | min (*a*, *b*) | [take the minimum of *a* and *b*] |
| *a* OR *b* means | max (*a, b*) | [take the maximum of *a* and *b*] |
| NOT *a* means | 1.0 - *a*. | |

For example, the expression (*Height* IS TALL) OR (*Height* IS SHORT) would be evaluated as max ( (*Height* IS TALL), (*Height* IS SHORT) ), where the membership functions for TALL and SHORT would give the degree of membership of *Height* in each set.

**Fuzzy Rules And Fuzzy Expert Systems**

Fuzzy rules look exactly like production rules in a traditional expert system. They have a premise consisting of one or more antecedents, and a conclusion consisting of one or more consequences. For example,

IF (Alpha IS SMALL) AND (Beta IS MEDIUM)
THEN Gamma IS BIG.

In this example, Alpha and Beta are input variables, Gamma is an output variable, and SMALL, MEDIUM and BIG are either conventional or fuzzy sets. A collection of these rules make up an expert system which takes inputs and produces outputs depending on which rules in the expert system "fire". In a traditional expert system, a rule will fire if its premise is TRUE. In a fuzzy expert system, a rule will fire if its premise evaluates to a nonzero belief level. When a rule fires, it contributes to the output of the expert system.

In a traditional expert system, the rules either fire completely or not at all. This is because a traditional expert system uses "yes/no," "black and white" logic to evaluate the premise of each rule. Typically, only one rule fires for a given group of inputs, so this one rule completely controls the output of the expert system.

In contrast, the rules in a fuzzy expert system fire to different degrees. Rather than an all or nothing response, the fuzzy rules produce "shades of gray" responses, depending on the degree of belief in the premise of each rule. In addition, more than one rule may fire for a given group of inputs, so the output of the expert system may be the combined result of several rules.

**How Do Fuzzy Rules Produce "Shades Of Gray" Responses?**

When a fuzzy rule fires, it fires to a certain degree depending on the belief level in each antecedent in the premise of the rule. The antecedents are evaluated using membership functions to produce belief levels, which are then combined using fuzzy operators to produce the final *output activation level*. Finally, the output activation level is used to either scale or clip the fuzzy output set. This entire process is called fuzzy inference. Clipping is called Max-Min inference, and scaling is called Max-Dot inference. See figure 4 below for an example of both methods of fuzzy inference. The higher the output activation level for a rule, the more it will contribute to the combined output of all the rules.

