NIST's Digital Signature Proposal

A Technical Review

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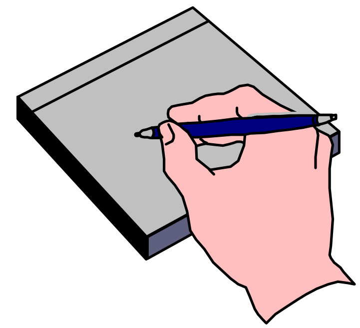
Introduction



Digital signatures

Signature, verification
with different keys

For authentication—of
message and signer

message

sign

private key

signature

verify

public key

valid or
invalid?

Users keep one key private, publish other.

Introduction (cont'd)

History

1976	Diffie, Hellman introduce digital signatures, suggest discrete logarithms as cryptographic problem.
1984	Elgamal proposes digital signature scheme based on discrete logarithms.
1989	Schnorr describes efficiency improvement for discrete-logarithm-based schemes.
1991	NIST announces DSS, a variant of Elgamal with Schnorr improvements.
1992	NIST revises DSS based on numerous comments.

Definition

System parameters

p: 512-bit prime [revised up to 1024 bits]

q: 160-bit prime factor of p-1

g: proper qth root of 1, mod p

Hash function

h: one-way map from message to 160-bit hash

Keys

y: 512-bit public key

x: 160-bit private key

$$y = g^x \bmod p$$

Definition (cont'd)

Signature

m: message

Signature = (r,s) where

$$r = (g^k \bmod p) \bmod q$$

$$s = k^{-1}(h(m)+xr) \bmod q$$

and k is 160-bit random integer.

Verification

Signature (r,s) for message m is valid if and only if:

$$r = (g^{h(m)t}\, y^{rt} \bmod p) \bmod q,$$

where $t = s^{-1} \bmod q$.

Definition (cont'd)

Why it works ...

If

$$r = (g^k \bmod p) \bmod q, \quad s = k^{-1}(h(m)+xr) \bmod q,$$
$$y = g^x \bmod p, \text{ and } t = s^{-1} \bmod q,$$

then

$$\begin{aligned} g^{h(m)t}\, y^{rt} \bmod p &= g^{h(m)t}\, [g^x]^{rt} \bmod p \\ &= g^{[h(m)+xr]t} \bmod p \\ &= g^{[ks]t} \bmod p \\ &= g^k \bmod p. \end{aligned}$$

So

$$(g^{h(m)t}\, y^{rt} \bmod p) \bmod q = (g^k \bmod p) \bmod q = r.$$

Definition (cont'd)

Comparison with other systems

Elgamal: no q

$$r = g^k \bmod p;$$

$$s = k^{-1}(h(m)+xr) \bmod p.$$

Schnorr: q, "zero knowledge" ideas

$$r = h(\langle g^k \bmod p, m\rangle);$$

$$s = (k+xr) \bmod q.$$

DSS adds q to Elgamal.

Performance

Signature

$$r = (g^k \bmod p) \bmod q$$
$$s = k^{-1}(h(m)+xr) \bmod q$$

Naive methods: 238.5 mod p multiplications
Sliding three-bit windows: 202
With precomputation (Brickell *et al*, 1991): 52

All but one mod q multiplication is *off line*.
Very good on-line speed, good off-line speed.

Performance (cont'd)

Verification

$$t = s^{-1} \bmod q$$

$$r \stackrel{?}{=} (g^{h(m)t}\, y^{rt} \bmod p) \bmod q$$

Naive methods: 477 mod p multiplications

Simultaneous two-bit windows: 246

With precomputation: 229

Fair speed.

Performance (cont'd)

Parameter generation

p: 512-bit prime

q: 160-bit prime factor of p-1

With trial division by primes ≤ 30, then base-2 pseudoprimality test: $56 \times 624 = 34944$ mod p multiplications

Key generation

$$y = g^x \bmod p$$

With precomputation: 52 mod p multiplications.

Performance (cont'd)

Comparison with other systems

In 512-bit modular multiplications, with 512-bit keys:

		RSA	Elgamal	DSS
*signing	*off-line*	n/a	624	**52**
	on-line	159	**1**	**< 1**
verification		**2 to 17**	689	229
parameter generation		n/a	34944	34944
*key generation		4452	624	52

For DSS, all computations that must be done in private (marked *) are fast—good in smart-card applications.

