The NRC Calculable Capacitor and Its Role in the SI

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Abstract

The National Research Council of Canada, in association with NMIA in Australia and BIPM in France, is establishing a calculable capacitor that represents a third generation of design. The calculable capacitor was original developed in the 1960s by Thompson in Australia and refined by others, notably Cutkosky at NBS. Its development caused one of the first major changes in the realization of SI electrical units in that it accurately derived a capacitance, only in terms of length and the permittivity of free space. Between 1969 and 1990 the calculable capacitor was the practical reference of the world's capacitance and resistance standards. While the discovery of the quantum Hall effect and its subsequent use as a resistance standard since1990 has lessened our dependence on calculable capacitors, they still remain important and independent determinations of the capacitance unit, the farad.

There are only a few operating calculable capacitors in the world and only a couple that can achieve uncertainties of order $5x10^{-8}$. This has been primarily due to inherent limitations in previous designs coupled with the difficulties in assessing a number of critical influences. While the dimensional tolerances of the new calculable capacitor are extremely demanding it is expected to achieve uncertainties of $\leq 10^{-8}$ and also has other properties that are unique.

We will describe the design considerations of this new design, outline some target specifications and uncertainties and review the progress to date. We will also present the contributions that such a calculable capacitor can make to SI units and to the determination of fundamental constants in light of the recently proposed changes to the SI by Mills et. al. [1].

1. Introduction

In 2005 the Electrical Standards Group of the National Research Council of Canada began a project to establish a calculable capacitor in Ottawa. This initiative is part of a larger project involving the National Metrology Institute of Australia (NMIA), in Sydney, Australia and the International Bureau of Weights and Measures (BIPM) in Sèvres, France and each of the three laboratories is planning to establish an independent calculable capacitor.

Since the 1960's calculable capacitors have had a major impact on electrical standards, having been the de facto reference for capacitance around the world. Perhaps more importantly they provided the SI reference for electrical resistance until the adoption of the quantum Hall effect in 1990. Despite their importance in electrical metrology there were very few operating calculable capacitors in the world. Furthermore, the number of operating calculable capacitors is declining as systems get older and are abandoned, being considered too difficult to maintain and operate or too expensive to rebuild or refurbish.

Recently, there have been proposals to fundamentally alter the SI system of units [2,3]. These proposals are presently being considered by different committees but there is an emerging expectation that the changes will be at least partly accepted and that they will be implemented some time in the not-too-distant future. Implementation of this proposal will not likely have a significant numerical effect on electrical measurements in that reference standards are not expected to undergo noticeable changes in value. However, the proposal will have profound effects on so-called absolute measurements; and the calculable capacitor is one of these measurements. In particular, the calculable capacitor will no longer determine a capacitance, at least not directly.

2. What is a Calculable Capacitor?

The calculable capacitor was original conceived by Thompson and Lampard [1] who proved that in cylindrical geometry in a vacuum, the mean cross capacitance between opposite segments of any closed conductor, that is divided into four segments, is given by a simple function that is only dependent on the cylindrical length and the fundamental constant ε_0, the permittivity of vacuum. Specifically the mean cross capacitance, C, is given by

$$C = (C1 + C2)/2 = (\varepsilon_0/\pi)\ \ln(2) \quad \text{F/m}$$

or about 1.95 pF/m, see Figure 1.

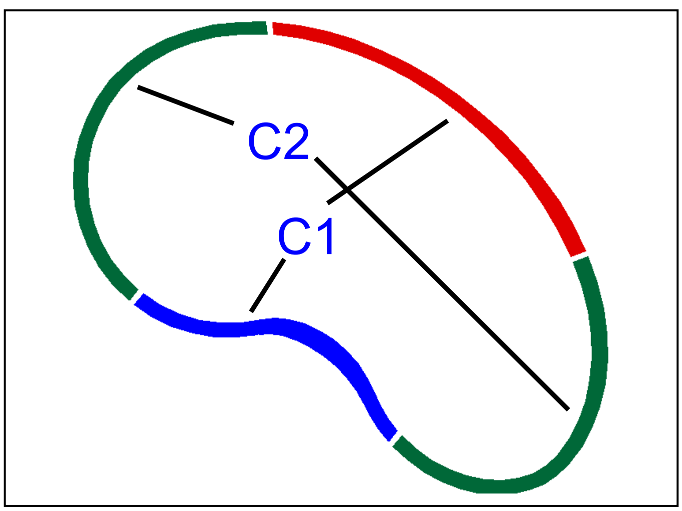

Figure 1. A cross sectional schematic of the cross capacitances of the Thompson-Lampard theorem.

