Formalizing Nomic: working on a theory of communication with modifiable rules of procedure
(extended abstract)

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ABSTRACT

In this paper we consider the following question: *how can we let computers play Nomic?* Nomic is an abstract game of rule-making and legislation. The idea behind Nomic is to change the rules of Nomic. This makes Nomic into an important case of a so-called self-modifying protocol. Formalizing Nomic is important because it gives insight in the workings of such protocols. By proving three modest claims, the paper attempts to provoke further research in this area.

1. Introduction

In this paper we consider the following question: *how can we let computers play Nomic?* In particular we are interested in formalizing Nomic and the protocol that regulates it.

Nomic [from the Greek νόμοζ (nómos), meaning 'law'] is an abstract game of rule-making and legislation, originally invented by Peter Suber (1980), and recently gaining popularity among a selected group of dedicated enthusiasts with access to international computer networks.[1]

Nomic is conceived and designed by Peter Suber, who presented it as a self-modifying game, based on reflexivity in law. The game was first published in Douglas Hofstadter's column *"Metamagical Themas"* (Hofstadter, 1982), and later in Hofstadter's book, by the same name (Hofstadter, 1985). Suber revised the rules and published them in his own book (Suber, 1990).

Here are a few initial rules of Nomic:

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[1]Accordingly, much of the information about Nomic presented here is fetched from the internet and similar information services. A Nomic FAQ list can be FTP'ed from `ftp.cse.unsw.edu.au` in the directory `/pub/doc/Nomic/FAQ`. (As of January 10, 1995.)

Rule 201.   Players shall alternate in clockwise order, taking one whole turn apiece. Turns may not be skipped or passed, and parts of turns may not be omitted. All players begin with zero points.

Rule 202.   One turn consists of two parts, in this order:

(1)   Proposing one rule change and having it voted on;

(2)   Throwing one die once and adding the number of its points on its face to one's score.

Rule 203.   A rule change is adopted if and only if the vote is unanimous among the eligible voters.

The idea behind Nomic is to change the rules of Nomic. The game can be completely different at the end than it was at the start. The basic play is explained in rule 202: a player proposes a rule change, all the players vote on it, and if the vote succeeds, the change is immediately incorporated into the game. An interesting point is that rule 202 itself can be changed. If a player changes this rule successfully, then the way you play Nomic changes, and the game proceeds from there. Nomic is completely self-reflexive. Every rule of Nomic can be changed, including the rule that says you can change rules. In principle, Nomic can become any other game.

Formalizing Nomic is important because it yields insight in the workings of modifiable communication protocols. It is expected that such protocols will be used in the next generation of distributed computer systems, particularly in the next generation of distributed knowledge-based systems, and multi-agent systems. The advantages of modifiable communication protocols are their flexibility, i.e., their capability of adapting to new situations, and their versatility, i.e., their capability of changing communication patterns 'on the run', without human intervention.

There are no former attempts of formalizing Nomic, or Nomic-like games. Most Nomic enthusiasts seem to enjoy playing Nomic in order to experience the possibilities of different kinds of lawmaking processes, and also to exercise their ingenuity in trying to discover loopholes in the rules, which give unusual results—mostly to the benefit of the player. (This is called "scamming".) The formal aspects of Nomic seem to be less attractive. There are a few researchers, though, that have attempted to formalize rule making and legislation. For example, Carl Page (1991) proposes a formalization of Robert's rules of order (Robert, 1971), in order to determine the communication flow of decision processes in multi-agent systems (Stary, 1991).[2]

The contribution of this paper is twofold:

1. The proposal to formalize Nomic and to start research in self-modifying protocol games.

2. Three easy claims about formalizing Nomic, that serve to encourage further research in the designated field.

In Section 4, it is shown that there are proposals for which the game reaches a 'dead' state. Furthermore, it is shown that it is possible to define the proposal space (i.e., the stock of admissible proposals) in such a way that it is possible to ensure that the game will never derail into a 'dead' state. Finally, in Sect. 6, it is shown that, in the presence of a fixed logical

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[2]Robert's rules of order (RRO) comprise an authoritative description of parliamentary process. In the United States, RRO became recognized as the *definition* of parliamentary rules rather than just a distillation of them.

repository, the proposal space can be directed by dialectical principles, in which case it grows nonmonotonically.

