6.4 Solving Absolute-Value Equations and Inequalities

What you should learn

GOAL 1 Solve absolute-value equations.

GOAL 2 Solve absolute-value inequalities.

Why you should learn it

▼ To solve **real-life** problems such as finding the wavelengths of different colors of fireworks in **Exs. 65–68.**

GOAL 1 SOLVING ABSOLUTE-VALUE EQUATIONS

You can solve some absolute-value equations using mental math. For instance, you learned in Lesson 2.1 that the equation $|x| = 8$ has *two* solutions: 8 and -8.

To solve absolute-value equations, you can use the fact that the expression inside the absolute value symbols can be either positive or negative.

EXAMPLE 1 Solving an Absolute-Value Equation

Solve $|x - 2| = 5$.

SOLUTION

Because $|x - 2| = 5$, the expression $x - 2$ can be equal to 5 or to -5.

$x - 2$ IS POSITIVE	$x - 2$ IS NEGATIVE
$\|x - 2\| = 5$	$\|x - 2\| = 5$
$x - 2 = +5$	$x - 2 = -5$
$x = 7$	$x = -3$

▶ The equation has two solutions: 7 and -3.

✓ **CHECK** Substitute both values into the original equation.

$|7 - 2| = |5| = 5$ $\qquad$ $|-3 - 2| = |-5| = 5$

EXAMPLE 2 Solving an Absolute-Value Equation

Solve $|2x - 7| - 5 = 4$.

SOLUTION

Isolate the absolute-value expression on one side of the equation.

$2x - 7$ IS POSITIVE	$2x - 7$ IS NEGATIVE
$\|2x - 7\| - 5 = 4$	$\|2x - 7\| - 5 = 4$
$\|2x - 7\| = 9$	$\|2x - 7\| = 9$
$2x - 7 = +9$	$2x - 7 = -9$
$2x = 16$	$2x = -2$
$x = 8$	$x = -1$

▶ The equation has two solutions: 8 and -1. Check these solutions in the original equation.

GOAL 2 SOLVING ABSOLUTE-VALUE INEQUALITIES

Recall that $|x|$ is the distance between x and 0. If $|x| < 8$, then any number between -8 and 8 is a solution of the inequality.

ACTIVITY Developing Concepts

Investigating Absolute-Value Inequalities

Use Guess, Check, and Revise to find values of x that satisfy each absolute-value inequality. Graph the solution set on a number line. Then use a compound inequality to describe the solution set.

1. $|x| < 2$ **2.** $|x + 2| \geq 1$ **3.** $|x - 3| \leq 2$

You can use these properties to solve absolute-value inequalities and equations.

SOLVING ABSOLUTE-VALUE EQUATIONS AND INEQUALITIES

Each absolute-value equation or inequality is rewritten as two equations or two inequalities joined by *and* or *or*.

- $|ax + b| < c$ **means** $ax + b < c$ *and* $ax + b > -c$.
- $|ax + b| \leq c$ **means** $ax + b \leq c$ *and* $ax + b \geq -c$.
- $|ax + b| = c$ **means** $ax + b = c$ *or* $ax + b = -c$.
- $|ax + b| > c$ **means** $ax + b > c$ *or* $ax + b < -c$.
- $|ax + b| \geq c$ **means** $ax + b \geq c$ *or* $ax + b \leq -c$.

Notice that when an absolute value is *less than* a number, the inequalities are connected by *and*. When an absolute value is *greater than* a number, the inequalities are connected by *or*.

EXAMPLE 3 *Solving an Absolute-Value Inequality*

Solve $|x - 4| < 3$.

SOLUTION

$x - 4$ IS POSITIVE	$x - 4$ IS NEGATIVE
$\|x - 4\| < 3$	$\|x - 4\| < 3$
$x - 4 < +3$	$x - 4 > -3$ ← Reverse inequality symbol.
$x < 7$	$x > 1$

The solution is all real numbers greater than 1 *and* less than 7, which can be written as $1 < x < 7$.

