CS340: Discrete Structures **Homework #4**

Name ____

Due: Beginning of Class Monday May 3, 2010.

Hand in hard copy. Staple all pages.

1. Find the partitioning induced by the following equivalence relation over the set **N**.

$$a \sim b \quad \text{iff} \quad a \bmod 4 = b \bmod 4.$$

2. Let $x \sim y$ iff x and y are nonempty lists over $\{a, b\}$ with the same tail.

a. The relation ~ is an equivalence relation because it is the kernel relation

of ____

b. List the elements in each of the following equivalence classes.

$[\langle a \rangle] =$ ____

$[\langle a, b \rangle] =$ ____

$[\langle a, a, b \rangle] =$ ____

3. Let $f : \mathbf{N} \to \mathbf{N}$ be defined by $f(n) = \lceil (n/4) \rceil$. Describe the partition on **N** induced by the kernel relation on f.

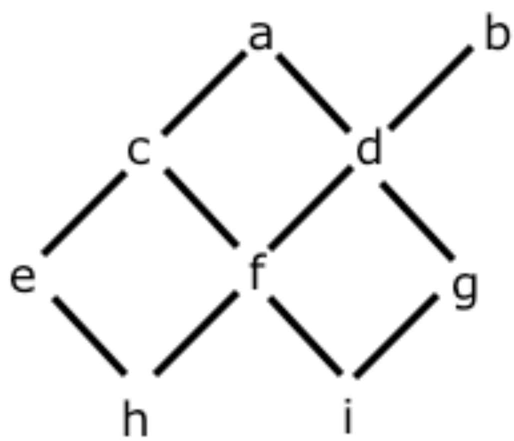
4. Consider a graph with this vertex set {a, b, c, d}. The graph has 5 edges which, when sorted by weight, are as follows:

{a, b}, {a, c}, {b, c}, {b, d}, {a, d}.

Use Kruskal's algorithm to find a minimal spanning tree T by showing the value of T and the corresponding equivalence classes at each step of the algorithm.

5. Let D = {2, 3, 6, 12, 24, 36} and for any x, y ∈ D let x < y mean x | y (i.e., x divides y). Draw the poset diagram for the partial order on D.

6. Given the following poset diagram for the set {A, B, C, D, E, F, G, H, I}.



Find each of the following items, where S = {C, D, F}.

a. The minimal elements of S: ________________________________

b. The maximal elements of S: ________________________________

c. The lower bounds of S: ________________________________

d. The upper bounds of S: ________________________________

e. The least upper bound of S: ________________________________

f. The greatest lower bound of S: ______________________________

7. Given the poset $\langle \mathbf{N} \times \mathbf{N}, < \rangle$, where $(a, b) < (c, d)$ means $a + b < c + d$. Write down a descending chain of maximum length that starts with (3, 2).

8. Write an inductive proof that the following statement is true for all natural numbers n.

$$2 + 6 + 10 + \ldots + (4n - 2) = 2n^2.$$

9. Write out an inductive proof of the following equation for all $n \in \mathrm{N}$.

$$3 + 5 + 7 + \ldots + (2n + 3) = (n + 1)(n + 3).$$