Epipolar Curves on Surfaces*

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Abstract

The epipolar parametrization of a surface is obtained by tracking its profiles in a sequence of images with known camera motion. Generally, this does not provide a global reconstruction and depends on the camera trajectory. This paper gives a systematic way of detecting the gaps left by the reconstruction process. We present an investigation of epipolar curves on the object surface and in a related 'spatio-temporal surface'. The epipolar curve has a cusp and the epipolar parametrization breaks down where the epipolar plane is tangent to the surface. This parametrization also breaks down where the epipolar curve is tangent to the critical set. A description is given for the geometry of all of the generic cases where the epipolar parametrization breaks down.

1 Introduction

This paper is concerned with some aspects of the reconstruction of a smooth surface M from a sequence of profiles (also called apparent contours, outlines and occluding contours) where the motion of the observer is known. Such a reconstruction was introduced in [6] for a simple class of motions, and was generalised in [1, 2, 15] to arbitrary motion. For the general motion case, the *epipolar correspondence* (as in stereo) played an important role in matching points from one profile to the next. In addition, this results in a useful parametrization of the surface. The purpose of this paper is to describe conditions when this parametrization does not exist.

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Given a smooth surface M and a curve $c(t)$ of camera centers, we have, for each t, a *critical set* or *contour generator* Σ_t on M consisting of those points $\mathbf{r}$ where the 'visual ray' or viewline from $c(t)$ to $\mathbf{r}$ is tangent to M (see Fig 1). The *epipolar plane* is the plane spanned by this ray and the tangent to the curve $\mathbf{c}$ of centers. In practice the epipolar plane is computed from a visual ray to one camera center and the baseline connecting the two centers [3, p.170].

On the surface M an *epipolar curve* through $\mathbf{r}$ is defined as one whose tangent is along the visual ray as shown in Fig. 1. As $\mathbf{c}$ moves with time, the visual ray slips along the epipolar curve. Note that this is almost never the intersection of the epipolar plane with the surface. In general, the critical sets and epipolar curves make a coordinate grid on M: a local parametrization $r(t, u)$ can be found in which the critical sets are given by t =constant and the epipolar curves by u = constant. It is this 'epipolar parametrization' of M which is used in [1, 2] to reconstruct M from its profiles, which are the images of the critical sets in a viewplane or viewsphere. In [14] it is shown that reconstruction from the epipolar parametrization is readily transformable into an optimal estimation problem. The epipolar parametrization also has another very interesting property: the viewlines associated with points $r(t, u)$ and $r(t+\delta t, u)$ of M will (being lines in space) generally not intersect. However, for the epipolar parametriza-

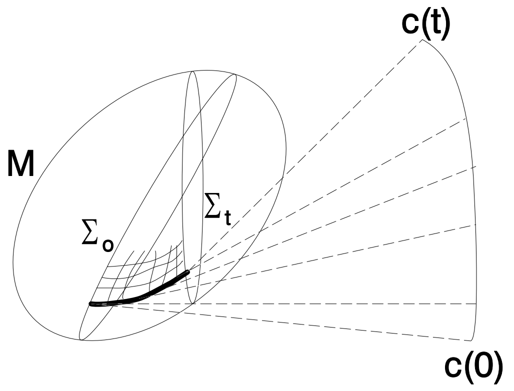

Figure 1: A surface M and segment of camera path from $\mathbf{c}(0)$ to $\mathbf{c}(t)$. Also shown are the two corresponding critical sets Σ_0, Σ_t, a segment of epipolar curve (drawn heavily), viewlines (dashed) tangent to the epipolar curve, and a local coordinate grid of critical sets and epipolar curves.

tion, the point at which these lines come *closest* to one another is (as $\delta t \to 0$) on the surface M. So this parametrization is the best one to choose if one uses the pseudo-intersection of viewlines as an approximation to the surface. In [8] we show that there is only one other parametrization besides the epipolar one for which this holds.

