CHAPTER 3

Chapter Summary

WHAT did you learn?

WHY did you learn it?

Solve a linear equation

- using addition and subtraction. (3.1) → Model and find increases and decreases. (p. 134)
- using multiplication and division. (3.2) → Find your distance from a thunderstorm. (p. 143)
- using two or more transformations. (3.3) → Find the temperature below Earth's surface. (p. 147)
- with variables on both sides. (3.4) → Decide whether a membership is economical. (p. 156)
- involving decimals. (3.6) → Find the expansion gap for a bridge. (p. 171)

Write an equation using equal ratios. (3.2) → Find unknown side lengths in similar triangles. (p. 141)

Solve a formula for one of its variables. (3.7) → Estimate the speed on Pathfinder's flight to Mars. (p. 175)

Rewrite an equation in function form, and find the output given the input. (3.7) → Prepare for graphing linear equations in two variables. (p. 176)

Use linear equations to model and solve real-life problems

- using a diagram. (3.5) → Design a high school yearbook. (p. 160)
- using a table or a graph to check. (3.5) → Understand when a gazelle is a safe distance from a cheetah. (p. 162)
- involving rates, ratios, and percents. (3.8) → Estimate how far a car can travel on a tank of gasoline. (p. 180)

How does Chapter 3 fit into the BIGGER PICTURE of algebra?

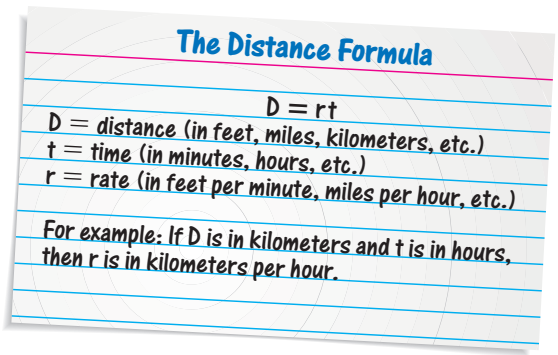
In Chapters 1 and 2 you used mental math to solve simple equations. In this chapter you learned systematic equation-solving techniques that allow you to solve more complicated equations such as $0.5(x - 4) - (x - 8) = 16$. These techniques are based on the rules of algebra and the methods for simplifying you learned in Chapter 2.

You will need to solve linear equations when you solve linear inequalities in Chapter 6 and systems of equations in Chapter 7.

STUDY STRATEGY

How did you use your formula cards?

One of the formula cards you made, using the **Study Strategy** on p. 130, may look like this one.



CHAPTER 3

Chapter Review

VOCABULARY

- equivalent equations, p. 132
- inverse operations, p. 132
- solution step, p. 133
- linear equation in one variable, p. 133
- properties of equality, p. 139
- ratio of *a* to *b*, p. 140
- similar triangles, p. 140
- identity, p. 155
- round-off error, p. 166
- formula, p. 174
- rate of *a* per *b*, p. 180
- unit rate, p. 180

3.1–3.2 SOLVING EQUATIONS USING ONE OPERATION

Examples on pp. 132–134, 138–141

EXAMPLES Use inverse operations to isolate the variable.

$y - 4 = -6$ — Write original equation.
$y = -2$ — Add 4 to each side.

$2 - x = 12$ — Write original equation.
$-x = 10$ — Subtract 2 from each side.
$x = -10$ — *x* is the opposite of 10.

$\frac{1}{8}m = -5$ — Write original equation.
$m = -40$ — Multiply each side by 8.

$-7n = 28$ — Write original equation.
$n = -4$ — Divide each side by −7.

Solve the equation.

1. $y - 15 = -4$ **2.** $-7 + x = -3$ **3.** $25 = -35 - c$ **4.** $-11 = z - (-15)$

5. $36 = \frac{h}{-12}$ **6.** $-\frac{2}{3}w = -70$ **7.** $6m = -72$ **8.** $\frac{y}{4} = \frac{15}{6}$

3.3 SOLVING MULTI-STEP EQUATIONS

Examples on pp. 145–147

EXAMPLE Solving some equations requires two or more transformations.

