Optimization

Louden: Finish Textbook (Chapters 1-8)

Basic Code Generation
Produces functional but poor code.

Goal: Improve the code as much as possible.
Dramatically improves code performance (e.g., 2X to 10X)

"*Optimization*" -- more likely "*Improvement*"

Machine-Independent v. **Machine-Dependent Optimizations**

Variety of techniques
Add as many optimization algorithms as possible
Some are VERY complex!
Do testing w/ sample programs to evaluate
which optimization strategies work best.

Different needs for different languages (FORTRAN)

Requirement: Correctness

Every optimization must be "safe"
Must not change the program output ... for any input.
Must not allow new errors or exceptions.

Goals of optimization:
- **Runtime Execution Speed!!!**
- **Other (e.g., Code Size, Power Consumption)**

Every optimization should improve the program
but may slow some programs!

Is optimization worth the effort?
Some algorithms may be difficult to implement.
Many programs run only once
Compiler used heavily during debugging.
Program is only run once or twice before being modified.
⇒ Compiler performance matters more.
But some programs are *computation-intensive*
More computation per time unit means more accurate results

The secret to getting programs to run faster?

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Use a better algorithm!

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Use a better algorithm!

Most optimizations done by a compiler are "*constant-factor*" speed-ups

(e.g., 25% faster)

Optimizations by the programmer:

- **Change the algorithm**
  $N^2 \rightarrow N \log N$
- **Profile the program and tweak the algorithm**
- **Misc. transformations**

Machine Independent Optimizations

- **Live Variable Analysis**
- **Common sub-expressions**
- **Eliminate unnecessary copying**
- **Loop transformations**

...etc...

Optimization transforms IR Code

Machine Dependent Optimizations

- **Effective Register Usage**
- **Select Best Target Instructions**
- **Select a schedule that executes quickly**
  ... given the CPU idiosynchracies
  (e.g., memory latencies, functional units, etc.)

Optimization transforms Target Code

Does the programmer trust the compiler to emit efficient code?

No:

Programmer will *mangle* the program to achieve greater efficiency.

Yes:

Programmer will concentrate on writing
- Clean, simple code
- Correct code
- Code that is easy to maintain

Is Optimization Necessary...

...assuming the programmer writes good, efficient code?

Source Code:

A[i] := B[i] + C[i];

Translation:

```
t1  := i * 4
t2  := B[t1]
t3  := i * 4
t4  := C[t3]
t5  := t2 + t4
t6  := i * 4
A[t6]  := t5
```

The compiler will insert many hidden operations
(often concerning pointers and address calculations)

"Local Transformations"
Within a single basic block

"Global Transformations"
Concern several basic blocks
(but typically within a single routine / control flow graph)

Local Common Sub-Expression Elimination

B_5
```
t6  := 4*i
x  := A[t6]
t7  := 4*i
t8  := 4*j
t9  := A[t8]
A[t7]  := t9
t10  := 4*j
A[t10]  := x
goto B2
```

→

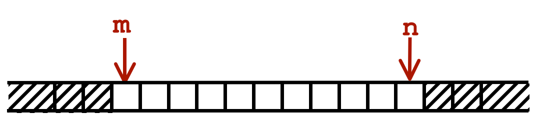
B_5
```
t6  := 4*i
x  := A[t6]

t8  := 4*j
t9  := A[t8]
A[t6]  := t9

A[t8]  := x
goto B2
```



```
procedure quicksort (m,n: int) is
   var i,j,v,x: int := 0;
   if (n ≤ m) then return; end;
   i := m - 1;
   j := n;
   v := A[n];
   while true do
      repeat
         i := i + 1;
      until A[i] ≥ v;
      repeat
         j := j - 1;
      until A[j] ≤ v;
      if i ≥ j then exit; end;
      x := A[i];
      A[i] := A[j];
      A[j] := x;
   end;
   x := A[i];
   A[i] := A[n];
   A[n] := x;
   quicksort (m,j);
   quicksort(i+1,n);
endProc;
```



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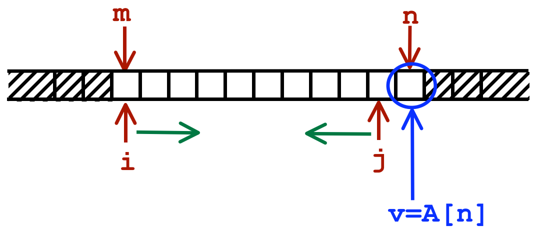


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      until A[i] ≥ v;
      repeat
         j := j - 1;
      until A[j] ≤ v;
      if i ≥ j then exit; end;
      x := A[i];
      A[i] := A[j];     Swap A[i] and A[j]
      A[j] := x;
   end;
   x := A[i];
   A[i] := A[n];
   A[n] := x;
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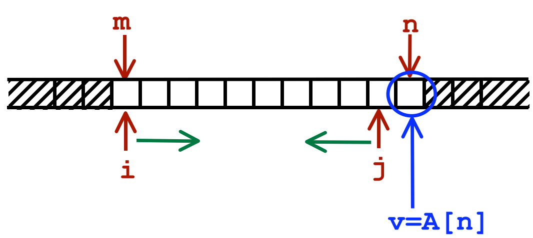

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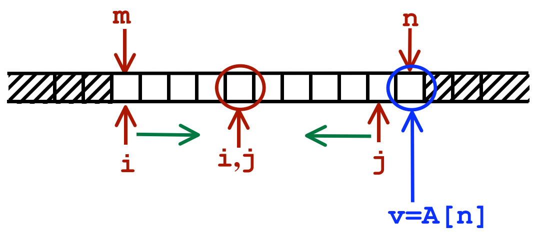
Swap A[i] and A[j]

Put "v" in the middle

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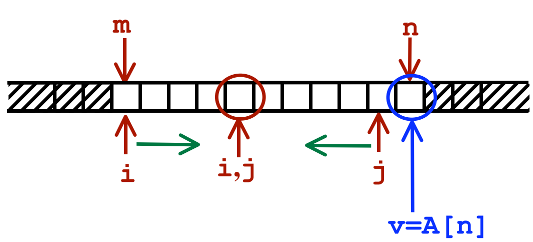

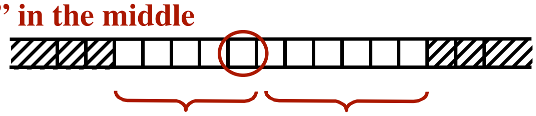

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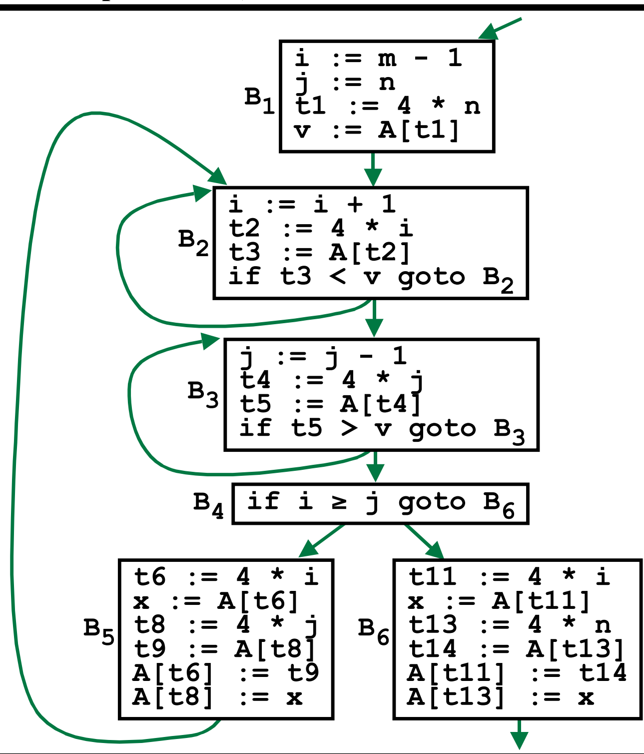


"Copy Propagation"

A "copy"

Any statement that uses the value of "x"

```
. . .
x := y
. . .
. . .
z := b + x
. . .
```


"Copy Propagation"

A "copy"

Any statement that uses the value of "x"

```
. . .                 . . .
x := y                x := y
. . .         →       . . .
. . .                 . . .
z := b + x            z := b + y
. . .                 . . .
```

Why perform this optimization?

“Copy Propagation”

A “copy”

Any statement that uses the value of “x’

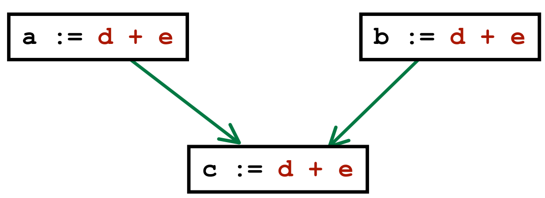
```
 . . .                      . . .
x := y          →          x := y
 . . .                      . . .
 . . .                      . . .
z := b + x                 z := b + y
 . . .                      . . .
```

Why perform this optimization?

The copy may become DEAD CODE.

We may delete the copy later!

