9.3 Graphing Quadratic Functions

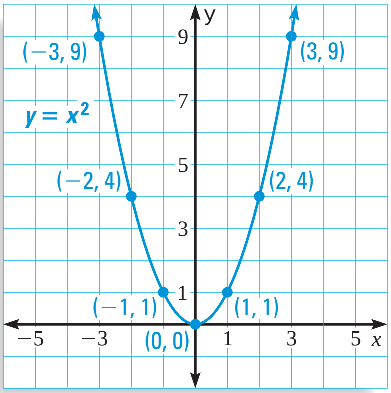
GOAL 1 SKETCHING A QUADRATIC FUNCTION

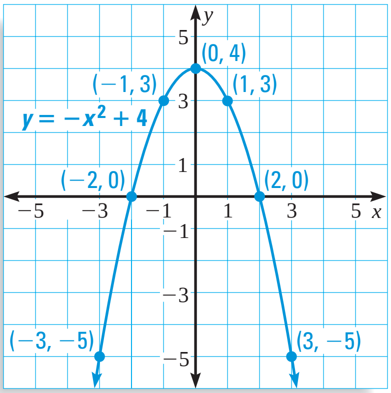
A **quadratic function** is a function that can be written in the **standard form**

$y = ax^2 + bx + c$, where $a \neq 0$.

Every quadratic function has a U-shaped graph called a **parabola**. If the leading coefficient a is positive, the parabola *opens up*. If the leading coefficient is negative, the parabola *opens down* in the shape of an upside down *U*.

The graph on the left has a positive leading coefficient so it opens up. The graph on the right has a negative leading coefficient so it opens down.





The **vertex** is the lowest point of a parabola that opens up and the highest point of a parabola that opens down. The parabola on the left has a vertex of (0, 0) and the parabola on the right has a vertex of (0, 4).

The line passing through the vertex that divides the parabola into two symmetric parts is called the **axis of symmetry**. The two symmetric parts are mirror images of each other.

GRAPH OF A QUADRATIC FUNCTION

The graph of $y = ax^2 + bx + c$ is a parabola.

- If a is positive, the parabola opens up.
- If a is negative, the parabola opens down.
- The vertex has an x-coordinate of $-\frac{b}{2a}$.
- The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

What you should learn

GOAL 1 Sketch the graph of a quadratic function.

GOAL 2 Use quadratic models in **real-life** settings, such as finding the winning distance of a shot put in **Example 3**.

Why you should learn it

▼ To model **real-life** parabolic situations, such as the height above the water of a jumping dolphin in **Exs. 65** and **66**.

REAL LIFE

GRAPHING A QUADRATIC FUNCTION

STEP 1 Find the x-coordinate of the vertex.

STEP 2 Make a table of values, using x-values to the left and right of the vertex.

STEP 3 Plot the points and connect them with a smooth curve to form a parabola.

EXAMPLE 1 *Graphing a Quadratic Function with a Positive a-value*

Sketch the graph of $y = x^2 - 2x - 3$.

SOLUTION

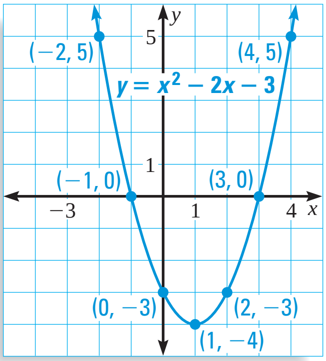
1. Find the x-coordinate of the vertex when $a = 1$ and $b = -2$.

$$-\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$

2. Make a table of values, using x-values to the left and right of $x = 1$.

x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

3. Plot the points. The vertex is $(1, -4)$ and the axis of symmetry is $x = 1$. Connect the points to form a parabola that opens up since a is positive.



EXAMPLE 2 *Graphing a Quadratic Function with a Negative a-value*

Sketch the graph of $y = -2x^2 - x + 2$.

