The Metrology Taboos

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Motto
"You can never solve a problem on the level on which it was created"
Einstein

ABSTRACT

Metrology's direct or indirect impact on society is as old as humankind, yet in modern times we seem to be dramatically more dependent on accurate measurements in our personal and professional life.

The very act of measurement perturbs the natural balance and the state of matter arrangement, making impossible for us to know the true value of a quantity. This paper presents comments and interpretation of some of the basic hypotheses and axioms in metrology and their societal influence. Areas such as: Dimensional equations; the homogeneity principle as a tool for creating the measurement model; minimum material principle used in mass metrology design; convention of signs, etc. are reconsidered, to better understand some fundamental concepts of a measuring system, and raise them to a new complexity, such as the case of measurement uncertainty, and the approach of new metrology complexities such as biotechnology, biometrics, etc.

The discussed topics were selected by their importance and although there is no apparent connection between them, they are part of the same field of measurement science.

INTRODUCTION

It is very obvious we measure today more than we measured yesterday and, we will measure in the future more than we measure today. Our everyday life is increasingly more dependent on technology and intertwined with the outputs of the high tech industry. We are creating different art, producing more goods and living longer by means of computer-ization, LASER-ization, cell-phone-ization, Internet-ization, and other "izations". We are surrounded by technical gadgets at home, in the car, at the office, and every place we go. All of these situations require measurements, starting with the design stage and continuing with production, usage, and disposal. The needed measurements are implicitly more accurate in our domestic and professional life: laser beams for wall alignment, humidity measurement for your studs, temperature in your oven, the letter scales, distance to obstacles, compass, and gas consumption in the car are just few examples. All of these need to deliver accurate information for our safety or for our entertainment; and this accuracy is achieved through measurements. So, it may not be an exaggeration to say that this avalanche, or, frenzy of information, features, and capabilities, depends on metrology at some point; and it may not be an exaggeration to say that all of these

Giga-possibilities are changing the landscape of our society, our personal behavior, interrelations with others, and society as a whole. METROLOGY plays an active and indispensable part in this techno-social reshaping.

It is from this angle, of advocating the increasing importance of metrology in our society, that this paper offers a refreshing look at some of the basic concepts, as well as, some of the tools imported by metrology from other domains. Armed with new tools, a new generation of professionals is coming on board and these concepts deserve undiminished attention.

Metrology as a science is by definition, conservative. Not in the negative sense of rejecting evolution and changes, but in the sense of laying a foundation and maintaining the same general concepts and postulates. We may call them the metrology taboos. It is the robustness of this foundation on which novelties can be comfortably added.

BASIC CONCEPTS REITERATED

1. True Value

The exact value of a quantity, considered at the time of evaluation, and under existing circumstances, is called the *true value* of that quantity.

The true value of a quantity is an ideal concept and generally cannot be determined. What we can always determine is the measured value of a quantity, inherently affected by an uncertainty. In practice, a measured value, accompanied by a small enough uncertainty, compared with the given application, may substitute for the true value. Such a close value, though negligibly different than true value, can substitute for it and is called *conventional true value*.

For example, when a measuring instrument of lower precision is compared with a measurement standard, the indicated value of the standard could be considered the conventional true value.

In conclusion, the true value of a quantity is unknown and cannot be rigorously determined; even its definition contains a degree of ambiguity.

Definitions (VIM 1984).

True value (of a quantity): the value which characterizes a quantity perfectly defined, in the conditions which exists when that quantity is considered.

Conventional true value (of a quantity): a value of a quantity which, for a given purpose, may be substituted for the true value.

2. Measurement Error

The result of a measurement is more or less close to the measurand true value. Every time we are performing a measurement, our process will be affected by measurement errors, the difference between the measured value and true value. Committing these errors is inevitable for multiple reasons: the imperfection of equipment and methods, environmental conditions variations, external disturbances, operator subjectivity, etc.

The measurement error is the deviation of the result of measurement from the conventional true value of the measurable quantity, and can be expressed in absolute or relative form. In practice, the conventional true value may be the average (mean) of a series of measurement results, a nominal value (i.e. inscription an a mass), and other reference values.

Definition (VIM 1984)

(absolute) error of measurement: the result of a measurement minus the (conventional) true value of the measurand.

