4.2 Graphing Linear Equations

What you should learn

GOAL 1 Graph a linear equation using a table or a list of values.

GOAL 2 Graph horizontal and vertical lines.

Why you should learn it

▼ To solve **real-life** problems such as finding the amount of calories burned while training for Triathlon in **Exs. 71–73**.

REAL LIFE

GOAL 1 GRAPHING A LINEAR EQUATION

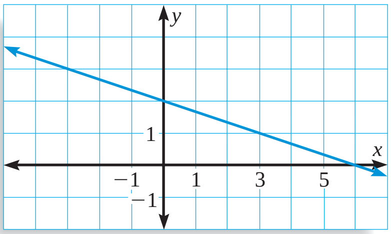
A **solution of an equation** in two variables x and y is an ordered pair (x, y) that makes the equation true. The **graph of an equation** in x and y is the set of *all* points (x, y) that are solutions of the equation. In this lesson you will see that the graph of a *linear* equation is a line.

EXAMPLE 1 *Verifying Solutions of an Equation*

Use the graph to decide whether the point lies on the graph of $x + 3y = 6$. Justify your answer algebraically.

a. (1, 2)

b. (−3, 3)



SOLUTION

a. The point (1, 2) is *not* on the graph of $x + 3y = 6$. This means that (1, 2) is not a solution. You can check this algebraically.

$x + 3y = 6$ — **Write original equation.**

$1 + 3(2) \stackrel{?}{=} 6$ — **Substitute 1 for x and 2 for y.**

$7 \neq 6$ — **Simplify. Not a true statement**

▶ (1, 2) is not a solution of the equation $x + 3y = 6$, so it is not on the graph.

b. The point (−3, 3) is on the graph of $x + 3y = 6$. This means that (−3, 3) is a solution. You can check this algebraically.

$x + 3y = 6$ — **Write original equation.**

$-3 + 3(3) \stackrel{?}{=} 6$ — **Substitute −3 for x and 3 for y.**

$6 = 6$ — **Simplify. True statement**

▶ (−3, 3) is a solution of the equation $x + 3y = 6$, so it is on the graph.

. . . .

In Example 1 the point (−3, 3) is on the graph of $x + 3y = 6$, but how many points does the graph have in all? The answer is that most graphs have too many points to list. Then how can you ever graph an equation? One way is to make a table or a list of a few values, plot enough solutions to recognize a pattern, and then connect the points. Even then, the graph extends without limit to the left of the smallest input and to the right of the largest input.

When you make a table of values to graph an equation, you may want to choose values for x that include negative values, zero, and positive values. This way you will see how the graph behaves to the left and right of the y-axis.

EXAMPLE 2 *Graphing an Equation*

Use a table of values to graph the equation $y + 2 = 3x$.

SOLUTION

STUDENT HELP

Look Back
For help with rewriting equations in function form, see p. 176.

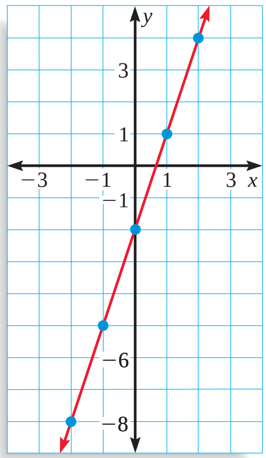
Rewrite the equation in function form by solving for y.

$y + 2 = 3x$ **Write original equation.**

$y = 3x - 2$ **Subtract 2 from each side.**

Choose a few values for x and make a table of values.

Choose x.	Substitute to find the corresponding y-value.
-2	$y = 3(-2) - 2 = -8$
-1	$y = 3(-1) - 2 = -5$
0	$y = 3(0) - 2 = -2$
1	$y = 3(1) - 2 = 1$
2	$y = 3(2) - 2 = 4$



With this table of values you have found the five solutions $(-2, -8)$, $(-1, -5)$, $(0, -2)$, $(1, 1)$, and $(2, 4)$.

Plot the points. Note that they appear to lie on a straight line.

▶ The line through the points is the graph of the equation.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

In Lesson 3.1 you saw examples of linear equations in one variable. The solution of an equation such as $2x - 1 = 3$ is a real number. Its graph is a point on the real number line. The equation $y + 2 = 3x$ in Example 2 is a linear equation in *two variables*. Its graph is a straight line.

