

# **The Significance on Consumer and Producer Risk for Uncorrected Product and Measurement Biases**

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## **Abstract**

Uncorrected product and measurement biases are routinely ignored in the calculation of consumer and producer risks. Unfortunately, this practice can unknowingly increase the magnitude of those risks to more than 50%. The users of measurement and test equipment that have been calibrated in that way will typically be accepting a much greater percentage of false accepts with potentially serious consequences.

Prior to the development of computerized tools to calculate the affect of product distribution and measurement system distribution bias on consumer and producer risk, the assumption was made that these distributions were centered about their respective performance specification nominal values. Universally applied guardbands have been based on that special case which resulted in erroneous calculations of consumer and producer risks.

Newly developed tools now permit accurate calculation of consumer and producer risks when either or both product and measurement distributions are biased. General cases can now be evaluated.

To enable the reader to easily determine the magnitude on consumer and producer risks, tables are provided (with and without guardbanding considerations) as a function of the most significant variables, including all relevant definitions.

## **1 Introduction**

It is impractical to develop tables for all values of consumer risk due to the large number of tables required to span the range of desired consumer and producer risks. The risk tables (without guardbanding) are therefore limited to maximum product biases for quartile levels of measurement biases (four, one-half sigma intervals of a two sigma measurement distribution). The risk tables (with guardbanding) are limited to guardbands for consumer risks of 0.80%, 1.60%, 2.26% and 5.00%.

A computer program must be used for risks and guardbands not included in the tables.

## **2 Variables**

Performance Specification – M&TE certified capability value.

Product Distribution Sigma – Standard uncertainty of a product measurand based on its history if historical data exists, or based on a current statistical evaluation if lacking historical data.

Measurement Distribution Sigma – Standard uncertainty of a measurement system measurand based on its history if historical data exists, or based on a current statistical evaluation if lacking historical data.

Sigma Model – Performance specification divided by the product distribution sigma.

Uncertainty Ratio – Product distribution sigma<sup>1</sup> divided by the measurement distribution sigma<sup>2</sup>

Product Bias – Displacement of the product distribution mean from its bilateral performance specification mid-point.

Measurement Bias – Displacement of the measurement distribution mean from its bilateral performance specification mid-point.

Consumer Risk – The unconditional probability that the true value of a measurand falls outside a given performance specification, but is measured to be within the guardband limit<sup>3</sup>.

Producer Risk – The unconditional probability that the true value of a measurand falls inside a given performance specification, but is measured to be outside the guardband limit.

## **3 Variable Data Acquisition**

### **3.1 Performance Specification**

This value is generally specified on the calibration label, certificate, calibration procedure, or derived from a current statistical evaluation. Regardless of the source, the performance specification should include the effects normally expected during the calibration interval.

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<sup>1</sup> The product distribution sigma, as defined here, is equivalent to the historical measurand estimate uncertainty on a standard uncertainty basis.

<sup>2</sup> The measurement distribution sigma, as defined here, is equivalent to the measurement system standard uncertainty.

<sup>3</sup> This definition is the particular one denoted as “ $\alpha$ ” in reference [1].

## **3.2 Product Distribution Sigma**

### **3.2.1 With Historical Data:<sup>4</sup>**

Calculate the standard deviation of the historical measurand estimate values. For the greatest statistical validity, all the estimate values should be of the same population. Values can be said to be of the same population if their means and standard deviations are not different by more than a specified confidence level. A histogram point plot can be used to verify the similarity to a single random distribution in the absence of a trend. An analysis of plotted points as a function of time can be used to determine if a trend exists. If a least square best fit line is calculated, evidence of a trend is indicated if the line slope rise or fall is greater than two times the standard deviation of the residuals. A monotonically increasing or decreasing trend indicates a systematically varying bias and therefore should not be treated the same as the values of a random variable. For single data points, a shift is considered to be significant if it shows evidence of belonging to a distribution displaced from another distribution with means separated by two standard deviations of the largest distribution can be considered evidence of not belonging to a single distribution. For averaged data points of the same distribution or of two suspect distributions, a  $t$  test of the means and an F test of the standard deviations can be used to provide evidence of belonging to a single distribution.

