Reliable multicast and the whiteboard

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1 Introduction

2 The whiteboard

3 Wb's reliable multicast

3.1 Repair/request algorithms for trees.

Trees, all links have distance 1, all interior nodes have fanout p. Level 0, 1, etc., relative to the failed link. Source distance j from failed link (that is, from the level 0 node).
Requests: [2d, 4d].
Level 0: time 0, timer [2j,4j], expiration [2j,4j].
Level 1: time 1, timer [2(j+1),4(j+1)], expiration 1+[2(j+1),4(j+1)].
Receives level 0 at 1+[2j,4j].
Receives other level 1s at 3+[2(j+1),4(j+1)].
Level i: time i, timer [2(j+i),4(j+i)].
expiration i+[2(j+i),4(j+i)].
For level k on branch through level h (for h¡i), receives level k at (i-h) + (k-h) + [2(j+k),4(j+k)]

Level i receives level 0's request at i+[2j,4j].
Level i no longer matters if i+4j¡i+2(j+i), that is, if j¡i.

Prob[level 0 suppresses one at level i] =
prob[X¡Y] for X uniform in i+[2j,4j], and Y uniform in i+[2(j+i),4(j+i)] =

$$1-\frac{(i-j)^2}{(2j(i+j))}.$$

Prob[level 0 suppresses all p^i at level i], for $i<j$? Distribution of first at level i = i + first of p^i, each uniform in [2(j+i),4(j+i)]. From Feller II, p. 24, in the limit the first of n independent random variables each uniformly distributed in [0,1] is exponentially distributed with expectation n^{-1}. (We approximate by 1/(n+1).)

So Prob[level 0 suppresses all p^i at level i] = Prob[X uniform in i+[2j,4j] is less than i + 2(j+i) + first of p^i uniform in interval of width 2(j+i) = i + 2(j+i) + exponential with expectation $1/\lambda$, for $\lambda=(p^i+1)/2(j+i)$.

This is Prob[[uniform in [0, 2j] is less than 2i + exponential with expectation $1/\lambda$. This is

$$\int_0^{2(j-i)}\frac{y+2i}{2j}\lambda e^{-\lambda y}dy+\int_{2(j-i)}^{\infty}\lambda e^{-\lambda y}dy=$$

$$\int_0^{2(j-i)}\frac{y}{2j}\lambda e^{-\lambda y}dy+\int_0^{2(j-i)}\frac{2i}{2j}\lambda e^{-\lambda y}dy+e^{-\lambda 2(j-i)}=$$

$$\frac{\lambda}{2j}\int_0^{2(j-i)}ye^{-\lambda y}dy+\frac{\lambda i}{j}\int_0^{2(j-i)}e^{-\lambda y}dy+e^{-\lambda 2(j-i)}=$$

$$\frac{\lambda}{2j}\left(\lambda^{-2}+\frac{-1-\lambda 2(j-i)}{\lambda^2 E^{\lambda 2(j-i)}}\right)+\frac{\lambda i}{j}\left(\lambda^{-1}-\frac{1}{\lambda E^{\lambda 2(j-i)}}\right)$$
$$+e^{-\lambda 2(j-i)}.$$

Prob[the first one at level i suppresses all other levels]?

We use the fact that for X and Y independent continuous random variables,

$$P[X\le Y]=\int_{-\infty}^{\infty}F_X(y)f_Y(y)dy$$

for distribution function F_X and density function f_Y.

Expected number of requests?

Prob[level 0 request is sent first].

We use the fact that, for X uniform in [a,c] and Y uniform in [b,d], and for $a<b<c<d$, the probability that Y is less than X is 1/2 times the probability that Y is in [b,c] times the probability that X is in [b,c]. This is $1/2(c-b)^2/((c-a)(d-b))$.

References

[Ch94] Cheriton

[J92] Jacobson

[JMF93] Jacobson

[J94] Jacobson

[M92] McCanne