Name _____

Due: Beginning of Class Monday April 5, 2010.

Hand in hard copy. Staple all pages.

1. Let S be the statement,

      "If x + y is prime, then x is prime or y is prime."

 a. The contrapositive of S is _____

 b. The converse of S is _____

2. Write out a proof of the following statement about the integers using only the definitions of even and odd together with algebra.
             If x is even and y is odd, then $x + y$ is odd.

3. Evaluate each expression.

   a. power($\{\varnothing, a, b\}$) = _____

   b. $\{a, b, c, d\} - (\{a, b, c\} \cap \{b, c, d, e\})$ =_____

   c. $\{a, b, c\} - (\{a, b, c, d\} - \{c, d, e\})$ = _____

   d. $\{a, b, c\} \oplus \{a, b, c\}$ =_____

   e. (Let **N** be the universe). $\{1, 2, 3\}' - \{3, 4, 5\}'$ = _____

   f. $\{a, b\} \times \{1, 2, 3\}$ = _____

   g. head($\langle b, c, d \rangle$) =_____

   h. tail($\langle a, b, c, d, e, f \rangle$) = _____

   i. cons($b, \langle c, b, a \rangle$) = _____

4. For each integer n let $A_n = \{x \mid x \in \mathbf{N} \text{ and } x \leq n\}$. Evaluate each of the following expressions.

a. $A_8 \cap A_3 = $ _____

b. $A_8 \cup A_3 = $ _____

c. $A_8 - A_3 = $ _____

d. $A_3 - A_8 = $ _____

5. Given the following facts about three sets $A, B,$ and C.

$|A \cap (B \cup C)| = 100,$

$|A \cap B| = 70,$

$|A \cap C| = 80,$

Find $|A \cap B \cap C|$.

6. Solve the language equation for L.

$\{a, b, ba\}L = \{a, ba, b, ab, bb, bab\}.$

7. Let L and M be two languages and let $x \in L*(ML)$. Describe the general form of x by writing it as a concatenation of strings, where each string is in either L or M.

8. Calculate the number of strings over the alphabet $\{a, b, c, d\}$ that have length 7, begin with a, b, or c, and contain at least one d.

9. Draw a minimal spanning tree for the following weighted graph.