GRAPHING A LINEAR EQUATION

STEP 1 Rewrite the equation in function form, if necessary.

STEP 2 Choose a few values of x and make a table of values.

STEP 3 Plot the points from the table of values. A line through these points is the graph of the equation.

EXAMPLE 3 *Graphing a Linear Equation*

Use a table of values to graph the equation $3x + 2y = 1$.

SOLUTION

1 Rewrite the equation in function form by solving for y. This will make it easier to make a table of values.

$$3x + 2y = 1 \quad \text{Write original equation.}$$

$$2y = -3x + 1 \quad \text{Subtract } 3x \text{ from each side.}$$

$$y = -\frac{3}{2}x + \frac{1}{2} \quad \text{Divide each side by 2.}$$

> **STUDENT HELP**
> **Skills Review**
> For help with fraction operations, see pp. 780–782.

2 Choose a few values of x and make a table of values.

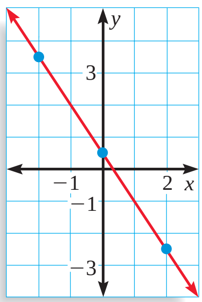
Choose x.	-2	0	2
Evaluate y.	$\frac{7}{2}$	$\frac{1}{2}$	$-\frac{5}{2}$

With this table of values you have found three solutions.

$$\left(-2, \frac{7}{2}\right), \left(0, \frac{1}{2}\right), \left(2, -\frac{5}{2}\right)$$

3 Plot the points and draw a line through them.

▶ The graph of $3x + 2y = 1$ is shown at the right.



REAL LIFE
Internet Access

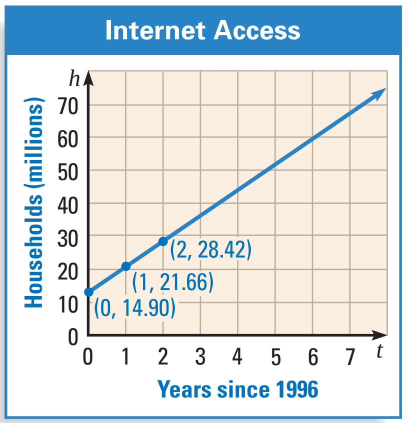
EXAMPLE 4 *Using the Graph of a Linear Model*

An Internet Service Provider estimates that the number of households h (in millions) with Internet access can be modeled by $h = 6.76t + 14.9$, where t represents the number of years since 1996. Graph this model. Describe the graph in the context of the real-life situation.

SOLUTION

Make a table of values. Use $0 \le t \le 6$ for 1996–2002.

t	h
0	14.90
1	21.66
2	28.42
3	35.18
4	41.94
5	48.70
6	55.46

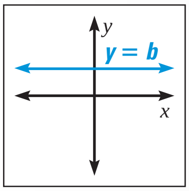


▶ From the table and the graph, you can see that the number of households with Internet access is projected to increase by about 7 million households per year.

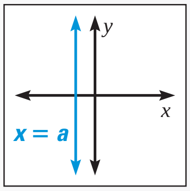
GOAL 2 HORIZONTAL AND VERTICAL LINES

All linear equations in x and y can be written in the form $Ax + By = C$. When $A = 0$ the equation reduces to $By = C$ and the graph is a horizontal line. When $B = 0$ the equation reduces to $Ax = C$ and the graph is a vertical line.

EQUATIONS OF HORIZONTAL AND VERTICAL LINES



In the coordinate plane, the graph of $y = b$ is a horizontal line.



In the coordinate plane, the graph of $x = a$ is a vertical line.

EXAMPLE 5 Graphing $y = b$

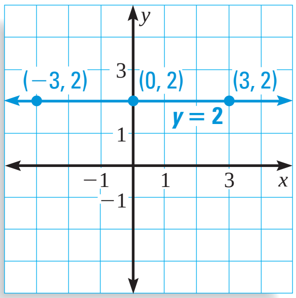
Graph the equation $y = 2$.

SOLUTION

The equation does not have x as a variable. The y-value is always 2, regardless of the value of x. For instance, here are some points that are solutions of the equation:

$(-3, 2), (0, 2), (3, 2).$

▶ The graph of the equation is a horizontal line 2 units above the x-axis.



