

# Overview of the Compiler

## Lexical Analysis

Break input into “*TOKENS*”

Source: `x = y + 1; /* incr x */ ...`

Tokens: `ID, EQUALS, ID, PLUS, INT, SEMI, ...`

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Break input into “*TOKENS*”

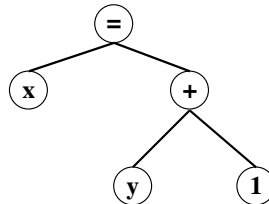
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## Syntax Analysis

Context-Free Grammar

Build a parse tree



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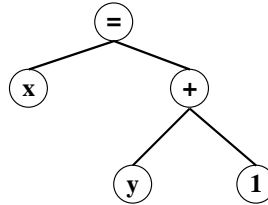
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## Syntax Analysis

Context-Free Grammar

Build a parse tree



## Semantic Analysis

Analyze types

Check for "semantic" errors

`x = y + true;`



## Symbol Table

One entry for each identifier

key	type	address
w	bool	50
x	int	54
y	double	58
...	...	...

## Introduction to Compiling - Part 1

### Symbol Table

One entry for each identifier

### Intermediate Code

Not machine specific

```
temp1 := x
temp2 := temp1 + 1
x := temp2
```

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Eliminate redundant data movement

Optimize “goto”s to other “goto” instructions

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Register Assignments

Machine Specific Code

```
mov.w    x, r5
add.w    #1, r5
mov.w    r5, x
```

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...	...	...

### Error Handling

Can't just abort! ... Find more errors!

Patch things up and keep going

Lexical Errors

Syntactic Errors

Semantic Errors

### Lexical Analysis

Token

A “word”

`if x == -123 then ...`

Types of tokens

ID	x foo ...
KEYWORD	if while ...
NUMBER	-123 4.0 ...
OPERATOR	+ == ( ) ; ...

“Lexeme”

The characters comprising a token.

`if x = -3.1415e37 then ...`

### Lexical Analysis

Token

A “word”

`if x == -123 then ...`

Types of tokens

ID	x foo ...
KEYWORD	if while ...
NUMBER	-123 4.0 ...
OPERATOR	+ == ( ) ; ...

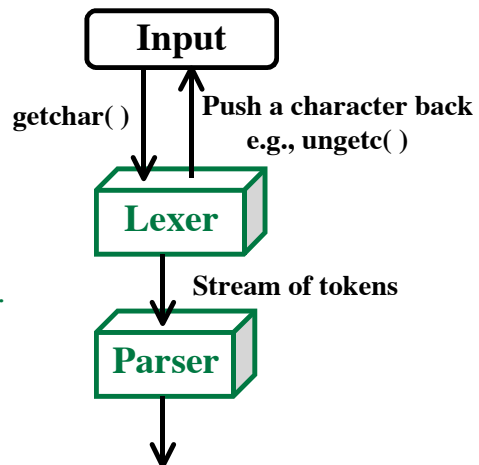
“Lexeme”

The characters comprising a token.

`if x = -3.1415e37 then ...`

### The Lexical Analysis Phase

“Lexer”  
“Scanner”



### “White Space”

Blanks, tabs, newlines

Ignored by lexer

⇒ Not seen by parser

### Comments

Identified by lexer

Treated like whitespace

⇒ Not seen by parser

## Parser Calls Lexer

To get next token

Lexer returns a single token

“type” (e.g., NUMBER)

“attribute” (e.g., -3.1415)

Example code:

```
#define NUM 1
#define ID 2
...
int getToken () {
    ...
    return NUM;
    ...
}
```

**Global Variable:**

`attribute`

**Side-effect:**

Sets this global variable  
whenever a NUM is seen.

### The Token Stream

**Source:**

```
if ( x == -3.1415 ) /* test x */ then ...
```

**Token Stream:**

```
< IF >
< LPAREN >
< ID, "x" >
< EQUALS >
< NUM, -3.14150000 >
< RPAREN >
< THEN >
...
```

Token: < NUM, -3.14150000 >



Option 1:

Lexer writes token stream out to a file.  
 Parser reads from this file.

More Likely:

Parser calls lexer when it needs a token.  
 Lexer returns one token at a time.