2. Voting about rule changes

For several reasons, voting is highly suitable for studying self-modifying protocol. Firstly, a collection of voting rules is a nice 'field of discourse' to work in. One can propose simple amendments about the quorum and the percentage of votes required to pass a proposal. Furthermore, the procedure of voting already resembles existing democratic voting procedures. The most important reason, however, for studying elementary voting games, is that voting protects the rules from weird proposals. If participants *feel* that the adoption of a proposal might work out unpleasantly, they do not have to underpin their (possibly intuitive) resistance with rational arguments, but have the possibility instead to cast their vote against the proposal in question. In this way, the integrity of the rules is maintained by common sense.

3. An elementary voting game

To become familiar with the peculiarities of self-referential rule making and legislation, let us consider the following elementary voting game.

- There are $1<n$ participants, submitting proposals in turn.
- The participants proceed by means of a shared protocol, denoted by R.[3]
- We have synchronous voting, in so-called *rounds*. Each round involves a submission, a voting, and if the vote succeeds, an adoption of the proposal submitted.
- There is a limited proposal space, denoted by $\mathcal{R}$. It is always the case that $R \subseteq \mathcal{R}$.
- Votes pass by 2/3 majority.

Let us suppose that players may choose to submit either one of the following four proposals:

$$p = \lceil voting\ by\ unanimous\ consent \rceil$$
$$q = \lceil all\ proposals\ pass \rceil$$
$$r = \lceil all\ future\ proposals\ must\ be\ written\ in\ Basque \rceil$$
$$s = \lceil stop\ the\ game \rceil$$

In the process of rule making and legislation, the set of voting rules R undergoes constant change and revision:

$$R_1 \rightarrow R_2 \rightarrow ... \rightarrow R_n \rightarrow ...$$

At any time, the voting rules R_i, $1 \leq i$ form a finite and consistent set of rules and regulations. Incorporating new proposals into this set might cause rule conflicts and inconsistencies. A rule conflict can be resolved by revising the rule set. For example, if the new proposal is inconsistent with a certain subset of the set of voting rules, one or more elements of that subset must be deleted to restore consistency. The maintenance of a consistent set of voting rules is a special case of the maintenance of a consistent set of knowledge. The latter has been studied by (Martins and Shapiro, 1986), (Gärdenfors, 1988), and others.

To keep voting rules consistent is a reasonable desideratum. There are more constraints to be put on voting rules and/or new proposals. For example:

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[3] We use R (instead of P), because the protocol is governed by rules. The letter R stands for 'the rules of the game'.

- proposals *must* be consistent with existing rules[4]
- proposals must be selected from a pre-defined proposal space
- proposals must be *invertible*: it must be possible to reverse the adoption of a proposal by adopting another proposal
- the size of the rule base may not exceed 10.000 characters

These and other constraints may help to structure the voting game. A structured voting game has less chance to derail into an undesired state.

4. Some principles

The following are a number of principles to get used to the idea of a modifiable protocol. I don't claim any rigorous theory.

