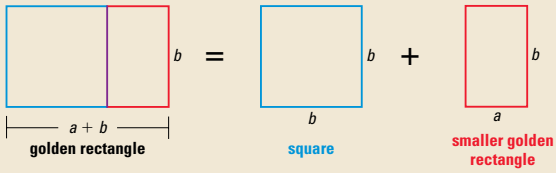
PROJECT
Applying Chapters 10–12

Investigating the Golden Ratio

OBJECTIVE **Explore what the golden ratio is and how it is used.**

Materials: graph paper, metric ruler, graphing calculator (optional)

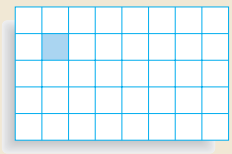
Over the centuries, the golden rectangle has fascinated artists, architects, and mathematicians. A golden rectangle has the special shape such that when a square is cut from one end, the ratio of length to width of the remaining rectangle is equal to the ratio of length to width of the original rectangle. This ratio, the *golden ratio*, is $\frac{1+\sqrt{5}}{2}$, or about 1.618034.



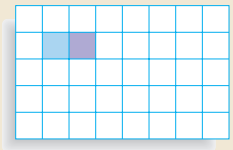
It is *not* possible to constuct a golden rectangle with integer sidelengths. However, it *is* possible to construct rectangles with integer sidelengths whose ratios of length to width approximate the golden ratio.

APPROXIMATING A GOLDEN RECTANGLE

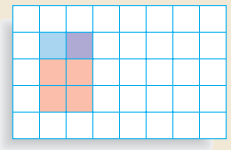
1 On graph paper, draw a 1-by-1 square.



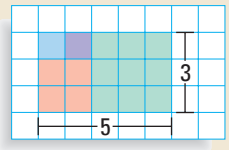
2 On one side of the square, add another 1-by-1 square.



3 Build a 2-by-2 square on the longest side of your 1-by-2 rectangle.



4 Build a 3-by-3 square on the longest side of your 2-by-3 rectangle.



INVESTIGATING THE GOLDEN RATIO

One property of golden rectangles is that their lengths and widths are proportional. For example, in the above diagram, $\frac{a+b}{b} = \frac{b}{a}$.

1. Copy and complete the table by using this property to predict the lengths and widths of the next four rectangles you can build on the drawing from **Step 4**.

length, *b*	3	5	?	?	?	?
width, *a*	2	3	?	?	?	?

2. Draw the next four rectangles to check your predictions.

3. Add a row to your table and record the ratio of the length to the width of the rectangles. Explain how the ratios are related to the golden ratio.

4. Let $r = \frac{b}{a}$ represent the golden ratio. Use $\frac{a+b}{b} = \frac{b}{a}$ to show that $r = \frac{1+\sqrt{5}}{2}$ if $a = 1$.

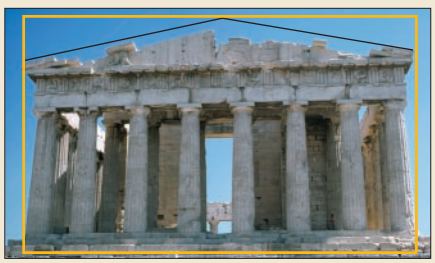
PRESENTING YOUR RESULTS

Write a report or make a poster to present your results.

- Include a sketch of a golden rectangle.
- Include the table you made in Exercise 1.
- Include your answers to Exercises 2–4.
- Describe what you learned about the golden ratio.

EXTENSIONS

- Before the pediment on top of the Parthenon in Athens was destroyed, the front of the building fit almost exactly into a golden rectangle. Do some research on the Parthenon to find out if the golden ratio was used in other aspects of its design.



- The golden rectangle can be found in the works of many artists and in art objects around the world, such as the proportions of a jar from the Ch'ing dynasty of China, shown at the right. Find pictures of works of art and see if you can find any golden rectangles in them.

- The golden ratio can also be found in nature. For example, the average chicken egg fits nicely inside a golden rectangle. Measure at least six chicken eggs and find the average of their ratios.

- Do some research and find other examples of the golden ratio in art, nature, and architecture.

- Find some rectangular objects around you that you think look nice. Measure them to see if they are golden rectangles. You might try a picture frame, a $1 bill, a business card, a 3-by-5 index card, or a TV screen.