> **STUDENT HELP**
>
> **Study Tip**
> The equations $y = 2$ and $0x + 1y = 2$ are equivalent. For any value of x, the ordered pair $(x, 2)$ is a solution of $y = 2$.

EXAMPLE 6 Graphing $x = a$

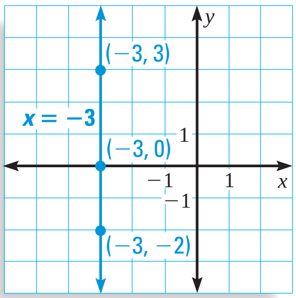
Graph the equation $x = -3$.

SOLUTION

The x-value is always -3, regardless of the value of y. For instance, here are some points that are solutions of the equation:

$(-3, -2), (-3, 0), (-3, 3).$

▶ The graph of the equation is a vertical line 3 units to the left of the y-axis.



Guided Practice

Vocabulary Check ✓

1. Complete the following sentence: An ordered pair that makes an equation in two variables true is called a(n) __?__.

Concept Check ✓

2. In Example 2, if you choose a value of x different from those in the table of values, will you find a solution that lies on the same line? Explain.

3. Decide whether the following statement is *true* or *false*. *The graph of the equation $x = 3$ is a horizontal line.* Explain.

Skill Check ✓

Use a table of values to graph the equation.

4. $6x - 3y = 12$ **5.** $x = 1.5$ **6.** $y = -2$

Tell whether the point is a solution of the equation $4x - y = 1$.

7. $(-1, 3)$ **8.** $(1, 3)$ **9.** $(1, 0)$ **10.** $(1, 4)$

11. **INTERNET ACCESS** Using the graph or the model in Example 4, estimate the number of households that had Internet access in 1999.

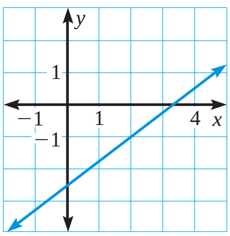
Practice and Applications

STUDENT HELP

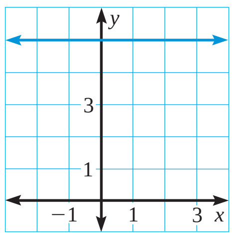
Extra Practice to help you master skills is on p. 800.

VERIFYING SOLUTIONS **Use the graph to decide whether the point lies on the graph of the line. Justify your answer algebraically.**

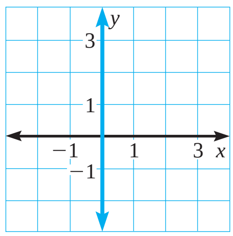
12. $3x - 4y = 10$
a. $(2, -1)$
b. $(-1, 2)$



13. $y = 5$
a. $(5, 0)$
b. $(0, 5)$



14. $x = 0$
a. $(0, 14)$
b. $(14, 0)$



CHECKING SOLUTIONS **Decide whether the given ordered pair is a solution of the equation.**

15. $2y - 4x = 8, (-2, 8)$

16. $-5x - 8y = 15, (-3, 0)$

17. $y = -2, (-2, -2)$

18. $x = -4, (1, -4)$

19. $6y - 3x = -9, (2, -1)$

20. $-2x - 9y = 7, (-1, -1)$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 12–20
Example 2: Exs. 21–55
Example 3: Exs. 21–55
Example 4: Exs. 67–70
Example 5: Exs. 44–55, 60–65
Example 6: Exs. 44–55, 60–65

FINDING SOLUTIONS **Find three different ordered pairs that are solutions of the equation.**

21. $y = 3x - 5$ **22.** $y = 7 - 4x$ **23.** $y = -2x - 6$

24. $x = 2$ **25.** $x = \frac{1}{2}$ **26.** $y = -6$

27. $y = \frac{1}{2}(4 - 2x)$ **28.** $y = 3(6x - 1)$ **29.** $y = 4\left(\frac{1}{2}x - 1\right)$

FUNCTION FORM **Rewrite the equation in function form.**

30. $-3x + y = 12$ **31.** $2x + 3y = 6$ **32.** $x + 4y = 48$

33. $5x + 5y = 19$ **34.** $\frac{1}{2}x + \frac{5}{2}y = 1$ **35.** $-x - y = 5$

