CS-581

Theory of Computation

Prof. Harry Porter

www.cs.pdx.edu/~harry/TheoryOfComp

COURSE DETAILS

E-MAIL: harry@cs.pdx.edu

TEXTBOOK: MICHAEL SIPSER
SECOND EDITION

PREREQUISITES: CS-311
Scan Chapter 0

OFFICE: FAB 115-06
Office Hours - After Class

MAILING LIST: MAILMAN
"PORTER CLASS LIST"

ATTENDANCE: REQUIRED.

READING ASSIGNMENT:
READ CHAPTER 1.

OUTLINE

WILL FOLLOW SIPSER TEXT CLOSELY.

CHAPTER 1:

- FINITE STATE MACHINES
- NONDETERMINISM
- REGULAR EXPRESSIONS

CHAPTER 2:

- CONTEXT-FREE LANGUAGES

CHAPTER 3:

- TURING MACHINES

CHAPTER 4:

- DECIDABILITY
- TURING RECOGNIZABILITY
- HALTING PROBLEM

CHAPTER 5:

- UNDECIDABLE PROBLEMS

CHAPTER 6:

- RECURSION THEOREM
- GÖDELS INCOMPLETENESS THEOREM

CHAPTER 7:

- TIME COMPLEXITY
- THE P=NP? QUESTION
- NP-COMPLETENESS.

CHAPTER 0: PREREQUISITE KNOWLEDGE

SKIM THIS CHAPTER.
REVIEW AS NECESSARY.

SETS

$\{a, b, c\}$

$\mathbb{N}$ = Natural Numbers = $\{1, 2, 3, \dots\}$

$\mathbb{Z}$ = Integers = $\{\dots, -2, -1, 0, +1, +2, \dots\}$

$\emptyset = \{\}$ empty set

$\cup$ Union

$\cap$ Intersection

$\overline{S}$ Complement

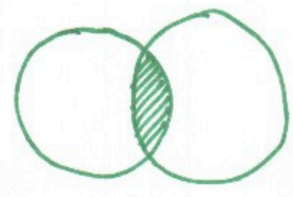
$S \times T$ Cross Product

$$\mathbb{N} \times \mathbb{N} = \{(i, j) \mid i, j \geq 1\}$$

$(4, 6)$ Tuples, Sequences

$P(S)$ Powerset

VENN DIAGRAMS



FUNCTIONS $f(a)$

$f: \text{Domain} \rightarrow \text{Range}$

Unary, Binary functions

Arity, k-ary functions

Infix, Prefix

PROPERTY (or "PREDICATE")

$P: \text{Domain} \rightarrow \{\text{TRUE}, \text{FALSE}\}$

RELATION $R: A \times A \times \ldots \times A \rightarrow \{\text{TRUE}, \text{FALSE}\}$

REFLEXIVE: $\forall x.\ x R x$

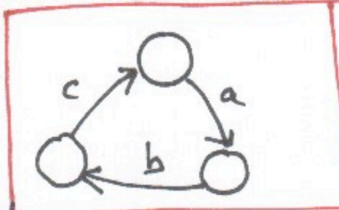
SYMMETRIC: $\forall x, y.\ x R y \Rightarrow y R x$

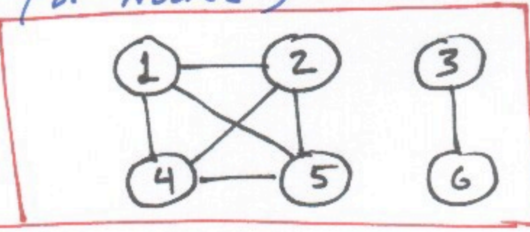
TRANSITIVE: $\forall x, y, z.\ x R y$ <u>and</u> $y R z$ <u>implies</u> $x R z$

GRAPHS

EDGES AND VERTICES (or "Nodes")

$G = (V, E)$





DIRECTED / UNDIRECTED EDGES

LABELED / UNLABELED NODES

SUBGRAPHS, CONNECTED COMPONENTS

PATHS, CYCLES

"IN-DEGREE"

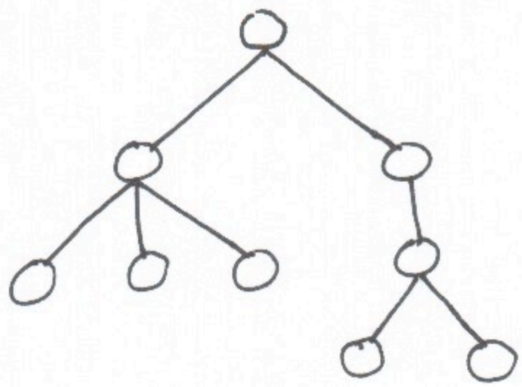
"OUT-DEGREE"