In practice the calculable capacitor is typically realized as four vertical metal rods whose axial centers are placed in a square pattern as in Figure 2. The metal rods, or electrodes, are carefully machined to be of round equal cross sections and to be straight over their length. The electrical length of the device is not the length of the rods but is defined by the separation of two grounded, smaller diameter conducting tubes placed parallel to and at the center of the defining square pattern. The ends of these guard tubes are semi-silvered reflectors that form a Fabry-Perot cavity. The electrical length is determined by optical interferometry using the Fabry-Perot cavity.

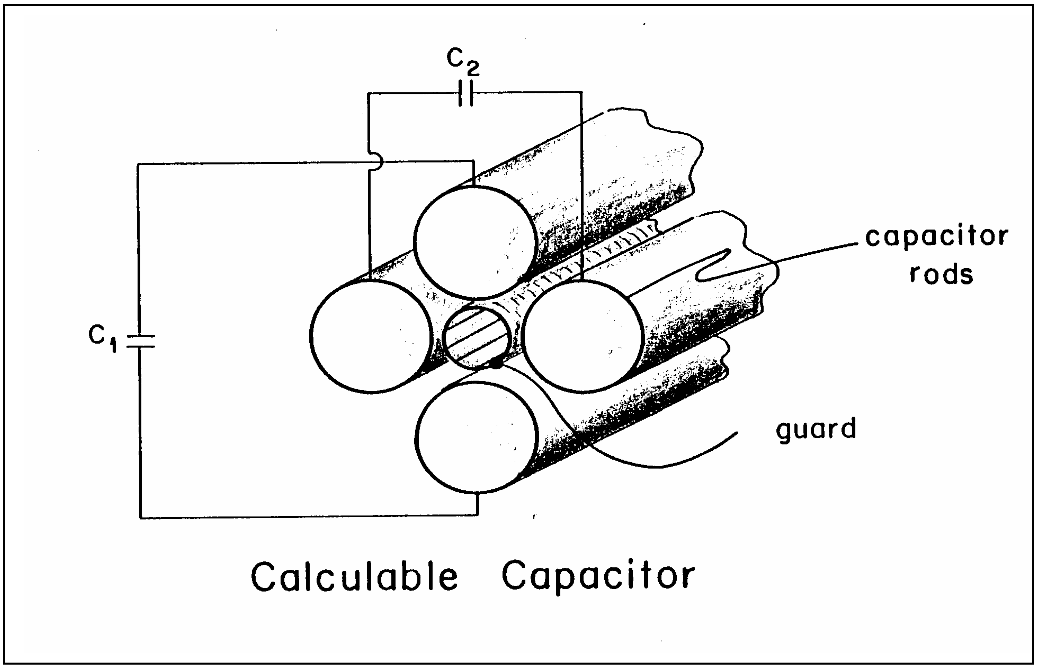

Figure 2. The four rods of a typical calculable capacitor with a grounded guard tube in the center.

As described so far, such a device would be prone to errors caused by the end effects of the guard tubes that define the electrical length. To avoid this problem the mean cross capacitance is measured for each of two different positions of one of the guard tubes. The difference between the two measured values can be related to the distance the guard tube is moved between the two positions, by means of the interferometer. This measurement is independent of any end effect so long as the general structure is symmetric.

One guard tube, usually called the fixed guard, is provided with an agile positional transducer so that the signal of the optical interferometer can be used to servo the distance between the two guards to an exact optical fringe. The fixed guard is typically positioned near one end but still inside the main electrodes. The other guard tube, the movable guard, is positioned either fairly close to the fixed guard or close to the other end of the main electrodes. A simple schematic is shown in Figure 3.

The entire assembly is placed in a vacuum and a stabilized laser of known wavelength in vacuum is used for the interferometry.

