6.2 Solving Multi-Step Linear Inequalities

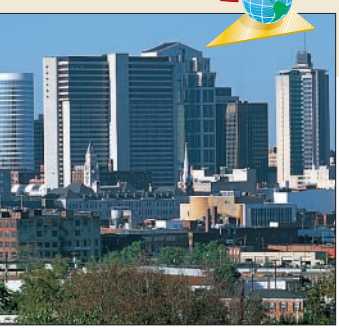
What you should learn

GOAL 1 Solve multi-step linear inequalities.

GOAL 2 Use linear inequalities to model and solve **real-life** problems, as in the business problem in **Example 4**.

Why you should learn it

▼ To describe **real-life** problems, such as city population growth in Nashville, Tennessee, in **Example 3**.



GOAL 1 SOLVING MULTI-STEP INEQUALITIES

In Lesson 6.1 you solved inequalities using one step. In this lesson you will learn how to solve linear inequalities using two or more steps.

EXAMPLE 1 *Using More than One Step*

Solve $2y - 5 < 7$.

SOLUTION

$2y - 5 < 7$ — Write original inequality.

$2y < 12$ — Add 5 to each side.

$y < 6$ — Divide each side by 2.

▶ The solution is all real numbers less than 6.

EXAMPLE 2 *Multiplying or Dividing by a Negative Number*

Solve the inequality.

a. $5 - x > 4$ **b.** $2x - 4 \geq 4x - 1$

SOLUTION

a. $5 - x > 4$ — Write original inequality.

$-x > -1$ — Subtract 5 from each side.

$(-x)(-1) < (-1)(-1)$ — Multiply each side by -1. *Reverse* inequality symbol.

$x < 1$ — Simplify.

▶ The solution is all real numbers less than 1.

b. $2x - 4 \geq 4x - 1$ — Write original inequality.

$2x \geq 4x + 3$ — Add 4 to each side.

$-2x \geq 3$ — Subtract $4x$ from each side.

$x \leq -\frac{3}{2}$ — Divide each side by -2. *Reverse* inequality symbol.

▶ The solution is all real numbers less than or equal to $-\frac{3}{2}$.

In part (b) of Example 2, you could begin by subtracting $2x$ from each side. Your solution would still be all real numbers less than or equal to $-\frac{3}{2}$.

STUDENT HELP

HOMEWORK HELP

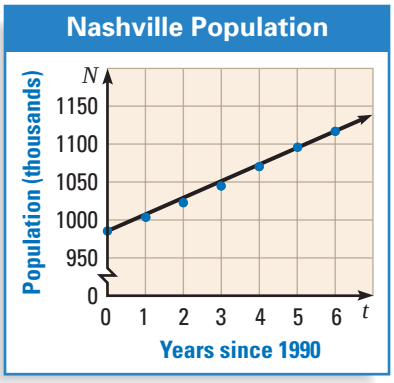
Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 MODELING AND SOLVING REAL-LIFE PROBLEMS

REAL LIFE Population

EXAMPLE 3 *Writing a Linear Model*

In 1990 Nashville, Tennessee, had a population of 985,000. From 1990 through 1996, the population increased at an average rate of about 22,000 per year. Write a linear model for the population of Nashville.



INTERNET **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

SOLUTION

To write a linear model of the population of Nashville, express the population as the 1990 population plus 22,000 per year.

Let N represent the population of Nashville.

Let t represent the number of years since 1990.

Population = 1990 population + 22,000 • Years since 1990

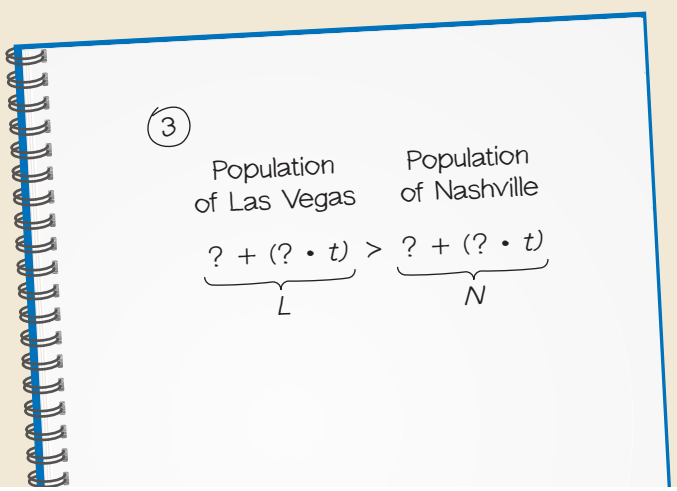
$$N = 985{,}000 + 22{,}000t$$

In the Activity below, you will compare populations of Nashville and Las Vegas.

