Optimised Decision-Making in Conformity Assessment

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Abstract

Wherever there are uncertainties, there are risks of incorrect decision-making in conformity assessment associated both with sampling and measurement. A decision theory approach, earlier used in analytical measurement, where the costs of testing are balanced against the costs associated with the consequences of incorrect decision-making is extended to more general measurements, especially legal metrology, in conformity assessment. This offers clearer procedures for setting and specifying tolerances and associated uncertainties; explanations of common rules in conformity assessment, including the 'shared risk' principle and maximum permissible uncertainties as well as in general facilitating acceptance of conformity by both customer and supplier.

1 Introduction

Few measurements are made for their own sake but rather provide often an invaluable quantitative basis for making decisions in science, technology and in society in general. While much effort has been expended in the last decade or so in giving guidance about and harmonising how to evaluate measurement uncertainty [1], arguably less attention has been paid to the treatment of uncertainty in conformity assessment and many questions remain as yet unanswered, such as a clear and objective rule for decision-making.

Test uncertainty can:

- lead to incorrect estimates of the consequences of entity error
- increase the risk of making incorrect decisions, such as failing a conforming entity or passing a non-conforming entity when the test result is close to a specification limit [2, 3].

Traditional treatment of the risk of incorrect decision-making associated with measurement uncertainty, including concepts such as 'shared risk' [4] and 'guard-banding' [5] in percentage terms, are arguably not always readily appreciated by the layman.

In seeking a criterion for 'fitness for purpose' in analytical measurement, a decision theory approach has been introduced by Thompson, Fearn and co-workers [6, 7], where the costs of analysis are balanced against the costs associated with the consequences of incorrect decision-making about analyte concentration. This is considered to be an important step, not only in analytical measurement but also in a wider measurement context, towards establishing clearer procedures for setting and specifying tolerances and associated uncertainties, and in facilitating acceptance of conformity by both customer and supplier.

An optimised uncertainty methodology in economic terms rather than just percentage risk offers increased clarity for the decision-maker, be he end-user, supplier or consumer. What appears to be statistically a certain sharing of risks between consumer and supplier in purely percentage terms may be rather unfair when different economic consequences are weighed in.

In the present review, we explore the introduction of an economic decision theory to legal metrology [§2] [2, 3]. An original discussion in economic terms of common rules in conformity assessment is presented, including the 'shared risk' principle and maximum permissible uncertainties [§2.2]. Finally, an analysis of subsequent verification of instruments in use in society leads [§2.3] to a new optimised uncertainty methodology [8, 9] based on attribute sampling which provides a complement to traditional statistical sampling plans [10, 11].

2 Optimised uncertainties in instrument testing in legal metrology

Legal metrology covers the measurement in society of many important quantities, such as the utilities (electricity, water, heat, gas, etc) but has recently been extended to other important societal measurements, such as the environment in the form of exhaust gas analysers. Conformity assessment in legal metrology is based (amongst other factors) on the testing of measurement instruments (rather than control of actual measurements in society).

2.1 Optimised uncertainties

In optimised uncertainty methodology [6, 7] models of how costs vary as a function of test uncertainty are formulated. Generally, test costs increase when attempting to reduce consequence costs – and *vice versa* – leading to a U-shaped cost curve as a function of test uncertainty and the possibility of identifying an optimal test uncertainty where costs are minimised.

An optimized uncertainty methodology could be applied with advantage to legal metrology since in many cases, clear and unambiguous cost estimates can be made [2, 3]. Costs associated with a particular type of instrument – say, a petroleum fuel dispenser – are normally a linear function of instrument error. Secondly, for many utilities but also increasingly in the environmental area, prices of commodities and emissions are known. Finally, in making a financial analysis in national economic terms, there is a direct relation between taxes levied on goods and transactions and the costs of regulation in legal metrology.

An annual tax cost for petroleum transactions, for example, is typically 4.8 G€/annum nationally, based on the average fuel consumption of the total number of vehicles in a nation [12], and a petrol price of 1.1 €/L taxed at 25% at the point of sale. There are typically 30000 petroleum dispensers nationally. The average tax levied on petroleum sales per dispenser is therefore 0.6 M€/annum. From the point of view of the legal metrology authority financed by government, this tax is perhaps one of the more relevant measures to be balanced against the cost of test and surveillance [2, 3].

