

# Ramsey-type results for Gallai colorings <sup>\*</sup>

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## Abstract

A Gallai-coloring (G-coloring) is a generalization of 2-colorings of edges of complete graphs: a G-coloring of a complete graph is an edge coloring such that no triangle is colored with three distinct colors.

Here we extend some results known earlier for 2-colorings to G-colorings. We prove that in every G-coloring of  $K_n$  there exists each of the following: 1. a monochromatic double star with at least  $\frac{3n+1}{4}$  vertices; 2. a monochromatic subgraph  $H$  such that all pairs of  $X \subset V(K_n)$  are at distance at most two in  $H$  where  $|X| \geq \lceil \frac{3n}{4} \rceil$ ; 3. a monochromatic diameter two subgraph with at least  $\lceil \frac{3n}{4} \rceil$  vertices.

We also investigate Ramsey numbers of graphs in G-colorings with a given number of colors. For any graph  $H$  let  $RG(r, H)$  be the minimum  $m$  such that in every G-coloring of  $K_m$  with  $r$  colors, there is a monochromatic copy of  $H$ . We show that for fixed  $H$ ,  $RG(r, H)$  is exponential in  $r$  if  $H$  is not bipartite; linear in  $r$  if  $H$  is bipartite but not a star; constant (does not depend on  $r$ ) if  $H$  is a star (and we determine its value). Somewhat surprisingly,  $RG(r, K_3)$  can be determined exactly.

## 1 Introduction, Gallai colorings.

We consider edge colorings of complete graphs in which no triangle is colored with three distinct colors. In [16] such colorings were called Gallai partitions, in [12] the term Gallai colorings was used for them. The reason for this terminology stems from its close connection to results of Gallai on comparability graphs [10]. The following theorem expresses the key property of Gallai colorings (abbreviated as *G-colorings* from now on). It is stated implicitly in [10] and appeared in various forms, [5], [6], [12]. The following formulation is from [12].

**Theorem 1.** *Any G-coloring can be obtained by substituting complete graphs with G-colorings into vertices of a 2-colored nontrivial complete graph called the base graph of the G-coloring.*

Substitution in Theorem 1 means replacements of vertices of the base graph by complete graphs so that all edges between replaced vertices retain their colors. It is important that the base graph is nontrivial, i.e. has at least two vertices. Theorem 1 is the main tool to prove results for G-colorings. For example - a first exercise in graph theory, (a remark of Erdős and Rado) is that any 2-colored complete graph has a monochromatic spanning tree. This remains true for G-colorings as proved directly in [1]. Another proof comes from Theorem 1: it is true for the base graph and substitution preserves connectivity. Another example - in fact a strengthening of the

previous remark: in every  $G$ -coloring of a complete graph there is a monochromatic spanning subgraph of diameter at most three. This follows immediately from its 2-color version, [19]. A third easy example is from [12]: in every  $G$ -coloring of  $K_n$  there is a monochromatic spanning tree with height at most two.

Sometimes more work is needed to extend a result from 2-colorings to  $G$ -colorings. A result of Burr - proving a conjecture of Bialostocki, Dierker and Voxman - says that there is a monochromatic spanning broom in every 2-coloring of a complete graph. Following Burr's nice argument on the base graph, it is possible to extend this result to  $G$ -colorings [12].

Theorem 1 is used to extend Lovász's perfect graph theorem to  $G$ -colorings, see [6], [16].  $G$ -colorings can also be considered as special cases of anti-Ramsey problems introduced in [9].

A *double star* is a tree obtained from two disjoint stars joining their centers with an edge  $e$ , that is called the base edge of the double star. Our first result here is

**Theorem 2.** *Every  $G$ -coloring of  $K_n$  contains a monochromatic double star with at least  $\frac{3n+1}{4}$  vertices. This is asymptotically best possible.*

The 2-color version of Theorem 2 was proved in [13], it slightly extends a special case of a result in [7]: in every 2-coloring of  $K_n$  there are two points,  $v, w$  and a color, say red, such that the size of the union of the closed neighborhoods of  $v, w$  in red is at least  $\frac{3n+1}{4}$ . The slight extension is that one can also guarantee that the edge  $vw$  is red. Theorem 2 is asymptotically best possible (see [7]). It is worth noting that stars behave differently: while 2-colorings of  $K_n$  obviously contain monochromatic stars of at least  $\frac{n}{2}$  vertices,  $G$ -colorings may contain only monochromatic stars with at most  $\frac{2n}{5} + 1$  vertices ([12]). Our next theorem extends a result of [11] from 2-colorings to  $G$ -colorings.