### Types of Token

	<u>Type</u>	<u>Attribute</u>
	ID	Ptr to String / Index into String Table
	INT	Integer value
	REAL	Double value
	CHAR	Char value
	STRING	Ptr to String / Index into String Table
<b>Keywords</b> {	WHILE	-
	IF	-
	ELSE	-
	...	-
<b>Operators/ Punctuation</b> {	PLUS	-
	MINUS	-
	LPAREN	-
	RPAREN	-
	SEMI	-
	...	-
	EOF	-

***Java Constants***

```

static final int
ID = 1;
INT = 2;
REAL = 3;
CHAR = 4;
STRING = 5;
WHILE = 6;
IF = 7;
...
EOF = 37;
        
```

## Identifiers

Letter { Letter | Digit }\*

A “Regular Expression”

“A letter followed by zero or more letters or digits.”

[May also want to include underscore: `my_val`]

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Letter { Letter | Digit }\*

A “Regular Expression”

“A letter followed by zero or more letters or digits.”

[May also want to include underscore: `my_val`]

Also called:

“Reserved Words”

## Keywords

A fixed set of words:

{ `if`, `while`, `do`, `return`, `else`, `break`, ... }

Look like identifiers, but they are not.

Identifiers:

Letter { Letter | Digit }\* - KEYWORDS



## The String Table

One entry per ID, KEYWORD

Each entry contains

- The lexeme (i.e., the string of characters)
- A type flag (ID, KEYWORD)

Implementation:

- Array (see next slide)
- Hash Table (faster lookup)

Goal:

Given a new lexeme...

Determine quickly:

- Is it a keyword? Which one?
- Is it an ID we've seen before?

*Identify equal IDs so the parser doesn't have to bother with string comparisons.*

### String Table Operations:

**lookup** (String) → int

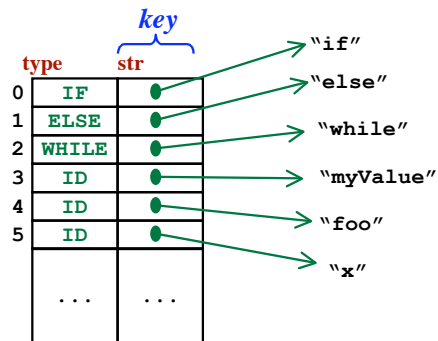
*Returns index of entry, or -1*

**insert** (String, type) → int

*Adds a new entry and returns its index*

### Keywords:

- Initialize the table
- Add keywords



## Introduction to Compiling - Part 1

```
function getToken () returns int
var c: char
    buffer: array of char
while true
    c = getChar ()
    if c == '\n'
        lineNumber++
    elseif c is whitespace
        -- nothing
    elseif isDigit (c)
        Read in zero or more digits.
        attribute = their value
        return INT
    elseif isLetter (c)
        Read in more letters and digits,
        placing them in buffer
        p = lookup (buffer)
        if p == -1
            p = insert (buffer, ID)
        endif
        attribute = p
        return p.type    -- ID, WHILE, IF, ELSE, ...
    elseif c = '+'
        return PLUS
    ...etc. for other operator symbols...
    else
        Error...
    endif
endwhile
endfunction
```

## Outline of Lexer

## Introduction to Compiling - Part 1

### Symbol Table (“Environment”)

*The same ID may have different meanings in one program.*

#### Example:

```
void foo (double x) { ... }
...
char x;
...
void x (int i) { ... }
```

## Symbol Table (“Environment”)

The same ID may have different meanings in one program.

*Example:*

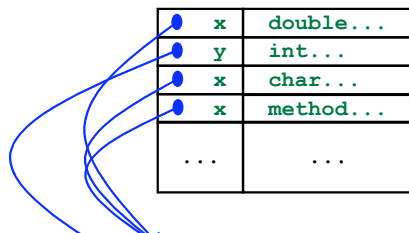
```
void foo (double x) { ... }
...
char x;
...
void x (int i) { ... }
```

*Symbol Table:*

x	double...
y	int...
x	char...
x	method...
...	...

*String Table:*

ID	"x"
ID	"y"
...	...



## Syntax Analysis

Context-Free Grammar (CFG)

Productions (“Grammar Rules”)

Meta Symbols

- “consists of”
- | “or”, alternatives
- ε epsilon, empty string

Non-terminals (**Expr, Term, Factor**)

Terminals

Lexical Tokens (**ID, NUMBER**)

Literals (**+, -, \*, /, (, )**)

Start Symbol (**Expr**)