Global Common Sub-expression Elimination



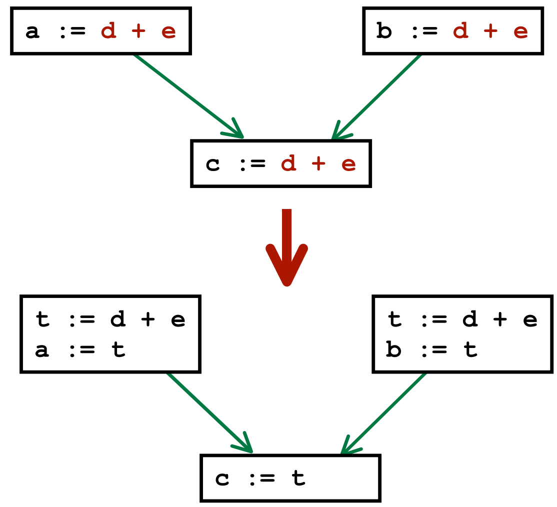
An expression...

Simple computation

Computed in several places

Global Common Sub-expression Elimination

An expression...
- Simple computation
- Computed in several places



Copies will be introduced during sub-expression elimination

Global Common Sub-expression Elimination

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- Simple computation
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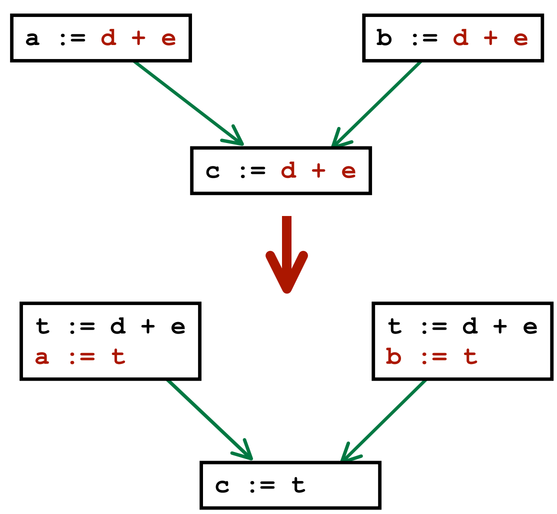
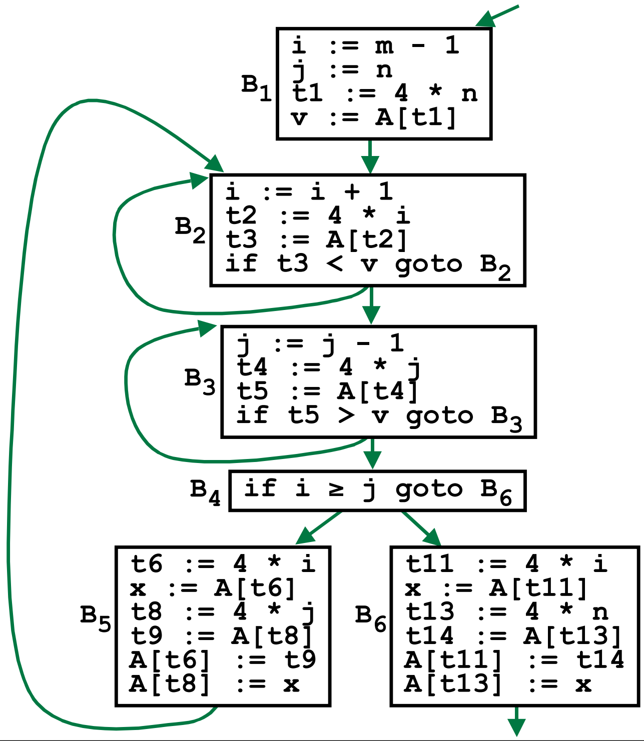


Copies will be introduced during sub-expression elimination

...but they may be DEAD CODE!



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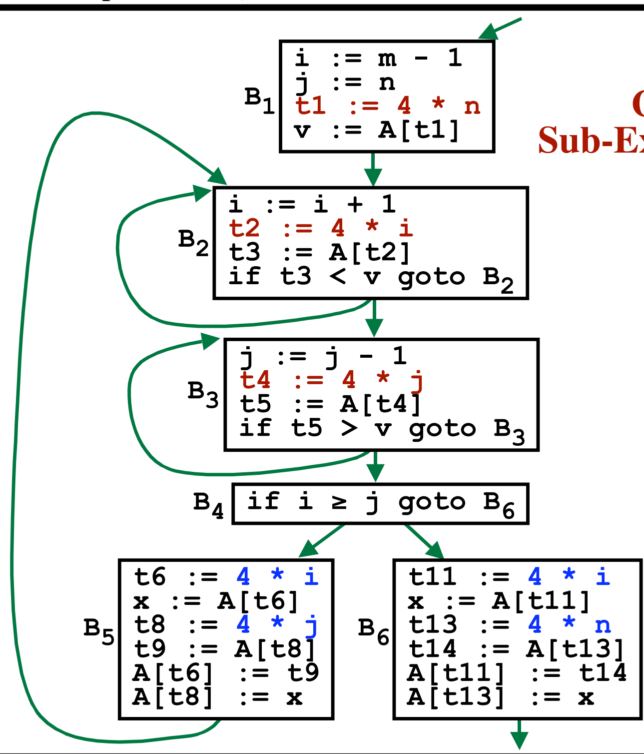

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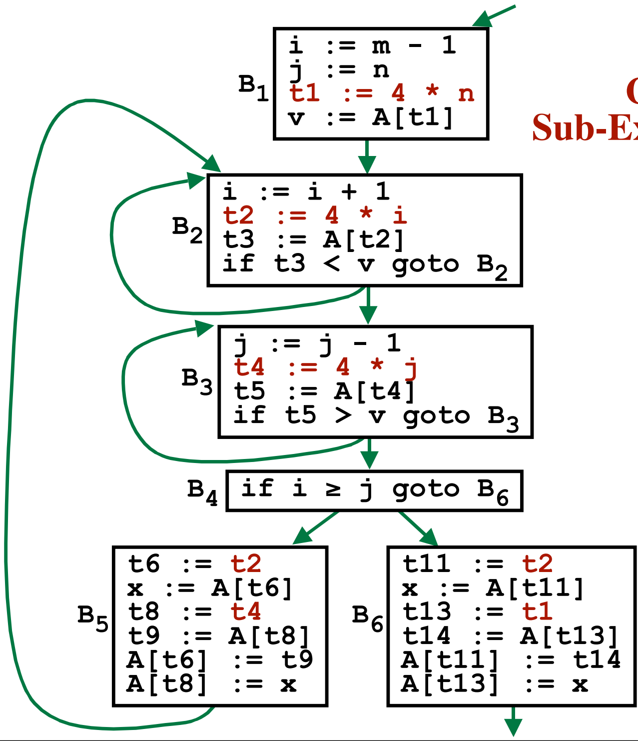
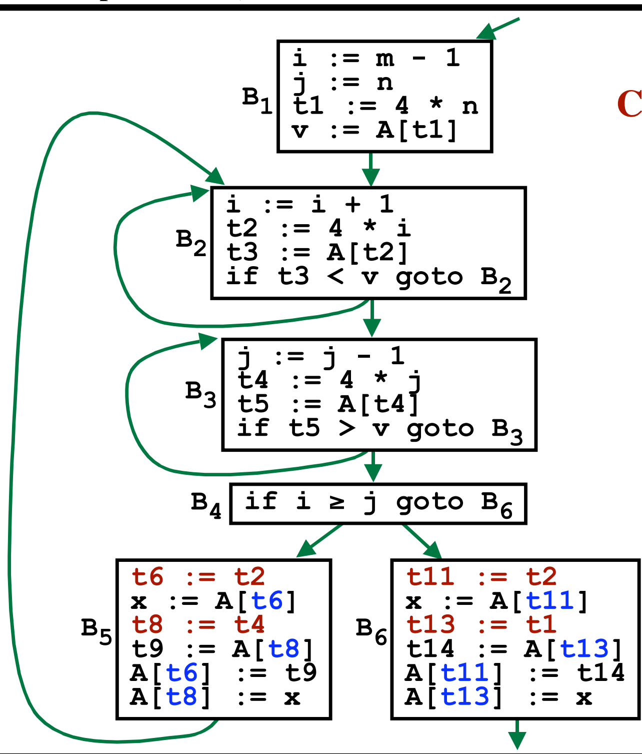


Copy Propagation



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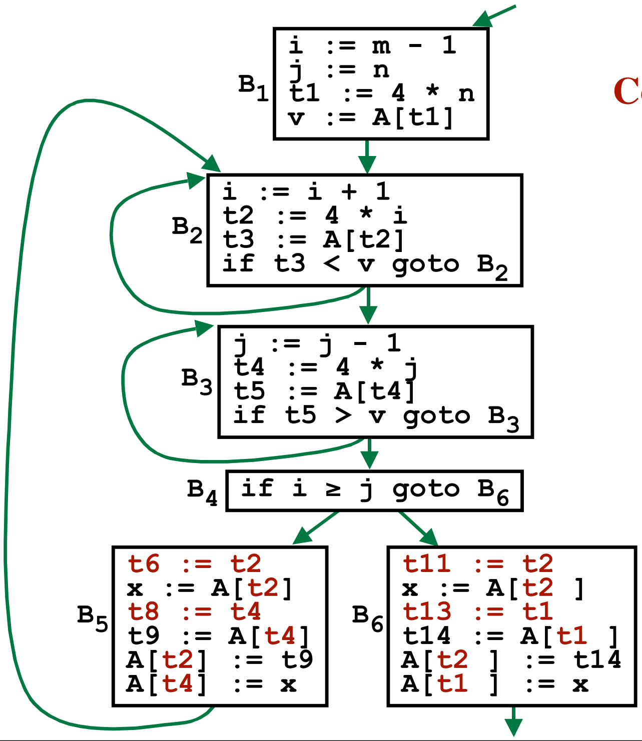