STUDENT HELP

Study Tip

When you rewrite an absolute-value inequality, reverse the inequality symbol in the inequality involving $-c$.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

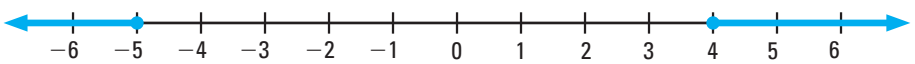
EXAMPLE 4 Solving an Absolute-Value Inequality

Solve $|2x + 1| - 3 \geq 6$. Then graph the solution.

SOLUTION

$2x + 1$ IS POSITIVE	$2x + 1$ IS NEGATIVE
$\|2x + 1\| - 3 \geq 6$	$\|2x + 1\| - 3 \geq 6$
$\|2x + 1\| \geq 9$	$\|2x + 1\| \geq 9$
$2x + 1 \geq +9$	$2x + 1 \leq -9$ ← Reverse inequality symbol.
$2x \geq 8$	$2x \leq -10$
$x \geq 4$	$x \leq -5$

The solution of $|2x + 1| - 3 \geq 6$ is all real numbers greater than or equal to 4 *or* less than or equal to -5, which can be written as the compound inequality $x \leq -5$ *or* $x \geq 4$.



FOCUS ON CAREERS

REAL LIFE QUALITY CONTROL INSPECTOR
A quality control inspector usually works in a manufacturing company. Experienced workers usually advance to more complex inspecting positions.

CAREER LINK
www.mcdougallittell.com

EXAMPLE 5 Writing an Absolute-Value Inequality

You work in the quality control department of a manufacturing company. The diameter of a drill bit must be between 0.62 inch and 0.63 inch.

a. Write an absolute-value inequality to represent this requirement.

b. A bit has a diameter of 0.623 inch. Does it meet the requirement?

SOLUTION

a. Let d represent the diameter (in inches) of the drill bit.

Write a compound inequality.

$$0.62 \leq d \leq 0.63$$

Find the halfway point: 0.625.

Subtract 0.625 from each part of the compound inequality.

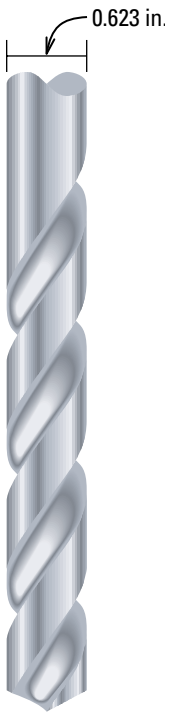
$$0.62 - 0.625 \leq d - 0.625 \leq 0.63 - 0.625$$

$$-0.005 \leq d - 0.625 \leq 0.005$$

Rewrite as an absolute-value inequality:

$$|d - 0.625| \leq 0.005$$

This inequality can be read as "the actual diameter must differ from 0.625 inch by no more than 0.005 inch."



b. Because $|0.623 - 0.625| \leq 0.005$, the bit does meet the requirement.

GUIDED PRACTICE

Vocabulary Check ✓

1. $|x + 3| = 8$ is an example of a(n) ? and $|x + 3| \leq 8$ is an example of a(n) ?.

Concept Check ✓

2. Explain each step you should use to solve $|x + 3| = 8$.
3. Explain why $|x - 5| < 2$ means that $x - 5$ is between -2 and 2.

Complete the sentence using the word *and* or the word *or*.

4. $|x - 5| < 2$ means $x - 5 < 2$? $x - 5 > -2$.
5. $|x - 5| > 2$ means $x - 5 > 2$? $x - 5 < -2$.

Skill Check ✓

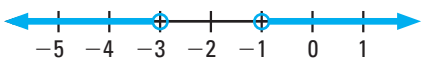
Solve the equation.

6. $|n| = 5$
7. $|a| = 0$
8. $|x + 3| = 6$
9. $|x - 4| = 10$
10. $|2n - 3| + 4 = 8$
11. $|3x + 2| + 2 = 5$

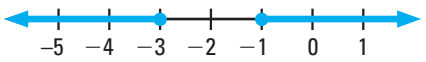
Match the inequality with the graph of its solution.

12. $|x + 2| \geq 1$
13. $|x + 2| \leq 1$
14. $|x + 2| > 1$

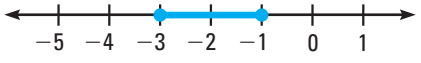
A.



B.



C.



Solve the inequality.