CLAIM 4.1. *There is a proposal $\perp$ such that, once $\perp \in R$, the development of R comes to a halt.*

Proof. Take $\perp = \lceil$ *all proposals are rejected* $\rceil$. Once this proposal is adopted and incorporated in the set of voting rules R, every proposal must be rejected henceforth. This means that every proposal to change R must be rejected as well. Hence, any further development of R comes to a halt. $\square$

The speed by which the set of rules changes, tends to increase along with the number of votes required to pass a proposal. At the one end of the spectrum lies the proposal $\perp$ for unconditional rejection. This proposal, once adopted, cause to 'freeze' the game into an immovable state. Even the proposal *'stop the game'* must be rejected, so that players are deemed to submit proposals forever! (Formally speaking.) At the other side of the spectrum lies the proposal for unconditional adoption. Once this proposal is adopted, the game 'breaks loose' into a rapid succession of unrelated states. Anything goes, until some proposal brings the game back into an easy fairway.

The alternation between 'calm' and 'turbulent' periods in Nomic can be adequately illustrated by running a computer simulation. In Figure 1 we see the graph of a simulation in which 15 participants collectively vote about either increasing or decreasing the number of votes required to pass a proposal.
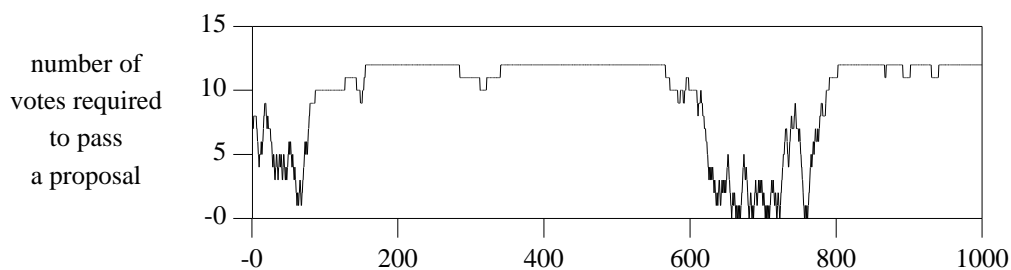

FIGURE 1. Voting about quotum

For example, if the quotum is near 15, say 14, then the chance that a next proposal will be

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[4] Suggested by Prof. John-Jules Ch. Meyer when I held a presentation on 'Open Protocol in Multi-agent Systems' at the Utrecht University (NL). Prof. Meyer refers to this option as *refinement*: new proposals may only refine existing rules.

voted for by 14 out of 15 participants is very low. We see that the game is in easy fairway near rounds number 200, 450, and 900.
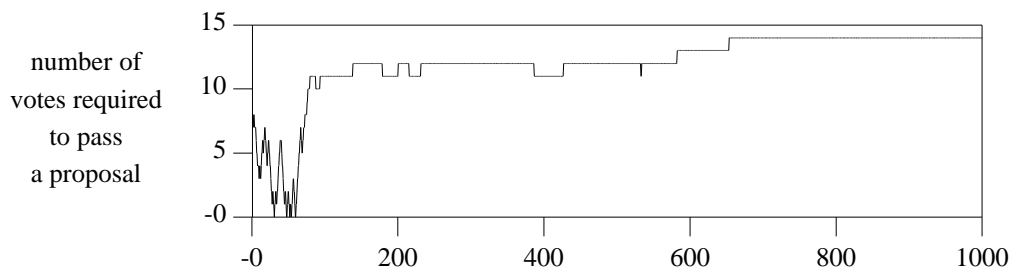
Many simulations proceed as follows:

FIGURE 2. Voting about quotum (another simulation)

Apparently, voting about the number of votes required to pass a proposal is a process that has the tendency to direct itself into calm fairways.

CLAIM 4.2. *If the proposal space $\mathcal{R}$ is equal to $\{s\}$, where $s = \lceil stop\ the\ game \rceil$, then the further development of R is predictable and may at any moment either be stopped or continued.*

Proof. The adoption of *s* may be deferred to any suitable moment. Once *s* is incorporated in R, the game stops in the succeeding round. □

If the proposal space (i.e., the stock of admissible proposals) consists of moderate proposals only, then one can almost prove that the game will never derail into a degenerate state. If, on the other hand, the proposal space contains a number of weird proposals (emptying the rule set, corrupting the rule set, etc.), the progression of the game is largely determined by the definition of voting maintained by the group.