$-2p - (-5) - 2p = 13$ — Write original equation.
$-4p + 5 = 13$ — Simplify.
$-4p = 8$ — Subtract 5 from each side.
$p = -2$ — Divide each side by −4.

Solve the equation.

9. $26 - 9p = -1$ **10.** $\frac{4}{5}c - 12 = -32$ **11.** $\frac{y}{4} + 2 = 0$ **12.** $-2(4 - x) - 7 = 5$

13. $6r - 2 - 9r = 1$ **14.** $16 = 5(1 - x)$ **15.** $-\frac{2}{3}(6 - 2a) = 6$ **16.** $n - 4(1 + 5n) = -2$

3.4 Solving Equations with Variables on Both Sides

Examples on pp. 154–156

EXAMPLES To solve, try to collect the variable terms on one side of the equation.

An equation with one solution:

$$-21d + 15 = -5d + 7$$
$$15 = 16d + 7$$
$$8 = 16d$$
$$\frac{1}{2} = d$$

An equation with no solution:

$$-3(2x - 5) = -(15 + 6x)$$
$$-6x + 15 = -15 - 6x$$
$$15 = -15$$

$15 \neq -15$ for any value of x, so the original equation has no solution.

An equation with many solutions:

$$2s - 5s + 11 = 2 - 3s + 9$$
$$-3s + 11 = 11 - 3s$$
$$11 = 11$$

The equation $11 = 11$ is always true, so all values of s are solutions.

Solve the equation if possible.

17. $9z + 24 = -3z$

18. $12 - 4h = -18 + 11h$

19. $24a - 8 - 10a = -2(4 - 7a)$

20. $9(-5 - r) = -10 - 2r$

21. $\frac{2}{3}(3x - 9) = 4(x + 6)$

22. $6m - 3 = 10 - 6(2 - m)$

3.5 Linear Equations and Problem Solving

Examples on pp. 160–162

EXAMPLE A diagram can help you to understand a problem. Two runners leave the starting line at the same time. The first runner crosses the finish line in 25 minutes and averages 12 kilometers per hour. The second runner crosses the finish line 5 minutes after the first runner. Find the second runner's speed.

VERBAL MODEL

$$\boxed{\text{Speed of first runner}} \cdot \boxed{\text{Time of first runner}} = \boxed{\text{Speed of second runner}} \cdot \boxed{\text{Time of second runner}}$$

LABELS

Speed of first runner = **12** (kilometers per hour)

Time of first runner = $\frac{5}{12}$ (hour)

Speed of second runner = x (kilometers per hour)

Time of second runner = $\frac{1}{2}$ (hour)

23. Write and solve the equation for the example. Then answer the question.

24. **TOMATOES** One tomato plant is 12 inches tall and grows $1\frac{1}{2}$ inches per week. Another tomato plant is 6 inches tall and grows 2 inches per week. When will the plants be the same height? Use a table or a graph to check.

3.6 Solving Decimal Equations

Examples on pp. 166–168

EXAMPLE For some equations, you need to give an approximate solution.

$3.45m = -2.93m - 2.95$ — Write original equation.

$6.38m = -2.95$ — Add $2.93m$ to each side.

$m \approx -0.462382445$ — Using a calculator, divide each side by 6.38.

$m \approx -0.46$ — Round answer. (Use hundredths, as in original equation.)

Solve the equation. Round the result to the nearest tenth.

25. $13.7t - 4.7 = 9.9 + 8.1t$ **26.** $4.6(2a + 3) = 3.7a - 0.4$ **27.** $-6(5.61x - 3.21) = 4.75$

3.7 Formulas and Functions

Examples on pp. 174–176

EXAMPLE You can solve the formula for the volume of a cone for the height h.