Notes 1. The term relates equally to:

- The indication
- The uncorrected result
- The corrected result

2. The known parts of the error of measurement may be compensated by applying appropriate corrections. The error of the corrected result can only be characterized by an uncertainty.

3. *"Absolute error"*, which has a sign, should not be confused with **absolute value of an error** which is the modulus of an error.

1st Metrology Postulate

Always the error of a measurement (E_M) is equal with the result of measurement (M_R) minus conventional true value (T_V), in that order.

$$E_M = M_R - T_V$$

In practice, we can state that measurement error (E) is the indication of the measurand (the unknown, U) minus indication of the standard (S), in that order.

$$E = U - S$$

3. Measurement Uncertainty

Uncertainty of a measurement can be defined as a range within which the true value is estimated to exist, with a specified probability. A particular application may require that a measurement be accompanied by an uncertainty estimate and report.

For example if a piston-cylinder fit requires the measurement of diameters with an uncertainty of ± 0.01 mm (0.0004"), measuring them with ± 0.1 mm (0.004") uncertainty would be useless.

2nd Metrology Postulate

A measurement result is only complete if a statement of the uncertainty in the measurement accompanies it.

For this reason, the measurement uncertainty is the most characteristic and decisive factor in assessing a measurement process. It could be said that uncertainty refers to, and characterizes a (measurement) process, however the error pertains to a measuring instrument. It is indispensable to have a correct assessment of the uncertainty of a measurement, particularly when the measurement result will be used to perform another measurement.
Measurement uncertainty is an important aspect of the metrology field, and with the more stringent reliability requirements, in fields such as health, bioengineering, food processing, nanotechnology, sensors, etc, its importance is proportionally higher.
For instance, during a manufacturing process, important decisions are made based on magnitude of uncertainty evaluated.

3rd Metrology Postulate

The error of a measurement figure (E_M), in module, added with the uncertainty figure (U), in module, should be equal or less than the figure of assigned tolerances (T) for that particular application.

$$|E_M| + |U| \leq |T|$$

Definition (VIM/1984)
Uncertainty of measurement: an estimate characterizing the range of values within which the true value of a measurand lies.

4. Expression of a Measurement Result

A measurement result could be expressed as in the following example. If the measuring of a length gives us the value of 200.06 mm and the measurement uncertainty is estimated as ±0.03 mm, the expression of the result of this measurement is

$$L = 200.06 \pm 0.03 \text{ mm}$$

What it means is that the true value of the measurand can be found between 200.03 and 200.09 mm, with sufficient probability and level of confidence.

4th Metrology Postulate

The most complete expression of a measurement result (M_R) contains the conventional true value (T_V) of a quantity, plus the measured departure from it (measured error with its sign E_M) plus and minus the uncertainty of that measurement (U)

$$M_R = T_V + (E_M) \pm U$$

PRINCIPLE OF DIMENSIONAL HOMOGENEITY

The equation

$$2 + 3 = 5$$

is mathematically correct, but if we consider two options of this equation:

1. 2 apples + 3 oranges = 5 bananas
2. 2 apples + 3 apples = 5 apples

The second option makes sense because each of the three terms in the equation has the same unit (therefore same dimensions). In the first option, despite the mathematical operation yielding the right answer, the equation does not make any sense. For an equation to be valid, it is absolutely necessary that *every term in the equation have the same dimensions.*

This is called the principle of dimensional homogeneity (see also VIM/1984 definition 1.04). The mathematical operation of addition is defined only for dimensionally consistent quantities.

Let us consider the following equation:

$$s = vt + 1/2at^2$$

There are three terms in the equation. In the left hand side there is one single term: 1(s). In the right hand side there are two terms: 1(vt) and $1/2(at^2)$.

The coefficient of a term is a number that informs us of how many such items are involved in the equation. The coefficient of the terms (s) and (vt) are implied to be 1.

Dim. s = [L]
Dim (vt) = $[L][T]^{-1}[T] = [L]$
Dim (at^2) = $[L][T]^{-2}[T]^2 = [L]$

Note that the coefficient ½ has no dimensions just as the coefficient 1 has no dimensions. Thus every term in the equation has the same dimensions. The equation is thus said to be dimensionally valid. The dimensions of the seven SI base quantities are:

1. L, length
2. M, mass
3. T, time
4. I, electrical current intensity
5. Θ, thermodynamic temperature
6. N, quantity of substance
7. J, luminous intensity

The dimensions of any SI derived quantities can be expressed as a product of the powers of L, M, T, I, Θ, N, J, such as:

$$\mathrm{Dim}X = L^a M^b T^c I^d \Theta^e N^f J^g$$

Where a, b, c, d, e, f, g are the *dimensions* of the SI base quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity, respectively. The exponents a, b, c, d, e, f and g are called "dimensional exponents."