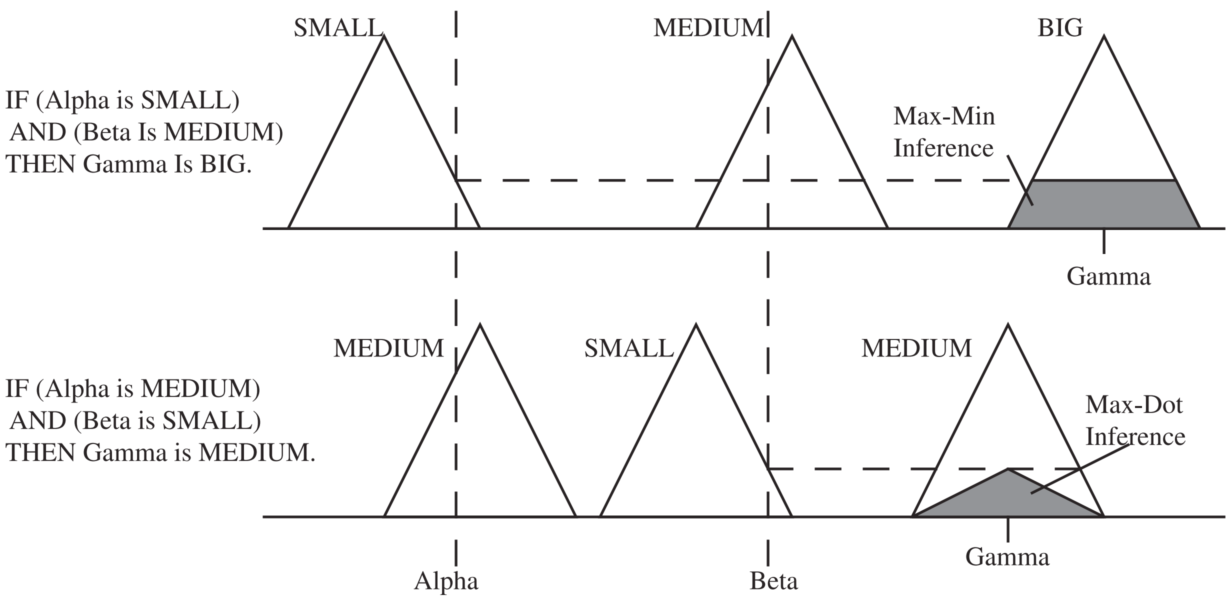


*Figure 4 - Fuzzy Inference Methods*

**How Do Fuzzy Rules Produce Combined Outputs?**

Once all of the fuzzy output sets have been computed, they are summed or unioned together to produce the combined fuzzy output set (see figure 5). In this example, the fuzzy output set BIG contributes more to the combined result because the output activation level associated with big was higher than that associated with MEDIUM.

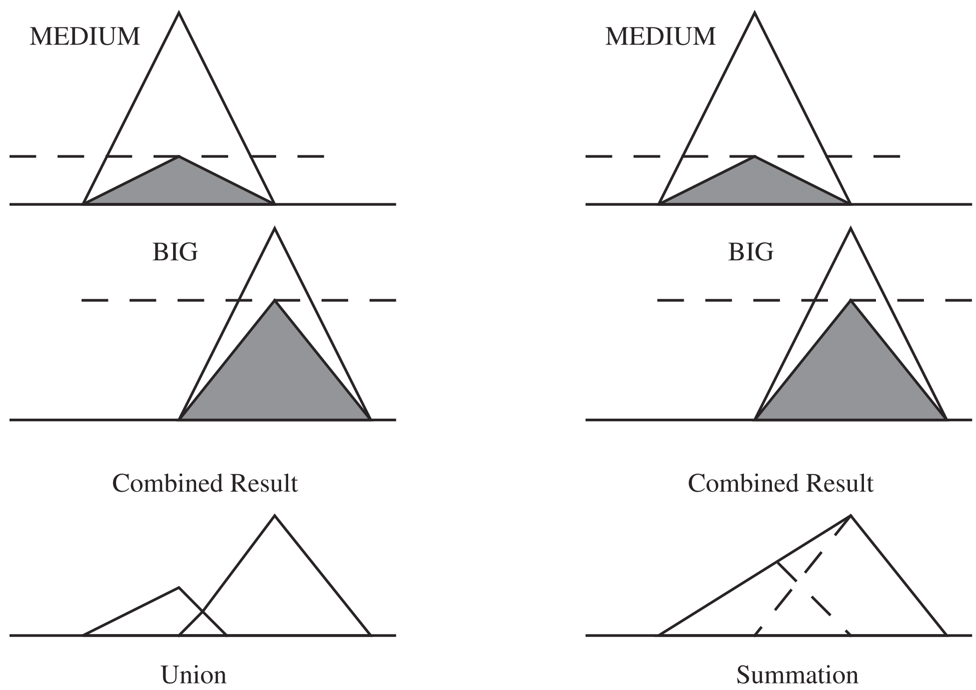


*Figure 5 - Combining Fuzzy Output Sets*

**How Does The Fuzzy Expert System Produce A Crisp Result?**

Real world systems require a crisp, numerical result. A fuzzy system takes the combined fuzzy output and converts it into a crisp, numerical result through a process called defuzzification. The procedure is mathematically complex; it involves finding the center of gravity, or the centroid, of the combined fuzzy output set. A simple way to visualize it is to imagine the combined fuzzy output set as a piece of cardboard. Now balance it on the edge of a razor blade. The point at which the cardboard balances on the razor blade is the center of gravity, and also the numerical value that best represents the fuzzy output set (see figure 6).

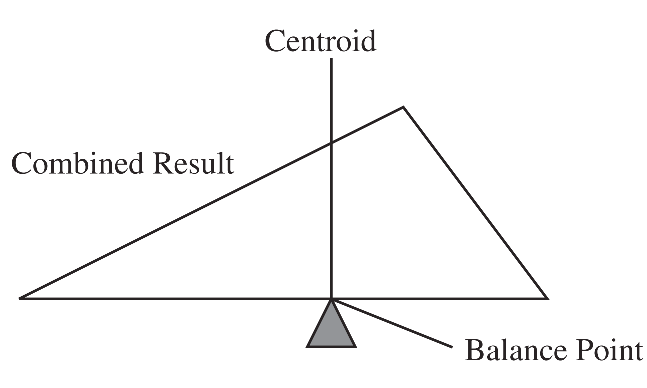


*Figure 6 - Computing A Crisp Result*