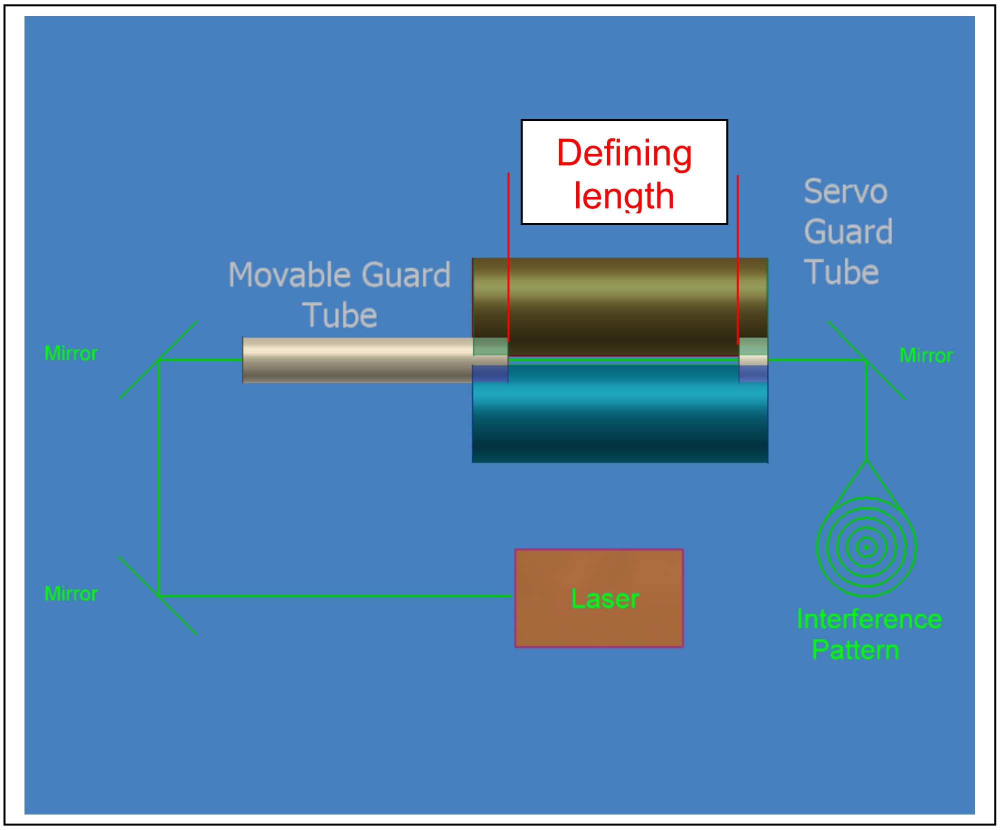

Figure 3. A schematic of a calculable capacitor

3. Calculable Capacitor Design Considerations

Less than twenty calculable capacitors have been constructed by different national measurement institutes around the world but only a very few are productive instruments. Most have fallen into disuse and are rarely operated primarily due to limitations of poor design. While the problems of different calculable capacitors vary, most can be generalized into three categories. Some have difficulty maintaining optical or electrical alignment during the movement of the central guard assembly. Others were not designed to allow adequate sensing and testing of critical dimensional or electrical properties. And some designs have poor set-ability and stability of the positioning of the guard assembly. The new design of calculable capacitor attempts to minimize all of these problems, and to provide an instrument that is reliable, stable, and easy to operate and yet provides excellent accuracy.

Much of the original design work for the new generation of calculable capacitors has been performed by G. Small and J. Fiander of NMIA and all three capacitors will employ many of their design elements. While each of the three systems will have similar construction of the main electrodes and movable guard assemblies, there will be differences in other areas. For example, the optical interferometers, capacitance measurement systems and reference standards are not planned to be the same in each of the three systems. The design is targeted for an overall fractional accuracy of at least 10^{-8} and with limits on specific individual elements to within 10^{-9}. To achieve this some of the following issues have been considered.

The dimensions of the main electrodes have been designed to optimize the rigidity of the system and yet to have sufficient operating length to provide a capacitance difference of 0.4 pF. The close-approach error (basically the interaction of two end effects when the two guards are very close) has been experimentally modeled. An optimized 'spike' has been developed to further reduce any possible small imperfections of the electrode surfaces. The obliquity error of the interferometry is related to the need to operate the Fabry-Perot through a small aperture at different spacing. Although this is only a small systematic error it is being carefully modeled for accurate correction. Careful attention has been given to fully characterizing the electrical connections and current paths within the calculable capacitor and this will be essential for accurate operation of the calculable capacitor over a frequency range of 200 Hz to 2 kHz.