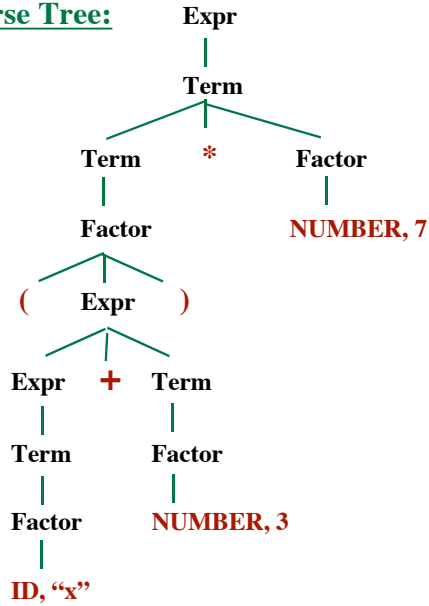
### The Classic Expression Grammar

- Expr → Expr + Term
- Expr → Expr - Term
- Expr → Term
- Term → Term \* Factor
- Term → Term / Factor
- Term → Factor
- Factor → NUMBER
- Factor → ID
- Factor → ( Expr )

## Parse Trees

Example:  $(x + 3) * 7$

Parse Tree:



Internal Nodes

Non-terminals

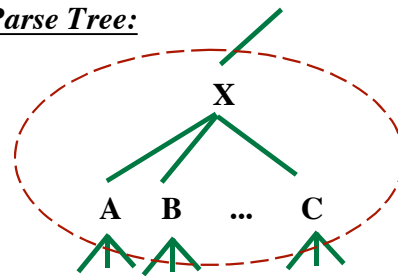
Leaves:

Terminals

A Rule (Production):

$X \rightarrow A B \dots C$

Parse Tree:



### Parse Trees

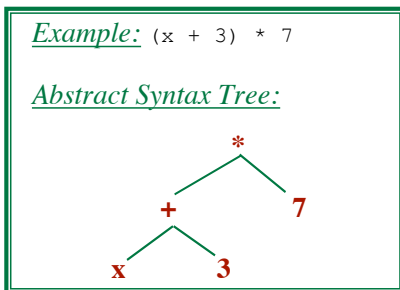
Match CFG Rules

### “Abstract Syntax Trees” (ASTs)

Capture only *essential* info.

Example:  $(x + 3) * 7$

Abstract Syntax Tree:

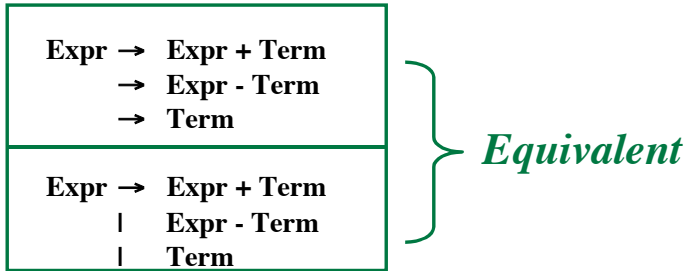


Fewer Nodes, Simpler

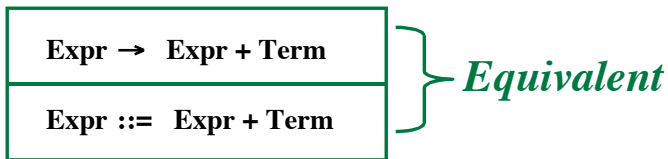
Project 1:

- Use Context-Free Grammar to Parse Expression
- Build an Abstract Syntax Tree
- Walk Abstract Syntax Tree to Evaluate Expression

**Alternative Right Sides:**



**Backus-Naur Notation:**



Epsilon

$\epsilon$  The empty string

Example:  
A  $\rightarrow$  **b** A |  $\epsilon$

Generates:

?

Epsilon

$\epsilon$  The empty string

Example:

$A \rightarrow \mathbf{b} A \mid \epsilon$

Generates:

$\epsilon$   
b  
bb  
bbb  
...

Epsilon

$\epsilon$  The empty string

Realistic CFG Example

Block  $\rightarrow$  “{” StmtList “}”  
StmtList  $\rightarrow$  Stmt StmtList  
 $\rightarrow \epsilon$   
Stmt  $\rightarrow$  **if** Expr **then** Stmt **else** Stmt  
 $\rightarrow$  ID “:=” Expr “;”  
 $\rightarrow$  **while** Expr **do** Stmt  
 $\rightarrow$  Block  
Expr  $\rightarrow$  ...

## Ambiguous Grammars

One string (“sentence”) has two or more parse trees!