Three stages of conformity assessment in legal metrology provide good examples of applying an optimized uncertainty methodology:

- when sampling by variables (instrument error):

- Type approval – 'producer' risk (Type I decision error),
- Initial verification – 'consumer' risk (Type II decision error)

- when sampling by attribute:
    - Subsequent verification

2.2 Initial verification and consumer risk

The overall testing costs, E_{np}[1] by variables of incorrectly accepting a non-conforming instrument on initial verification when the uncertainty interval [13] of a test result $y_{measure}$ partially (< 50%) lies outside the region of permissible values can be estimated as follows:

$$E_{np} = \frac{D_{np}}{u_{measure}^2} + C_{np} \cdot \int_{USL}^{+\infty} y \cdot \frac{1}{\sqrt{2\pi} \cdot u_{measure}} \cdot e^{-\frac{(y - y_{measure})^2}{2 \cdot u_{measure}^2}} \cdot dy \qquad (1)$$

assuming that the distribution in instrument error, y, under test is dominated by measurement uncertainties, $u_{measure}$ and follows a Normal Gaussian form. The consequence cost C_{np} of a non-conforming instrument being incorrectly passed at initial verification is 4.8 10^9€/annum as estimated in §2.1 directly from the national taxed transactions. Account of how costs vary linearly with instrument error is included in the integral over a distribution of instrument error. Test costs are typically D_{np} = 200 €/instrument at the actual standard measurement uncertainty, $u_{measure}$ and modeled as varying with the squared, inverse uncertainty.

The costs associated with consumer risk in this example of initial verification of a measurement instrument can be modelled as a function of test uncertainty with equation (1) and the results are shown for three different levels (dashed lines) of instrument error in Figure 1:

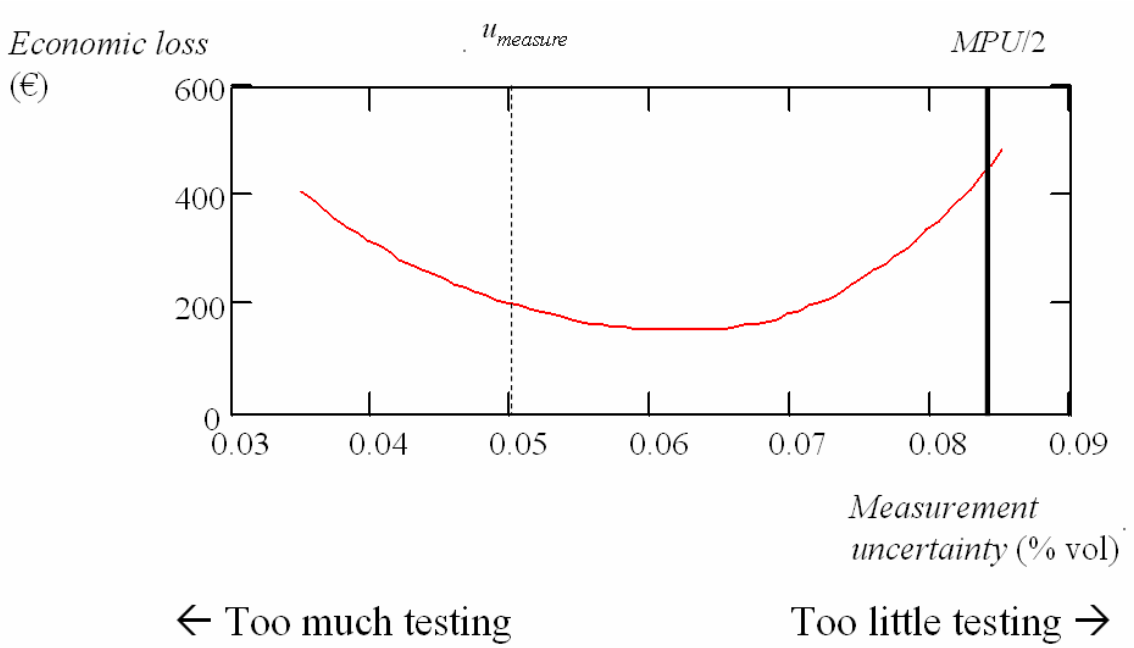

Figure 2 Consumer risk costs (€/instrument) expressed as *Economic loss*, E_{np}, as a function of standard measurement uncertainty, $u_{measure}$, (% volume) for a petroleum dispenser (MID MI-005 [14]) at one level of instrument error below $USL = + MPE$ [equation (1)]

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[1] subscript '*np*' stands for 'non-conforming passed', that is the incorrect acceptance of non-conforming product

Opportunities to optimize test uncertainties at a level corresponding to a minimum in overall costs are evident from Figure 1. At an instrument error $y_{measure}$ of about $USL - 0.5 \cdot u_{measure}$, if the standard measurement uncertainty can be reduced below the actual value of $u_{measure}$ = 0.05%, costs will rise further at lower uncertainty levels[2]. Consumer risk costs with increasing measurement uncertainty on the other hand will first decrease to a minimum (of about 170 €) at 0.06% uncertainty, thereafter increase more rapidly towards higher uncertainties including *MPU*, so that measurement effort could in this case be reduced with advantage compared with the actual level of test uncertainty.

Examples of application of these methodologies to typical decision-making in the context of legal metrology are given [2] covering all 10 categories of measuring instruments ranging from water meters, automatic weighing instruments, to exhaust gas analysers [8] covered by the European Measurement Instrument Directive MID [14].