The fabrication of the main electrodes, as well as the mechanism for moving the central guard tube assembly has required extensive development. New grinding and polishing techniques have been developed and further refined using a custom-made device for capacitivly characterizing the dimensional profile of each electrode. It is expected that the combination of these efforts, while not yet completed, will achieve dimensional profiles of the main electrodes which are cylindrically round and vertically straight to within 100 nm over the electrically active length of the electrode ~ 0.3 m.

The electrical measurement system is currently being designed. It features a comparison of the calculable capacitor's change in capacitance of 0.4 pF with a reference 10 pF capacitor. It is being designed to be invariant with respect to the variable shunt capacitance to ground of the calculable capacitor in its two operating positions. This should greatly reduce the time needed to re-balance the measurement bridge after changing operating positions.

4. The Present SI

The SI system of units, despite a long history, is an evolving system that regularly undergoes changes to keep pace with the developments of science. The SI was only introduced after the signing of the Convention of the Metre in 1875. In 1889 there were only four base units, the kilogram, the metre, the second and the Kelvin and all were based on artifacts. Even the second was referenced to an artifact, the rotation of the earth about its axis. It is important to realize that the principal aim of the SI was to ensure a consistent set of units that could be used worldwide.

It was not until 1928 with the adoption of the definition of the ampere that the first fundamental constant, μ_0, the permeability of vacuum was incorporated into the SI. It was done surreptitiously and without any direct mention of the terms 'fundamental constant' or 'permeability of vacuum' but its numerical value of $4\pi x 10^{-7}$ H/m was fixed exactly within the definition of the ampere.

Another unique feature of the definition of the ampere was the implicit use of a physical principal to link the electrical units to the mechanical units. In this case it was the equivalence of mechanical and electrical force while in the case of the volt it was the equivalence of mechanical and electrical power.

Other fundamental constants have also been introduced into the SI. The hyperfine splitting of ^{133}Cs was incorporated into the definition of the second in 1967 and the speed of light, *c*, was fixed in 1983 effectively re-defining the metre and altering the status and intent of the original base units.

The other base units, the candela and the mole were also introduced relatively recently but these have less impact on this particular discussion. The present SI can be represented as shown in Figure 4. The defining base units and fundamental constants are at the top of the figure and derived units are below. Not all of the SI units or their inter-relationships are shown but the more important ones are indicated.