Security

Goals

Given message m, find a signature.

Or, better yet,

Given public key y, find private key $x = \log_g y$.
This is the *discrete logarithm problem*.

Security (cont'd)

***q*-based approaches**

Example: Baby-step/giant-step method (Shanks)

1. Tabulate (u,$g^{u\sqrt{q}}$ mod p) for all u, $0 \le u < \sqrt{q}$. $\sqrt{q}$ time, $\sqrt{q}$ space.
2. For each instance, find v, where $0 \le v < \sqrt{q}$, such that yg^{-v} mod p is in the table, i.e.,

$$yg^{-v} \bmod p = g^{u\sqrt{q}} \bmod p.$$

Then $x = u\sqrt{q}+v$. $\sqrt{q}$ time.

Other methods find x in $\sqrt{q}$ time, constant space, without a table.

Security (cont'd)

***p*-based approaches**

Example: Index calculus (Adleman *et al*)

Define $L(p) = \exp((1+\varepsilon)\sqrt{(\log p)\,(\log \log p)})$

1. "Tabulate" $(s, \log_g s)$ for all prime s, $2 \le s \le L(p)$.
   $L(p)$ time, $L(p)$ space.
2. For each instance, find v such that all factors of $yg^{-v} \bmod p$ are in the table, i.e.,

$$yg^{-v} \bmod p = s_1^{e_1} \times \cdots \times s_k^{e_k} .$$

   Then $x = e_1 \log_g s_1 + \cdots + e_k \log_g s_k + v$. $L(p)$ time.

Improved methods find v in $\sqrt{L(p)}$ time, $\sqrt{L(p)}$ space, with similar time for table.

Security (cont'd)

***p*-based approaches (cont'd)**

Example: Number field sieve (Gordon, 1991)

Time

$$\exp((2.08+\varepsilon)(\log p)^{1/3} (\log \log p)^{2/3})$$

Asymptotically faster than index calculus, not yet practical.

For special p, especially effective.

Note: Attacks based on both p and q are unexplored.

Security (cont'd)

Cost in MIPS-years

Based on $L(p)$ as instruction count (Rivest, 1991):

$\log_2 p$	$L(p)$	MIPS years
512	6.7×10^{19}	2.1×10^{6}
576	1.7×10^{21}	5.5×10^{7}
	...	
960	3.7×10^{28}	1.2×10^{15}
1024	4.4×10^{29}	1.4×10^{16}

Security is comparable to RSA's.

$2^{80} \approx 1.2\times10^{24}$ (not directly comparable).

Security (cont'd)

Other attacks

Random number recovery: If k is known, then r and x can be computed:

$$r = (g^k \bmod p) \bmod q;$$
$$x = r^{-1}(ks - h(m)) \bmod q.$$

Weak random number generator may reveal x.

Hash function attacks: Finding messages with the same hash should take time 2^{80}. May take less time if h is weak.

Special methods: Forging signatures is not known to require discrete logarithms, but neither are alternative methods known.

Trap Doors

What is a trap door?

Given an algorithm for the forward function, it is computationally infeasible to find a simply computed inverse. Only through knowledge of certain *trap-door information* ... can one easily find the easily computed inverse. (Diffie & Hellman, 1976)

Trap door makes a hard inverse easy.

Does DSS have a trap door?

Whoever selects *system parameters*, may select *system* trap door.

Trap Doors (cont'd)

System trap door

p of special form (Haber & Lenstra, 1991)

$$p = \sum_{i=0}^{d} p_i m^i$$

where m is an integer and d, $p_0, \ldots, p_d$ are small

Number field sieve especially effective:

$$\exp((1.00475+\varepsilon)(\log p)^{2/5} (\log \log p)^{3/5})$$

But the smaller the ε, the more obvious the trap door (Gordon, 1992).

Trap Doors (cont'd)

Avoiding system trap doors

1. Choose unique parameters.
2. Trust the one who selects system parameters.

NIST recommends generating p with hash function.

Conclusions

DSS: Digital Signature Standard

Based on discrete logarithms

Variant of ElGamal, Schnorr

Performance

Very good on-line signature speed, fair verification speed

Security

Strong to very strong, by current estimates

Trap doors

Possible, but easily avoided