### **3.2.2 Without Historical Data:**

For the first calibration, calculate the standard deviation of at least four measurements under repeatability conditions of the current measurand estimate values. This standard deviation or standard deviation of the mean is defined to be equivalent to the product distribution sigma. For the second calibration, calculate the standard deviation of all the measurements of the first calibration with a single measurement of the second calibration. For the third calibration, calculate the standard deviation of all the measurements of the first calibration with the single measurements of the second and third calibrations. Continue this process until the number of calibrations equals one more than the sample size of the first calibration. For that calibration and the following calibrations, calculate the standard deviation of the average of the measurements of the first calibration and single measurements of the succeeding calibrations. For example, if the first calibration sample size was 4, the sample sizes for the following calibrations would be 5, 6, 7 and 5 for the second, third, fourth and fifth calibrations, respectively.

If a multiple sample size for the first calibration is not practical, assume the product distribution sigma is equal to one-half of the performance specification, e.g., for a performance specification of  $\pm 2$ , the product distribution sigma would be  $\pm 1$ . Continue with this assumption until the fifth calibration, for which, the product distribution sigma is defined to be equal to the standard deviation of the single measurement values of each of the total number of calibrations.

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<sup>4</sup> A minimum of four calibrations is required to qualify. Until then, calculate the product distribution sigma as described under “Without Historical Data”.

### **3.3 Measurement Distribution Sigma**

#### **3.3.1 With Historical Data:<sup>5</sup>**

Calculate the standard deviation of the historical measurand estimate values. For the greatest statistical validity, all the estimate values should be of the same population. Values can be said to be of the same population if their means and standard deviations are not different by more than a specified confidence level. A histogram point plot can be used to verify the similarity to a single random distribution in the absence of a trend. An analysis of plotted points as a function of time can be used to determine if a trend exists. If a least square best fit line is calculated, evidence of a trend is indicated if the line slope rise or fall is greater than two times the standard deviation of the residuals. A monotonically increasing or decreasing trend indicates a systematically varying bias and therefore should not be treated the same as the values of a random variable. For single data points, a shift is considered to be significant if it shows evidence of belonging to a distribution displaced from another distribution with means separated by two standard deviations of the largest distribution can be considered evidence of not belonging to a single distribution.

For averaged data points of the same distribution or of two suspect distributions, a  $t$  test of the means and an  $F$  test of the standard deviations can be used to provide evidence of belonging to a single distribution.

#### **3.3.2 Without Historical Data:**

For the first calibration, calculate the standard deviation of at least four measurements under repeatability conditions of the current measurand estimate values. This standard deviation or standard deviation of the mean is defined to be equivalent to the measurement distribution sigma.

For the second calibration, calculate the standard deviation of all the measurements of the first calibration with a single measurement of the second calibration.

For the third calibration, calculate the standard deviation of all the measurements of the first calibration with the single measurements of the second and third calibrations.

Continue this process until the number of calibrations equals one more than the sample size of the first calibration.

For that calibration and the following calibrations, calculate the standard deviation of the average of the measurements of the first calibration and single measurements of the succeeding calibrations.

For example, if the first calibration sample size was 4, the sample sizes for the following calibrations would be 5, 6, 7 and 5 for the second, third, fourth and fifth calibrations, respectively.

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<sup>5</sup> A minimum of five calibrations is required to qualify. Until then, calculate the product distribution sigma as described under “Without Historical Data”.

If a multiple sample size for the first calibration is not practical, assume the product distribution sigma is equal to one-half of the performance specification, e.g., for a performance specification of  $\pm 2$ , the measurement distribution sigma would be  $\pm 1$ . Continue with this assumption until the fifth calibration, for which, the measurement distribution sigma is defined to be equal to the standard deviation of the single measurement values of each of the total number of calibrations.

### **3.4 Sigma Model**

Divide the performance specification by the product distribution sigma.

### **3.5 Uncertainty Ratio**

Divide the product distribution sigma by the measurement distribution sigma.

### **3.6 Product Bias**

Randomly varying distribution: Calculate the average measurand estimate value.

Trending distribution: Use the last measurand estimate value.

First calibration: Calculate the average measurand estimate value or use the measurand estimate value, as applicable.

### **3.7 Measurement Bias**

Randomly varying distribution: Calculate the average measurand estimate value.

Trending distribution: Use the last measurand estimate value.

First calibration: Calculate the average measurand estimate value or use the measurand estimate value, as applicable.

## **4 Risk Tables (without guardbands)**

A description of table 1 contents follows.

Column 1 contains the sigma model.

Column 2 contains the uncertainty ratio.