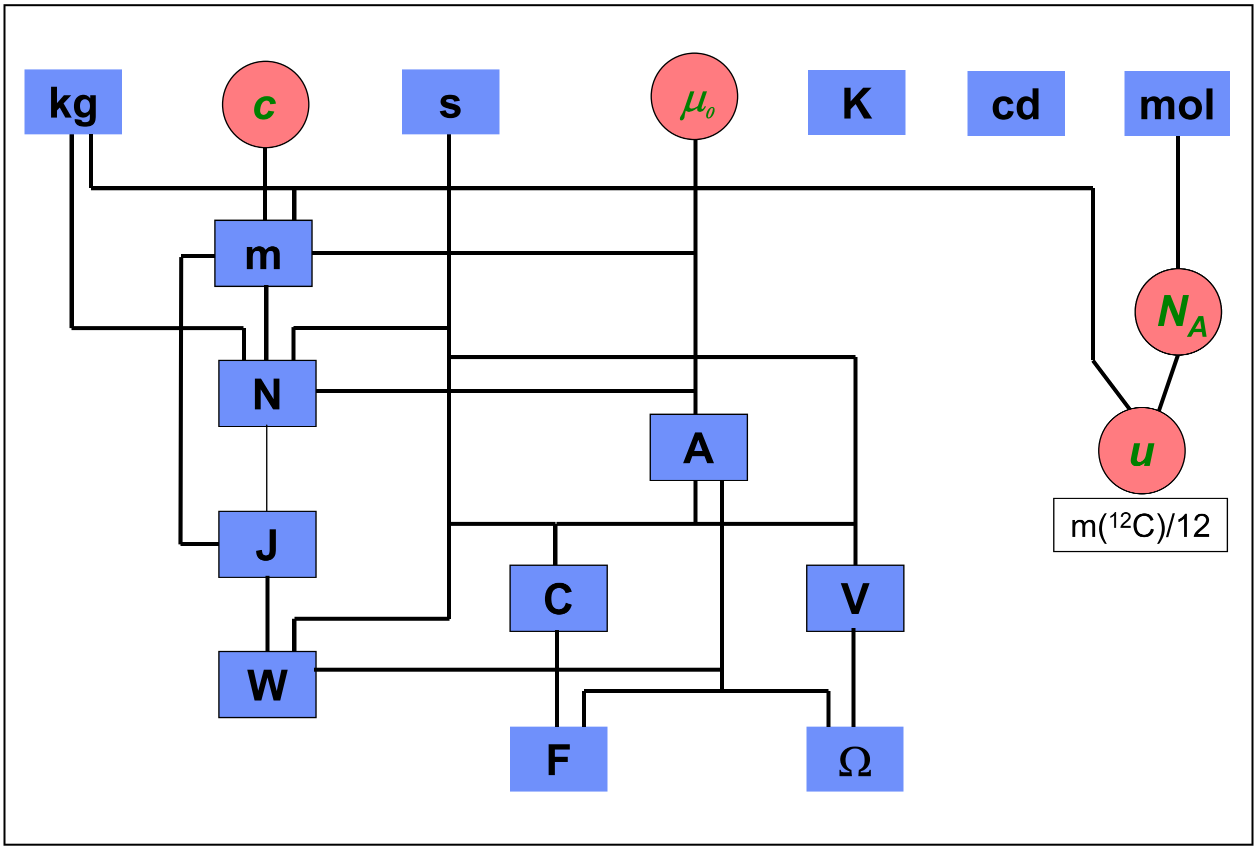

Figure 4. A schematic of the present SI, its base and derived units and defining fundamental constants.

The calculable capacitor was developed in the mid-late 1960s and resulted in a direct means of realizing the farad (and the ohm) without directly applying the principles of equivalence of electrical force or power. The discovery was never incorporated into the SI definitions but was used at the highest levels as a reference of both capacitance and resistance.

In 1990 there were again changes in the electrical units, this time not in their definitions but in the practical ways that national metrology laboratories realized consistent units of voltage and resistance. These were the adoption of the Josephson and quantum Hall effects with the conventional values of K_{J-90} and R_{K-90} as representation of the SI volt and ohm. This situation is schematically shown in Figure 5 which also shows the relationship of the calculable capacitor to the present SI.