The epipolar curves on M have the striking property that their osculating planes are precisely epipolar planes. The epipolar plane becomes tangent to M at points which we shall identify as 'frontier points' (§2.2), and it is precisely at such points that the epipolar curve becomes singular. The 'frontier,' which is the locus of such points of M, plays an important role in this paper, since it is along the frontier that the epipolar parametrization breaks down. To examine the situation at the frontier we introduce a 'spatio-temporal surface' in §2.1.

There are other places where the epipolar parametrization breaks down. First, there are points where the epipolar curve and critical set become tangent. This results in a profile with a cusp (or higher singularity) and is also reflected in the geometry of the epipolar curve: it has a zero of curvature. Second, the critical sets themselves can become singular (having an isolated point or a crossing). A more complete version of this paper will appear in [8], and some of the results have appeared in [7].

2 Critical sets and the spatio-temporal surface

Let M be a smooth surface in 3-space, without boundary, and let $c(t)$ and $\mathbf{r}$ be as above. We use $\mathbf{n}$ to denote the unit (say outward) normal at $\mathbf{r}$. Then the *critical set* Σ_t corresponding to 'time' t is the set of points $\mathbf{r}$ of M satisfying

$$(r - c(t)) \cdot n = 0. \tag{1}$$

In this paper we are mostly concerned with 'local' results, that is, we can assume our surface M to be parametrized: $(u, v) \to r(u, v)$. Notice that the equation (1) then becomes one equation in three variables t, u, v, defining a surface in 3-space.

Definition 2.1 *The surface $\widetilde{M}$ defined by*

$$(r(u, v) - c(t)) \cdot n(u, v) = 0 \tag{2}$$

is called the spatio-temporal surface. *It is associated with M and the motion $\mathbf{c}$ of the camera center. Compare [4]. There is a natural projection $\pi : \widetilde{M} \to M$ given by $\pi(u, v, t) = r(u, v)$.*

Thus $\widetilde{M}$ can be regarded as the union of the critical sets on M, spread out in the t-direction. On $\widetilde{M}$ we call the sets t = constant *lifted critical sets*, $\widetilde{\Sigma}_t$. We shall often use the (u, v) parameter space of M in place of M itself. Then π becomes $(u, v, t) \to (u, v)$. Clearly the epipolar curves can be parametrized by t, and when the tangent is in the t direction on $\widetilde{M}$ the epipolar curve on M will have a cusp.

In order to understand how $\widetilde{M}$ relates to M, consider a paraboloid as shown at the top of Figure 2. The u, v parameter space is shown in the middle Figure 2. The critical sets in the parameter space are circles tangent to the two frontier lines $u = 0, 2$. The spatio-temporal surface $\widetilde{M}$ is shown at the bottom of Figure 2. The u, v parameter space is shown as the bottom of the bounding box, and this time the epipolar curves are drawn. Note that they have cusps along the frontier in the u, v plane but not on $\widetilde{M}$. If one were to draw the frontier on M, it would consist of two parabolas above the lines $u = 0, 2$. The epipolar curves would have cusps on these two parabolas.

Definition 2.2 *The* frontier F *of* M*, relative to the given motion* $\mathbf{c}$*, is the set of points of* M *for which* $c_t \cdot n = 0$*. This can also be described as*

1. *the envelope of critical sets on* M *(i.e., the locus of intersections of 'adjacent' critical sets on* M*);*

2. *the set of points of* M *where the epipolar plane (assumed defined) is the tangent plane to* M*: the set of 'epipolar tangency points'.*

3. *the set of 'critical values' of the map* π *in Definition 2.1, i.e. the points of* M *under the 'fold line' of the projection* π *. Instead of* π *here we can equivalently use the linear 'vertical' projection of* $\widetilde{M}$ *to the parameter space of* M *:* $(u, v, t) \rightarrow (u, v)$*.*