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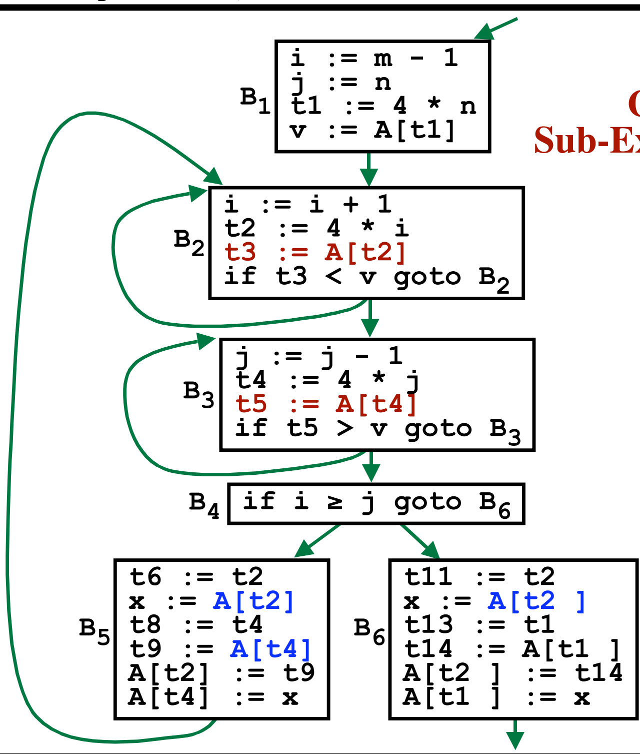

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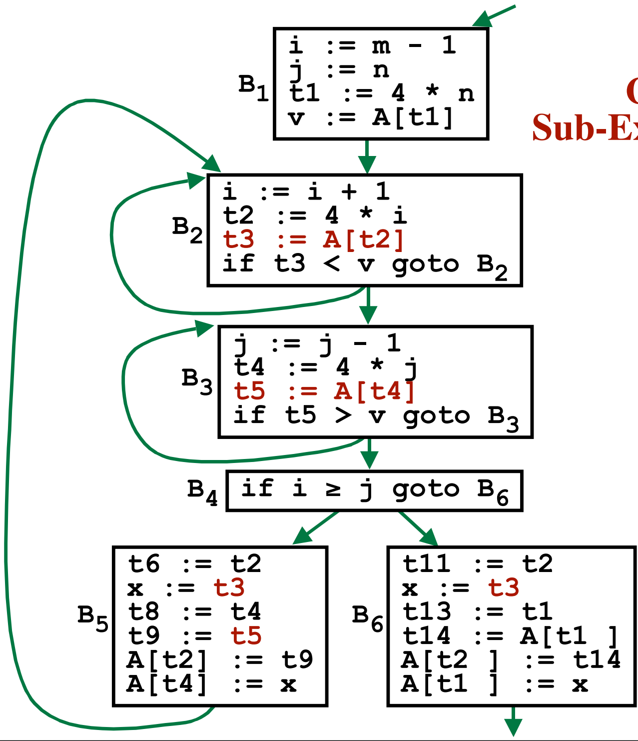

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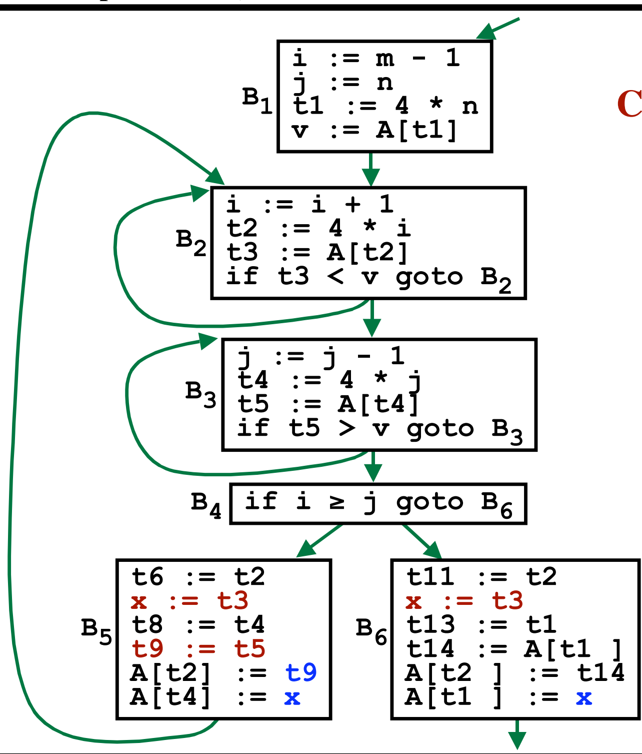

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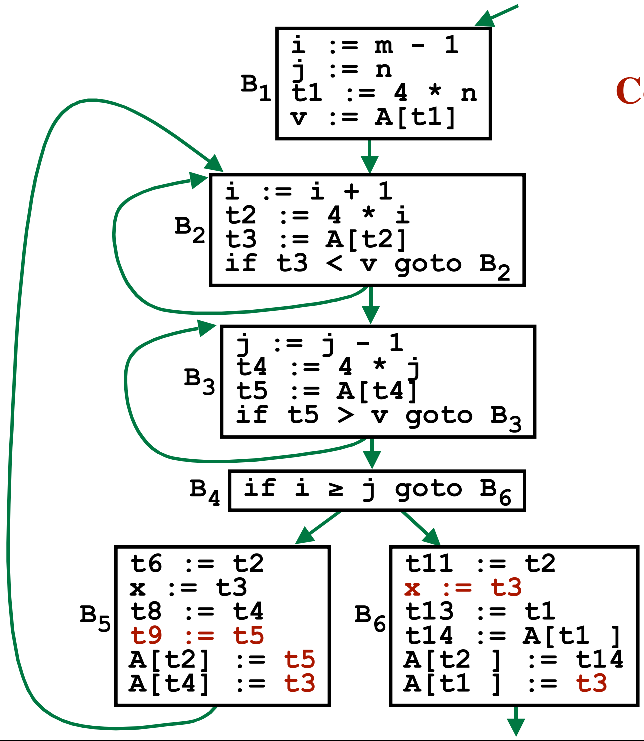
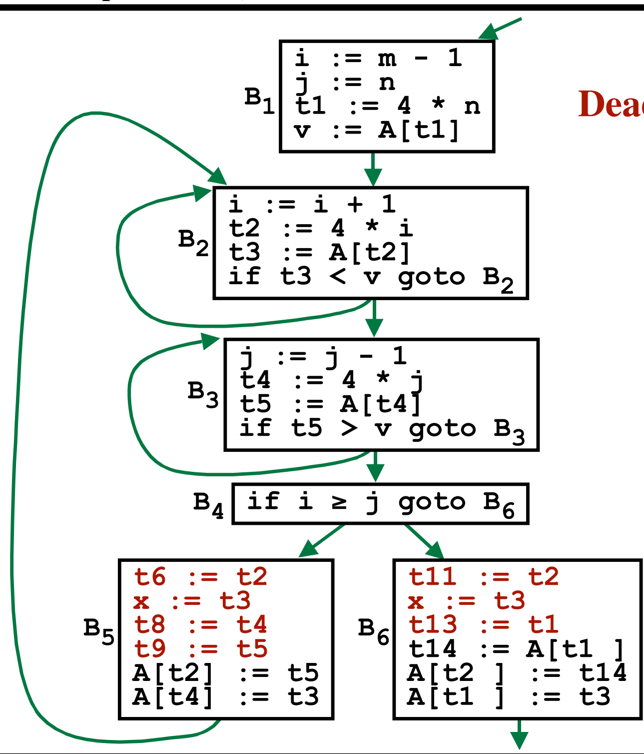
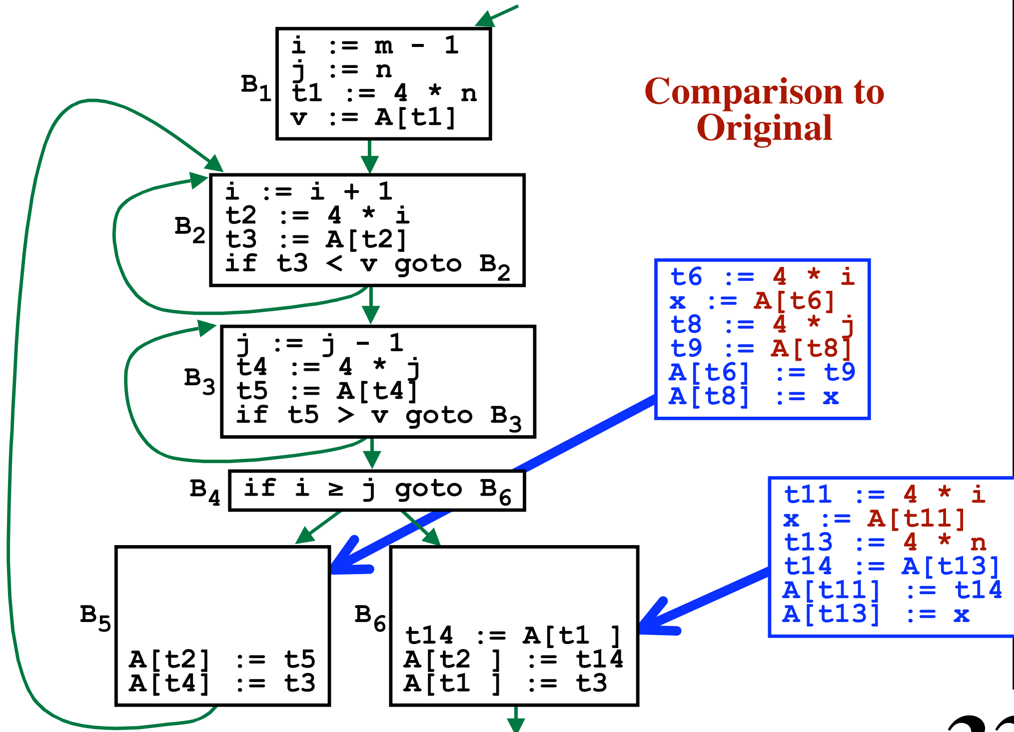


