CS340: Discrete Structures **Homework #2**

Name ____

Due: Beginning of Class Monday April 12, 2010.

Hand in hard copy. Staple all pages.

1. Evaluate each expression.

a. floor(x) $\neq$ ceiling(x) if and only if ________________________________

b. gcd(236, 112) = ____

c. (–12) mod 7 = ____

d. $\log_2(1024)$ = ____

e. map(gcd, dist(2, seq(4))) = __________________________________

f. If $f(x) = x$ mod 5, then map(f, seq(6)) = ________________________

2. Let $f : \mathbf{N}_9 \rightarrow \mathbf{N}_9$ be defined by $f(x) = 3x$ mod 9. Evaluate each expression.

a. $f^{-1}(\{0, 3\})$ = ____

b. $f(\{1, 2, 4\})$ = ____

c. range(f) = ____

3. Let $f(x) = 3x^2$ and let $g(x, y) = 2x + y$. Find an expression that uses f and g to represent the following expression.

$$6a^2 + 3b^4$$

4. Express each of the following function definitions as a **composition** of known functions from the set {seq, dist, pairs, map, +, –, *, cons, head, tail}.

a. $f(n, g) = \langle g(0), g(1), ..., g(n)\rangle$.

b. $f(n) = \langle \log_2(1), \log_2(2), ..., \log_2(\text{n} + 1)\rangle$.

5. Let $f : \mathbf{N} \rightarrow \mathbf{N}$ be defined by $f(x) = x \bmod 12$.

a. Show that f is not surjective.

b. Show that f is not injective.

6. Show that the set S of odd integers and the set $\mathbf{Z}$ of integers have the same cardinality. (i.e., find a bijection between the two sets.)

7. Let $f : \mathbf{N}_7 \rightarrow \mathbf{N}_7$ be defined by $f(x) = (4x + 3) \bmod 7$.

a. (Fill in the blank.) f is a bijection because gcd(___________) = 1.

b. Find a formula for f^{-1}, the inverse of f.

8. Let S = {one, two, three, four, five, six, seven, eight} and suppose that

$h: S \to \mathbf{N}_8$ is the hash function defined by

$h(x) = \text{length}(x) \bmod 8,$

where length(x) is the number of letters in x. Use h to place each element of S into the following hash table starting with one, then two, and so on until eight. Resolve collisions by **linear probing with a gap of 3.**

0	
1	
2	
3	
4	
5	
6	
7	