Apparently, there is a fine line between 'safe' and 'unsafe' proposal spaces.[5]

5. Rational support for rule changes

Writing an agent that is able to play Nomic with other agents is difficult because, in 'ordinary' Nomic, a vote expresses the player's sympathy with the proposal in question. If a player feels that the proposal will have undesired effects once passed (either for him, individually, or for the game as a whole), he (or she) will vote against. Such a vote may *or may not* be the result of a player's careful analysis of the pro's and con's of adopting the proposal in question. Voting is often the result of rational *and* intuitive processes.

How can we let agents in Nomic vote in a sensible way? Intuition is not ruled out (cf. neural networks), but for simplicity I propose to let go on intuition, and to concentrate on rational voting schemes for artificial agents only.

A choice for rationality means that writing an artificial agent that is able to play Nomic involves writing a 'rational voting module' for that agent, that computes the agent's vote with

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[5]I hope that a few researchers will rise to the bait, and grasp the opportunity to formulate characterizations of 'safe' proposal spaces, or will survey the boundaries of legitimate proposal. Both are important prerequisites for a comprehensive theory of self-modifying protocol games.

respect to a proposal. My reasoning is that rationality involves rational support, that rational support involves reasoning, that reasoning involves dialectic (i.e., exchange of arguments *pro* and *con*), and that dialectic involves arguments. In this way, we arrive at a scenario where agents do not vote *on* a proposal but reason *about* a proposal. In summary, my opinion is that 'rational voting' or 'voting with arguments' is the way to go in simple versions of automated Nomic.

For example, consider the print below. It suggests a wide-area network disputation record. The agents involved deliberate on a modification of the quorum.

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AGENT_1 submits proposal LOWER_QUORUM -- proposal considered by AGENT_1 & AGENT_2 & AGENT_3
(QUORUM = 2, SO THAT IS OK)

Disputing LOWER_QUORUM

  1. (1) | AGENT_1 searches arguments for LOWER_QUORUM ...
  2. (1) | ... found 1
  3. (1) | LOWER_QUORUM <= MANY_AGENTS_BUSY DECISION_IS_NEEDED
  4. (2) || AGENT_2 accepts AGENT_1's attempt to establish LOWER_QUORUM
  5. (2) || checking whether this argument can be defeated .. yes
  6. (2) || whether it in fact has been defeated ............ no
  7. (2) || whether the contrary has been proven ............ no
  8. (2) || whether the argument submitted is pending ....... no
  9. (2) || AGENT_2 tries to block AGENT_1's establishment of LOWER_QUORUM
 10. (2) || by inspecting sub-arguments:
 11. (2) || initial target for AGENT_2: argument supporting LOWER_QUORUM [at 3]
 12. (2) || AGENT_2 searches arguments for NOT LOWER_QUORUM ...
 13. (2) || ... found 1
 14. (2) || NOT LOWER_QUORUM <-
 15. (2) ||                RAISE_QUORUM <=
 16. (2) ||                              POTENTIAL_CONSPIRACY <= NOT COMMITMENT
 17. (3) ||| AGENT_3 receives AGENT_2's attempt to establish NOT LOWER_QUORUM, checking
 18. (3) ||| whether it is as strong as AGENT_1's nr. 1 [at 3] ... no:
 19. (3) ||| argument nr. 1 [at 14] is too weak to defeat AGENT_1's
 20. (3) ||| argument for LOWER_QUORUM [at 3]
 21. (2) || AGENT_2 loses argument nr. 1 [at 14] against LOWER_QUORUM
 22. (2) || no more arguments against LOWER_QUORUM
 23. (2) || AGENT_2 backtracks
 24. (2) || new target for AGENT_2: sub-argument supporting MANY_AGENTS_BUSY [at 3]
 25. (2) || passed over because MANY_AGENTS_BUSY is a fact
 26. (2) || AGENT_2 backtracks
 27. (2) || new target for AGENT_2: sub-argument supporting DECISION_IS_NEEDED [at 3]
 28. (2) || passed over because DECISION_IS_NEEDED is a fact
 29. (2) || AGENT_2 backtracks
 30. (2) || inspected all sub-arguments of AGENT_1's argument for LOWER_QUORUM
 31. (2) || in effect, AGENT_2 agrees to AGENT_1's nr. 1 [at 3]
 32. (1) | AGENT_1 wins main-argument nr. 1 [at 3] for LOWER_QUORUM