Column 3 contains the number of product sigma the mean of the measurement system distribution is biased relative to its certified bilateral performance specification nominal.

Column 4 contains the number of product sigma the mean of the product measurand distribution is biased relative to its bilateral performance specification nominal. This value will be zero for the first group of five rows. The non-zero product biases of the middle and last group of five rows are those that result in the greatest (worst-case) producer risks at zero measurement bias.

Column 5 contains the value of producer risk for the parameter values in columns 1 through 4.

Column 6 contains the number of product sigma the mean of the product measurand distribution is biased relative to its bilateral performance specification nominal. This value will be zero for the first group of five rows. The non-zero product biases of the middle and last group of five rows are those that result in the greatest (worst-case) consumer risks at zero measurement bias

Column 7 contains the value of consumer risk for the parameter values in columns 1, 2, 3 and 6.

Column 8 contains the number of product sigma the mean of the measurement system distribution is biased relative to its certified bilateral performance specification nominal.

Column 9 contains the number of product sigma the mean of the product measurand distribution is biased relative to its bilateral performance specification nominal. This value will be zero for the first group of five rows. The non-zero product biases of the middle and last group of five rows are those that result in the greatest (worst-case) producer risks at zero measurement bias.

Column 10 contains the value of producer risk for the parameter values in columns 1, 2, 8 and 9.

Column 11 contains the number of product sigma the mean of the product measurand distribution is biased relative to its bilateral performance specification nominal. This value will be zero for the first group of five rows. The non-zero product biases of the middle and last group of five rows are those that result in the greatest (worst-case) consumer risks at zero measurement bias

Column 12 contains the value of consumer risk for the parameter values in columns 1, 2, 8 and 11.

1	2	3	4	5	6	7	8	9	10	11	12
$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
4	4	0.000	0	0.006	0	0.002	0.000	0	0.006	0	0.002
4	4	-0.125	3.845	3.360	4.155	6.880	0.125	3.845	4.592	4.155	1.960
4	4	-0.500	-3.845	6.880	-4.155	0.084	0.500	-3.845	1.960	-4.155	19.348

Table 1 illustrates how product biases (PB) and measurement biases (MB) affect consumer risk (CR) and producer risk (PR) for various sigma models (s) and uncertainty ratios (UR). The biases are expressed in units of product sigma, and the risks are expressed in percent. Guardbanding is not a consideration. A description of how the table is to be interpreted follows. All descriptions are based on the first table section, but can be similarly applied to the other sections.

1. The CR is 0.002% for a PB of 0 product sigma and a MB of 0 product sigma.
2. The PR is 0.006% for a PB of 0 and MB of 0.

3. The CR is 19.348% for a PB of 4.155 and MB of  $-0.500$ .
4. The PR is 1.960% for a PB of 3.845 and MB of  $-0.500$ .
5. The CR is 3.946% for a PB of 4.155 and MB of 0.
6. The PR is 3.946% for a PB of 3.845 and MB of 0.

The least intuitive observations for the same sigma model and uncertainty ratio are:

1. Consumer risks are equal for  $-MB/0PB$  and  $+MB/0PB$ .
2. Producer risks are equal for  $-MB/0PB$  and  $+MB/0PB$ .
3. Consumer risks are equal for  $-MB/+PB$  and  $+MB/-PB$ .
4. Producer risks are equal for  $-MB/+PB$  and  $+MB/-PB$ .
5. Consumer risks are equal for  $-MB/-PB$  and  $+MB/+PB$ .
6. Producer risks are equal for  $-MB/-PB$  and  $+MB/+PB$ .

Note 1: The non-zero product biases specified in the table are those that result in the greatest (worst-case) risks at zero measurement bias.

Note 2: Although the measurement biases are expressed in units of product sigma, they are equivalent to quartiles of the respective 2 sigma measurement distribution, i.e., a MB of 0.125 product sigma is equivalent to 0.500 measurement sigma, for an uncertainty ratio of 4:1. Similarly, a MB of 2/3 product sigma is equivalent to 2.000 measurement sigma, for an uncertainty ratio of 3:1, a MB of 0.750 product sigma is equivalent to 1.500 measurement sigma for an uncertainty ratio of 2:1, and a MB of 2.000 product sigma is equivalent to 2.000 measurement sigma, for an uncertainty ratio of 1:1.

Note 3: All values are based on a guardband factor of 1.

