High Precise Thermo-anemometer.

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Abstract: The presented work describes a high accuracy method of gas flow velocity measurement based on the phase shift of thermal wave measurements at some manifold wave frequencies. The flow velocity is derived from the linear relationship between phase shift and wave frequency. The method employs a continuous, sine thermal wave rather than a short pulse. Applied probe consists of two thin wires. A transmitter of thermal wave and operates in CTA mode. Wave detector place downstream to the transmitter operates in CC mode. Presented analysis, formulas necessary to flow velocity calculation as well as requirements for probe and range of velocity to be measured. Some of the results of the testing in a laminar air flow for flow velocity ranging from 20 cm/s to 200 cm/s was compared. Achieved accuracy of the velocity measurements was better than 0.4 cm/s.

1. Introduction

The idea of flow velocity measurement by means thermal wave is based on measurement the 'phase velocity' of thermal wave introduced in flowing gas. It is sensible if 'phase velocity' of wave is equal to flow velocity. The well known pulsed wire anemometry technique (PWA) consists of measuring the time of flight a tracer of heated air from a transmitter (wave source) to wave detector. To meet above condition the detector is placed near by the transmitter. A consequence of such arrangement is necessity of a probe calibration (see e.g. Bruun[1]) In current paper we proposed a different approach to the problem. The task is to find a requirements for a transmitter-detector arrangement as well as wave frequency in order to flow velocity could be measured with sufficient accuracy without previous calibration of a probe. Presented method employs a continuous, sine thermal wave rather than a short pulse. A flow velocity is determined on the base of the phase shift of the wave on two detectors placed downstream to the transmitter. If detector is distant of Δx from the wave source, the following elementary relationship is valid:

$$\Delta\varphi = \frac{\omega \Delta x}{U_W}, \quad (1)$$

where $\Delta\varphi$ denotes phase shift, ω – wave angular frequency and U_W – flow velocity.

2. Outline of the theory

Since the wave transmitter is a thin wire we consider a linear, infinite wave source place perpendicularly to the flow velocity vector. In coordinates system as in Fig.1, the temperature distribution is governed by the equation of heat conduction:

$$\frac{\partial T}{\partial t} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - U\frac{\partial T}{\partial x} + \frac{1}{\rho c} Q_0\,\delta(x-0)\delta(y-0)\exp(i\omega t). \quad (2)$$

where T-temperature, ρ-density, c-specific heat, κ-thermal conductivity, t-time, U-flow velocity $Q(t) = Q_0\delta(x-0)\delta(y-0)\exp(i\omega t)$ - periodic heat intensity of a source. It has been assumed that the density, specific heat and thermal conductivity of gas and flow velocity remain constant. The solution to above equation was given by Kiełbasa [2]:

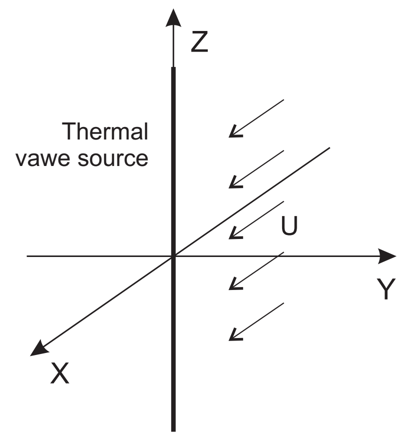

Figure 1. Flow velocity and wave source in coordinate system.

$$T(x,y,t) = T_0 \exp\left(\frac{Ux}{2\kappa} - i\omega t\right) K_0\left(AQ_x + iBQ_r\right), \quad (3)$$

where K_0-modified Bessel function of the second order, $T_0 = \frac{Q_0}{2\pi c\rho\kappa}$, and dimensionless variables $Q_x = \frac{xU}{2\kappa}, Q_r = \frac{rU}{2\kappa}, P = \frac{4\kappa\omega}{U^2}\ A = \sqrt{\frac{1}{2}\left(\sqrt{1+P^2}+1\right)}, B = \sqrt{\frac{1}{2}\left(\sqrt{1+P^2}-1\right)}$. Equation 3 represents solution of Eq.2 in steady state. The time t_0 which elapsed from the moment when the source was activated so that an influence of initial conditions on can be neglected is [2,3]:

$$t_0 \geq 24\frac{\kappa}{U_{MIN}^2} \quad (4)$$

Using expansion of Bessel function in series: $K_0(\zeta) = \sqrt{\frac{\pi}{2\xi}}\exp(-\zeta)\left(1 - \frac{1}{8\zeta} + \frac{9}{2!(8\zeta)^2} + \ldots\right)$

and considering only first term leads to solution in form:

$$T(x,y,t) = \frac{\Theta}{\sqrt[8]{1+P^2}\sqrt{Q_r}}\exp(i\omega t + \varphi), \quad (5)$$

where the phase φ is:

$$\varphi(r,\omega,\kappa,U) = \frac{Ur}{2\kappa}\sqrt{\frac{1}{2}\left(\sqrt{1+P^2}-1\right)} + \frac{1}{4}\arctan P. \quad (6)$$