The frontier points do *not* remain stationary as the camera moves, but slip along the frontier. An exception to this rule is motion where $\mathbf{c}$ moves in a plane P. If P is tangent to M then any point of tangency is a stationary frontier point, but other frontier points may move around. For orthographic projection, motion in any plane P has stationary frontier points where the tangent plane to M is parallel to P. This is used by Kutulakos and Dyer [10] for producing a set of stationary points. Joshi *et al.* used these points to compute structure from unkown motion [9]

3 Breakdown of the epipolar parametrization

The (local) epipolar parametrization of M near $\mathbf{r}$ breaks down when any of the following occur:

(i) The critical set and epipolar curve on M are smooth and tangent to one another (the profile has a singularity, Fig 3),

(ii) The critical set on M is singular (Fig 4),

(iii) The critical sets form an envelope on M(frontier points, Fig 2).

Figures 2-4 show some of the cases of the three types of events listed above. The figures are drawn in the (u, v) parameter space. The critical sets are labeled Σ (thin lines), the epipolar curves are E (thicker lines), and the line of cusps when present is L (thick line). The line of cusps is where the epipolar curves and critical sets are tangent. Typically, the profile ends at these points. The line of cusps forms part of the *natural boundary*, where the surface profile is occluded. The other part of the natural boundary is associated with T-junctions [8].

4 Conclusion

The epipolar parametrization of a surface M has been shown elsewhere to be very useful in the reconstruction process. We present the cases where the epipolar parametrization cannot be obtained, namely, at the frontier, at a singularity of the profile, e.g. a cusp point, and where the critical set is singular. We have shown that at the frontier we cannot parametrize M using critical sets as parameter curves, but that the epipolar curves can be understood using the 'spatio-temporal surface' $\widetilde{M}$, which is (except at a (profile) cusp point on the frontier) parametrized locally by lifted critical sets and lifted epipolar curves. In these cases we have found the detailed structure of the epipolar curves around the point at which the epipolar parametrization breaks down. When this occurs, it is possible to use other parametrizations which may even include the critical sets.

In general, the reconstruction of surfaces from profiles leaves gaps. These gaps are bounded by frontier curves or natural boundaries. Making multiple passes over the surface can reduce these gaps, but may not eliminate them if the object has concavities or the camera trajectory is limited by environmental constraints. Note that the frontier curve can be reconstructed by triangulation. Once one has determined where the frontier and natural boundary curves are, there are three ways to fill in the gaps. As in [11], one can actively move the camera to a trajectory that will reconstruct some missing parts of the surface (if this is possible). One could also apply different sensors or use information from surface markings and texture (as in multiframe stereo). A third approach is to use other information which is implicitly present in the profiles. The profile of a surface from a given view determines a bounding cone. The intersection of these cones from different viewpoints can be used to construct the bounding volume for an object [12, 13]. Merging of the appropriate pieces of these two surfaces along the frontier and the natural boundary can produce a closed surface which uses all of the information available from the profiles. Additional information from surface markings can potentially be combined for visible regions which are not covered by the critical sets