The principle of dimensional homogeneity can by applied in two ways: to verify an existing equation as described above, or to create a new equation.

Use of dimensional homogeneity to create a new equation.

Dimensional homogeneity can be successfully used to find a relationship between quantities, including the model equation for a measurement uncertainty calculation.

Given that a quantity (f) depends upon certain other quantities (x, y, z), it is possible to use the principle of dimensional homogeneity to form a connection statement showing in what manner f is connected to the quantities x, y and z.

To find the formula for a free fall body, we know, from experimental observations, the distance (space) covered depends on gravitational acceleration and time.

$$s = kg^a t^b$$

Where

k = dimensionless coefficient
g = gravitational acceleration
t = time
The dimensional equation will be:

$$L^1 = (LT^{-2})^a T^b = L^a T^{-2a+b}$$

Applying the homogeneity principle, we need to equate the exponents:

$$a = 1$$
$$b-2a = 0$$
$$b = 2a = 2$$

Therefore

$$s = kgt^{-2}$$

where coefficient k is determined through experiments.

Disadvantages of the dimensional homogeneity method

1. The numerical coefficient of any term cannot be verified by this method. For example the value of k in the above equation cannot be found out.
2. The existence of additional terms cannot be verified by this method.

Consider the equation

$$s = vt + 1/2at^2$$

If one of the terms on the right hand side were omitted, the equation would still be dimensionally valid. Hence, equations which have the same dimensions on both sides, are not necessarily correct in all respects.

3. Quantities having the same dimensions are not necessarily comparable.

   Both torque and work have the same dimensions $[M][L]^2[T]^{-2}$. If an equation stating that torque is equal to work is dimensionally verified, it will be found to be dimensionally correct. But since torque and work are intrinsically different kinds of quantities, the equation would make as little practical sense as to say that oranges are equal to bananas.

4. The method of equating dimensions on both sides in order to form an equation, cannot be used to solve equations based on logarithmic or trigonometric functions.

   Consider the equation

$$y = 2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi t}{T}$$

   in which x and λ are both lengths while t and T are both times and y and a are also lengths.

   The method of dimensions cannot verify the correctness of whether the right hand side should contain a ratio of lengths or a ratio of energies or electric currents. However, the dimensions method can verify that x/λ or t/T should be without units and that a should have the same dimensions as y.

5. If a variable depends on more than three independent variables, then the equation cannot be solved by the method of dimensions using only [L], [M] and [T]. In order to solve for one unknown (x, say,) one equation is enough. But, to solve for two unknowns (x and y), two equations are needed. Similarly, to solve for 'n' unknowns, 'n' number of equations are necessary. Equating powers of each [L], [M] and [T], only three equations are available. Thus, it is possible to solve for the dependence of the function on up to three independent variables, but no more.

THE MINIMUM MATERIAL PRINCIPLE FOR WEIGHTS

In the mass measurement field, designing the mass artifacts for weighing (the weights) requires attention in two directions:

- the weights denomination

The weights denomination needs to be chosen as such as we may be able to create any load with a minimum number of weights.
The most used denominations are 5321, 52211, 51111
In relation with this minimum number of used weights is the amount of material the weights are made of.

- the minimum material for the weights corresponding to a minimum surface is ideally achieved with a cylindrical shape; that is the volume of a sphere can be contained by a cylinder with a smaller surface. This is also convenient technologically as cylinders are machined much easier than other shapes and their measurement is also facilitated.

CONVENTION OF SIGNS

- The measurement error always has the sign resulting from the first metrology postulate.

If the reading on the unknown quantity is larger than the reading on standard we have a positive (plus) error.
If the reading on the unknown quantity is smaller than the reading on standard we have a negative (minus) error.

- The correction is always the measurement error with the changed sign.

A positive error give becomes a negative correction and vice-versa.

CONCLUSIONS
There are more "untouchable" conventions in metrology. This paper mentioned some of the more transparent basic principles and concepts with the declared purpose of strengthening and exposing them to facilitate positive development through discussions and debates.

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