1 Introduction

This document provides information on the various coordinate systems used in the experiment, the measurements taken and the methods used to find coordinate transformations. Actual values of the parameters are also provided.

The experiments were all performed using Martin Marietta's Autonomous Land Vehicle (ALV). The ALV is equipped with cameras, digitizers and computers. For our experiment we used a color camera (Sony) and a Vicom digitizer. Some important parameters of the camera and digitizer are provided in the file camera_info.dat .

The ALV also has an inertial platform - or Land Navigation System (LNS) on board. The LNS consists of 3 gyroscopes and an odometer (see [] for more details).

Some of this information is taken from a Martin-Marietta draft document.

2 Notation

A point will be denoted by its position vector $\mathbf{p} = (x, y, z)^T$. We will use lowercase letters in boldface to denote vectors and uppercase letters in boldface to denote matrices.

An angle will be denoted by specifying an axis (e.g. X and the rotation α about it. eg (X,α) . The right-hand rule is used to determine the sign of the angle. Thus for example the positive angle is the direction in which a right-hand screw turns when pointing in the direction of the axis.

2.1 Homogeneous Coordinates

A position vector $\mathbf{p}$ is represented in homogeneous coordinates as $\mathbf{p} = (x, y, z, 1)^T$.

A translation $\mathbf{T}_{vw}$ which takes a vector $\mathbf{p}_v$ in the v coordinate system to a vector $\mathbf{p}_w$ in the w coordinate system is specified by the homogeneous matrix

$$\mathbf{T}_{vw} = \begin{matrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{matrix} \tag{1}$$

In matrix notation this is

$$\mathbf{p}_w = \mathbf{T}_{vw}\mathbf{p}_v \tag{2}$$

Or equivalently

$$\mathbf{p}_w = \mathbf{p}_v + (x, y, z)^T \tag{3}$$

Homogeneous rotation matrices are represented by the axis and angle about which the rotation occurs. Thus for example a rotation (X, α) is represented as:

$$\mathbf{R}_{X,\alpha} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha & 0 \\ 0 & sin\alpha & cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \tag{4}$$

a rotation (Y, β) as:

$$\mathbf{R}_{Y,\beta} = \begin{matrix} cos\beta & 0 & sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\beta & 0 & cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \tag{5}$$

and a rotation (Z, γ) as:

$$\mathbf{R}_{Z,\gamma} = \begin{matrix} cos\gamma & -sin\gamma & 0 & 0 \\ sin\gamma & cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \tag{6}$$

An arbitrary 3-d rotation $\mathbf{R}$ could be represented by a rotation (X, α) followed by a rotation (Y, β) followed by another rotation (Z, γ)

$$\mathbf{R} = \mathbf{R}_{Z,\gamma}\mathbf{R}_{Y,\beta}\mathbf{R}_{X,\alpha} \tag{7}$$

A homogeneous rotation $\mathbf{R}_{vw}$ would thus map a vector $\mathbf{p}_v$ in a coordinate system v into the vector $\mathbf{p}_w$ in the rotated coordinate system w

$$\mathbf{p}_w = \mathbf{R}_{vw}\mathbf{p}_v \tag{8}$$

3 Coordinate Systems

There are four coordinate systems to consider.

1. A world coordinate system defined by (x_w, y_w, z_w). The y-axis points North, the x-axis points East and the z-axis is parallel to the direction of gravity ([]).

2. A vehicle coordinate system (x_v, y_v, z_v). The x-axis points in the direction of forward travel of the vehicle, the y-axis points to the left of the vehicle and the z-axis points upwards. The LNS provides measurements of the vehicle's coordinate system with respect to the world coordinate system in the following manner []:

   (a) It gives the translation $\mathbf{t}_{vw} = (x_w, y_w, z_w)$ of the origin of the vehicle coordinate system with respect to the world coordinate system.

   (b) It provides the vehicle azimuth angle ψ - i.e the angle from the world y-axis to the vehicle x-axis.