Dead Code Elimination



Comparison to Original



Consider This Example

```
debug := FALSE;
...
if (debug) then print (...) endif
```

Unreachable? Can this code be eliminated?

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debug := FALSE;
...
if (debug) then print (...) endif
```

Unreachable? Can this code be eliminated?

Data Flow Analysis

"Which computations can reach which points"

Only one DEFINITION of "debug" can reach this USE.

Must have the value "false", so okay to optimize the IF statement.

Global Common Sub-Expression Elimination.

Copy Propagation.

Live-Variable Anaylsis.

Constant Folding

"If we know the value of a variable at compile-time, we may go ahead and perform the computation."

Dead-Code Elimination

"Eliminate code that is unreachable."

"Eliminate code that compute DEAD variables."

Optimizing Loops

The 90-10 Rule

"90% of execution time is spent in 10% of the code."

Try to move code out of loops

Identify Loops

Nesting of loops

"inner loops"

"outer loops"

GOAL:

Move Code "Outward"

Make Loops Run Faster

Loop-Invariant Computations

```
while i <= MAX-1 do
       ...
       j := i * (MIN+1);
       ...
end;
```

Assume MAX and MIN are not altered in the loop

If an expression is computed within a loop
and
It does not depend on variables that change in the loop

CS-322 Optimization, Part 1

Loop-Invariant Computations

```
while i <= MAX-1 do
       ...
       j := i * (MIN+1);
       ...
end;
```

These computations are "loop-invariant"

If an expression is computed within a loop
and
It does not depend on variables that change in the loop
then
Move it to just before the loop!

Loop-Invariant Computations

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These computations are "loop-invariant"

If an expression is computed within a loop

and

It does not depend on variables that change in the loop

then

Move it to just before the loop!

```
t1 := MAX-1;
t2 := MIN-1;
while i <= t1 do
        ...
        j := i * t2;
        ...
end;
```


Induction Variables

Definition:

Loop counters that move together (in "lock-step") during loop execution.

Example Source:

```
loop
  ...
  i := i + 1;
  ... A[i]...
  ...
endloop
```

Translation to IR:

```
...
i := i + 1
t := i * 4
... A[t]...
...
```

Relationship:

t = i*4

Note:

```
t := i*4
```

Establishes the relationship.

Also: We know this relationship exists directly after

```
t := i*4
```

is executed.

i	t
1	4
2	8
3	12
4	16
5	20
⋮	

Some Reasoning

Assume t = i*4 here.

```
•
•
•
i  := i + 1
t  := i * 4
•
•
•
```


Some Reasoning

Assume t = i*4 here.

Then, after "i" is incremented, the relationship will be
t = (i-1) * 4

```
•
•
•
i  := i + 1
t  := i * 4
•
•
•
```

Some Reasoning

Assume t = i*4 here.

Then, after "i" is incremented, the relationship will be
    t = (i-1) * 4
Rewriting:
    t = i*4 - 4
Or:
    i = (t + 4) / 4

```
   :
i  := i + 1
t  := i * 4
   :
```


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Then, after "i" is incremented, the relationship will be
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Use this value of "i" to compute the new "t" as a function of the old "t".

```
t  := i  * 4
t  := [ (t + 4) / 4 ] * 4
```

```
   :
i  := i + 1
t  := i * 4
   :
```

Some Reasoning

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Rewriting:

```
t := t + 4
```

```
  :
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  :
```


CS-322 Optimization, Part 1

Some Reasoning

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Rewriting:

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t := t + 4
```

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i := i + 1
t := i * 4
  :
```


Conclusion:

It is okay to replace `t := i * 4`

by: `t := t + 4`

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```

```
  :
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  :
```
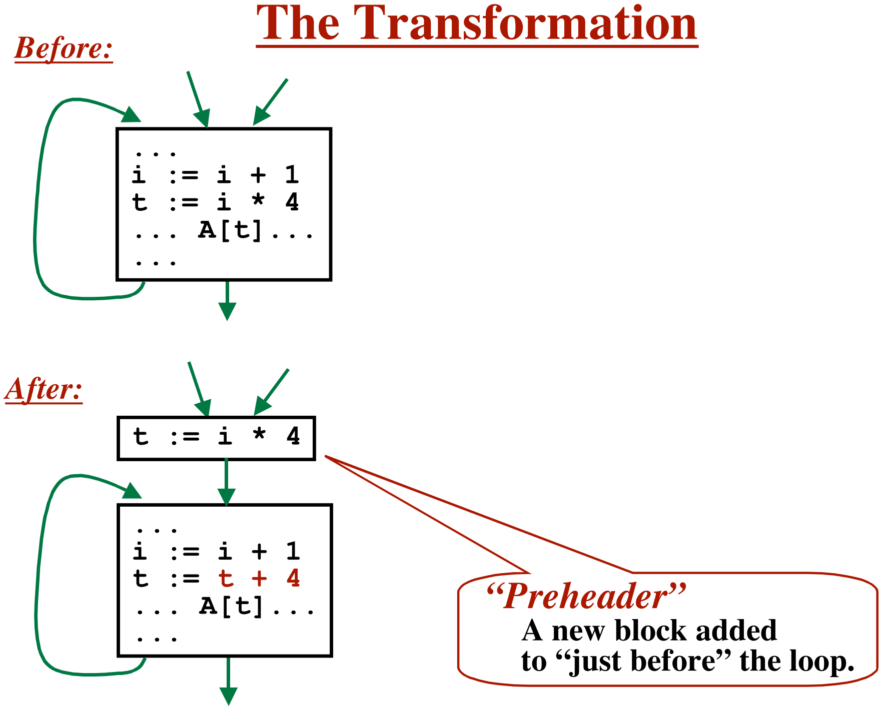

Conclusion:

It is okay to replace `t := i * 4`
by: `t := t + 4`

But don’t forget to establish t = i*4 *before the loop begins!*

The Transformation



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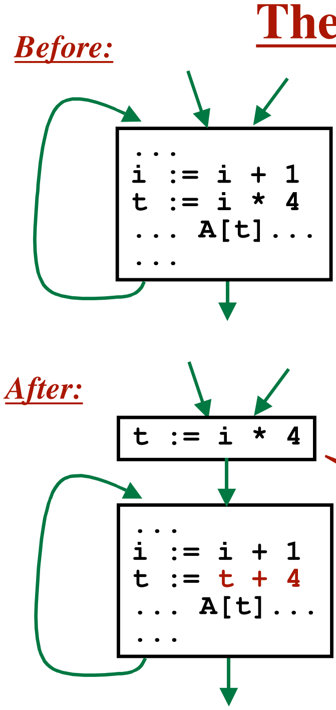
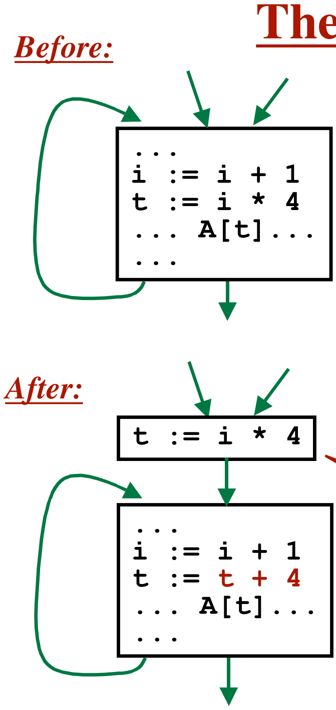


Must also check:

"t" and "i" are not changed elsewhere in the loop

"Preheader"
A new block added to "just before" the loop.

The Transformation



Must also check:

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Benefit:

The definition of i may become DEAD.

... *Eliminate it altogether!*

"Preheader"
A new block added to "just before" the loop.

Definitions

"*Reduction in Strength*"

A costly operation is replaced by a cheaper operation.

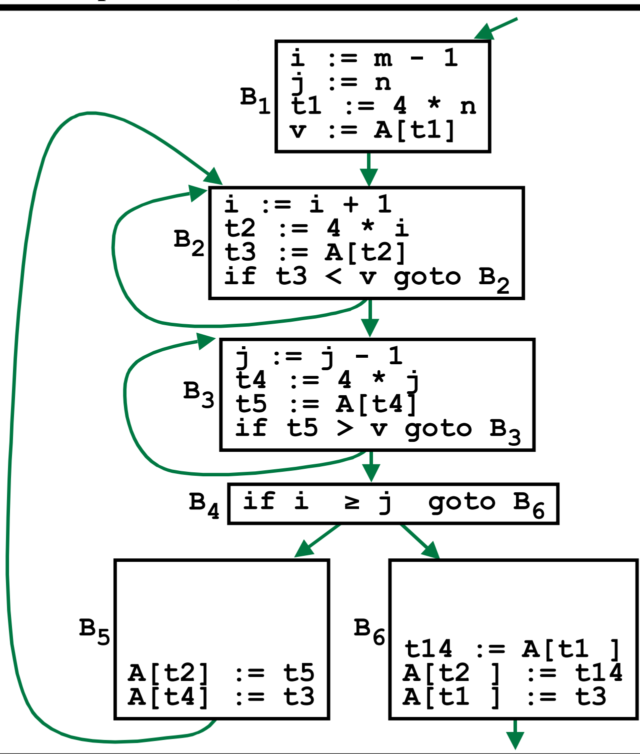
```
t := i * 4
    ↓
t := t + 4
```

"*Constant Folding*"

If all operands to an operator are constants...
evaluate the operator at compile-time.