$V = \frac{1}{3}\pi r^2 h$ — Write original formula.

$3V = \pi r^2 h$ — Multiply each side by 3.

$\frac{3V}{\pi r^2} = h$ — Divide each side by πr^2.

In Exercises 28–30, solve for the indicated variable.

28. $S = 2\pi rh$ for h **29.** $S = \pi s(R + r)$ for r **30.** $R = \frac{pV}{nT}$ for p

31. Rewrite the equation $3x + 2y - 4 = 2(5 - y)$ so that y is a function of x. Then use the result to find y when $x = -2$, 0, 1, and 5.

3.8 Rates, Ratios, and Percents

Examples on pp. 180–182

EXAMPLE For a United States flag, the ratio $\frac{\text{length}}{\text{width}}$ is $\frac{1.9}{1}$. How long is a 5-inch-wide flag?

length of small flag → x / width of small flag → 5 $= \frac{1.9}{1}$ ← standard length to width ratio

The solution of the equation is $x = 9.5$, so the flag is 9.5 inches long.

32. You earn \$210 in 40 hours. At this rate, how much do you earn in 55 hours?

33. A cab driver typically receives about 15% of the fare charged as a tip. To earn a total of \$30 in tips, about how much would a driver need to collect in fares?

34. Convert 470 Ethiopian birrs to U.S. dollars. (1 dollar is 7.821 birrs.)

CHAPTER 3

Chapter Test

Solve the equation if possible.

1. $2 + x = 8$

2. $19 = a - 4$

3. $-3y = -18$

4. $\frac{x}{4} = 5$

5. $17 = 5 - 3p$

6. $-\frac{3}{4}x - 2 = -8$

7. $\frac{5}{3}(9 - w) = -10$

8. $-3(x - 2) = x$

9. $-5r - 6 + 4r = -r + 2$

10. $-4y - (5y + 6) = -7y + 3$

Solve the equation. Round the result to the nearest hundredth.

11. $13.2x + 4.3 = 2(2.7x - 3.6)$

12. $-4(2.5x + 8.7) = (1.4 - 9.2x)(6)$

In Exercises 13 and 14, solve for the indicated variable.

13. $C = 2\pi r$, r

14. $S = B + \frac{1}{2}Pl$, l

15. Rewrite $3x + 4y = 15 + 6y$ so that y is a function of x.

16. Use the result in Exercise 15 to find y when $x = -1$, 0, and 2.

17. How many feet are in 3.5 kilometers? (Hint: 1 km $\approx$ 3281 ft)

18. **SHOVELING SNOW** You shovel snow. You charge \$7 per driveway and earn \$42. Let x represent the number of driveways you shoveled. Which of the following equations is an algebraic model for the situation?

A. $42x = 7$ **B.** $\frac{1}{7}x = 42$ **C.** $7x = 42$ **D.** $\frac{1}{42}x = 7$

EARNINGS In Exercises 19 and 20, your cousin earns about \$25 per week baby-sitting and receives one \$5 bonus. You earn about \$15 per week mowing lawns and \$12 per week running errands. After working the same number of weeks, you have \$11 more than your cousin.

19. Write and solve an equation to find how many weeks you worked.

20. Check your solution in Exercise 19 with a table or a graph.

21. **SAVINGS INTEREST** You invest \$400. After one year, the total of the investment is \$414.40. Use the formula $A = P + Prt$ to find the annual simple interest rate for the investment, where A is the total of the investment, P is the principal (amount invested), r is the annual simple interest rate, and t is the time in years.

In Exercises 22 and 23, write and solve an equation to answer the question.

22. **VOLUNTEER WORK** You stuffed 108 envelopes in 45 minutes. At this rate, how many envelopes can you stuff in 2 hours?

23. **WAGES** After an 8% increase in your wages, you receive \$.94 more per hour. About how much did you receive per hour before the increase in your wages?