**Theorem 3.** *In every  $G$ -coloring of  $K_n$  there is a color, say red, and a subgraph  $H \subset G$  such that  $|V(H)| \geq \lceil \frac{3n}{4} \rceil$  and between any pair of vertices in  $V(H)$  there is a red path of length at most two in  $G$ . This is best possible for every  $n$ .*

Notice that the subgraph  $H$  in Theorem 3 is not necessarily a diameter two red subgraph because the midpoints of the connecting 2-paths can be in  $V(K_n) \setminus V(H)$ . However, such a stronger result is proved by Erdős and Fowler [8] for 2-colorings.

**Theorem 4.** *(Erdős, Fowler) In every 2-coloring of  $K_n$  there is a monochromatic diameter two subgraph with at least  $\lceil \frac{3n}{4} \rceil$  vertices. This is best possible for every  $n$ .*

The Erdős - Fowler theorem can also be extended to  $G$ -colorings.

**Theorem 5.** *In every  $G$ -coloring of  $K_n$  there is a monochromatic diameter two subgraph with at least  $\lceil \frac{3n}{4} \rceil$  vertices. This is best possible for every  $n$ .*

The construction in [11],[8] shows that Theorems 3, 4, 5 are sharp: consider a 2-coloring of  $K_4$  with both color classes isomorphic to  $P_4$ . Then substitute nearly equal vertex sets into this coloring with a total of  $n$  vertices. (The colorings within the substituted parts are arbitrary.)

Since for every  $n$  there is a (canonical)  $G$ -coloring of  $K_n$  where each of the  $n - 1$  color classes is a star, for most  $H$  there is a  $G$ -coloring of  $K_n$  which does not have a monochromatic copy of  $H$ . However, we may define for any graph  $H$  a kind of restricted Ramsey number,  $RG(r, H)$ , the minimum  $m$  such that in every  $G$ -coloring of  $K_m$  with  $r$  colors, there is a monochromatic copy of  $H$ .

It turns out that some classical Ramsey numbers that seem hopelessly difficult to determine even asymptotically, behave nicely if we restrict ourselves to  $G$ -colorings with  $r$ -colors. For example, the Ramsey number of a triangle in  $r$ -colorings,  $R(r, K_3)$  is known to be only between bounds far apart ( $c^r$  and  $\lfloor er! \rfloor + 1$ , see for example in [18]) but it is not hard to determine  $RG(r, K_3)$  *exactly* as follows.

**Theorem 6.**

$$RG(r, K_3) = \begin{cases} 5^k + 1 & \text{for } r = 2k \\ 2 \times 5^k + 1 & \text{for } r = 2k + 1 \end{cases}$$

It is worth noting that there are several “extremal” colorings for Theorem 6. For example, let  $G_1$  be a black edge and let  $G_2$  be the  $K_5$  partitioned into a red and a blue pentagon. The graphs  $H_1, H_2$  obtained by substituting  $G_1$  ( $G_2$ ) into vertices of  $G_2$  ( $G_1$ ) have essentially different 3-colorings and both are extremal for  $r = 3$  in Theorem 6.

Although one can easily determine some more exact values of  $RG(r, H)$  for small graphs  $H$ , we conclude with a general result and a very special one.