```
t := 100 * 4
    ↓
t := 400
```


Our Example, so far...



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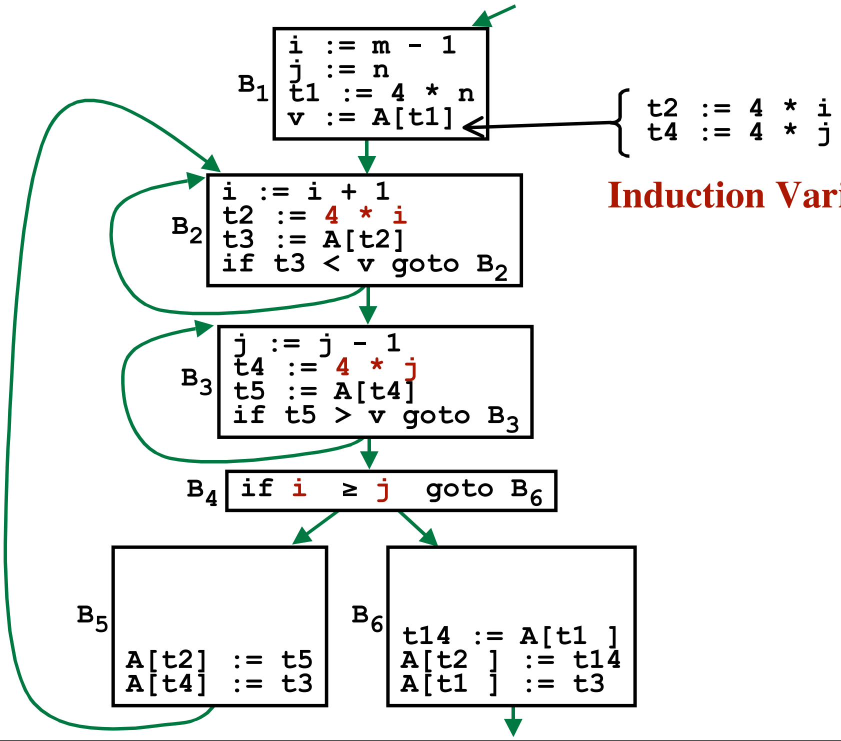

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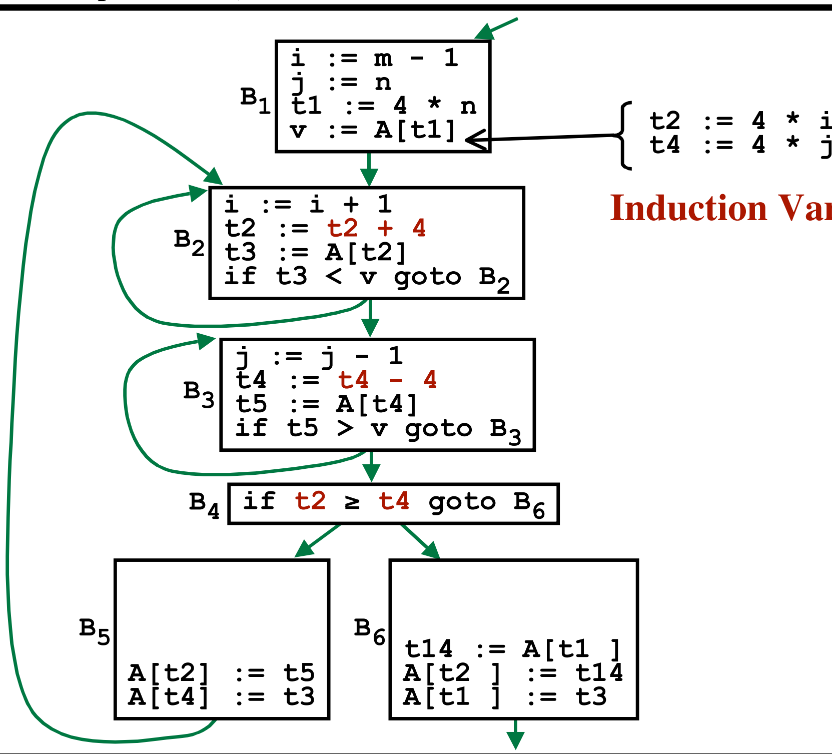
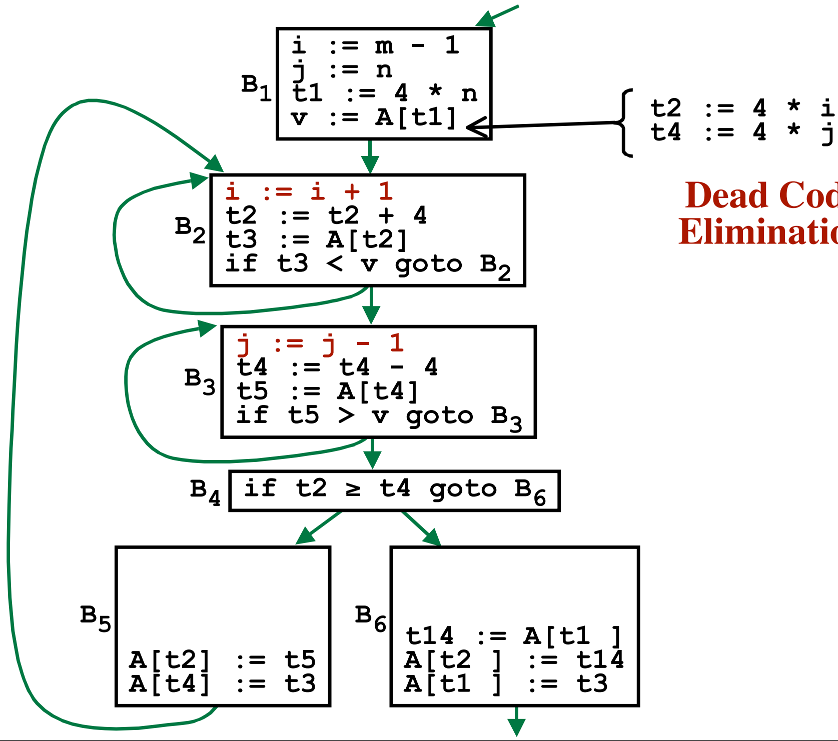


Dead Code Elimination



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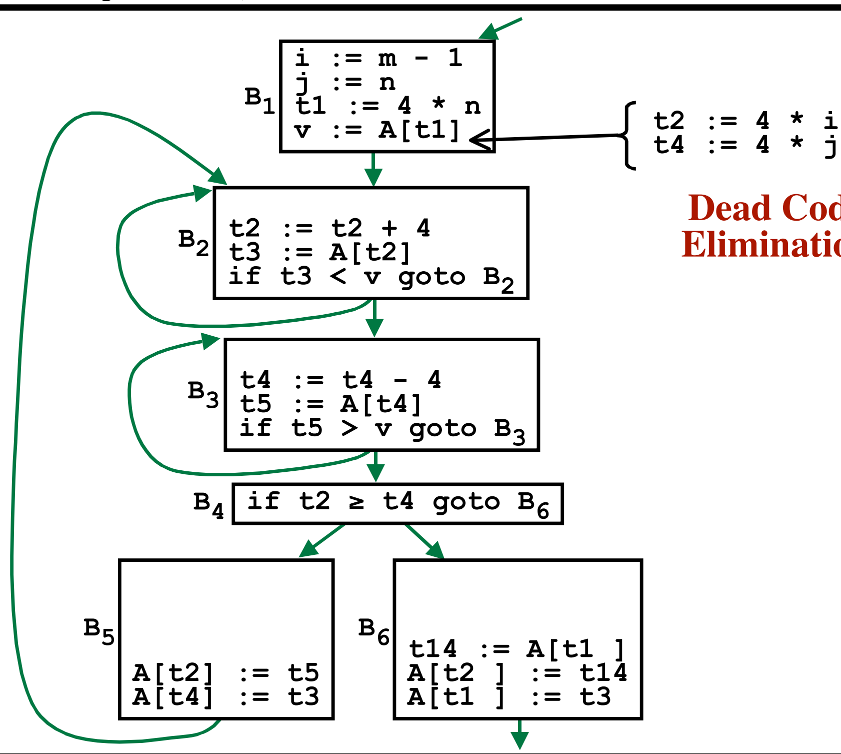
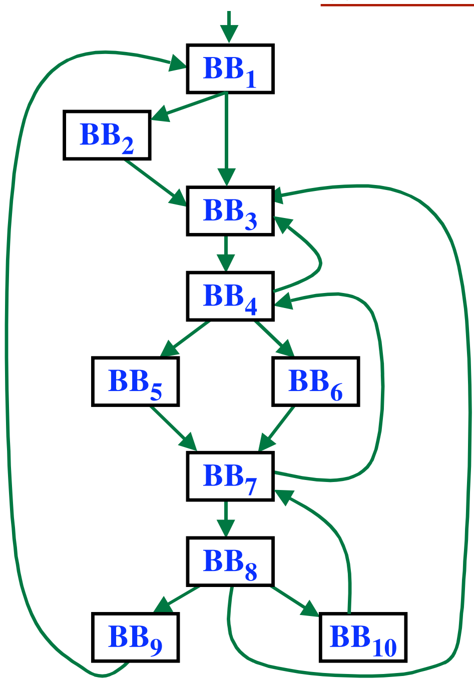


Where are the Loops?



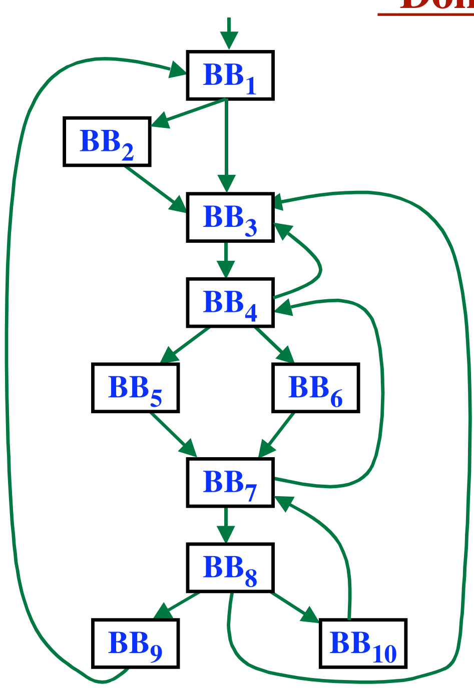
Given:

A Control Flow Graph
(Each node is a Basic Block)

Goal:

Locate the loops.

"Dominates"



Definition:

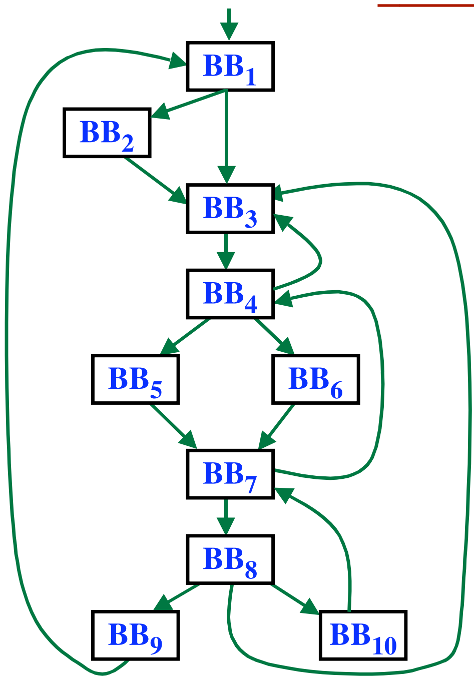
A relation between nodes...

> Node D "dominates" node N
> D dom N
> If every path from the initial node to N must go through D.

The entry node in a loop will dominate all nodes in the loop.

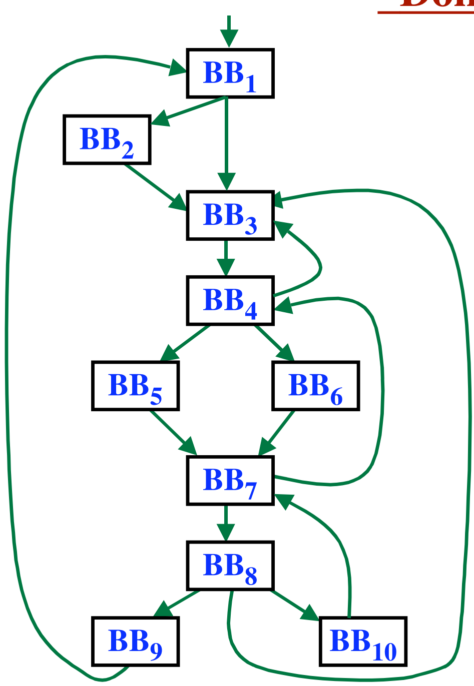
Every node dominates itself.

“Dominates”



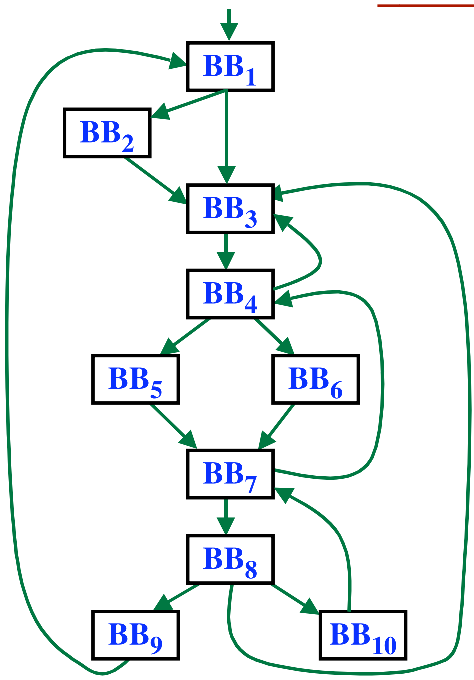
Node	Dominates
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

“Dominates”



Node	Dominates
1	All
2	
3	
4	
5	
6	
7	
8	
9	
10	

"Dominates"



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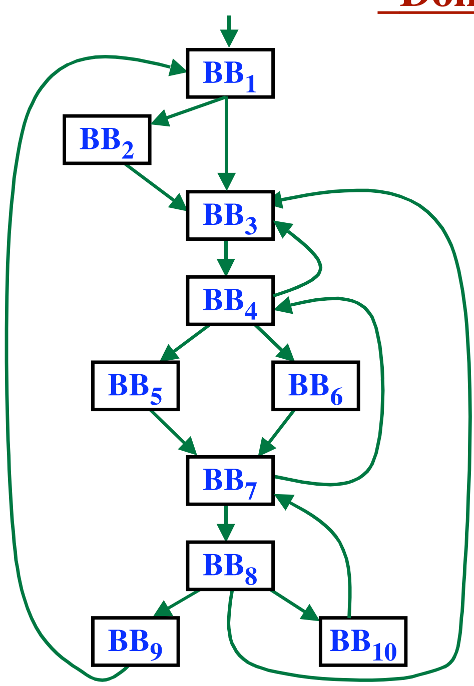