Example:

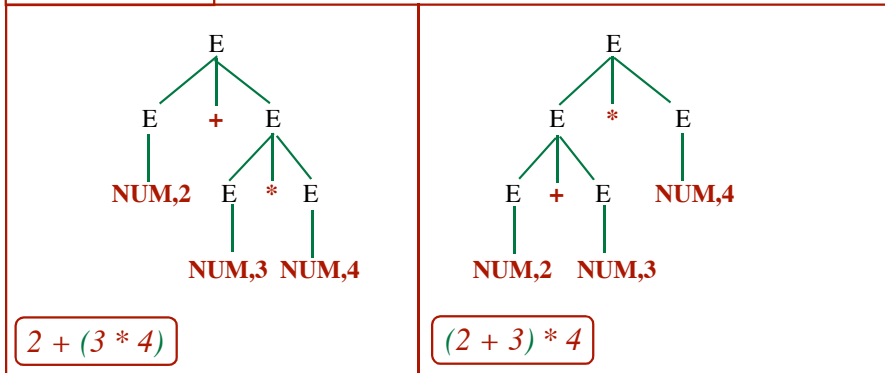
$E \rightarrow E + E$   
 $\rightarrow E * E$   
 $\rightarrow \text{NUM}$

Source:

$2 + 3 * 4$

*Want  
To  
Avoid!*

*Parse Trees:*



### Associativity

What does  $5 - 4 - 3$  mean?

$(5 - 4) - 3$

$5 - (4 - 3)$

### Precedence

### Normal Conventions

## Introduction to Compiling - Part 1

### Associativity

What does  $5 - 4 - 3$  mean?

$(5 - 4) - 3$  ← “Left Associative”

$5 - (4 - 3)$  ← “Right Associative”

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$(x = y) = z$

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$(1 + 2) * 3$  ← “Plus has higher precedence”

$1 + (2 * 3)$  ← “Multiplication has higher precedence”

### Normal Conventions

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What does  $5 - 4 - 3$  mean?  
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What does  $x = y = z$  mean?  
 $(x = y) = z$   
 $x = (y = z)$

### Precedence

What does  $1 + 2 * 3$  mean?  
 $(1 + 2) * 3$  ← “Plus has higher precedence”  
 $1 + (2 * 3)$  ← “Multiplication has higher precedence”

### Normal Conventions

Left Associative:  $+ - * /$   
Right Associative:  $= ^$   
Precedence:  
 $^$  ← highest  
 $* /$   
 $+ -$   
 $=$  ← lowest

**Assignment Operator:**  
 $x = y;$   
 $x := y;$   
**Exponentiation Operator:**  
 $x = y ^ 3;$

## Parsing Algorithms

Assume we have a grammar...

### “Parser”

#### Input:

- String of tokens

#### Output:

- Accept / Reject if errors
- Build a parse tree (or build AST)
- Execute “semantic actions” (e.g., to check program)
- Print good error messages (when source has errors)

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### “Parser Generators”

Input:

- A Grammar

Output:

- A Parser (e.g., a Java code file)

Any context-free grammar can be recognized!!!  
*A parser can be built.*

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Worst-case (Nasty grammars):  $O(N^3)$  time

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Typical Programming Languages:  $O(N)$  time (“linear”)

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Major Approaches:

- **Top-Down Algorithms**  
Simpler  
Better Error Messages
- **Bottom-Up Algorithms**  
Faster  
More General  
More Complex

**Recursive Descent Parsing**

A “Top-Down” Algorithm

Main Idea:

*For each non-terminal, write a routine.*

CFG Rule:

$A \rightarrow X Y Z$

Routine:

```
function ParseA () {  
  ParseX ();  
  ParseY ();  
  ParseZ ();  
}
```

The routines may be recursive.

Parse Tree is not constructed  
... explicitly.

Invocation of recursive routines = Implicit Parse Tree

## Handling Terminals

Global Variable

`var nextToken: TokenType`*“Look-ahead” Token*

```

function MustHave (t: TokenType) {
  if nextToken == t then
    nextToken = getToken ();
  else
    error;
  endif
}

```

A CFG Rule:  $\text{Expr} \rightarrow \text{Term} + \text{Expr}$ 

```

function ParseExpr () {
  ParseTerm ();
  MustHave (PLUS);
  ParseExpr ();
}

```

## Handling Alternatives

Factor  $\rightarrow$  NUM  
 $\rightarrow$  ( Expr )  
 $\rightarrow$  ID

```

function ParseFactor () {
  if nextToken == NUM then
    MustHave (NUM);
  elseif nextToken == '(' then
    MustHave ('(');
    ParseExpr ();
    MustHave (')');
  elseif nextToken == ID then
    MustHave (ID);
  else
    error "Problems in Factor";
  endif
}