**Theorem 7.** *Assume that  $H$  is a fixed graph without isolated vertices. Then  $RG(r, H)$  is exponential in  $r$  if  $H$  is not bipartite and linear in  $r$  if  $H$  is bipartite and not a star.*

**Theorem 8.** *If  $H = K_{1,p}$  is a star and  $r \geq 3$  then  $RG(r, H) = \frac{5p-1}{2}$  for odd  $p$ ,  $RG(r, H) = \frac{5p}{2} - 3$  for even  $p$ .*

For completeness of the star case, notice that for  $H = K_{1,p}$  we have trivially  $RG(1, H) = R(1, H) = p + 1$  and  $RG(2, H) = R(2, H)$  can be determined easily ( $2p - 1$  for even  $p$  and  $2p$  for odd  $p$ , [14]). It is also worth noting that while  $RG(r, H)$  is constant (does not depend on  $r$ ),  $R(r, H)$  is linear in  $r$  (and in  $p$ ), see [4].

## 2 Extending 2-coloring results to weighted complete graphs.

A straightforward method to prove Theorems 2, 3 and 5 is to extend their corresponding 2-color versions to graphs with positive integer vertex weights. That is what we shall do in the next three theorems. Let  $w(x)$  denote the weight of  $x \in V(G)$  and for  $X \subset V(G)$ ,  $w(X) = \sum_{x \in X} w(x)$ . The weight of a subgraph  $H \subset G$  is  $w(V(H))$ .

**Theorem 9.** *Every 2-coloring of a weighted complete graph  $K$  contains a monochromatic double star with weight at least  $\frac{3w(K)+1}{4}$ .*

**Theorem 10.** *For every 2-coloring of a weighted complete graph  $K$  there is a color, say red, and a subgraph  $H \subset K$  such that  $|w(H)| \geq \lceil \frac{3w(K)}{4} \rceil$  and between any pair of vertices in  $V(H)$  there is a red path of length at most two in  $K$ .*

**Theorem 11.** *Every 2-coloring of a weighted complete graph  $K$  contains a monochromatic diameter two subgraph with weight at least  $\lceil \frac{3w(K)}{4} \rceil$ .*

Before proving the theorems above, we show how Theorems 2, 3, 5 follow from them. Suppose that a G-coloring is given on  $K_n$ . By Theorem 1 this coloring comes by substitutions into a 2-colored nontrivial base graph. Use the cardinalities of the substituted vertex sets to weigh the vertices of the base graph and apply theorems 9, 10, 11 to get a monochromatic subgraph  $H$  with the required portion of the total weight  $n$ . Then the proofs can be finished by observing that the properties P1: “having a spanning monochromatic double star”, P2: “there is a monochromatic path of length at most two between any pair of vertices in  $V(H)$ ” and P3: “having a spanning monochromatic diameter two subgraph” are preserved by substitutions - i.e.  $H$  corresponds to a monochromatic subgraph of  $K_n$  that has  $w(H)$  vertices.

**Proof of Theorem 9:** Let  $G_1, G_2$  denote the graphs spanned by the edges of color 1,2, respectively. The open neighborhood of a vertex  $x \in V(G_i)$  is denoted by  $\Gamma_i(x)$ . Let  $v \in V(K)$  and  $A = \Gamma_1(v), B = \Gamma_2(v)$ . The graphs  $G_1, G_2$  decompose the complete bipartite graph  $[A, B]$  into bipartite graphs  $H_1, H_2$ . We shall prove the following claim. Either there exists  $a \in A$  such that

$$w(B \setminus \Gamma_1(a)) \leq \frac{w(A) + w(B)}{4}$$

or there exists  $b \in B$  such that

$$w(A \setminus \Gamma_2(b)) \leq \frac{w(A) + w(B)}{4}.$$

To prove the claim, suppose that  $w(B \setminus \Gamma_1(a)) \geq t, w(A \setminus \Gamma_2(b)) \geq t$  holds for all  $a \in A, b \in B$ .

Then

$$\begin{aligned}
w(A)w(B) &= \sum_{a \in A, b \in B} w(a)w(b) = \sum_{ab \in E(H_1)} w(a)w(b) + \sum_{ab \in E(H_2)} w(a)w(b) = \\
&= \sum_{a \in A} w(a) \sum_{b \in B \cap \Gamma_1(a)} w(b) + \sum_{b \in B} w(b) \sum_{a \in A \cap \Gamma_2(b)} w(a) \leq \\
&\leq \sum_{a \in A} w(a)(w(B) - t) + \sum_{b \in B} w(b)(w(A) - t) = \\
&= w(A)(w(B) - t) + w(B)(w(A) - t) = 2w(A)w(B) - t(w(A) + w(B))
\end{aligned}$$

therefore  $t(w(A) + w(B)) \leq w(A)w(B)$  and

$$t \leq \frac{w(A)w(B)}{w(A) + w(B)} \leq \frac{(w(A) + w(B))^2}{4(w(A) + w(B))} = \frac{w(A) + w(B)}{4},$$

proving the claim.