SOLUTION

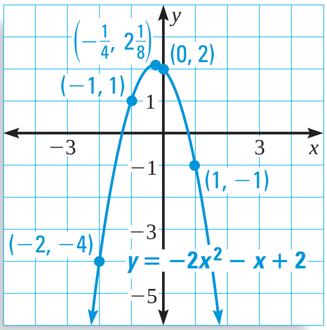
1. Find the x-coordinate of the vertex when $a = -2$ and $b = -1$.

$$-\frac{b}{2a} = -\frac{-1}{2(-2)} = -\frac{1}{4}$$

2. Make a table of values, using x-values to the left and right of $x = -\frac{1}{4}$.

x	-2	-1	$-\frac{1}{4}$	0	1	2
y	-4	1	$2\frac{1}{8}$	2	-1	-8

3. Plot the points. The vertex is $\left(-\frac{1}{4}, 2\frac{1}{8}\right)$ and the axis of symmetry is $x = -\frac{1}{4}$. Connect the points to form a parabola that opens down since a is negative.



STUDENT HELP

Study Tip
If the x-coordinate of the vertex is a fraction, you can still choose whole numbers when you make a table.

GOAL 2 USING QUADRATIC MODELS IN REAL LIFE

When an object has little air resistance, its path through the air can be approximated by a parabola.

EXAMPLE 3 *Using a Quadratic Model*

TRACK AND FIELD Natalya Lisovskaya holds the world record for the women's shot put. The path of her record-breaking throw can be modeled by $y = -0.01347x^2 + 0.9325x + 5.5$, where x is the horizontal distance in feet and y is the height (in feet). The initial height is represented by 5.5, the height at which the shot (a 4-kilogram metal ball) was released.

a. What was the maximum height (in feet) of the shot thrown by Lisovskaya?

b. What was the distance of the throw to the nearest hundredth of a foot?

SOLUTION

a. The maximum height of the throw occurred at the vertex of the parabolic path. Find the x-coordinate of the vertex. Use $a = -0.01347$ and $b = 0.9325$.

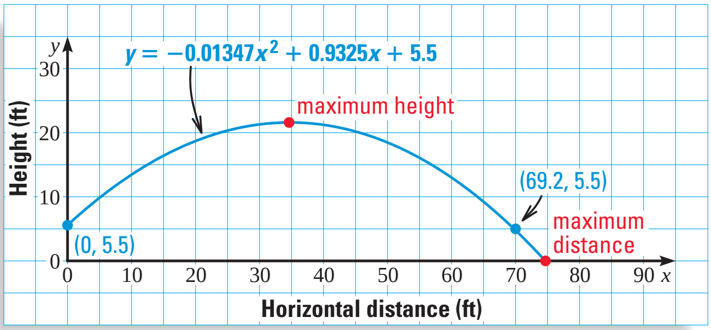
$$-\frac{b}{2a} = -\frac{0.9325}{2(-0.01347)} \approx 34.61$$

Substitute 34.61 for x in the model to find the maximum height.

$$y = -0.01347(34.61)^2 + 0.9325(34.61) + 5.5 \approx 21.6$$

▶ The maximum height of the shot was about 21.6 feet.

b. To find the distance of the throw, sketch the parabolic path of the shot. Because a is negative, the graph opens down. Use (35, 22) as the vertex. The y-intercept is 5.5. Sketch a symmetric curve.



The distance of the throw is the x-value that yields a y-value of 0. The graph shows that the distance was between 74 and 75 feet. You can use a table to refine this estimate.

Distance, *x*	74.5 ft	74.6 ft	74.7 ft	74.8 ft
Height, *y*	0.209 ft	0.102 ft	−0.006 ft	−0.114 ft

▶ The shot hit the ground at a distance between 74.6 and 74.7 feet.

FOCUS ON PEOPLE

REAL LIFE

NATALYA LISOVSKAYA
In 1987, the women's world record in shot put was set by Lisovskaya. She also won an Olympic gold medal in 1988.

GUIDED PRACTICE

Vocabulary Check ✓

1. Identify the values of a, b, and c for the quadratic function in standard form $y = -5x^2 + 7x - 4$.

2. Why is the vertical line that passes through the vertex of a parabola called the axis of symmetry?

Concept Check ✓

3. Explain how you can decide whether the graph of $y = 3x^2 + 2x - 4$ opens up or down.

4. Find the coordinates of the vertex of the graph of $y = 2x^2 + 4x - 2$.

Skill Check ✓

Tell whether the graph opens up or down. Write an equation of the axis of symmetry.