Note 4: Based on a maximum consumer risk of 0.80%, for a 4 sigma model and an uncertainty ratio of 4:1 (first table), it can be seen that measurement values up to and including the specification limit are acceptable for the entire top group of five, i.e., for all measurement biases (shown in italics). It can also be seen that for the worst-case possibility of product bias, only measurement biases slightly less than  $-0.250$  product sigma and slightly greater than  $0.250$  product sigma are acceptable for the middle group of five. None of the bottom group of five is acceptable.

Table 1. Sigma Models of 4 to 1, and Uncertainty Ratios of 4 to 1 without Guardbanding.

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
4	4	0.000	0	0.006	0	0.002	0.000	0	0.006	0	0.002
4	4	-0.125	0	0.006	0	0.002	0.125	0	0.006	0	0.002
4	4	-0.250	0	0.006	0	0.002	0.250	0	0.006	0	0.002
4	4	-0.375	0	0.006	0	0.002	0.375	0	0.006	0	0.002
4	4	-0.500	0	0.007	0	0.003	0.500	0	0.007	0	0.003
4	4	0.000	3.845	3.946	4.155	3.946	0.000	3.845	3.946	4.155	3.946
4	4	-0.125	3.845	3.360	4.155	6.880	0.125	3.845	4.592	4.155	1.960
4	4	-0.250	3.845	2.835	4.155	10.629	0.250	3.845	5.297	4.155	0.826
4	4	-0.375	3.845	2.369	4.155	14.892	0.375	3.845	6.061	4.155	0.291
4	4	-0.500	3.845	1.960	4.155	19.348	0.500	3.845	6.880	4.155	0.084
4	4	0.000	-3.845	3.946	-4.155	3.946	0.000	-3.845	3.946	-4.155	3.946
4	4	-0.125	-3.845	4.592	-4.155	1.960	0.125	-3.845	3.360	-4.155	6.880
4	4	-0.250	-3.845	5.297	-4.155	0.826	0.250	-3.845	2.835	-4.155	10.629
4	4	-0.375	-3.845	6.061	-4.155	0.291	0.375	-3.845	2.369	-4.155	14.892
4	4	-0.500	-3.845	6.880	-4.155	0.084	0.500	-3.845	1.960	-4.155	19.348

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
4	3	0	0	0.010	0	0.002	0	0	0.010	0	0.002
4	3	-1/6	0	0.011	0	0.002	1/6	0	0.011	0	0.002
4	3	-1/3	0	0.012	0	0.002	1/3	0	0.012	0	0.002
4	3	-1/2	0	0.013	0	0.003	1/2	0	0.013	0	0.003
4	3	-2/3	0	0.016	0	0.003	2/3	0	0.016	0	0.003
4	3	0	3.795	5.227	4.205	5.227	0	3.795	5.227	4.205	5.227
4	3	-1/6	3.795	3.898	4.205	9.095	1/6	3.795	6.382	4.205	2.599
4	3	-1/3	3.795	3.101	4.205	13.999	1/3	3.795	7.674	4.205	1.097
4	3	-1/2	3.795	2.599	4.205	19.509	1/2	3.795	9.095	4.205	0.386
4	3	-2/3	3.795	1.987	4.205	25.158	2/3	3.795	10.633	4.205	0.112
4	3	0	-3.795	5.227	-4.205	5.227	0	-3.795	5.227	-4.205	5.227
4	3	-1/6	-3.795	6.382	-4.205	2.599	1/6	-3.795	3.898	-4.205	9.095
4	3	-1/3	-3.795	7.674	-4.205	1.097	1/3	-3.795	3.101	-4.205	13.999
4	3	-1/2	-3.795	9.095	-4.205	0.386	1/2	-3.795	2.599	-4.205	19.509
4	3	-2/3	-3.795	10.633	-4.205	0.112	2/3	-3.795	1.987	-4.205	25.158