ACTIVITY Developing Concepts

Investigating Problem Solving

1. In 1990 Las Vegas, Nevada, had a population of 853,000. From 1990 through 1996, the population increased at an average rate of about 58,000 per year. Write a linear model for the population of Las Vegas. Let L represent the population of Las Vegas.
2. Graph your linear model from **Step 1**. Graph the linear model from Example 3 in the same coordinate plane. According to the models, in what year will the populations be the same?
3. Write and solve a linear inequality to find the year that the population of Las Vegas exceeded the population of Nashville.
4. In which years was the population of Las Vegas less than the population of Nashville?
5. Write an inequality to represent the numbers of years y (since 1990) in which the population of Las Vegas was less than the population of Nashville.



REAL LIFE *Fly Fishing*

EXAMPLE 4 *Writing and Using a Linear Model*

You see an advertisement for instructions on how to tie flies for fishing. The cost of materials for each fly is \$.15. You plan to sell each fly for \$.58, and you want to make a profit of at least \$200. How many flies will you need to tie and sell?

SOLUTION

Your total expenses will be \$.15 per fly, plus \$15 for the instruction book.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Price per fly}} \cdot \boxed{\text{Number of flies sold}} - \boxed{\text{Total expenses}} \geq \boxed{\text{Desired profit}}$$

LABELS

Price per fly = **0.58** (dollars per fly)

Number of flies sold = x (flies)

Total expenses = $\mathbf{0.15}x + \mathbf{15}$ (dollars)

Desired profit = **200** (dollars)

ALGEBRAIC MODEL

$$\mathbf{0.58}\,x - (\mathbf{0.15}x + \mathbf{15}) \geq \mathbf{200} \quad \text{Write algebraic model.}$$

$$0.58x - 0.15x - 15 \geq 200 \quad \text{Use distributive property.}$$

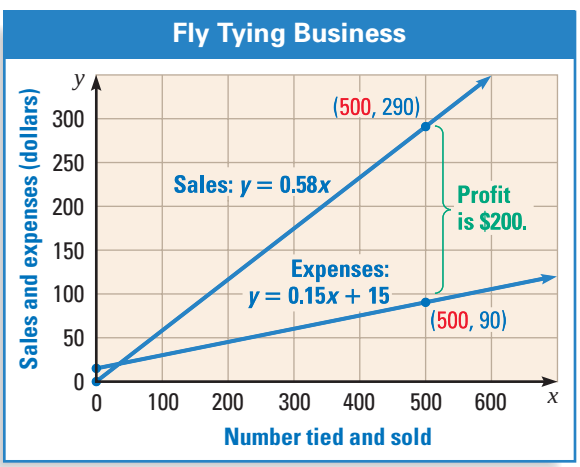
$$0.43x - 15 \geq 200 \quad \text{Combine like terms.}$$

$$0.43x \geq 215 \quad \text{Add 15 to each side.}$$

$$x \geq 500 \quad \text{Divide each side by 0.43.}$$

▶ You need to tie and sell at least 500 flies.

✓ **CHECK** You can check your result graphically by graphing equations for sales and for expenses separately.



FOCUS ON APPLICATIONS

REAL LIFE **FLY FISHING** In fly fishing, flies are made of feathers, hair, or other materials. They are intended to look like insects.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain why an inequality such as $3a + 6 \geq 0$ is called a *multi-step* inequality. Give another example of a multi-step inequality.