```

## A Problem...

Stmt  $\rightarrow$  "{" StmtList "  
 $\rightarrow$  AssignStmt  
 $\rightarrow$  IfStmt  
 $\rightarrow$  WhileStmt

```

function ParseStmt () {
  if nextToken == '{' then
    MustHave ('\{');
    ParseStmtList ();
    MustHave ('\}');
  else
    ... ? ...   ParseAssignStmt ();
                ParseIfStmt ();
                ParseWhileStmt ();
  endIf
}
  
```

A  $\rightarrow$  X ...  
 $\rightarrow$  Y ...  
 $\rightarrow$  Z ...

*Two or more rules start  
with non-terminals...  
Which function to call???*

## First Sets

Let  $\alpha$  be a sequence of terminal and non-terminal symbols.

Consider all strings that  $\alpha$  can generate.

Define: **FIRST**( $\alpha$ ) = the set of...

"All tokens that can appear first in such strings"

Example:

Stmt  $\rightarrow$  "{" StmtList "  
 $\rightarrow$  AssignStmt  
 $\rightarrow$  IfStmt  
 $\rightarrow$  WhileStmt

**FIRST ( Stmt ) = { "{", ID, IF, WHILE }**

## Example

A  $\rightarrow$  B C  
     $\rightarrow$  x  
B  $\rightarrow$  y z  
     $\rightarrow$  D w  
C  $\rightarrow$  u  
D  $\rightarrow$  v  
     $\rightarrow$   $\epsilon$

FIRST ( D ) = { ? }

## Example

A  $\rightarrow$  B C  
     $\rightarrow$  x  
B  $\rightarrow$  y z  
     $\rightarrow$  D w  
C  $\rightarrow$  u  
D  $\rightarrow$  v  
     $\rightarrow$   $\epsilon$

FIRST ( D ) = { v,  $\epsilon$  }  
FIRST ( C ) =



Example

$$\begin{aligned}
 A &\rightarrow B C \\
 &\rightarrow \underline{x} \\
 B &\rightarrow \underline{y} \underline{z} \\
 &\rightarrow D \underline{w} \\
 C &\rightarrow \underline{u} \\
 D &\rightarrow \underline{v} \\
 &\rightarrow \varepsilon
 \end{aligned}$$

$$\text{FIRST}(D) = \{ \underline{v}, \varepsilon \}$$

$$\text{FIRST}(C) = \{ \underline{u} \}$$

$$\text{FIRST}(D\underline{w}) =$$
Example

$$\begin{aligned}
 A &\rightarrow B C \\
 &\rightarrow \underline{x} \\
 B &\rightarrow \underline{y} \underline{z} \\
 &\rightarrow D \underline{w} \\
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 D &\rightarrow \underline{v} \\
 &\rightarrow \varepsilon
 \end{aligned}$$

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$$\text{FIRST}(C) = \{ \underline{u} \}$$

$$\text{FIRST}(D\underline{w}) = \{ \underline{v}, \underline{w} \}$$

$$\text{FIRST}(B) =$$

Example

$A \rightarrow B C$   
 $\rightarrow \underline{x}$   
 $B \rightarrow \underline{y} \underline{z}$   
 $\rightarrow D \underline{w}$   
 $C \rightarrow \underline{u}$   
 $D \rightarrow \underline{v}$   
 $\rightarrow \epsilon$

$FIRST(D) = \{ \underline{v}, \epsilon \}$   
 $FIRST(C) = \{ \underline{u} \}$   
 $FIRST(D\underline{w}) = \{ \underline{v}, \underline{w} \}$   
 $FIRST(B) = \{ \underline{y}, \underline{y}, \underline{w} \}$   
 $FIRST(BC) =$

Example

$A \rightarrow B C$   
 $\rightarrow \underline{x}$   
 $B \rightarrow \underline{y} \underline{z}$   
 $\rightarrow D \underline{w}$   
 $C \rightarrow \underline{u}$   
 $D \rightarrow \underline{v}$   
 $\rightarrow \epsilon$

$FIRST(D) = \{ \underline{v}, \epsilon \}$   
 $FIRST(C) = \{ \underline{u} \}$   
 $FIRST(D\underline{w}) = \{ \underline{v}, \underline{w} \}$   
 $FIRST(B) = \{ \underline{y}, \underline{y}, \underline{w} \}$   
 $FIRST(BC) = \{ \underline{y}, \underline{y}, \underline{w} \}$   
 $FIRST(A) =$

