

Breadth Exam in Numerical Analysis (Fall 2005)

NOTE: This is a two hour closed book exam. Please write on one side of each answer sheet only. Please mark your answer sheets with consecutive numbers. Please do not write your name on any page of the answer sheet, just write your assigned code.

1. (20 points) A table of values (x_k, y_k) , where $k = 0, 1, \dots, m$, is obtained from an experiment. When plotted on semilogarithmic graph paper, the points lie nearly on a straight line, implying that $y \approx e^{ax+b}$. Suggest a simple procedure for obtaining parameters a and b .
2. (20 points) The most commonly used method to compute a QR factorization of a matrix A is the classic Gram-Schmidt procedure. Show that QR factorization will not breakdown if and only if the column vectors of A are linearly independent.
3. (20 points) (i) Construct a polynomial of degree ≤ 4 that interpolates the data

x	0	1	2	3	4
y	1	-1	1	-1	1

- (ii) What would be the corresponding polynomial if all values y_i were equal 1 (instead of ± 1 as above)? (You can get p_4 easily without any computations.)
4. (20 points) Suppose we have an iterative process so that the computed approximations $\{x_n\}_{n=0}^{\infty}$ converge to the solution α . Suppose also that the convergence is faster than a linear one, i.e., that

$$|x_{n+1} - \alpha| \leq c_n |x_n - \alpha| \quad \text{with} \quad \lim_{n \rightarrow \infty} c_n = 0.$$

Show that then

$$\lim_{n \rightarrow \infty} \frac{|x_n - \alpha|}{|x_n - x_{n+1}|} = 1,$$

i.e., $|x_{n+1} - x_n|$ is a good approximation to $|x_n - \alpha|$.

5. (20 points) Find a, b, c such that the rule $af(x) + bf(x+h) + cf(x+2h)$ approximates the second derivative $f''(x)$ with the error $O(h^p)$ for as large p as possible. What is the value of p ?