15. $|x + 6| < 4$
16. $|x - 2| > 9$
17. $|3x + 1| \leq 5$
18. **MANUFACTURING** In Example 5, suppose the diameter of the drill bit could be 0.5 inch, plus or minus as much as 0.005 inch. Write an absolute value inequality to represent this requirement.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 802.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 19–39
Example 2: Exs. 19–39

continued on p. 357

SOLVING EQUATIONS **Solve the equation.**

19. $|x| = 7$
20. $|x| = 10$
21. $|x| = 25$
22. $|x - 4| = 6$
23. $|x + 5| = 11$
24. $|x + 8| = 9$
25. $|x - 5| = 2$
26. $|x - 1| = 4$
27. $|x + 3| = 9$
28. $|x + 1| = 6$
29. $|x - 3.2| = 7$
30. $|x + 5| = 6.5$
31. $|4x - 2| = 22$
32. $|6x - 4| = 2$
33. $|3x + 5| = 23$
34. $|5 - 4x| - 3 = 4$
35. $|2x - 4| - 8 = 10$
36. $|7x + 3| + 2 = 33$
37. $|x + 3.6| = 4.6$
38. $|x - 1.2| - 2 = 5$
39. $\left|x - \frac{1}{2}\right| = \frac{5}{2}$

STUDENT HELP

HOMEWORK HELP
continued from p. 356

Example 3: Exs. 40–60
Example 4: Exs. 40–60
Example 5: Exs. 61, 62

SOLVING INEQUALITIES **Solve the inequality.**

40. $|x + 3| < 8$ **41.** $|2x - 9| \leq 11$ **42.** $|x + 10| \geq 20$

43. $|x - 2.2| > 3$ **44.** $|4x + 2| - 1 < 5$ **45.** $|5x - 15| - 4 \geq 21$

SOLVING AND GRAPHING **Solve the inequality. Then graph the solution.**

46. $|9 + x| \leq 7$ **47.** $|4 - x| < 5$ **48.** $|x + 12| > 36$

49. $|x - 3| \leq 17$ **50.** $|x + 5| \geq 1$ **51.** $|x + 3| \geq 8$

52. $|10 - 4x| \leq 2$ **53.** $|2x + 3| > 4$ **54.** $|x + 2| - 5 \geq 8$

55. $|3 + x| + 7 < 10$ **56.** $|3x + 2| - 1 \geq 10$ **57.** $|5x + 1| - 8 \leq 16$

58. $|5x + 3| - 4 \geq 9$ **59.** $|2x + 5| - 1 < 6$ **60.** $|3x - 9| + 2 > 7$

61. BASKETBALL On your basketball team, the starting players' scoring averages are between 8 and 22 points per game. Write an absolute-value inequality describing the scoring averages for the players.

62. TEST SCORES The test scores in your class range from 60 to 100. Write an absolute-value inequality describing the range of the test scores.

63. CAR MILEAGE Your car averages 28 miles per gallon in the city. The actual mileage varies from the average by at most 4 miles per gallon. Write an absolute-value inequality that shows the range for the mileage your car gets.

64. BOXING The cruiser weight division in boxing is centered at 183 pounds. A boxer's weight can be as much as 7 pounds more than or less than 183 pounds. Write an absolute-value inequality for this weight requirement.

FIREWORKS **When a firework star bursts, the color of the "stars" is determined by the chemical compounds in the firework. The wavelengths for different colors in the spectrum are shown below.**

Color	Wavelength, w
Ultraviolet	$w < 400$
Violet	$400 \leq w < 424$
Blue	$424 \leq w < 491$
Green	$491 \leq w < 575$
Yellow	$575 \leq w < 585$
Orange	$585 \leq w < 647$
Red	$647 \leq w < 700$
Infrared	$700 \leq w$

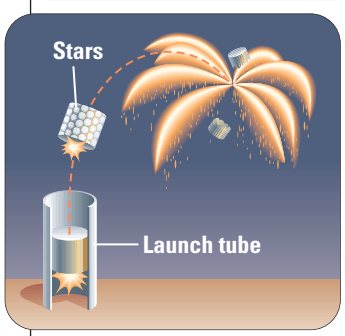
65. A firework star contains strontium. When it is burned, strontium emits light at wavelengths given by $|w - 643| < 38$. What colors could the star be?