If the relative error of above approximation has to be not greater than the expected value ε the following inequality must be satisfied: $\frac{1}{|8\zeta|} < \varepsilon$ Substituting the argument of Bessel function accordingly to Eq. 3 and substituting parameters A, Q_x, Q_r by its explicit form yields [3]:

$$r_{MIN} > \frac{1}{4\varepsilon\sqrt[4]{\left(\frac{U}{\kappa}\right)^4 + \left(\frac{4\omega}{\kappa}\right)^2}}. \tag{7}$$

It appears that r_{MIN} is a decreasing function of flow velocity and wave frequency. For example, introducing U_{MIN} =10 cm/s , $\omega_{MIN} = 2\pi*5$ rad/s and for the air at room temperature κ=0.18cm^2/s, we get $r_{MIN} \geq 4.5*10^{-3}\ \varepsilon^{-1}$cm. After neglecting the term with arctan in Eq.6 we get:

$$\Delta\varphi = \frac{U\Delta x}{2\kappa}\sqrt{\frac{1}{2}\left(\sqrt{1+P^2}-1\right)}. \tag{8}$$

Comparing Eq.1 with Eq.8 yields:

$$U_W = U\sqrt{\frac{1}{2}\left(1+\sqrt{1+P^2}\right)}. \tag{9}$$

Notice, that U is involved in P. Equation (8) states the condition when wave velocity U_W can be treated as equal to flow velocity U : $\frac{16\kappa^2\omega^2}{U^2}$ <<1. If this inequality is satisfied the inner root in Eq. 8 can be replaced by $1+\frac{1}{2}P^2$, and hence:

$$\Delta\varphi = \frac{\omega\Delta r}{U} \tag{10}$$

Above formula is the basis to determine flow velocity. The velocity U can be derived in another way. Differentiation $\Delta\varphi$ expressed by Eq.7 with respect to ω gives:

$$\frac{d(\Delta\varphi)}{d\omega} = \frac{\partial(\Delta\varphi)}{\partial P}\frac{\partial P}{\partial\omega} = \frac{\sqrt{2}}{2}\frac{\Delta r}{U}\sqrt{\frac{\sqrt{1+P^2}+1}{1+P^2}} \tag{11}$$

For P tends to zero we get:

$$\frac{d(\Delta\varphi)}{d\omega} = \frac{\Delta r}{U}. \tag{12}$$

Applying a set of succeeding various frequencies of the wave and rewriting Eq. 10 we get:

$$\Delta\varphi_i = \frac{\omega_i\Delta r}{U}. \tag{13}$$

This formula shows that flow velocity can be calculated simply by means of linear regression. If the detector is placed along y-axis (see Fig.1) the registered temperature T_l is can be expressed as an integral of gas temperature T with respect to y:

$$T_l(x,t,l) = \frac{1}{l}\int_{-\frac{1}{2}l}^{+\frac{1}{2}l} T(x,y,t)dy \ . \tag{14}$$

After calculus we obtain formula for phase shift of the temperature signals registered by two detectors [4]:

$$\Delta\varphi_l = \frac{U\Delta x}{2\sqrt{2}\kappa}\sqrt{\sqrt{1+P^2}-1} \ . \tag{15}$$

Above equation displays the same behavior like Eq. 8. Substitution the inner square root by initial two terms in expansions leads to Eq. 10. Differentiation with respect to ω and substitution zero variable P for zero gives Eq. 12.

3. The measuring instrument

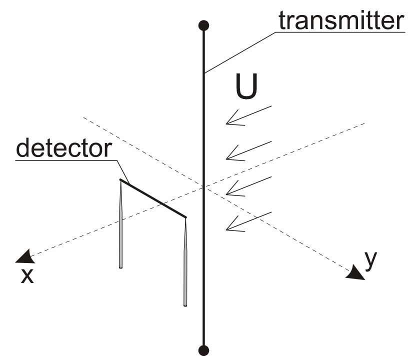

Figure 2. Arrangement of the probe with respect to flow.

Applied probe consists of three thin wires. Coplanar detectors were placed downstream perpendicularly to the transmitter (Fig.2). The probe was orientated so that the measured flow velocity is perpendicular to the wires. The distance between the transmitter and the first detector was 5.0mm, and between the detectors 5mm. The wave transmitter was built from a tungsten wire of 6mm length and of 8µ diameter, the wave detector was a tungsten wire of 1.5mm length and of 5µ diameter. The transmitter operated in digital controlled CTA system, which forced the temperature of the transmitter to be sine function of the time, with provided frequency and amplitude, regardless on the flow velocity. The applied frequencies were from 30 to 60Hz, the amplitude of the wave was $\pm 25\%$ from 1.5 base overheating ratio of the transmitter. The detectors operated CC mode as resistance thermometers. The voltage signals from the detectors were sent to an A/D converter and were stored in computer memory. To calculate the phase shift, signals were decomposed in Fourier series using FFT, and the phase shift of first harmonic component was measured. Afterwards, the computer-controlled CTA system switched the voltage supplying transmitter to a subsequent frequency.