Concept Check ✓

2. Describe the steps you could use to solve the inequality $-3y + 2 > 11$.

ERROR ANALYSIS **Describe and correct the error.**

3.

$$-4y + 10 < 15$$
$$-4y < 5$$
$$\frac{-4y}{-4} < \frac{5}{-4}$$
$$y < -\frac{5}{4}$$

4.

$$6x - 4 \geq 2x + 1$$
$$6x \geq 2x - 3$$
$$4x \geq -3$$
$$x \geq -\frac{3}{4}$$

Skill Check ✓

Solve the inequality.

5. $y + 2 > -1$ **6.** $-2x < -14$ **7.** $-4x \geq -12$

8. $4y - 3 < 13$ **9.** $5x + 12 \leq 62$ **10.** $10 - c \geq 6$

11. $2 - x > 6$ **12.** $3x + 2 \leq 7x$ **13.** $2x - 1 > 6x + 2$

14. **BUSINESS** In Example 4, how many flies will you need to tie and sell to make a profit of at least \$150?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 802.

SOLVING INEQUALITIES **Solve the inequality.**

15. $x + 5 > -13$ **16.** $15 - x < 7$ **17.** $-5 \leq 6x - 12$

18. $-6 + 5x < 19$ **19.** $7 - 3x \leq 16$ **20.** $-3x - 0.4 > 0.8$

21. $6x + 5 < 23$ **22.** $-17 > 5x - 2$ **23.** $-x + 9 \geq 14$

24. $4x - 1 \leq -17$ **25.** $12 > -2x - 6$ **26.** $-x - 4 > 3x - 2$

27. $\frac{2}{3}x + 3 \geq 11$ **28.** $6 \geq \frac{7}{3}x - 1$ **29.** $-\frac{1}{2}x + 3 < 7$

30. $3x + 1.2 < -7x - 1.3$ **31.** $x + 3 \leq 2(x - 4)$

32. $2x + 10 \geq 7(x + 1)$ **33.** $-x + 4 < 2(x - 8)$

34. $-x + 6 > -(2x + 4)$ **35.** $-2(x + 3) < 4x - 7$

36. **POPULATION** In 1990 Tampa, Florida, had a population of 280,015. From 1990 through 1996, the population increased at an average rate of about 850 per year. Write a linear model for the population of Tampa.

▶ Source: U.S. Bureau of the Census

37. **AMUSEMENT PARK** An amusement park charges \$5 for admission and \$1.25 for each ride. You go to the park with \$25. Write an inequality that represents the possible number of rides you can go on. What is the maximum number of rides you can go on?

STUDENT HELP

HOMEWORK HELP

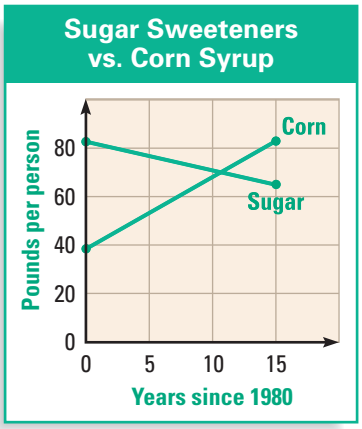
Example 1: Exs. 15–35
Example 2: Exs. 15–35
Example 3: Ex. 36
Example 4: Exs. 37, 39

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 39.

38. SWEETENERS From 1980 to 1995, the per capita consumption of sugar sweeteners dropped from 83.6 pounds to 65.5 pounds. The per capita consumption of corn syrup rose from 38.2 pounds to 83.2 pounds. For which years did the per capita consumption of corn syrup exceed sugar sweeteners?

39. PIZZA You have \$18.50 to spend on pizza. It costs \$14 plus \$.75 for each additional topping, tax included. Solve an inequality to find the maximum number of toppings the pizza can have.



Source: U.S. Department of Agriculture

FOCUS ON APPLICATIONS

REAL LIFE
ANIMATED FILMS begin with animators' drawings. Artists trace the drawings onto sheets of acetate called *cels* and then paint the cels.

ANIMATED FILMS **In Exercises 40–42, you are working as an animator. Each frame in an animated feature film takes at least one hour to draw. When projected, 35-millimeter film runs at 24 frames per second. A $2\frac{1}{2}$-hour movie has about 216,000 frames.**

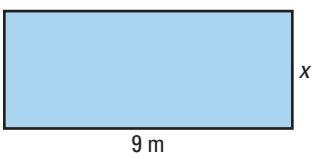
40. Write an inequality that describes the number of hours it would take to draw the frames needed for a $1\frac{1}{2}$-hour animated feature film.