66. A firework star contains a copper compound. When it is burned, the compound emits light at wavelengths given by $|w - 455| < 23$. Determine the color of the star.

67. A firework star contains barium chlorate. When it is burned, barium chlorate emits light at wavelengths given by $|w - 519.5| < 12.5$. What color is the star?

68. A firework star contains a sodium compound. When it is burned, the compound emits light at wavelengths given by $|w - 600| < 5$. Determine the color of the star.

FOCUS ON APPLICATIONS



REAL LIFE FIREWORKS
The diagram above shows what happens when a firework is launched.

INTERNET APPLICATION LINK
www.mcdougallittell.com

Test Preparation

69. MULTIPLE CHOICE Solve $|x - 7| < 6$.

(A) $-6 < x < 6$ (B) $-7 < x < 7$ (C) $1 < x < 13$ (D) $-1 < x < 13$

70. MULTIPLE CHOICE Solve $|3x + 3| > 12$.

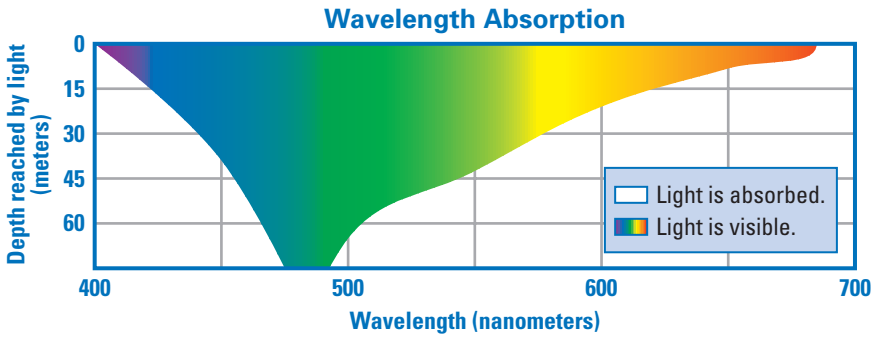
(A) $-5 < x < 3$ (B) $3 < x < -5$ (C) $x > 3$ or $x < -5$ (D) $x > 3$

71. MULTIPLE CHOICE Solve $|2x - 4| \leq 3$.

(A) $2 \leq x \leq 7$ (B) $\frac{1}{2} \leq x \leq \frac{7}{2}$ (C) $-\frac{7}{2} \leq x \leq \frac{1}{2}$ (D) $-\frac{7}{2} \leq x \leq \frac{7}{2}$

Challenge

SCIENCE CONNECTION **In Exercises 72 and 73, use the diagram below. It shows how light is absorbed in seawater.**



72. Write an absolute-value inequality approximating the wavelengths (w) of light that reach a depth of 30 meters in seawater.

EXTRA CHALLENGE
www.mcdougallittell.com

73. Write an absolute-value inequality approximating the wavelengths (w) of light that reach a depth of 60 meters in seawater.

MIXED REVIEW

MATRICES **Find the sum or difference of the matrices.** (Review 2.4)

74. $\begin{bmatrix} -2 & 7 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ -1 & -5 \end{bmatrix}$ **75.** $\begin{bmatrix} -4 & -9 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} -12 & 8 \\ -10 & -5 \end{bmatrix}$

SLOPE-INTERCEPT FORM **Rewrite the equation in slope-intercept form.** (Review 3.7, 4.6 for 6.5)

76. $x + 5y = 20$ **77.** $6x + 9y = 36$ **78.** $3x - 7y = 42$

EQUATIONS **Graph the equation.** (Review 4.6)

79. $x = -1$ **80.** $3y = 15$ **81.** $x + y = 7$

WRITING EQUATIONS **Write the point-slope form of the equation of the line that passes through the point and has the given slope. Then rewrite the equation in slope-intercept form.** (Review 5.5)

82. $(0, 4)$, $m = 3$ **83.** $(2, -5)$, $m = -2$ **84.** $(-3, 1)$, $m = \frac{2}{3}$

85. TRAVELING It is 368 miles from New York City to Pittsburgh. You make the trip in $6\frac{1}{2}$ hours. What was your average speed? Round your answer to the nearest mile per hour. (Review 1.1)