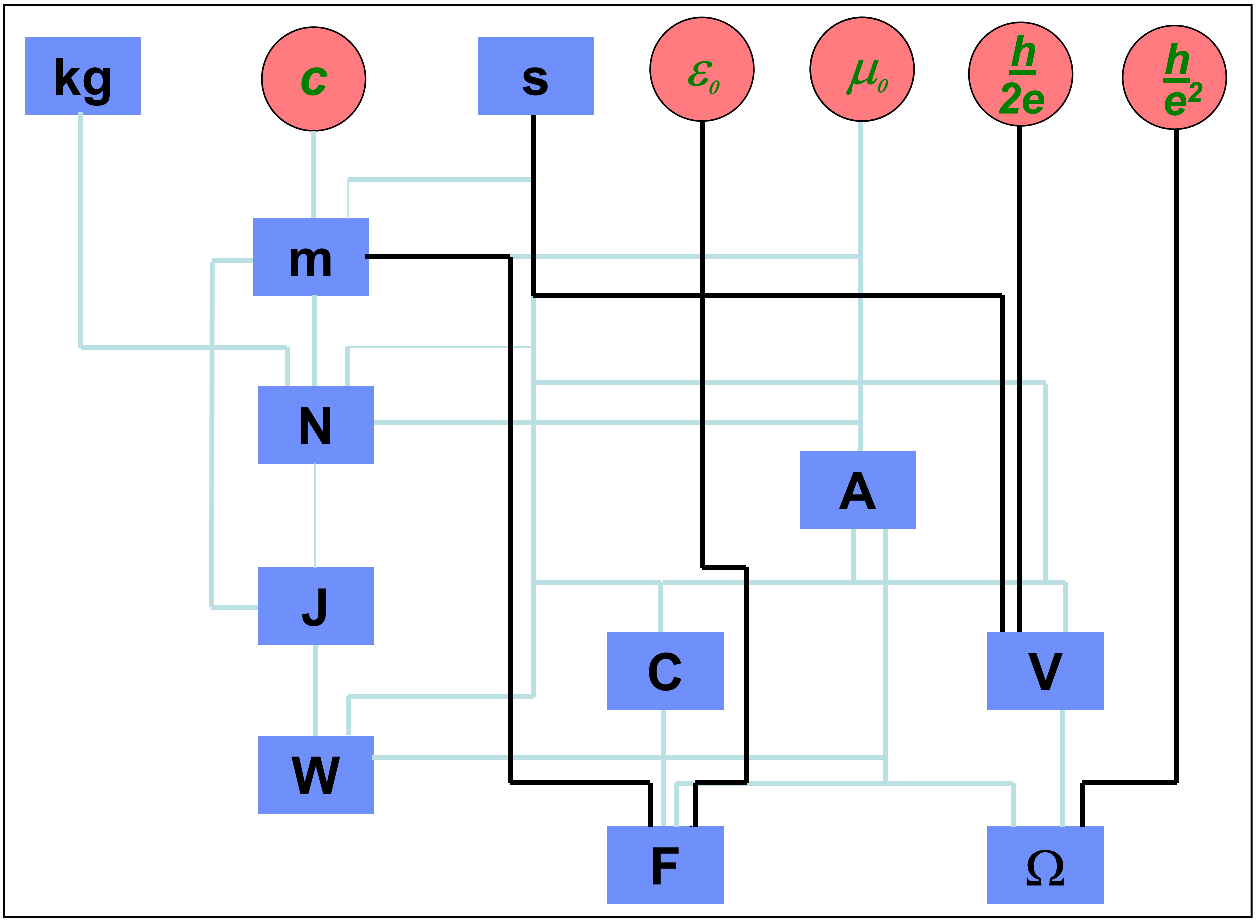

Figure 5. A schematic of the present SI, but highlighting the practical means to realize farad, volt and ohm with the calculable capacitor, Josephson volt and the quantum Hall resistance.

5. The New SI and its effect on Calculable Capacitors

Recent publications by Mills et. al. [2,3] have recommended that a substantial change be made to the SI system of units which would formally introduce four fundamental constants with exactly defined values and with no uncertainties. This would be similar to what was done in 1983 with the speed of light and the definition of the metre but would affect the kilogram, ampere, kelvin and mole. The four fundamental constants are the Planck constant h, the elementary charge e, the Boltzman constant k, and the Avogardo constant N_A. This proposal is being seriously considered and initial recommendations concerning this subject have been already been issued by the CODATA Task Group on Fundamental Constants and the Consultative Committees of Units, Mass and Electricity and Magnetism.

By assigning exact values to Planck constant h, the elementary charge e, the Boltzman constant k, and the Avogardo constant N_A and incorporating them directly into the SI, both the definitions of the existing SI units and their inter-relationships will change. Figure 6 is a diagram of the structure and hierarchy of this newly proposed SI system.