BINARY RELATION ≡ DIRECTED GRAPH

R(a, b) = TRUE

aRb

$\boxed{\text{TREES}}$

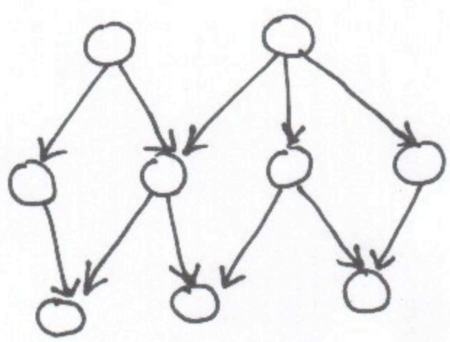


GRAPH WITH DIRECTED EDGES

NO CYCLES

ROOT NODE

DAG: Shared Parents Allowed



STRINGS AND LANGUAGES

Σ Alphabet, Set of symbols

$\Sigma = \{a, b, c, d\}$ finite set!

STRING

FINITE SEQUENCE OF SYMBOLS

ϵ EMPTY STRING (EPSILON, also "ε")

LENGTH

$w = abaa$

$|w| = |abaa| = 4$

$|\epsilon| = 0$

w^R Reverse of w

wz Concatenation

"LANGUAGE" = A SET OF STRINGS (often an <u>infinite</u> set.)

BOOLEAN LOGIC

AND $\wedge$ CONJUNCTION

OR $\vee$ DISJUNCTION

NOT $\neg$ NEGATION (Also: $\overline{P}$)

$\oplus$ EXCLUSIVE-OR

$\leftrightarrow$ EQUALITY (Also: $\Leftrightarrow$, $=$)

$\rightarrow$ IMPLIES, IMPLICATION (Also: $\Rightarrow$)

DISTRIBUTION LAWS:

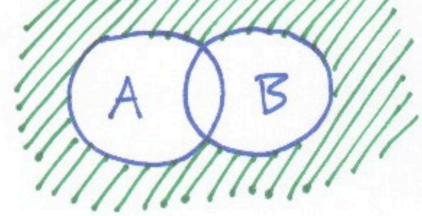
$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

DEMORGAN'S LAWS

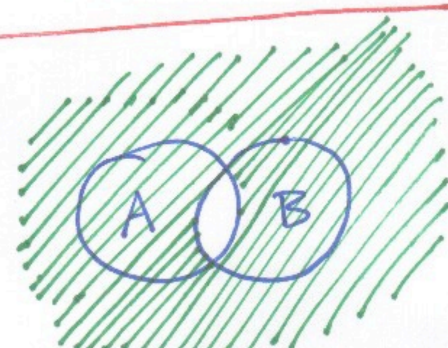
$$\neg(A \vee B) = (\neg A) \wedge (\neg B)$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$



$$\neg(A \wedge B) = (\neg A) \vee (\neg B)$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$



DEFINITIONS

THEOREMS

PROOFS

LEMMAS PROVED IN ISOLATION.
PART OF A LARGER PROOF.

COROLLARY A TRUE STATEMENT DERIVED EASILY FROM THE MAIN THEOREM.

CONJECTURES UNPROVEN; POSSIBLY TRUE

P iff Q (Also: $P \Leftrightarrow Q$)

MUST PROVE FORWARD DIRECTION

$P \Rightarrow Q$ "P only if Q"

MUST PROVE REVERSE DIRECTION

$Q \Rightarrow P$ "P if Q"

PROOF BY CONSTRUCTION

THEOREM: "x exists; There is an x."

PROOF: Show how to build an x.

PROOF BY CONTRADICTION

THEOREM: "P is true"

PROOF:
- Assume P is false.
- Do some logical reasoning.
- Conclude the truth of something known to be false (an ABSURDITY)

PROOF BY INDUCTION

THEOREM: "P is true for all... [numbers]."

PROOF:

BASIS CASE: Show $P(0)$ is true.

INDUCTIVE STEP:

Assume $P(i)$ is true.
(The "INDUCTIVE HYPOTHESIS")

Use logical reasoning to show $P(i+1)$ is true.

Conclude P is true for all $i \geq 0$