GRAPHING EQUATIONS **Use a table of values to graph the equation.**

36. $y = -x + 4$ **37.** $y = -2x + 5$ **38.** $y = -(3 - x)$ **39.** $y = -2(x - 6)$

40. $y = 3x + 2$ **41.** $y = 4x - 1$ **42.** $y = \frac{4}{3}x + 2$ **43.** $y = -\frac{3}{4}x + 1$

44. $x = 9$ **45.** $y = -1$ **46.** $y = 0$ **47.** $y = -3x + 1$

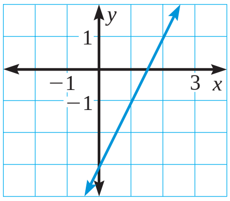
48. $x = 0$ **49.** $x - 2y = 6$ **50.** $4x + 4y = 2$ **51.** $x + 2y = -8$

MATCHING EQUATIONS WITH GRAPHS **Match the equation with its graph.**

A. $x = -2$ **B.** $2x - y = 3$

C. $6x + 3y = 0$ **D.** $-x + 2y = 6$

52.



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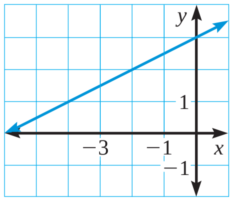

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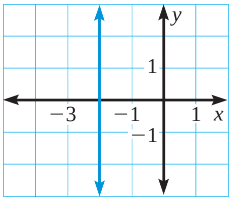

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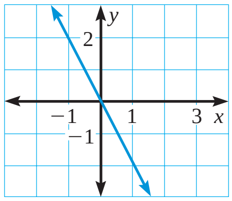


56. CRITICAL THINKING Robin always finds at least three different ordered pairs when making a table to graph an equation. Why do you think she does this?

STUDENT HELP

Look Back
For help with scatter plots, see p. 204.

PLOTTING POINTS **In Exercises 57–59, use the ordered pairs below.**
$(-3, -9), (-2, -7), (-1.6, -5.5), (0.4, -2.2), (3.1, 3.2), (5.2, 7.4)$

57. Use a graphing calculator or a computer to graph the ordered pairs.

58. The line that fits most of the points is the graph of $y = 2x - 3$. One of the points does not appear to fall on the line with the others. Which point is it?

59. Show algebraically that one point does not lie on this line.

POINTS OF INTERSECTION **In Exercises 60–65, graph the two lines in the same coordinate plane. Then find the coordinates of the point at which the lines cross.**

60. $x = -5, y = 2$ **61.** $y = 11, x = -8$ **62.** $y = -6, x = 1$

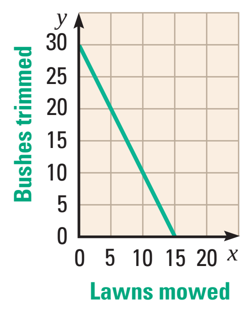
63. $x = 4, y = -4$ **64.** $y = -1, x = 0$ **65.** $x = 3, y = 0$

66. VISUAL THINKING Name a point that would be on the graphs of both $x = -3$ and $y = 2$. Sketch both graphs to verify your answer.

LANDSCAPING BUSINESS **In Exercises 67–70, use the following information.**

One summer you charge \$20 to mow a lawn and \$10 to trim bushes. You want to make \$300 in one week. An algebraic model for your earnings is $20x + 10y = 300$, where x is the number of lawns you mow and y is the number of bushes you trim.

Summer Earnings



67. Solve the equation for y.

68. Use the equation in function form from Exercise 67 to make a table of values for $x = 5$, $x = 10$, and $x = 15$.

69. Look at the graph at the right. Does the line shown appear to pass through the points from your table of values?

70. If you do not trim any bushes during the week, how many lawns will you have to mow to earn \$300?

TRAINING FOR A TRIATHLON **In Exercises 71–73, Mary Gordon is training for a triathlon. Like most triathletes she regularly trains in two of the three events every day. On Saturdays she expects to burn about 800 calories during her workout by running and swimming.**

Running:	7.1 calories per minute
Swimming:	10.1 calories per minute
Bicycling:	6.2 calories per minute

71. Copy and complete the model below. Let x represent the number of minutes she spends running, and let y represent the number of minutes she spends swimming.