Selecting either  $a$  or  $b$  from the claim and selecting the base edge of the double star accordingly as  $va$  or  $vb$ , we get a monochromatic double star whose weight is at least

$$w(A) + w(B) + w(v) - \frac{w(A) + w(B)}{4} = \frac{3w(K) + w(v)}{4}.$$

□

**Proof of Theorem 10:** The proof follows the argument in [11]. Consider a red-blue coloring of the edges of a weighted complete graph  $K$ . An edge  $e = xy$  in  $E(K)$  is called a red (blue) *spanner* if each vertex of  $K$  is adjacent in red (blue) to at least one vertex of  $\{x, y\}$ .

**Lemma 1.** *Assume that in a 2-colored complete graph there is at least one red and at least one blue spanner edge. Then the red and blue spanner edges form vertex disjoint bipartite graphs.*

**Proof:** Suppose that  $xy$  is a red spanner and  $zy$  is a blue spanner. Then the edge  $xz$  can not have a color, showing that the red and blue spanners form vertex disjoint subgraphs.

Assume that  $C$  is a cycle formed by red spanner edges. Let  $e$  be a blue spanner edge,  $e$  is vertex disjoint from  $C$ . Each vertex of  $C$  is adjacent to some end of  $e$  in blue from the definition of  $e$ . But two consecutive vertices of  $C$  can not be adjacent in blue to the same end of  $e$  because the edges of  $C$  are red spanners. This is possible only if  $C$  is an even cycle, thus the red spanners form a bipartite graph. Applying

the same argument to cycles formed by blue spanner edges, the proof of the lemma is finished.  $\square$

Now the proof of the theorem is finished as follows. If there is no spanner in one of the colors then  $K$  has diameter two in the other color, so  $H = K$  satisfies the requirements of the theorem. Otherwise, by Lemma 1, the spanner edges form two vertex disjoint bipartite graphs. Among their four partite classes select one, say  $Z$ , with the smallest weight (resolving ties arbitrarily), w.l.o.g.  $Z$  is a partite class of the bipartite graph formed by the red spanner edges. Consider  $X = V(K) \setminus Z$ . Clearly  $|w(X)| \geq \lceil \frac{3w(K)}{4} \rceil$  from the definition of  $Z$ . On the other hand, for  $x, y \in X$  either  $xy$  is blue or - since  $xy$  can not be a red spanner edge - there exists  $z \in V(K)$  such that both  $zx, zy$  are blue. Thus the blue color and  $X$  satisfy the requirements.  $\square$

**Proof of Theorem 11:** The proof is implicitly in the Erdős - Fowler proof of Theorem 4 in [8]. With an argument that is similar but much more involved than the one used in the proof of Theorem 10, Erdős and Fowler proved that - unless there is a spanner in some of the colors - the vertex set of  $K$  has a partition into four parts such that the deletion of any of these parts leaves a monochromatic graph of diameter two. Therefore, deleting the part with the smallest weight proves the theorem.  $\square$

### 3 G-colorings with $r$ colors.

**Proof of Theorem 6:** Let  $f(r)$  denote the function one less than the claimed value of  $RG(r, K_3)$ . Observe that

$$f(r) \geq 2f(r-1) \tag{1}$$

for  $r \geq 2$  with equality for odd  $r$ , and

$$f(r) = 5f(r-2) \tag{2}$$

for  $r \geq 3$ .