5. $y = x^2 + 4x - 1$ **6.** $y = 3x^2 + 8x - 6$ **7.** $y = x^2 + 7x - 1$

8. $y = -x^2 - 4x + 2$ **9.** $y = 5x^2 - 2x + 4$ **10.** $y = -x^2 + 4$

Sketch the graph of the function. Label the vertex.

11. $y = -3x^2$ **12.** $y = -3x^2 + 6x + 2$ **13.** $y = -5x^2 + 10$

14. $y = x^2 + 4x + 7$ **15.** $y = x^2 - 6x + 8$ **16.** $y = 5x^2 + 5x - 2$

17. $y = -4x^2 - 4x + 12$ **18.** $y = 3x^2 - 6x + 1$ **19.** $y = 2x^2 - 8x + 3$

20. **BASKETBALL** You throw a basketball whose path can be modeled by $y = -16x^2 + 15x + 6$, where x represents time (in seconds) and y represents height of the basketball (in feet).

a. What is the maximum height that the basketball reaches?

b. In how many seconds will the basketball hit the ground if no one catches it?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

PREPARING TO GRAPH **Complete these steps for the function.**

a. Tell whether the graph of the function opens up or down.

b. Find the coordinates of the vertex.

c. Write an equation of the axis of symmetry.

21. $y = 2x^2$ **22.** $y = -7x^2$ **23.** $y = 6x^2$ **24.** $y = \frac{1}{2}x^2$

25. $y = -5x^2$ **26.** $y = -4x^2$ **27.** $y = -16x^2$ **28.** $y = 5x^2 - x$

29. $y = 2x^2 - 10x$ **30.** $y = -7x^2 + 2x$ **31.** $y = -10x^2 + 12x$

32. $y = 6x^2 + 2x + 4$ **33.** $y = 5x^2 + 10x + 7$ **34.** $y = -4x^2 - 4x + 8$

35. $y = 2x^2 - 7x - 8$ **36.** $y = 2x^2 + 7x - 21$ **37.** $y = -x^2 + 8x + 32$

38. $y = \frac{1}{2}x^2 + 3x - 7$ **39.** $y = 4x^2 + \frac{1}{4}x - 8$ **40.** $y = -10x^2 + 5x - 3$

41. $y = 0.78x^2 - 4x - 8$ **42.** $y = 3.5x^2 + 2x - 8$ **43.** $y = -10x^2 - 7x + 2.66$

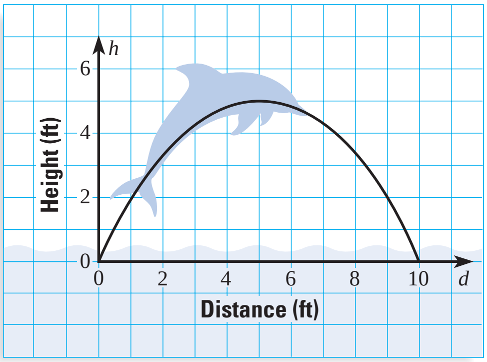
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 21–64
Example 2: Exs. 21–64
Example 3: Exs. 65–75

SKETCHING GRAPHS **Sketch the graph of the function. Label the vertex.**

44. $y = x^2$
45. $y = -2x^2$
46. $y = 4x^2$
47. $y = x^2 + 4x - 1$
48. $y = -3x^2 + 6x - 9$
49. $y = 4x^2 + 8x - 3$
50. $y = 2x^2 - x$
51. $y = 6x^2 - 4x$
52. $y = 3x^2 - 2x$
53. $y = x^2 + x + 4$
54. $y = x^2 + x + \frac{1}{4}$
55. $y = 3x^2 - 2x - 1$
56. $y = 2x^2 + 6x - 5$
57. $y = -3x^2 - 2x - 1$
58. $y = -4x^2 + 32x - 20$
59. $y = -4x^2 + 4x + 7$
60. $y = -3x^2 - 3x + 4$
61. $y = -2x^2 + 6x - 5$
62. $y = -\frac{1}{3}x^2 + 2x - 3$
63. $y = -\frac{1}{2}x^2 - 4x + 6$
64. $y = -\frac{1}{4}x^2 - x - 1$

DOLPHIN **In Exercises 65 and 66, use the following information.** A bottlenose dolphin jumps out of the water. The path the dolphin travels can be modeled by $h = -0.2d^2 + 2d$, where h represents the height of the dolphin and d represents horizontal distance.



