Name ____

Due: Beginning of Class Monday April 19, 2010.

Hand in hard copy. Staple all pages.

1. Write **countable** or **uncountable** to indicate the cardinality of each set.

a. Rational numbers ____

b. Positive real numbers ____

c. $\mathbf{N} \cup \mathbf{N}$ ____

d. power($\mathbf{N}$) ____

e. $\mathbf{N} \times \mathbf{N} \times \mathbf{N}$ ____

f. $\{a, b\}^*$ ____

2. Write an inductive definition for each set.

a. $S = \{a\}^* \times \{b\}^*$. Assume that the basis case is: $(\Lambda, \Lambda) \in S$.

b. $S = \{\langle 1\rangle, \langle 3, 1\rangle, \langle 5, 3, 1\rangle, \langle 7, 5, 3, 1\rangle, \ldots\}$.

3. Show each step in the calculation of $f(47)$, where f is defined by

$f(0) = 0$

$f(n) = f(\text{floor}(n/3)) + n$

4. Write a recursive definition for the following function.

$f(n) = 4 + 6 + \ldots + (2n + 4)$, where $n \in \mathbf{N}$.

6. Write a recursive definition for the procedure *leaves*, where for a binary tree T, let *leaves*(T) be a procedure to print out the leaves of T as they occur from left to right.

7. For each of the following relations, write down the properties that the relation satisfies from the list: *reflexive, symmetric, transitive, irreflexive, antisymmetric*.

a. isParentOf, over the set of people.

b. $\neq$, over the set **N** of natural numbers.

c. isSubsetOf, over a collection of sets.

8. Given the following binary relations over $\{a, b, c, d\}$.

$R = \{(a, b), (b, c), (c, c), (d, c)\}$

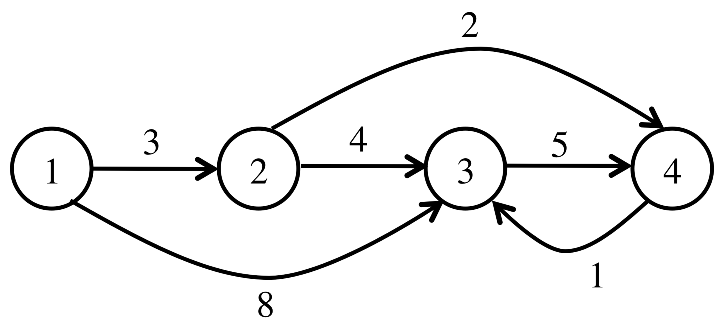
$S = \{(b, a), (c, b), (c, d)\}$

a. Find $R \circ S$

b. Find $S \circ R$

9. Find the transitive closure of $R = \{(1, 2), (3, 1), (3, 2), (2, 4)\}$.

10. Given the following weighted graph.



a. Draw a matrix that can be used to look up the length of shortest paths between any two points.

b. Draw a path matrix that can be used to compute the shortest path between any two points.