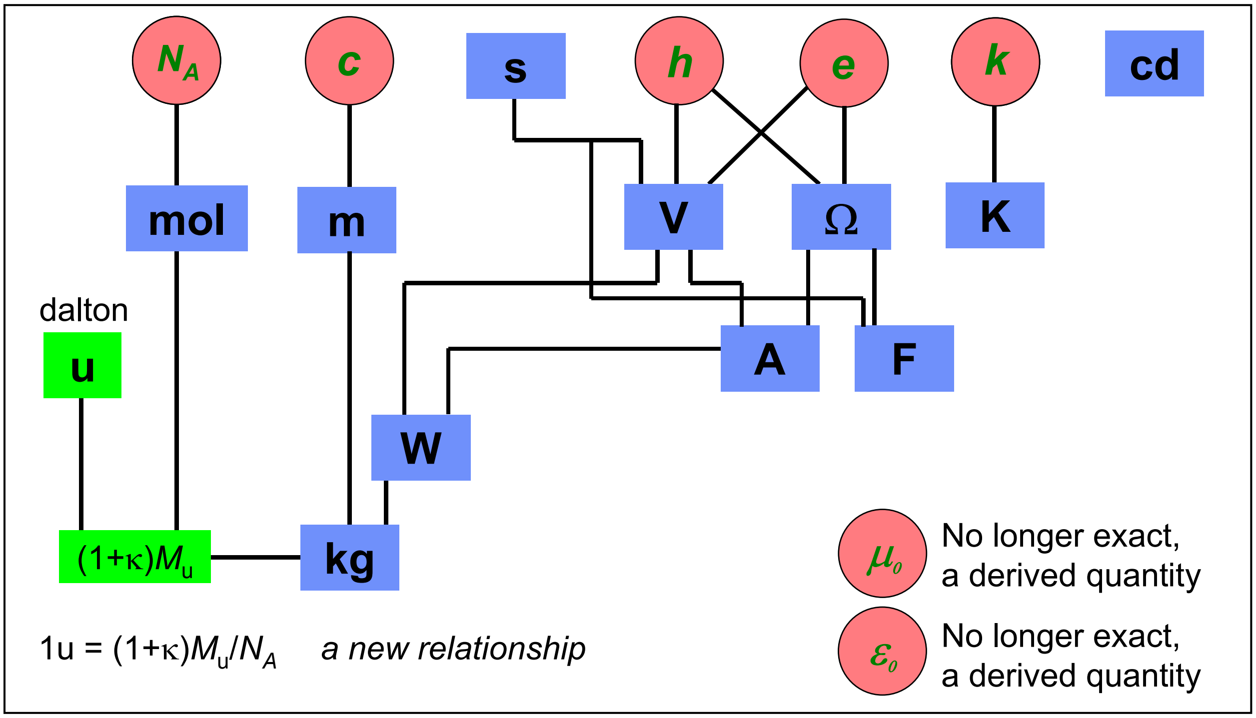

Figure 6. A schematic of the proposed SI and its relationship to the defining fundamental constants and common SI units.

There is insufficient time to detail all of the impacts that would result from such a change in the SI however the following is a list of the advantages expected.

- Units with longer term invariance than present artifact units.
- Improved uncertainties in many other fundamental constants [2,3,4].
- Many units, particularly electrical, will have lower uncertainties and better consistency with other units.
- The present volt and ohm representations will become legitimate SI units rather than the practical implementations.
- Closer ties with the scientific and chemical communities by implementing units that are more reasonably related to their needs.
- Better utilization of the extreme scaling linearity available from Josephson devices, cryogenic current comparators and quantum Hall resistors.
- The new system should serve well into the future, until we change physics.
- No significant discontinuous changes are expected at implementation.

However there are also some possible disadvantages as well.

- Prototype kilogram will have a relative uncertainty. (~0.02 x10^{-6})
- Triple point of water will have an uncertainty. (~0.25 mK)
- One mol of ^{12}C does not exactly = 12 g
- μ_0 and ε_0 will no longer be exact and will have uncertainties ($<10^{-8}$).

- The new definitions may not be as easily comprehended by the general public.
- Many textbooks and other publications will no longer be precisely correct.
- Effort is needed to explain the changes and educate the public.

Of particular interest to this paper is the change that occurs in the interpretation of the calculable capacitor results. In this new SI the ohm would typically be realized with the quantum hall effect and with the fixed values of h and e. Using various ratio bridges, a quadrature bridge which relates a capacitance and resistance at a specific frequency and a resistance of known calculated frequency dependence, a capacitance can be related to the dc resistance of the quantum Hall effect. This determines a capacitance in the new SI. Note that there is no need to know the value of μ_0, the permeability of vacuum or ε_0, the permittivity of free space and in fact they are not yet determined within the proposed SI.

If one now uses a calculable capacitor and the SI metre to generate a capacitance that is then compared to the SI capacitance derived above, then one will have derived a value for ε_0. Remember that the calculable capacitor relationship is, $C = (\varepsilon_0/\pi)\ln(2)$ F/m. Furthermore since $c^2\varepsilon_0\mu_0 = 1$ and since $e^2 = 2\varepsilon_0 hc\alpha$ and because h, c and e all have a fixed values then this set of measurements also derived values for μ_0 and the fine structure constant, α. While there are other methods that can be used to derive a value for ε_0, the calculable capacitor will no longer be able to directly determine an SI capacitance. In this circumstance the use of the term 'calculable capacitor' will probably be avoided and the system will instead be referred to as the Thompson-Lampard capacitance.

References

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2. Mills, P. Mohr, T. Quinn, B. Taylor and E. Williams, "Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to Implementing CIPM recommendation 1(CI-2005)", Metrologia, 2006
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4. P. Mohr and B. Taylor, "CODATA recommended values of the fundamental constants: 2002", Reviews of Modern Physics, 77, 1, Jan. 2005.