41. You are part of a team of 36 artists, each working 40 hours per week for 45 weeks per year. How many years would it take your team to draw the $1\frac{1}{2}$-hour animated feature film?

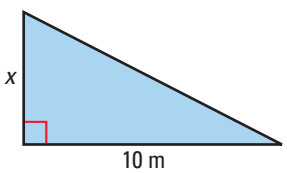
42. LOGICAL REASONING If you doubled the size of your team, how would that affect the time it would take to draw the film?

GEOMETRY CONNECTION **Write an inequality for the values of *x*.**

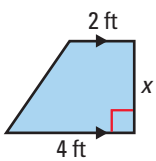
43. Area $>$ 36 square meters



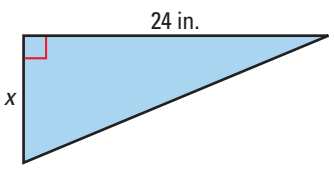
44. Area $\geq$ 25 square meters



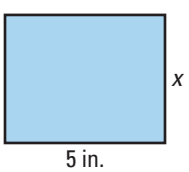
45. Area $<$ 30 square feet



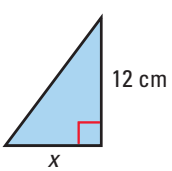
46. Area $<$ 144 square inches



47. Area $\leq$ 25 square inches



48. Area $>$ 60 square centimeters



Test Preparation

49. MULTIPLE CHOICE At a grocery store, four oranges cost \$2.39 and kiwi fruit costs \$.69 each. If you have \$5 to spend and you buy four oranges, which inequality represents the number of kiwi fruit you can buy?

(A) $x \leq 3$ (B) $x \geq 3$ (C) $x > 3$ (D) $x < 7$

50. MULTIPLE CHOICE The Glee Club budgeted \$250 for food for the annual Spaghetti Supper. Each meal costs \$1.75 to prepare. Which inequality represents the number of meals that can be prepared without going over the budget?

(A) $x \leq 143$ (B) $x \geq 143$ (C) $x \leq 142$ (D) $x \geq 142$

51. MULTIPLE CHOICE For Park College's basketball games, it costs \$15 to attend. A season's pass costs \$170. At most, how many games could you attend at the \$15 price before spending more than the cost of a season's pass?

(A) 10 (B) 11 (C) 15 (D) 17

★ Challenge

52. GRAPHICAL REASONING Use the graph of $y = \frac{2}{3}x - 2$ to solve the inequality $\frac{2}{3}x - 2 < 0$. Explain your reasoning.

EXTRA CHALLENGE
www.mcdougallittell.com

53. GRAPHICAL REASONING Use the graph of $y = -2x + 4$ to solve the inequality $-2x + 4 > 0$. Explain your reasoning.

MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 1.3, 2.7)

54. $(a + 4) - 8$ when $a = 7$

55. $3x + 2$ when $x = -4$

56. $b^3 - 5$ when $b = 2$

57. $2(r + s)$ when $r = 2$ and $s = 4$

58. $x^2 - 3x$ when $x = 5$

59. $\frac{a^2}{8 - b}$ when $a = 4$ and $b = -9$

TRANSLATING VERBAL SENTENCES **Translate the verbal sentence into an equation or an inequality.** (Review 1.5 for 6.3)

60. Sarah's height S is 4 inches more than Joanne's height J.

61. The number of tickets t sold this week is more than twice the number of tickets l sold last week.

WRITING INTEGERS **In Exercises 62 and 63, write an integer to represent the situation.** (Review 2.1)

62. a loss of \$12

63. three feet above ground

64. SHOPPING TRIP You buy a pair of shoes for \$42.99, a shirt for \$14.50, and a pair of jeans for \$29.99. You have a coupon for \$10 off your purchase. How much did you spend? (Review 1.3)

65. PROBABILITY You put 20 slips of paper in a hat. Eight slips are yellow, five slips are blue, and the rest are red. Without looking, you reach into the hat and choose a slip of paper. What is the probability that the slip of paper is not blue? (Review 2.8)