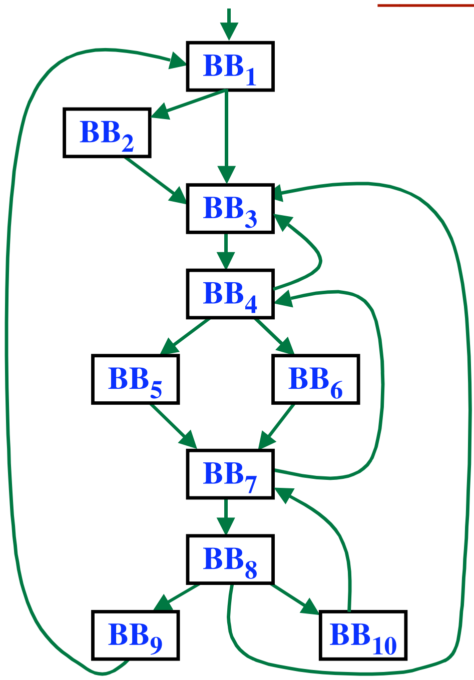


Node	Dominates
1	All
2	2
3	3 - 10
4	
5	
6	
7	
8	
9	
10	

"Dominates"



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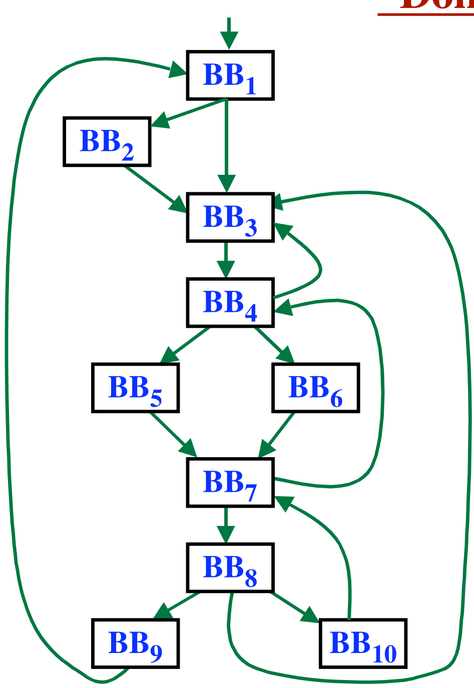
CS-322 Optimization, Part 1

"Dominates"



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10	

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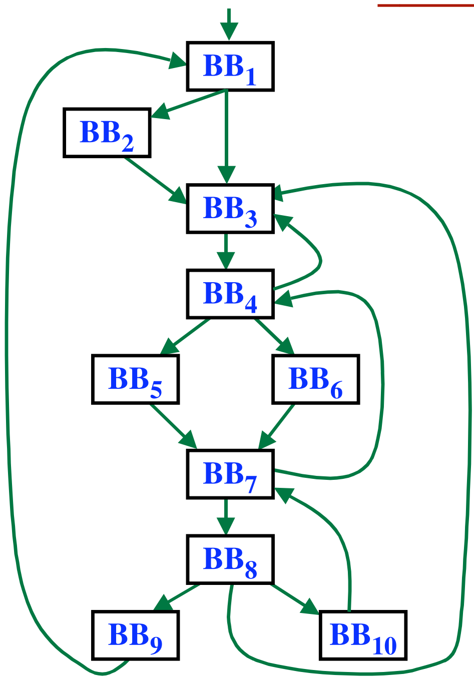
"Dominates"



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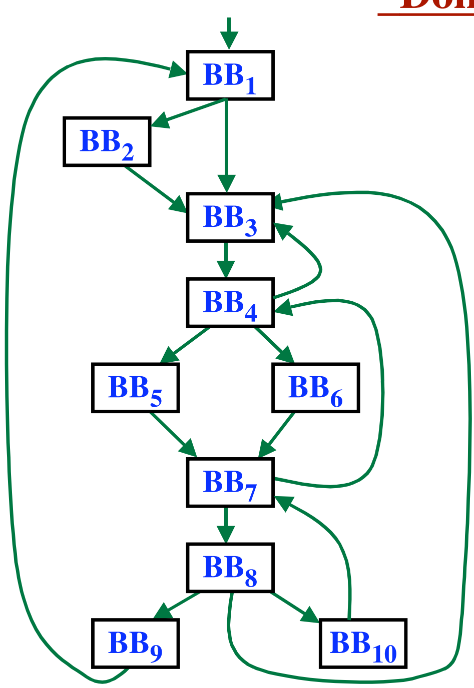
"Dominates"



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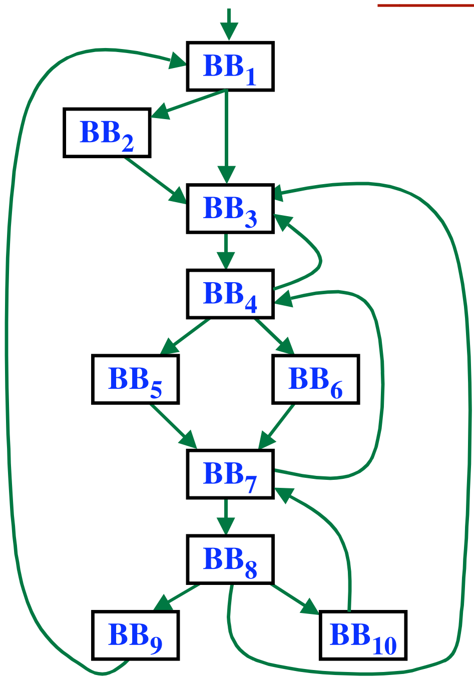

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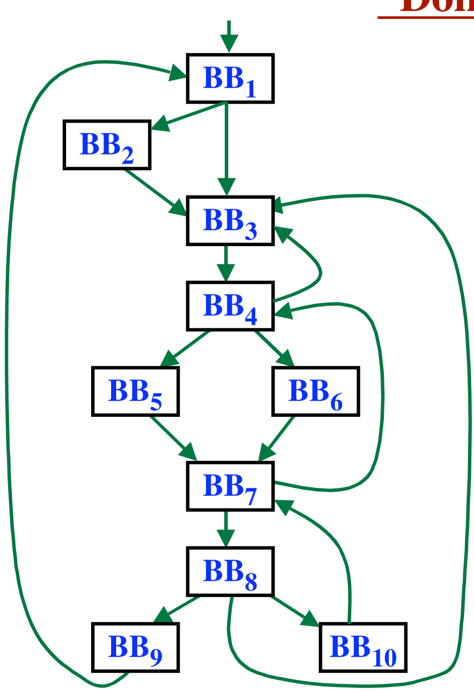

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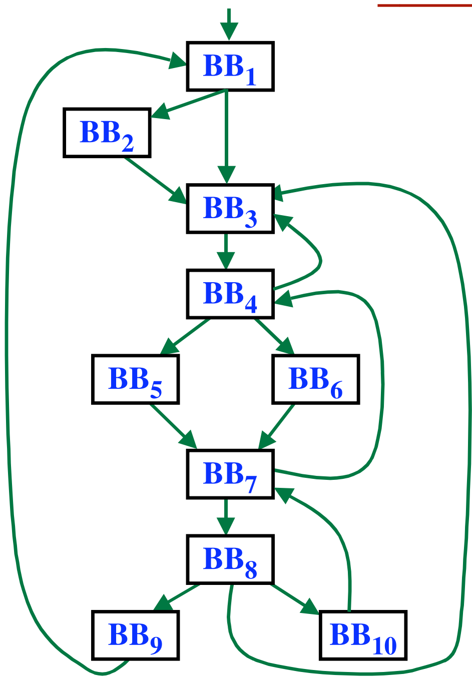

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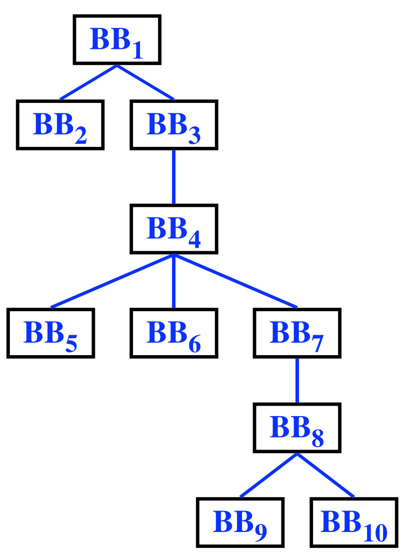
Dominator Tree



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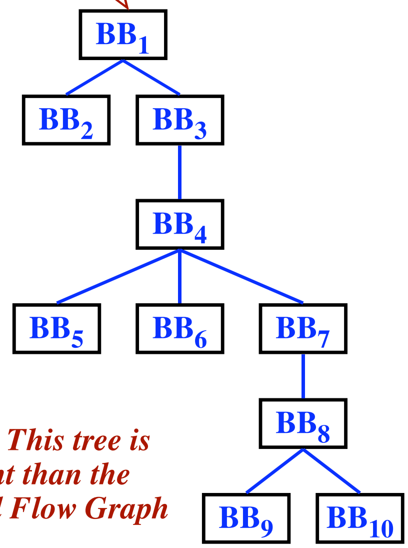
Dominator Tree

Initial node will be the "root" of the dominator tree



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NOTE: This tree is different than the Control Flow Graph

The Definition of "Natural Loops"

What is a loop anyway?