4. Results of experiments

The measurements were performed in air at room temperature. Fig.3 presents the measured phase shift with respect to frequency at several flow velocities. The diagrams display better linearity of the phase shift for low velocities. The uncertainty of the measured velocity was roughly evaluated by means logarithm derivative. The relative uncertainty of Δx did not exceed $1*10^{-3}$. The relative uncertainty of the regression line slope did not exceed $3*10^{-3}$. Some results of measurements are

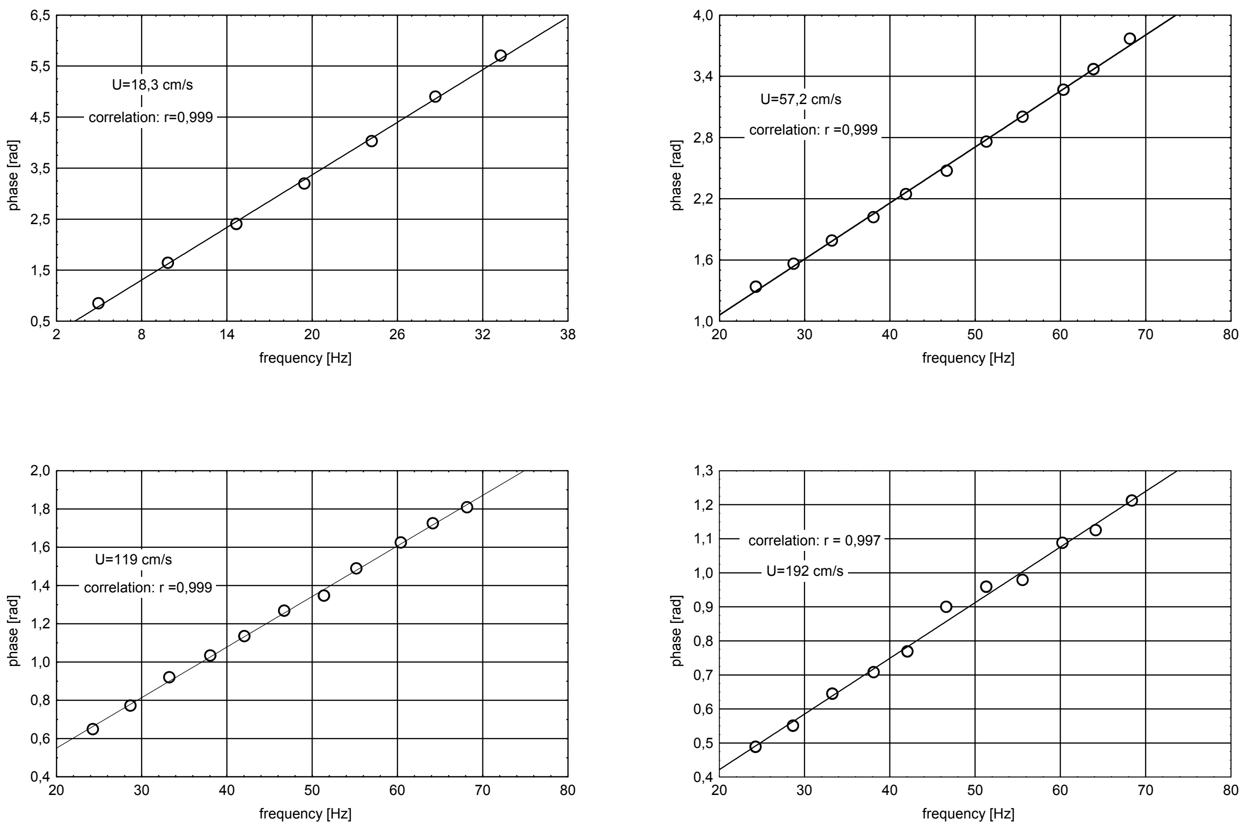

Figure 3. The phase shift vs wave frequency at values of flow velocity.

placed in Tab.1. However the uncertainty increases with velocity, the relative accuracy remains on the same level.

Tab.1 .The measured flow velocity and its uncertainty

flow velocity [cm/s]		frequency points	uncertainty [cm/s]
established	measured		
20	18.9	7	0.3
58	57.2	11	0.9
120	119	11	1.6
190	192	11	5.6

5. Conclusions

Presented method can be used as an additional technique of low laminar flows measurements. Since, after fulfilling definite conditions the probe doesn't need calibration, it can be applied to flows with manifold composition of flowing gas as long as this conditions remains valid. A criterion of the measurement correctness is linearity of measured dependence between the phase shift and wave frequency. In this meaning presented technique can be called as ‘self-testing’. Since maintaining measurements at subsequent frequencies requires time, the flow should be steady or changes very slowly.

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