$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
4	2	0.00	0	0.031	0	0.002	0.00	0	0.031	0	0.002
4	2	-0.25	0	0.033	0	0.002	0.25	0	0.033	0	0.002
4	2	-0.50	0	0.043	0	0.002	0.50	0	0.043	0	0.002
4	2	-0.75	0	0.060	0	0.002	0.75	0	0.060	0	0.002
4	2	-1.00	0	0.086	0	0.002	1.00	0	0.086	0	0.002
4	2	0.00	3.695	7.705	4.305	7.705	0.00	3.695	7.705	4.305	7.705
4	2	-0.25	3.695	5.551	4.305	13.329	0.25	3.695	10.309	4.305	3.845
4	2	-0.50	3.695	3.845	4.305	20.363	0.50	3.695	13.326	4.305	1.623
4	2	-0.75	3.695	2.554	4.305	27.962	0.75	3.695	16.700	4.305	0.572
4	2	-1.00	3.695	1.625	4.305	35.422	1.00	3.695	20.332	4.305	0.166
4	2	0.00	-3.695	7.705	-4.305	7.705	0.00	-3.695	7.705	-4.305	7.705
4	2	-0.25	-3.695	10.309	-4.305	3.845	0.25	-3.695	5.551	-4.305	13.329
4	2	-0.50	-3.695	13.326	-4.305	1.623	0.50	-3.695	3.845	-4.305	20.363
4	2	-0.75	-3.695	16.700	-4.305	0.572	0.75	-3.695	2.554	-4.305	27.962
4	2	-1.00	-3.695	20.332	-4.305	0.166	1.00	-3.695	1.625	-4.305	35.422

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
4	1	0.00	0	0.464	0	0.003	0.00	0	0.464	0	0.003
4	1	-0.50	0	0.736	0	0.003	0.50	0	0.736	0	0.003
4	1	-1.00	0	1.712	0	0.003	1.00	0	1.712	0	0.003
4	1	-1.50	0	3.857	0	0.003	1.50	0	3.857	0	0.003
4	1	-2.00	0	7.863	0	0.003	2.00	0	7.863	0	0.003
4	1	0.00	3.440	14.231	4.560	14.231	0.00	3.440	14.231	4.560	14.231
4	1	-0.50	3.440	7.209	4.560	24.047	0.50	3.440	24.047	4.560	7.209
4	1	-1.00	3.440	3.074	4.560	35.468	1.00	3.440	35.468	4.560	3.074
4	1	-1.50	3.440	1.088	4.560	46.664	1.50	3.440	46.664	4.560	1.088
4	1	-2.00	3.440	0.322	4.560	56.024	2.00	3.440	56.024	4.560	0.316
4	1	0.00	-3.440	14.231	-4.560	14.231	0.00	-3.440	14.231	-4.560	14.231
4	1	-0.50	-3.440	24.047	-4.560	7.209	0.50	-3.440	7.209	-4.560	24.047
4	1	-1.00	-3.440	35.468	-4.560	3.074	1.00	-3.440	3.074	-4.560	35.468
4	1	-1.50	-3.440	46.664	-4.560	1.088	1.50	-3.440	1.088	-4.560	46.664
4	1	-2.00	-3.440	56.024	-4.560	0.316	2.00	-3.440	0.322	-4.560	56.024

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
3	4	0.000	0	0.149	0	0.058	0.000	0	0.149	0	0.058
3	4	-0.125	0	0.151	0	0.063	0.125	0	0.151	0	0.063
3	4	-0.250	0	0.157	0	0.074	0.250	0	0.157	0	0.074
3	4	-0.375	0	0.166	0	0.089	0.375	0	0.166	0	0.089
3	4	-0.500	0	0.179	0	0.103	0.500	0	0.179	0	0.103
3	4	0.000	2.845	3.946	3.155	3.946	0.000	2.845	3.946	3.155	3.946
3	4	-0.125	2.845	3.360	3.155	6.880	0.125	2.845	4.592	3.155	1.960
3	4	-0.250	2.845	2.835	3.155	10.629	0.250	2.845	5.297	3.155	0.826
3	4	-0.375	2.845	2.369	3.155	14.892	0.375	2.845	6.061	3.155	0.291
3	4	-0.500	2.845	1.959	3.155	19.349	0.500	2.845	6.883	3.155	0.084
3	4	0.000	-2.845	3.946	-3.155	3.946	0.000	-2.845	3.946	-3.155	3.946
3	4	-0.125	-2.845	4.592	-3.155	1.960	0.125	-2.845	3.360	-3.155	6.880
3	4	-0.250	-2.845	5.297	-3.155	0.826	0.250	-2.845	2.835	-3.155	10.629
3	4	-0.375	-2.845	6.061	-3.155	0.291	0.375	-2.845	2.369	-3.155	14.892
3	4	-0.500	-2.845	6.883	-3.155	0.084	0.500	-2.845	1.959	-3.155	19.349