- Must have a single entry point

    The Header Node

    (The Header dominates all nodes in the loop.)
- Must be a path back to the header.

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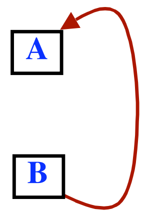

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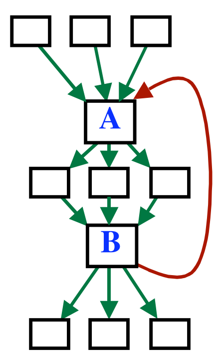

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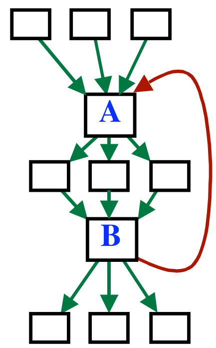

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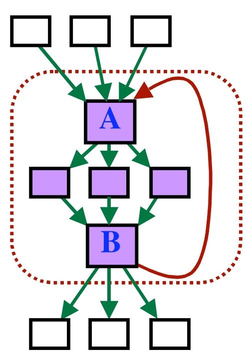

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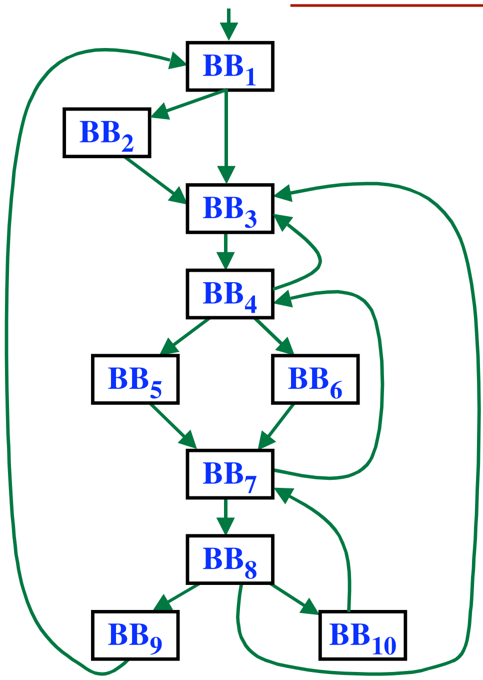

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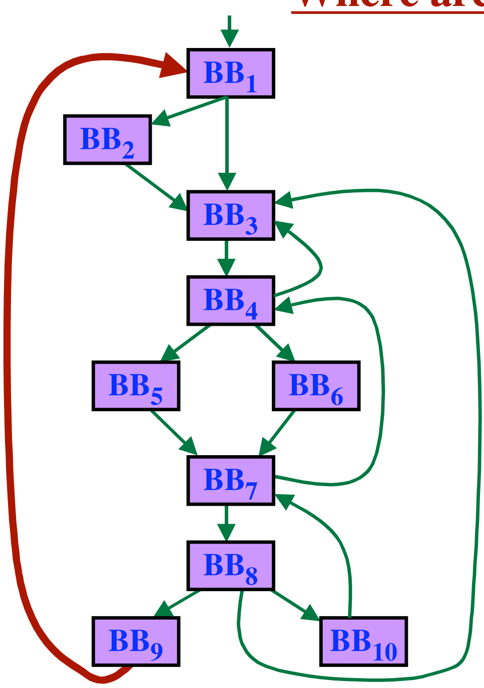

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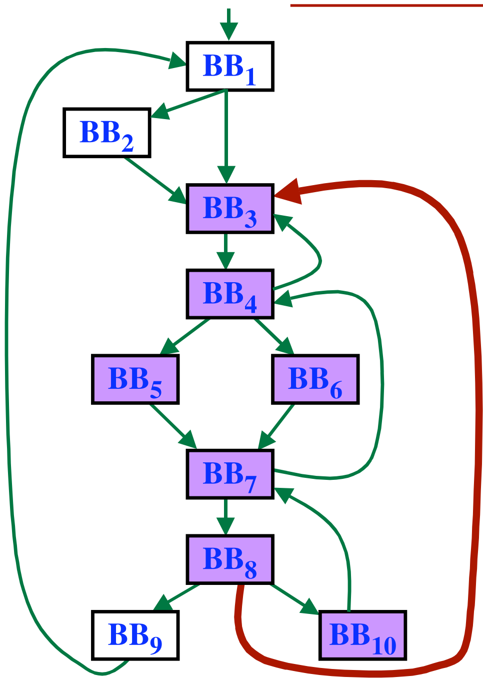

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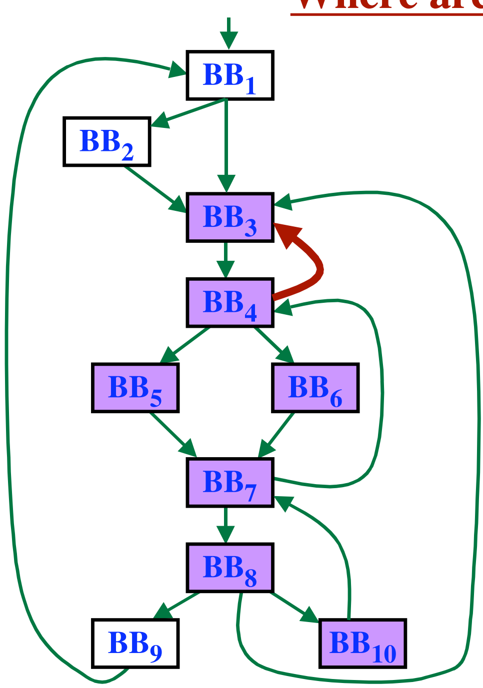

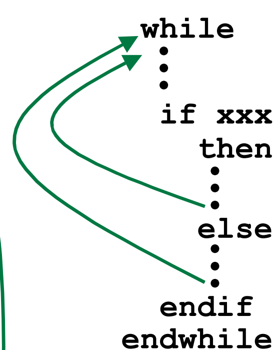

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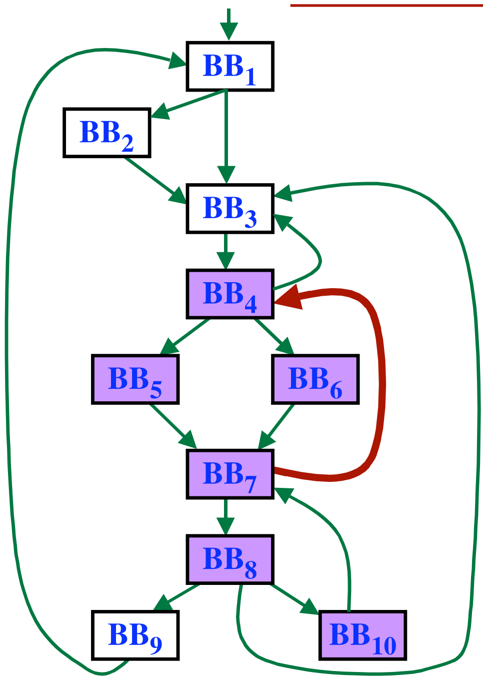

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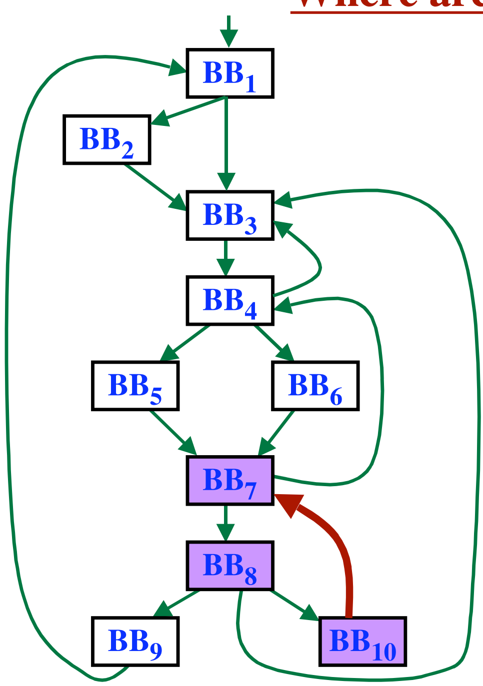


An Algorithm to Find a Natural Loop

Input: A Control Flow Graph
A Back-Edge, B→A

Output: Result = Set of nodes in the natural loop