Example

$A \rightarrow B C$   
 $\rightarrow \underline{x}$   
 $B \rightarrow \underline{y} \underline{z}$   
 $\rightarrow D \underline{w}$   
 $C \rightarrow \underline{u}$   
 $D \rightarrow \underline{v}$   
 $\rightarrow \epsilon$

$FIRST(D) = \{ \underline{v}, \epsilon \}$   
 $FIRST(C) = \{ \underline{u} \}$   
 $FIRST(D\underline{w}) = \{ \underline{v}, \underline{w} \}$   
 $FIRST(B) = \{ \underline{y}, \underline{v}, \underline{w} \}$   
 $FIRST(BC) = \{ \underline{y}, \underline{v}, \underline{w} \}$   
 $FIRST(A) = \{ \underline{x}, \underline{y}, \underline{v}, \underline{w} \}$

For this rule

$A \rightarrow X \dots$   
 $\rightarrow Y \dots$   
 $\rightarrow Z \dots$

Create this code

```

if nextToken ∈ FIRST (X...) then
  ParseX...();
elseif nextToken ∈ FIRST (Y...) then
  ParseY...();
elseif nextToken ∈ FIRST (Z...) then
  ParseZ...();
else
  Error
endif

```

For this rule

Stmt → “{” StmtList “}”  
→ AssignStmt  
→ IfStmt  
→ WhileStmt

Create this code

```
function ParseStmt () {  
  if nextToken == LBRACE then  
    Scan a token;  
    ParseStmtList ();  
    MustHave (RBRACE);  
  elseif nextToken == ID then  
    ParseAssignStmt ();  
  elseif nextToken == IF then  
    ParseIfStmt ();  
  elseif nextToken == WHILE then  
    ParseWhileStmt ();  
  else  
    Error  
  endif  
endFunction
```

Problem:

A → X ...  
→ Y ...  
→ Z ...

What if

$\text{FIRST}(X...) \cap \text{FIRST}(Y...) \neq \emptyset$

Solutions:

- Try to rewrite grammar rules.
- Don't use Recursive-Descent Parsing  
“LR Parsing”

## Left Recursion

*Grammar Example:*

Expr  $\rightarrow$  Expr + Term  
 $\rightarrow$  Term

*Source:*

“2 + 3 + 4 + 5”

*Means:*

“((2 + 3) + 4) + 5”

Left Associativity

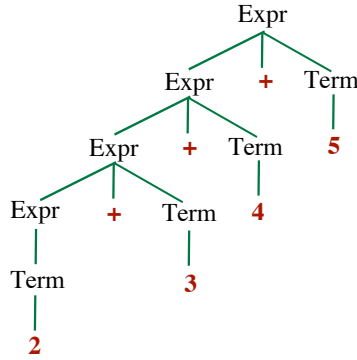
*General form:*

$A \rightarrow A \alpha \mid \beta$

$\beta$   
 $\beta\alpha$   
 $\beta\alpha\alpha$   
 $\beta\alpha\alpha\alpha$   
 ...

**Notation:**

A, B, C, ... Non-terminal symbols  
 $\alpha, \beta, \gamma, \dots$  Strings of arbitrary symbols



## Right Recursion

*Grammar Example:*

Expr  $\rightarrow$  Term + Expr  
 $\rightarrow$  Term

*Source:*

“2 + 3 + 4 + 5”

*Means:*

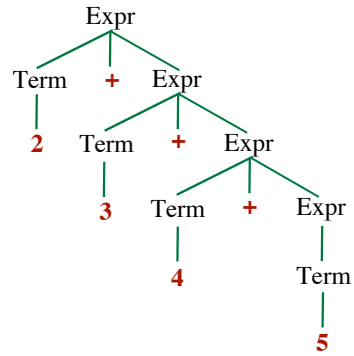
“2 + (3 + (4 + 5))”

Right Associativity

*General form:*

$A \rightarrow \alpha A \mid \beta$

$\beta$   
 $\alpha\beta$   
 $\alpha\alpha\beta$   
 $\alpha\alpha\alpha\beta$   
 ...



### Problem: Left Recursive Rules

*Before:*

$\text{Expr} \rightarrow \text{Term} + \text{Expr}$

*Now Consider:*

$\text{Expr} \rightarrow \text{Expr} + \text{Term}$

```
function ParseExpr () {  
    ParseExpr ();  
    MustHave (PLUS);  
    ParseTerm ();  
endFunction
```

What is the problem?

### Problem: Left Recursive Rules

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What is the problem?