To show that  $RG(r, K_3) > f(r)$  let  $G_1$  be a 1-colored  $K_2$  and let  $G_2$  be a 2-colored  $K_5$  with both colors forming a pentagon. Recursively construct  $G_r$  for odd  $r \geq 3$  by substituting two identically colored  $G_{r-1}$ 's into the two vertices of  $G_1$  (colored with a different color). Similarly, for even  $r \geq 4$ , let  $G_r$  be defined by substituting five identically colored  $G_{r-2}$ 's into the vertices of  $G_2$  (colored with two different colors). The  $r$ -coloring defined on  $G_r$  is a G-coloring, clearly has  $f(r)$  vertices and contains no monochromatic triangles.

We prove by induction that if a G-coloring of  $K$  with  $r$ -colors and without monochromatic triangles is given then  $|V(K)| \leq f(r)$ . Using Theorem 1, the coloring of  $K$  can be obtained by substitution into a 2-colored nontrivial base graph  $B$ . In our case clearly  $2 \leq |V(B)| \leq 5$ .

**Case 1:**  $|V(B)| = 2$ . Since there are no monochromatic triangles, the graphs substituted can not contain any edge colored with the color of the base edge, therefore, by induction, they have at most  $f(r - 1)$  vertices. Thus

$$|V(K)| \leq 2f(r - 1) \leq f(r)$$

using (1).

**Case 2:**  $|V(B)| = 3$ . The base graph has no monochromatic triangle so it has an edge  $b_1b_2$  whose color is used only once (as a color on a base edge). Then the graphs substituted into  $b_1, b_2$  must be colored with at most  $r - 2$  colors and the graph substituted into the third vertex must be colored with at most  $r - 1$  colors. Thus

$$|V(K)| \leq 2f(r - 2) + f(r - 1) \leq f(r - 1) + f(r - 1) = 2f(r - 1) \leq f(r)$$

using (1) twice.

**Case 3:**  $4 \leq |V(B)| \leq 5$ . The base graph has no monochromatic triangle so each vertex in the base is incident to edges of both colors. Therefore

$$|V(K)| \leq |V(B)|f(r - 2) \leq 5f(r - 2) = f(r)$$

using (2).  $\square$

**Proof of Theorem 7:** First we give an upper bound on  $RG(r, H)$  that is exponential in  $r$  by showing  $RG(r, H) \leq t^{(n-1)r+1}$  where  $t = R(2, H) - 1$  and  $n = |V(H)|$ . We shall assume that  $|V(H)| \geq 3$  therefore  $n \geq 3, t \geq 2$ . Suppose indirectly that a G-coloring with  $r$  colors is given on  $K$ ,  $|V(K)| \geq t^{(n-1)r+1}$  but there is no monochromatic  $H$ . The base graph  $B$  of this coloring has no monochromatic  $H$  therefore  $|V(B)| \leq R(2, H) - 1 = t$ . This implies that some of the graphs, say  $G_1$ , substituted into  $B$  has at least  $t^{(n-1)r}$  vertices. Let  $v_1$  be an arbitrary vertex of  $K$  not in  $V(G_1)$ . Note that every edge from  $v_1$  to  $V(G_1)$  has the same color. Iterating this process with  $G_1$  in the role of  $K$ , one can define a sequence of vertices  $v_1, v_2, \dots, v_{(n-1)r+1}$  such that for every fixed  $i$  and  $j > i$  the colors of the edges  $v_i, v_j$  are the same. By the pigeonhole principle there is a subsequence of  $n$  vertices spanning a monochromatic complete subgraph  $K_n \subset K$  and clearly  $H$  is a monochromatic subgraph of  $K_n$  - a contradiction. Thus, for any - in particular non-bipartite -  $H$  we proved an upper bound exponential in  $r$ .

For a bipartite  $H$  assume that both color classes of  $H$  have at most  $n$  vertices. We show that  $RG(r, H) \leq pt(n - 1)$ , where  $p = (n - 1)r + 2$  ( and  $t$  is as defined earlier), providing an upper bound linear in  $r$ . Indeed, suppose indirectly that a G-coloring with  $r$  colors is given on  $K$ ,  $|V(K)| \geq pt(n - 1)$  but there is no monochromatic  $H$ . The base graph of the G-coloring has at most  $t$  vertices, otherwise we have a monochromatic  $H$ . Applying the same argument as in the previous paragraph, we