LOWER_QUORUM established in dispute

AGENT_1 & AGENT_2 & AGENT_3 decide to LOWER_QUORUM
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Not every proposal need to be the conclusion of an argument. If it is, we call a proposal *rational*. Thus, a proposal is rational if it is supported by one or more arguments. Moreover, not every rational proposal emerges victorious in dispute. We call a proposal *warranted* if it is the conclusion of an irrefutable argument, i.e., an argument that survives dispute successfully. A collection of proposals is called warranted if each of its elements is warranted. For instance, the proposal LOWER_QUORUM has survived dispute successfully, because its supporting argument is irrefutable. It is therefore a warranted proposal.

6. Another principle

My last claim characterizes the directive impact of dialectic on the rules of procedure.

CLAIM 6.1. *If proposals must be conclusions of arguments that are constructed with the help of a fixed logical repository, then the proposal space $\mathcal{R}$ is a fixed collection of rational proposals. Furthermore, if proposals are adopted if and only if they are supported by irrefutable arguments, then the set of procedural rules R grows nonmonotonically over time, and is warranted.*

Proof. By definition, a rational claim is supported by one or more arguments. Further, the set $\mathcal{R}$ is fixed, because we have assumed a fixed logical repository. A fixed logical repository implies fixed rules, fixed arguments, and, hence, fixed rational claims. Further, a warranted claim is by definition an element $r \in R$ that is supported by one or more irrefutable arguments. Remains to show that R grows nonmonotonically over time. This phenomenon can be summoned by fabricating a so-called self-contradictory point of order. One example of such a self-contradictory point of order is $r = \lceil remote\ opposition \rceil$, combined with the situation in which the local host has arguments for remote opposition, while remote opponents adduce stronger arguments against remote opposition. The initiator then alternates between the propositions r and $\neg r$. As a result, the set R grows nonmonotonically. $\square$

7. Conclusion

The intention of this paper has been to stimulate further research in formal theories of communication, in which the protocol is modifiable by the users of that protocol. I have done so by proposing to work on a formalization of Nomic. This is by no means an easy endeavor. By pointing at a few problems that one may encounter during formalization, and by proving three modest claims about self-modifying protocol games, I hope to have challenged at least a few researchers to improve my results, thus heading collectively towards a rigorous theory of modifiable protocol.

References

1. Hofstadter, D.R., (June 1982). About Nomic: A Heroic Game That Explores the Reflexivity of the Law, in: *Scientific American*, No. 246, pp. 16-28.

2. Hofstadter, D.R., (1985). *Metamagical Themas: questing for the Essence of Mind and Pattern,* Basic Books, New York.

3. Suber, P., (1990). *The Paradox of Self-Amendment, A Study of Logic, Law, Omnipotence, and Change.,* Peter Lang Publishing.

4. Page, C.V., (1991). Principles for Democratic Control of Bounded-Rational, Distributed, Knowledge Agents, in: *Proceedings of the European Simulation Multiconference*, pp. 359-361.

5. Robert, H.M., (1971). *Robert's rules of order Revised—The classic 1915 edition,* Morrow paperback editions, New York. Foreword by General Henry M. Robert III.

6. Stary, C., (1991). Modeling Decision Support for Rational Agents, in: *Proceedings of the European Simulation Multiconference*, ed. E. Mosekilde, pp. 351-356.