VERBAL MODEL $\boxed{\text{Calories burned while running}} \cdot \boxed{?} + \boxed{?} \cdot \boxed{\text{Swimming Time}} = \boxed{\text{Total calories burned}}$

LABELS

Calories burned while running $= \boxed{?}$ (calories/minute)

Running time $= \boxed{x}$ (minutes)

$\boxed{?} = 10.1$ (calories/minute)

$\boxed{?} = \boxed{y}$ (minutes)

Total calories burned $= 800$ (calories)

ALGEBRAIC MODEL $\boxed{?} \cdot x + \boxed{?} \cdot y = 800$ **Write a linear model.**

72. Make a table of values and graph the equation from Exercise 71.

73. If Mary Gordon spends 45 minutes running, about how many minutes will she have to spend swimming to burn 800 calories?

FOCUS ON APPLICATIONS

REAL LIFE **TRIATHLON** In 1998, over 1500 people competed in Hawaii's Ironman Triathlon World Championship.

INTERNET **APPLICATION LINK** www.mcdougallittell.com

CHOOSING A MODEL **In Exercises 74 and 75, decide whether a graph of the data would be points that appear to lie on a *horizontal line*, a *vertical line*, or *neither*. Explain. Let the *x*-axis represent time.**

74. The number of books read by Avery each year from 1999 to 2005.

75. The number of senators in the United States Congress each year from 1991 to 2000.

Test Preparation

76. MULTIPLE CHOICE Which point lies on the graph of $2x + 5y = 6$?

(A) $(-2, 2)$ (B) $(3, 0)$ (C) $(0, 3)$ (D) both A and B

77. MULTIPLE CHOICE Which point does *not* lie on the graph of $y = 3$?

(A) $(0, 3)$ (B) $(-3, 3)$ (C) $(3, -3)$ (D) $\left(\frac{1}{3}, 3\right)$

78. MULTIPLE CHOICE The ordered pair $(-3, 5)$ is a solution of __?__.

(A) $y = 5$ (B) $x = 5$ (C) $y = \frac{1}{2}x - 2$ (D) $y = -\frac{1}{2}x - 2$

★ Challenge

MISINTERPRETING GRAPHS **Even though they provide an accurate representation of data, some graphs are easy to misinterpret. In Exercises 79 and 80, use the following.**

Based on actual and projected data from 1995 to 2000, a linear model for a company's profit P is $P = 3{,}005{,}000 - 900t$, where t represents the number of years since 1995.

79. Sketch two graphs of the model. In the first graph, divide the vertical axis into \$1,000,000 intervals. In the second graph start at \$3,000,000 and use \$1000 intervals.

80. CRITICAL THINKING What are the advantages and disadvantages of each graph?

EXTRA CHALLENGE
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MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 2.2)

81. $5 + 2 + (-3)$ **82.** $-6 + (-14) + 8$ **83.** $-18 + (-10) + (-1)$

84. $-\frac{1}{3} + 6 + \frac{1}{3}$ **85.** $\frac{4}{7} + \frac{3}{7} - 1$ **86.** $-\frac{7}{9} + \frac{1}{3} + 2$

MATRICES **Find the sum of the matrices.** (Review 2.4)

87. $\begin{bmatrix} 1 & 6 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} 15 & -3 \\ 0 & 16 \end{bmatrix}$ **88.** $\begin{bmatrix} 2 & 5 \\ 3 & 10 \end{bmatrix} + \begin{bmatrix} -14 & -5 \\ 12 & 7 \end{bmatrix}$

89. $\begin{bmatrix} 4 & 10 \\ -1 & 9 \end{bmatrix} + \begin{bmatrix} -20 & 40 \\ -8 & 10 \end{bmatrix}$ **90.** $\begin{bmatrix} 5 & -2 \\ 5 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -16 \\ 11 & -3 \end{bmatrix}$

SOLVING EQUATIONS **Solve the equation.** (Review 3.2 for 4.3)

91. $-2z = -26$ **92.** $9x = 3$ **93.** $6p = -96$ **94.** $24 = 8c$

95. $\frac{2}{3}t = -10$ **96.** $-\frac{p}{7} = -9$ **97.** $\frac{n}{15} = \frac{3}{5}$ **98.** $\frac{c}{6} = \frac{2}{3}$