*Infinite Recursion!!!*

## Introduction to Compiling - Part 1

### More Generally:

$A \rightarrow A \dots$   
 $\rightarrow \dots$

```
function ParseA () {  
    ParseA ();  
    ...  
endFunction
```

### Solution:

- Rewrite rules to eliminate left-recursion.
- Build parser from revised rules.

### Example:

```
Expr  $\rightarrow$  Expr + Term  
       $\rightarrow$  Term
```

```
Term  
Term + Term  
Term + Term + Term  
Term + Term + Term + Term  
...
```

## Introduction to Compiling - Part 1

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 $\rightarrow \dots$

```
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### Solution:

- Rewrite rules to eliminate left-recursion.
- Build parser from revised rules.

### Example:

```
Expr  $\rightarrow$  Expr + Term  
       $\rightarrow$  Term
```

```
Expr  $\rightarrow$  Term + Expr  
       $\rightarrow$  Term
```

```
Term  
Term + Term  
Term + Term + Term  
Term + Term + Term + Term  
...
```

## A General Formula for Eliminating Left-Recursion

Given:

$$A \rightarrow A \alpha \mid \beta$$

Introduce a new non-terminal (say "R").

Rewrite rules:

$$A \rightarrow \beta R$$

$$R \rightarrow \alpha R \mid \epsilon$$

**Notation:**

A, B, C, ... Non-terminal symbols

$\alpha, \beta, \gamma, \dots$  Strings of arbitrary symbols

Example:

Given:

$$\begin{aligned} \text{Expr} &\rightarrow \text{Expr} + \text{Term} \\ &\rightarrow \text{Term} \end{aligned}$$

Rewrite as:

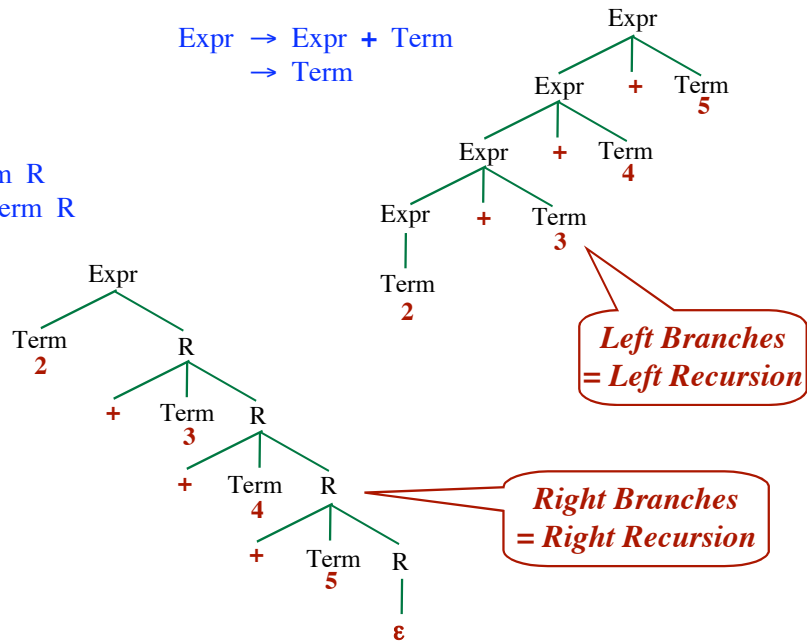
$$\begin{aligned} \text{Expr} &\rightarrow \text{Term} R \\ R &\rightarrow + \text{Term} R \\ &\rightarrow \epsilon \end{aligned}$$

$$\begin{aligned} A &= \text{Expr} \\ \alpha &= + \text{Term} \\ \beta &= \text{Term} \end{aligned}$$

## Example: 2 + 3 + 4 + 5

$$\begin{aligned} \text{Expr} &\rightarrow \text{Expr} + \text{Term} \\ &\rightarrow \text{Term} \end{aligned}$$

$$\begin{aligned} \text{Expr} &\rightarrow \text{Term} R \\ R &\rightarrow + \text{Term} R \\ &\rightarrow \epsilon \end{aligned}$$

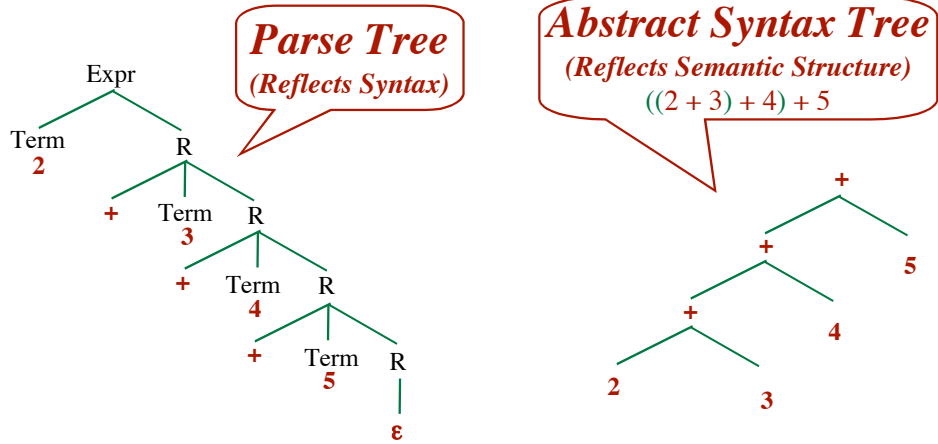




New Grammar Rules:

Expr → Term R  
 R → + Term R  
 → ε

Source: "2+3+4+5"



**Project 1: "E" Language**

The Grammar:

Expr → Term  
 → Term + Term  
 → Term - Term  
 → **set** ID = Expr  
 Term → ID  
 → NUM  
 → "(" Expr ")"

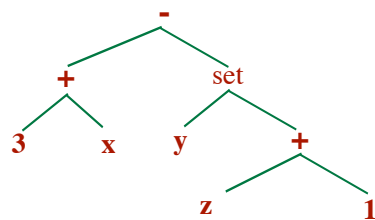
**Semantics:**

- Evaluate z+1
- Modify y
- Use that value in Expr

Source:

"( 3 + x ) - ( **set** y = z + 1 )"

Abstract Syntax Tree:



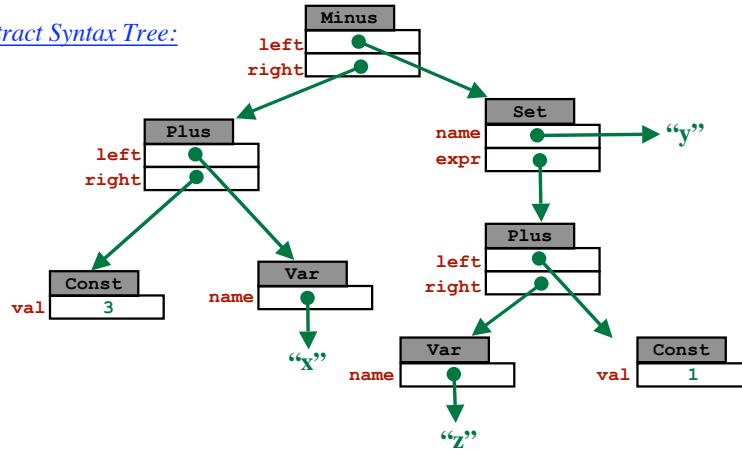
## Representation in Java

Classes:

- Plus
- Minus
- Set
- Var
- Const

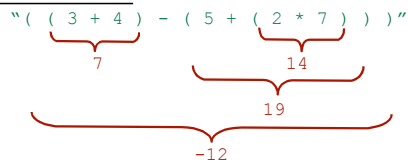
“(3 + x) - (set y = z + 1)”

Abstract Syntax Tree:



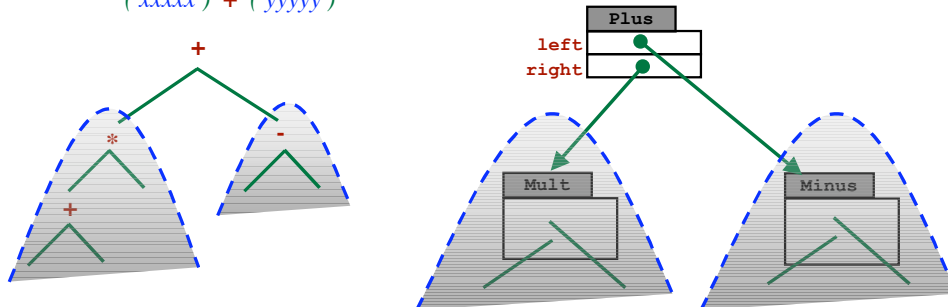
## Expression Evaluation

How to evaluate



- Evaluate sub-expressions first.
- Then evaluate the operation.

(xxxx) + (yyyy)



- A Recursive Method:

Node . Eval ( ValueEnv ) → IntegerResult