   (c) The vehicle pitch angle γ is the magnitude of the rotation about the y-axis of the vehicle i.e. (Y_v, γ)

   (d) The roll of the vehicle η is given by the magnitude of the angle about the x-axis of the vehicle. i.e. (X_v, η)

3. A theodolite coordinate system (x_t, y_t, z_t). The origin is defined to be at theodolite-M and the negative x-axis in the direction of theodolite-D. The y-axis is in the same horizontal plane as the x-axis (horizontal with respect to gravity). The z-axis is parallel to the direction of gravity and points upward. (Note that the z-axes of the world and theodolite coordinate systems point in the same direction - thus any coordinate conversion between them essentially reduces to a 2-D problem).

4. A camera coordinate system (x_c, y_c, z_c). The origin is defined to be at the optic centre of the camera. The z-axis is perpendicular to the camera with its positive direction given by the right-hand rule.

4 Coordinate Transformations

All coordinate transformations will be expressed in terms of homogeneous matrices. This allows us to specify the translation and rotation using just one matrix eg. The vehicle to world coordinate transformation is given by:

$$\mathbf{p}_w = \mathbf{H}_{vw}\mathbf{p}_v \tag{9}$$

where the coordinates of a point p are given in the vehicle coordinate system by $\mathbf{p}_v$ and in the world coordinate system by $\mathbf{p}_w$. $\mathbf{H}_{vw}$ is the homogeneous transformation which takes the vector $\mathbf{p}_v$ to $\mathbf{p}_w$.

Notation In homogeneous coordinates the vector $\mathbf{p}_v = (x_v, y_v, z_v, 1)$. (We will use $\mathbf{r}_v = (x_v, y_v, z_v)$ to indicate the corresponding 3-D position vector). The same convention will be followed for all coordinate systems.

4.1 Camera to Vehicle Transformation

$$\mathbf{p}_c = \mathbf{H}_{cv}\mathbf{p}_c \tag{10}$$

where

1. $\mathbf{p}_c$ is the position vector of p in camera coordinates.

2. $\mathbf{p}_v$ is the position vector of p in vehicle coordinates.

3. $\mathbf{H}_{cv}$ is the homogeneous matrix which takes $\mathbf{p}_c$ to $\mathbf{p}_v$.

   Since the camera was rigidly fixed with respect to the vehicle, this transformation remains the same across all the sequences. The values of the matrix are provided in the file cam_to_veh_homo.dat .

4.2 Vehicle to World Transformation

$$\mathbf{p}_w = \mathbf{H}_{vw}\mathbf{p}_v \tag{11}$$

where

(a) $\mathbf{p}_w$ is the position vector of p in world coordinates.

(b) $\mathbf{p}_v$ is the position vector of p in vehicle coordinates.

(c) $\mathbf{H}_{vw}$ is the homogeneous matrix which takes $\mathbf{p}_v$ to $\mathbf{p}_w$

The values of $\mathbf{H}_{vw}$ are provided in the appropriate directory in the file veh_to_world_homo.dat .

4.3 World to Theodolite Coordinates

Let us assume that we have the 3-d coordinates of n points in world and theodolite coordinates. Then we need to solve the 2-D absolute orientation problem which maps a point (x_w, y_w) to (x_t, y_t) (note that the world and theodolite coordinate systems have their z-axes parallel to each other). Although the z-axes are parallel their origins are shifted with respect to each other. This is also solved for to find the transformation.

$$\mathbf{p}_t = \mathbf{H}_{wt}\mathbf{p}_w \tag{12}$$

where

(a) $\mathbf{p}_t$ is the position vector of p in theodolite coordinates.

(b) $\mathbf{p}_w$ is the position vector of p in world coordinates.

(c) $\mathbf{H}_{wt}$ is the homogeneous matrix thich takes $\mathbf{p}_w$ to $\mathbf{p}_t$

The values of this homogeneous matrix are provided in the appropriate directories in the file world_to_theo_homo.dat