65. What is the maximum height the dolphin reaches?

66. How far did the dolphin jump?

FOCUS ON APPLICATIONS

WATER ARC To celebrate the engineering feat of reversing the flow of the Chicago River, the Water Arc was built on the hundredth anniversary of this event.

WATER ARC **In Exercises 67 and 68, use the following information.** On one of the banks of the Chicago River, there is a water cannon, called the Water Arc, that sprays recirculated water across the river. The path of the Water Arc is given by the model

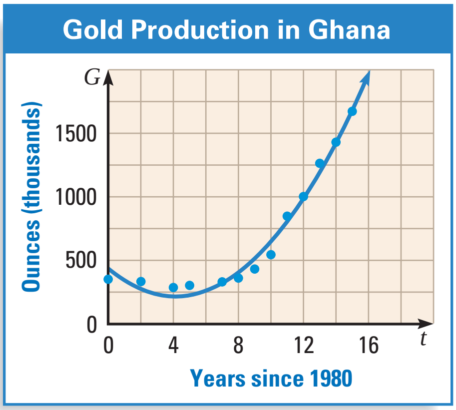
$$y = -0.006x^2 + 1.2x + 10$$

where x is the distance (in feet) across the river, y is the height of the arc (in feet), and 10 is the number of feet the cannon is above the river.

67. What is the maximum height of the water sprayed from the Water Arc?

68. How far across the river does the water land?

GOLD PRODUCTION **In Exercises 69–71, use the following information.** In Ghana from 1980 to 1995, the annual production of gold G in thousands of ounces can be modeled by $G = 12t^2 - 103t + 434$, where t is the number of years since 1980.



69. From 1980 to 1995, during which years was the production of gold in Ghana decreasing?

70. From 1980 to 1995, during which years was the production of gold increasing?

71. How are the questions asked in Exercises 69 and 70 related to the vertex of the graph?

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 72–75.

TABLE TENNIS **In Exercises 72–75, use the following information.**
Suppose a table-tennis ball is hit in such a way that its path can be modeled by $h = -4.9t^2 + 2.07t$, where h is the height in meters above the table and t is the time in seconds.

72. Estimate the maximum height reached by the table-tennis ball. Round to the nearest tenth.

73. About how many seconds did it take for the table-tennis ball to reach its maximum height after its initial bounce? Round to the nearest tenth.

74. About how many seconds did it take for the table-tennis ball to travel from the initial bounce to land on the other side of the net? Round to the nearest tenth.

75. **CRITICAL THINKING** What factors would change the path of the table-tennis ball? What combination of factors would result in the table-tennis ball bouncing the highest? What combination of factors would result in the table-tennis ball bouncing the lowest?

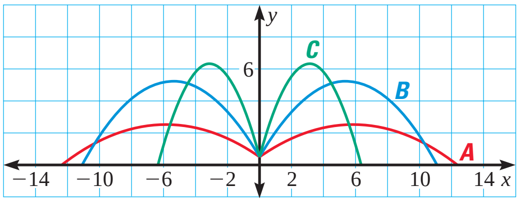
Test Preparation

76. **MULTI-STEP PROBLEM** A sprinkler can eject water at an angle of 35°, 60°, or 75° with the ground. For these settings, the paths of the water can be modeled by the equations below where x and y are measured in feet.