7. Gärdenfors, P., (1988). *Knowledge in Flux: Modelling the dynamics of epistemic states,* Bradford book, MIT Press, London.

8. Martins, J.P. and Shapiro, S.C., (1986). Theoretical Foundations for Belief Revision,

in: *Proceedings of the First Conference on Theoretical Aspects of Reasoning about Knowledge*, ed. J.Y. Halpern, Bradford book, pp. 383-398.

9.   Vreeswijk, G.A.W. and Wan, F.D.M., Open Protocol in Multi-agent Systems: Notes and Premature Formalization, (1994). submitted, Vakgroep Informatica (FdAW), Rijksuniversiteit Limburg, Maastricht, The Netherlands. Available via anonymous ftp from ftp.cs.rulimburg.nl :/pub/papers/vreeswyk.

10.   Vreeswijk, G.A.W., (1993). Defeasible Dialectics: A Controversy-Oriented Approach towards Defeasible Argumentation, in: *The Journal of Logic and Computation*, Vol. 3, No. 3, pp. 3-27. Available via anonymous ftp from ftp.cs.rulimburg.nl :/pub/papers/vreeswyk.

Initial rules of Nomic

1. Immutable rules

101.   All players must always abide by all the rules then in effect, in the form in which they are then in effect. The rules in the Initial Set are in effect whenever a game begins. The Initial Set consists of Rules 101-116 (immutable) and 201-213 (mutable).

102.   Initially, rules in the 100's are immutable and rules in the 200's are mutable. Rules subsequently enacted or transmuted (i.e. changed from immutable to mutable or vice versa) may be immutable or mutable regardless of their numbers, and rules in the Initial Set may be transmuted regardless of their numbers.

103.   A rule change is any of the following:

    (1)   The enactment, repeal, or amendment of a mutable rule;

    (2)   The enactment, repeal, or amendment of an amendment;

    (3)   The transmutation of an immutable rule into a mutable rule, or vice versa.

    (Note: This definition implies that, at least initially, all new rules are mutable. Immutable rules, as long as they are immutable, may not be amended or repealed; mutable rules, as long as they are mutable, may be amended or repealed. No rule is absolutely immune to change.)

104.   All rule changes proposed in the proper way shall be voted on. They will be adopted if and only if they receive the required number of votes.

105.   Every player is an eligible voter. Every eligible voter must participate in every vote on rule changes.

106.   Any proposed rule change must be written down before it is voted on. If adopted, it must guide play in the form in which it was voted on.

107.   No rule change may take effect earlier than the moment of the completion of the vote that adopted it, even if its wording explicitly states otherwise. No rule change may have retroactive application.

108.   Each proposed rule change shall be given a rank-order number (ordinal number) for reference. The numbers shall begin with 301, and each rule change proposed in the proper way shall receive the next successive integer, whether or not the proposal is adopted.

    If a rule is repealed and then re-enacted, it receives the ordinal number of the proposal to re-enact it. If a rule is amended or transmuted, it receives the ordinal number of the proposal to amend or transmute it. If an amendment is amended or repealed, the entire rule of which it is a part receives the ordinal number of the proposal to amend or repeal the amendment.

109.   Rule changes that transmute immutable rules into mutable rules may be adopted if and only if the vote is unanimous among the eligible voters.

110.   Mutable rules that are inconsistent in any way with some immutable rule (except by proposing to transmute it) are wholly void and without effect. They do not implicitly transmute immutable rules into mutable rules and at the same time amend them.

Rule changes that transmute immutable rules into mutable rules will be effective if and only if they explicitly state their transmuting effect.