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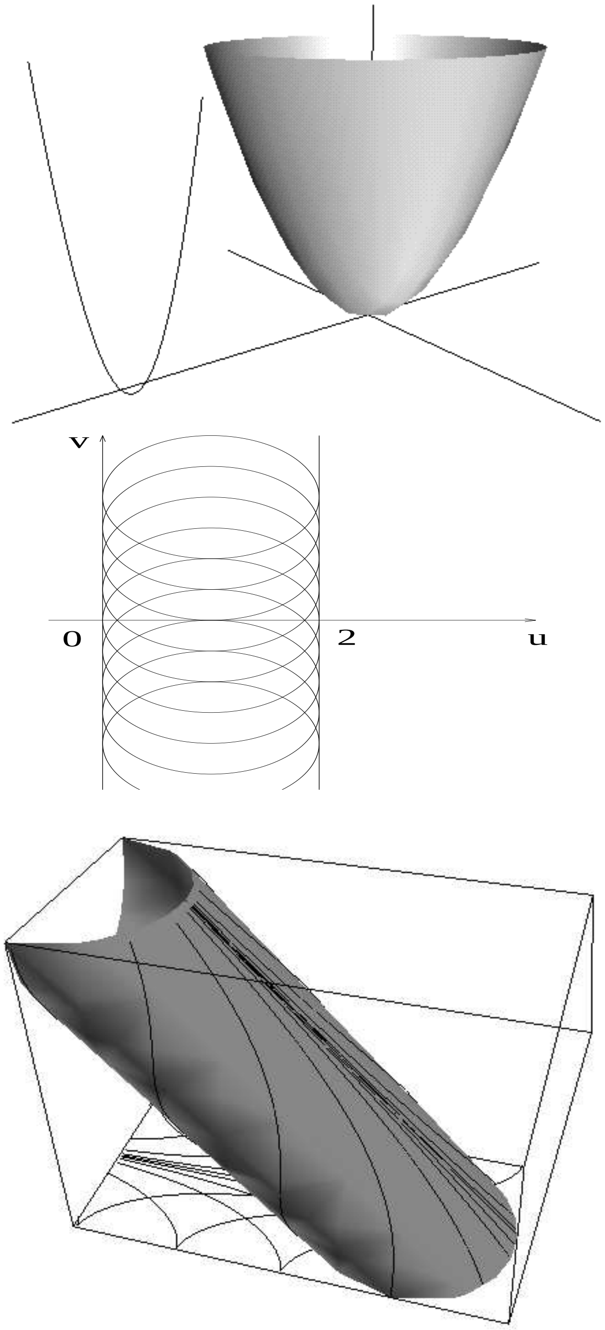

Figure 2: Top: A paraboloid and camera motion. Middle: The critical sets (circles) and their envelope in the u, v parameter space (case (iii)). Bottom: The spatio-temporal surface.

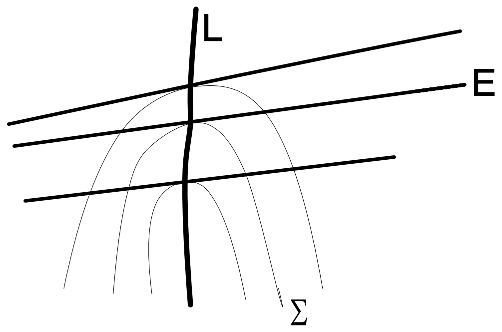

Figure 3: Cusp: case (i), epipolar curves and critical sets are tangent. For an opaque surface, all the lines to one side of the line of cusps L are occluded, thus the profiles and epipolar curves end at L. E is an epipolar curve and Σ a critical set. The other possibility for case (i) is a swallowtail event [8].

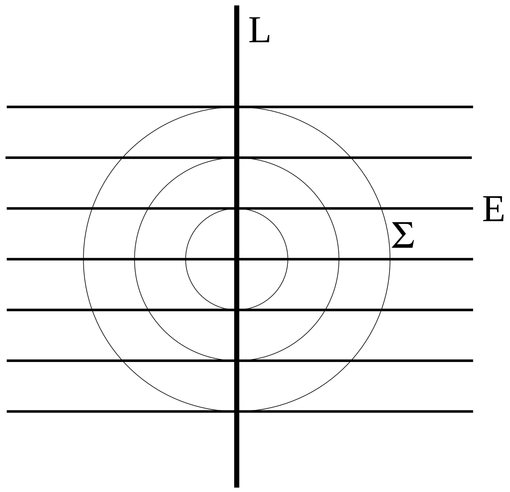

Figure 4: Lips: case (ii), the critical set is singular. For an opaque surface all to one side of L is occluded. The other possibility for case (ii) is a beaks event [8].