

Name _____

Due: Beginning of Class Monday May 10, 2010.*Hand in hard copy. Staple all pages.*

1. For any nonempty set S let $f : S \times \text{lists}(S) \rightarrow \mathbf{N}$ be defined as follows:

$$f(x, L) = \begin{array}{l} \text{if } L = \langle \rangle \text{ then } 0 \\ \text{else if } x = \text{head}(L) \text{ then } 1 + f(x, \text{tail}(L)) \\ \text{else } f(x, \text{tail}(L)). \end{array}$$

Let $P(L) = "f(x, L) \text{ is the number of occurrences of } x \text{ in } L."$ Write out an induction proof that $P(L)$ is true for all lists L .

2. Find the unknown quantity for each of the following problems.

a. A solution to a problem has 63 possible outcomes. An algorithm to solve the problem has a ternary decision tree of depth d . What is the smallest value that d could be?

b. A solution to a problem has x possible outcomes. An algorithm to solve the problem has a binary decision tree of depth 5. What can the value of x be?

c. A solution to a problem has 100 possible outcomes. An algorithm to solve the problem has an n -way decision tree of depth 4. What is the smallest value that n could be?

3. Use known closed forms and summation facts to find a closed form for each of the following expressions:

a. $\sum_{i=0}^n (3i + 2)$

b. $2 \sum_{i=1}^n 3^{i+1}$

4. Find an expression for the number of times, in terms of the natural number n , that S is executed in the following algorithm.

```
 $i := 0;$   
while  $i < n$  do  
   $j := i;$   
  while  $j < n$  do  
     $S;$   
     $j := j + 1;$   
  endWhile  
   $i := i + 1;$   
endWhile
```

5. For each of the following problems, find an expression to represent the answer (DO NOT EVALUATE IT).

- a. How many ways can 7 people be arranged in a row?
- b. How many ways can 5 people be arranged in a row when the people are chosen from a set of 12 people?
- c. How many different sets of 5 cans of soda that can be gotten from a machine that dispenses 4 kinds of soda?
- d. How many strings of length 12 over $\{a, b, c, d\}$ contain 2 a 's, 2 b 's, 5 c 's, and 3 d 's?

6. Given the following procedure P defined for all natural numbers n .

```
 $P(n)$ : if  $n = 0$  then  
     $C(0)$   
else  
     $C(n)$ ;  
     $P(n - 1)$ ;  
endIf
```

Let a_n denote the number of times that a certain operation is executed during the execution of $P(n)$. Suppose that $C(n)$ executes the operation $2n$ times. Write down a recurrence to describe a_n . Do not solve it.

7. Given the following recurrence.

$$a_0 = 1$$
$$a_n = 3a_{n-1} + 4n.$$

Solve the recurrence for a_n by cancellation or substitution. Leave the answer in summation form. You do not have to find a closed form for the answer.