111. If a rule change as proposed is unclear, ambiguous, paradoxical, or clearly destructive of play, or if it arguably consists of two or more rule changes compounded or is an amendment that makes no difference, or if it is otherwise of questionable value, then the other players may suggest amendments or argue against the proposal before the vote. A reasonable amount of time must be allowed for this debate. The proponent decides the final form in which the proposal is to be voted on and decides the time to end debate and vote. The only cure for a bad proposal is prevention: a negative vote.

112. The state of affairs that constitutes winning may not be changed from achieving n points to any other state of affairs. However, the magnitude of n and the means of earning points may be changed, and rules that establish a winner when play cannot continue may be enacted and (while they are mutable) be amended or repealed.

113. A player always has the option to forfeit the game rather than continue to play or incur a game penalty. No penalty worse than losing, in the judgement of the player to incur it, may be imposed.

114. There must always be at least one mutable rule. The adoption of rule changes must never become completely impermissible.

115. Rule changes that affect rules needed to allow or apply rule changes are as permissible as other rule changes. Even rule changes that amend or repeal their own authority are permissible. No rule change or type of move is impermissible solely on account of the self-reference or self-application of a rule.

116. Whatever is not explicitly prohibited or regulated by a rule is permitted and unregulated, with the sole exception of changing the rules, which is permitted only when a rule or set of rules explicitly or implicitly permits it.

2. *Mutable rules*

201. Players shall alternate in clockwise order, taking one whole turn apiece. Turns may not be skipped or passed, and parts of turns may not be omitted. All players begin with zero points.

202. One turn consists of two parts, in this order:

(1) Proposing one rule change and having it voted on;

(2) Throwing one die once and adding the number of its points on its face to one's score.

203. A rule change is adopted if and only if the vote is unanimous among the eligible voters.

204. If and when rule changes can be adopted without unanimity, the players who vote agains winning proposals shall receive 10 points apiece.

205. An adopted rule change takes full effect at the moment of the completion of the vote that adopted it.

206. When a proposed rule change is defeated, the player who proposed it loses 10 points.

207. Each player always has exactly one vote.

208. The winner is the first player to achieve 100 (positive) points.

209. At no time may there be more than 25 mutable rules.

210. Players may not conspire or consult on the making of future rule changes unless they are teammates.

211. If two or more mutable rules conflict with one another, or if two or more immutable rules conflict with one another, then the rule with the lowest ordinal number takes precedence.

    If at least one of the rules in conflict explicitly says of itself that it defers to another rule (or type of rule) or takes precedence over another rule (or type of rule), then such provisions shall supersede the numerical method for determining precedence.

    If two or more rules claim to take precedence over one another or to defer to one another, then the numerical method must again govern.

212. If players disagree about the legality of a move or the interpretation or application of a rule, then the player preceding the one moving is to be the Judge and to decide the question. Disagreement, for the purposes of this rule, may be created by the insistence of any player. Such a process is called *invoking judgement*.

    When judgement has been invoked, the next player may not begin his or her turn own turn without the consent of a majority of the other players.

    The Judge's judgement may be overruled only by a unanimous vote of the other players, taken before the next turn is begun. If a Judge's judgement is overruled, the player preceding the Judge in the playing order becomes the new Judge for the question, except that no player is to be Judge during his own turn or during the turn of a teammate.

    Unless a Judge is overruled, one Judge settles all questions arising from the game until the next turn is begun, including questions as to his or her own legitimacy and jurisdiction as Judge.

    New Judges are not bound by the decisions of old Judges. New Judges may, however, settle only those questions on which the players currently disagree and that affect the completion of the turn in which judgement was invoked. All decisions by Judges shall be in accordance with all the rules then in effect; but when the rules are silent, inconsistent, or unclear on the point at issue, then the Judge's only guides shall be common morality, common logic, and the spirit of the game.

213. If the rules are changed so that further play is impossible, or if the legality of a move is impossible to determine with finality, or if by the Judge's best reasoning, not overruled, a move appears equally legal and illegal, then the first player who is unable to complete a turn is the winner.

    This rule takes precedence over every other rule determining the winner.