```
Stack := empty
ResultSet := {A}
Insert (B)
while NotEmpty (Stack) do
  M := Pop (Stack)
  for each predecessor of P of M do
    Insert (P)
  endfor
endwhile
```

```
procedure Insert(X)
  if X is not in ResultSet then
    Add X to ResultSet
    Push X onto Stack
  endif
```

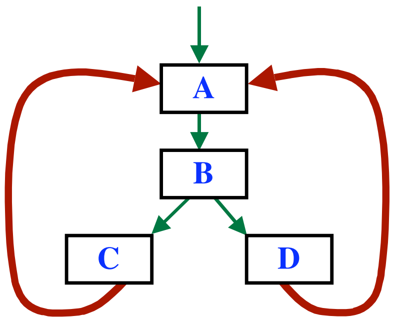

Inner / Outer Loops

A loop is a set of nodes.

Given two Natural Loops...
Either...
- The loops are disjoint, or
- One loop is contained in (i.e., nested) within the other, or
- Both loops have the same header.

If two loops have the same header...
They will be the same loop (same set of nodes)

Loops with Multiple Back-Edges

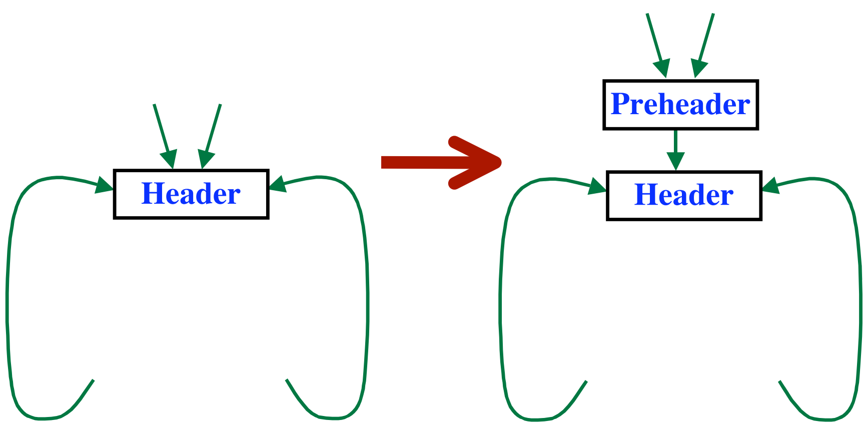



```
while (...) do
   ...A...
   ...B...
   if ... then
      ...C...
   else
      ...D...
   endif
endwhile
```

Which path is traversed most frequently?
Undecideable...

Must treat as equally probable.

Loop "Preheader"



We can place loop-invariant computations in the preheader.

Reducible Control Flow Graphs

Definition:
In a reducible control flow graph, all loops have a single entry point.

Structured programming constructs
⇒ The control flow graph is reducible.
⇒ All loops are natural.

In a reducible flow graph...
We have only...
• Forward Edges
These form an acyclic graph.
All nodes can be reached via
forward edges from initial node.
• Back Edges
The HEAD dominates the TAIL
• No "Cross Edges"