2.3 Optimised attribute sampling

Of considerable practical and economic importance is to set and test compliance of product to a specified limit, SL_p, of fraction non-conforming product. An optimised uncertainty methodology – analogous to that familiar in quantitative decision-making by variable described above [10, 11] – can be introduced in which costs of attribute sampling are balanced against the risks and consequence costs of exceeding SL_p.

An optimised attribute sampling uncertainty, u_{sample}, is estimated as that which gives a minimum in overall costs $E_{USL,attr}$, (the sum of costs of testing and the consequences of incorrect decisions) by balancing consequence, C_{USL} and sampling costs D_{np} with the following formula:

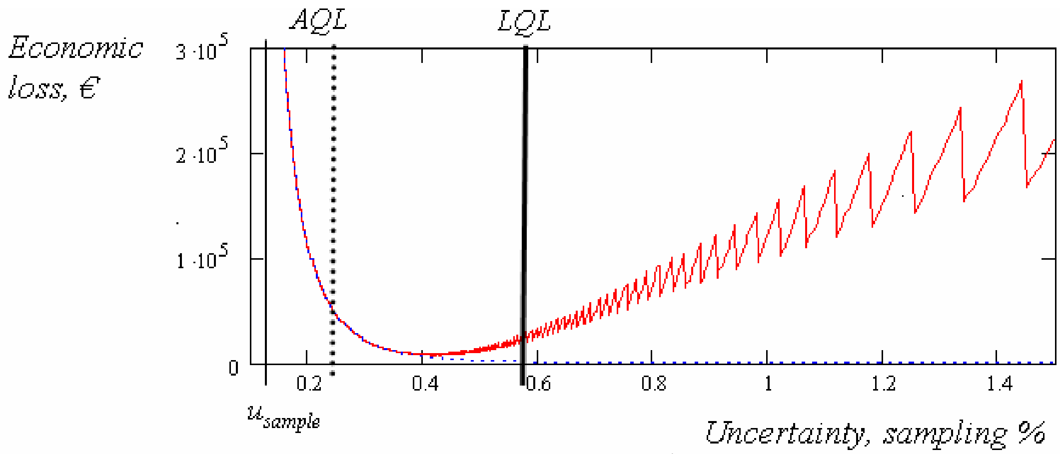
$$E_{USL,attr} = \frac{D_{np}}{u_{sample}^2} + C_{USL} \cdot \Phi_{binomial}(d, n, SL_{\hat{p}}) \qquad (2)$$

where a specification limit, $SL_{\hat{p}}$, is set for the fraction non-conforming entities exceeding a product error (upper) specification limit, from a sample of size n and actual observed number of non-conforming entities, d. Consequence costs are modelled as scaling with the integrated (or cumulative) probability Φ that the specification limit is exceeded, while test costs are assumed to vary inversely proportional to the squared measurement (standard) uncertainty [6, 15 - 17].

An evaluation is made of Eq. (2) with the consequence costs of C_{USL} = + 0.91 M€, estimated as the cost associated with 3.59% of instruments found in a national survey as exceeding the upper specification limit on instrument error, based in turn on the economic analysis of section §2.1 above. It might in this case be that a national decision puts a cap on excessive tax revenue due to over-estimated fuel metering of say + 1.23 M€. The corresponding limit on the fraction of non-conforming instruments would thus be $SL_{\hat{p}}$ = 4.85%. Further a test cost, D_{np}, of 40€ per instrument at the actual standard measurement uncertainty, $u_{measure}$ is assumed and modelled as varying with the squared, inverse uncertainty. This allows predictions of how

[2] There are of course practical limits to how far test uncertainties can be reduced.

overall costs vary with sampling uncertainty and sample size (assuming an infinite population[3]).



Figures 2 Costs as function of attribute sample uncertainty[4] for fuel dispensers.

Figure 2 illustrates how overall costs vary with the number of instruments sampled (assuming an infinite population) and how an optimised sampling uncertainty [9] can be identified in relation to traditional sampling planning limits *AQL* and *LQL*.

This new optimised sampling uncertainty methodology, extends traditional attribute sampling plans to include economic assessments of the costs of measuring, testing and sampling together with the costs of incorrect decision-making.

3 Conclusion

The development of general procedures about how to make decisions in conformity assessment in general in the presence of measurement uncertainty is a topic of international study.

It has been shown that an optimised uncertainty methodology can be applied to minimising costs in testing and sampling by both variable and attribute, as exemplified in the conformity assessment of measurement instruments in legal metrology. An original discussion in economic terms has also been given of common rules in conformity assessment, including the 'shared risk' principle and maximum permissible uncertainties.This economic approach is a complement to traditional sampling plans and treatment of risks in decision-making.

It is hoped that this will stimulate a more harmonised approach to measurement uncertainty in decision-making in future, in both the instrument standards and normative documents in the context of legal metrology as well as in the broader conformity assessment area (such as health, safety, environmental protection and fair trading).

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[3] A hypergeometric distribution can be used when sampling from a finite population [15].
[4] Oscillations in the curve are an artefact of the symbolic integral evaluation of equation (9)

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