35°: $y = -0.06x^2 + 0.70x + 0.5$

60°: $y = -0.16x^2 + 1.73x + 0.5$

75°: $y = -0.60x^2 + 3.73x + 0.5$



a. Find the maximum height of the water for each setting.

b. Find how far from the sprinkler the water reaches for each setting.

c. **CRITICAL THINKING** Do you think there is an angle setting for the sprinkler that will reach farther than any of the settings above? How do the angle and reach represented by the graph of $y = -0.08x^2 + x + 0.5$ compare with the others? What angle setting would reach the least distance?

★ Challenge

77. **UNDERSTANDING GRAPHS** Sketch the graphs of the three functions in the same coordinate plane. Describe how the three graphs are related.

a. $y = x^2 + x + 1$	**b.** $y = x^2 - 1x + 1$	**c.** $y = x^2 - x + 1$
$y = \frac{1}{2}x^2 + x + 1$	$y = x^2 - 5x + 1$	$y = x^2 - x + 3$
$y = 2x^2 + x + 1$	$y = x^2 - 10x + 1$	$y = x^2 - x - 2$

78. How does a change in the value of a change the graph of $y = ax^2 + bx + c$?

79. How does a change in the value of b change the graph of $y = ax^2 + bx + c$?

80. How does a change in the value of c change the graph of $y = ax^2 + bx + c$?

EXTRA CHALLENGE
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MIXED REVIEW

GRAPHING Write the equation in slope-intercept form, and then graph the equation. Label the *x*- and *y*-intercepts on the graph. (Review 4.6 for 9.4)

81. $-3x + y + 6 = 0$

82. $-x + y - 7 = 0$

83. $4x + 2y - 12 = 0$

84. $x + 2y - 7 = 5x + 1$

GRAPHING LINEAR INEQUALITIES Graph the system of linear inequalities. (Review 7.6)

85. $x - 3y \geq 3$, $x - 3y \leq 12$

86. $x + y \leq 5$, $x \geq 2$, $y \geq 0$

87. $x + y < 10$, $2x + y > 10$, $x - y < 2$

SIMPLIFYING EXPRESSIONS Simplify. Write your answer as a power or as an expression containing powers. (Review 8.1)

88. $4^5 \cdot 4^8$

89. $(3^3)^2$

90. $(3^6)^3$

91. $a \cdot a^5$

92. $(3b^4)^2$

93. $6x \cdot (6x)^2$

94. $(3t)^3(-t^4)$

95. $(-3a^2b^2)^3$

SCIENTIFIC NOTATION Rewrite the number in scientific notation. (Review 8.4)

96. 0.0012

97. 987,000

98. 3,984,328

99. 1,229,000,000

100. 0.000432

101. 0.00999

QUIZ 1

Self-Test for Lessons 9.1–9.3

Evaluate the expression. Give the exact value if possible. Otherwise, approximate to the nearest hundredth. (Review 9.1)

1. $\sqrt{144}$

2. $-\sqrt{196}$

3. $-\sqrt{676}$

4. $-\sqrt{27}$

5. $\sqrt{6}$

6. $\sqrt{1.5}$

7. $\sqrt{0.16}$

8. $\sqrt{2.25}$

Solve the equation. (Review 9.1)

9. $x^2 = 169$

10. $4x^2 = 64$

11. $12x^2 = 120$

12. $-6x^2 = -48$

Simplify the expression. (Review 9.2)

13. $\sqrt{18}$

14. $\sqrt{5} \cdot \sqrt{20}$

15. $\dfrac{2\sqrt{121}}{\sqrt{4}}$

16. $\sqrt{\dfrac{45}{36}}$

Tell whether the graph of the function opens up or down. Find the coordinates of the vertex. Write the equation of the axis of symmetry of the function. (Review 9.3)

17. $y = x^2 + 2x - 11$

18. $y = 2x^2 - 8x - 6$

19. $y = 3x^2 + 6x - 10$

20. $y = \frac{1}{2}x^2 + 5x - 3$

21. $y = 7x^2 - 7x + 7$

22. $y = x^2 + 9x$

Sketch a graph of the function. (Review 9.3)

23. $y = -x^2 + 5x - 5$

24. $y = 3x^2 + 3x + 1$

25. $y = -2x^2 + x - 3$