find that there is a graph  $G_1$ , substituted to some vertex of the base graph, such that  $|V(G_1)| \geq \frac{|V(K)|}{t} \geq p(n-1)$ . If  $|V(K) \setminus V(G_1)| \geq 2n-1$  then - by the pigeonhole principle - we can select  $X \subset V(K) \setminus V(G_1)$  so that  $|X| = n$  and  $[X, V(G_1)]$  is a monochromatic complete bipartite graph - this graph contains  $H$  and the proof is finished. We conclude that  $|V(G_1)| \geq pt(n-1) - 2(n-1) = (pt-2)(n-1)$ . Select  $v_1 \in V(K) \setminus V(G_1)$  and iterate the argument: into some vertex of the base graph of the G-coloring on  $G_1$  a graph  $G_2$  is substituted with at least  $\frac{|V(G_1)|}{t} \geq (p-1)(n-1)$  vertices. Selecting  $v_2 \in V(G_1) \setminus V(G_2)$  we continue until  $T = \{v_1, v_2, \dots, v_{p-1}\}$  is defined. There is still at least  $2(n-1) > n$  vertices in  $G_{p-1}$  thus selecting  $Y \subset V(G_{p-1})$  with  $|Y| = n$ , we have a complete bipartite graph  $[Y, T]$  such that from each  $v \in T$  all edges from  $Y$  to  $v$  are colored with the same color. Since  $|T| = p-1 = (n-1)r+1$ , by the pigeonhole principle there is  $Z \subset T$  such that  $|Z| = n$  and  $[Y, Z]$  is a monochromatic complete bipartite graph which obviously contains a monochromatic  $H$  - a contradiction. Thus, for bipartite  $H$  we have an upper bound linear in  $r$ .

Lower bounds of the same order of magnitude can be easily given. For a non-bipartite  $H$  it is obvious that  $RG(r, H) > 2^r$  because we can easily define a suitable G-coloring with  $r$  colors by repeatedly joining with a new color two identically colored complete graphs of the same size. In fact, for certain graphs  $H$  a more refined lower bound can be given as follows. Suppose that  $H$  is a connected graph without equivalent vertices (two vertices are equivalent if their open neighborhoods coincide). Then

$$RG(r, H) > \begin{cases} t^k & \text{for } r = 2k \\ (n-1)t^k & \text{for } r = 2k+1 \end{cases} \quad (3)$$

where  $t = R(2, H) - 1, n = |V(H)|$  as defined above. Note that for  $H = K_3$  we get the lower bound of Theorem 6. To see that (3) is right, call a G-coloring of a complete graph  $K$  with  $r$  colors optimal if it has no monochromatic  $H$  and  $|V(K)|$  agrees with the formula in (3). It is obvious that there are optimal colorings for  $r = 1, 2$ . Suppose that  $G$  is optimal for  $r-2$  colors. Take an optimal 2-colored base graph  $B$  colored with colors distinct from the colors of  $G$ . Substitute  $G$  into each vertex in  $B$ . The resulting graph is an optimal coloring with  $r$  colors.

If  $H$  is bipartite and not a star, it contains two independent edges. Then we have  $RG(r, H) > r+1$  because the canonical G-coloring of  $K_{r+1}$  with  $r$  colors (where color class  $i$  is a star with  $i$  edges) does not have a monochromatic  $H$ .  $\square$

**Proof of Theorem 8:** Assume  $H = K_{1,p}, r \geq 3$ . We use a construction and a result from [12]. To see that the claimed values of  $RG(r, H)$  can not be lowered, let  $C$  be a  $K_5$  colored with red and blue so that both color classes form a pentagon. For odd  $p$  substitute a green  $K_{\frac{p-1}{2}}$  to each vertex of  $C$ . For even  $p$  substitute  $K_{\frac{p}{2}}$  into one vertex of  $C$  and  $K_{\frac{p}{2}-1}$  to the other four vertices of  $C$ . The claimed upper bound for

$RG(r, H)$  follows immediately from the following result of [12]: any  $G$ -coloring of  $K$  contains a monochromatic star with at least  $\frac{2|V(K)|}{5}$  edges.  $\square$

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