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
3	3	0	0	0.242	0	0.069	0	0	0.242	0	0.069
3	3	-1/6	0	0.248	0	0.074	1/6	0	0.248	0	0.074
3	3	-1/3	0	0.266	0	0.086	1/3	0	0.266	0	0.086
3	3	-1/2	0	0.297	0	0.100	1/2	0	0.297	0	0.100
3	3	-2/3	0	0.339	0	0.113	2/3	0	0.339	0	0.113
3	3	0	2.795	5.227	3.205	5.227	0	2.795	5.227	3.205	5.227
3	3	-1/6	2.795	4.213	3.205	9.095	1/6	2.795	6.382	3.205	2.599
3	3	-1/3	2.795	3.338	3.205	13.999	1/3	2.795	7.674	3.205	1.097
3	3	-1/2	2.795	2.599	3.205	19.509	1/2	2.795	9.095	3.205	0.386
3	3	-2/3	2.795	1.987	3.205	25.158	2/3	2.795	10.633	3.205	0.112
3	3	0	-2.795	5.227	-3.205	5.227	0	-2.795	5.227	-3.205	5.227
3	3	-1/6	-2.795	6.382	-3.205	2.599	1/6	-2.795	4.213	-3.205	9.095
3	3	-1/3	-2.795	7.674	-3.205	1.097	1/3	-2.795	3.338	-3.205	13.999
3	3	-1/2	-2.795	9.095	-3.205	0.386	1/2	-2.795	2.599	-3.205	19.509
3	3	-2/3	-2.795	10.633	-3.205	0.112	2/3	-2.795	1.987	-3.205	25.158

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
3	2	0.00	0	0.544	0	0.084	0.00	0	0.544	0	0.084
3	2	-0.25	0	0.581	0	0.089	0.25	0	0.581	0	0.089
3	2	-0.50	0	0.697	0	0.100	0.50	0	0.697	0	0.100
3	2	-0.75	0	0.895	0	0.112	0.75	0	0.895	0	0.112
3	2	-1.00	0	1.184	0	0.122	1.00	0	1.184	0	0.122
3	2	0.00	2.695	7.705	3.305	7.705	0.00	2.695	7.705	3.305	7.705
3	2	-0.25	2.695	5.551	3.305	13.329	0.25	2.695	10.309	3.305	3.845
3	2	-0.50	2.695	3.845	3.305	20.320	0.50	2.695	13.326	3.305	1.625
3	2	-0.75	2.695	2.554	3.305	27.962	0.75	2.695	16.700	3.305	0.572
3	2	-1.00	2.695	1.625	3.305	35.379	1.00	2.695	20.320	3.305	0.167
3	2	0.00	-2.695	7.705	-3.305	7.705	0.00	-2.695	7.705	-3.305	7.705
3	2	-0.25	-2.695	10.309	-3.305	3.845	0.25	-2.695	5.551	-3.305	13.329
3	2	-0.50	-2.695	13.326	-3.305	1.625	0.50	-2.695	3.845	-3.305	20.320
3	2	-0.75	-2.695	16.700	-3.305	0.572	0.75	-2.695	2.554	-3.305	27.962
3	2	-1.00	-2.695	20.320	-3.305	0.167	1.00	-2.695	1.625	-3.305	35.379

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
3	1	0.00	0	3.226	0	0.106	0.00	0	3.226	0	0.106
3	1	-0.50	0	4.361	0	0.109	0.50	0	4.361	0	0.109
3	1	-1.00	0	7.946	0	0.117	1.00	0	7.946	0	0.117
3	1	-1.50	0	14.370	0	0.124	1.50	0	14.370	0	0.124
3	1	-2.00	0	23.856	0	0.130	2.00	0	23.856	0	0.130
3	1	0.00	2.440	14.237	3.560	14.231	0.00	2.440	14.237	3.560	14.231
3	1	-0.50	2.440	7.233	3.560	24.047	0.50	2.440	24.048	3.560	7.209
3	1	-1.00	2.440	3.159	3.560	35.468	1.00	2.440	35.468	3.560	3.074
3	1	-1.50	2.440	1.354	3.560	46.633	1.50	2.440	46.664	3.560	1.088
3	1	-2.00	2.440	1.065	3.560	56.024	2.00	2.440	56.024	3.560	0.316
3	1	0.00	-2.440	14.237	-3.560	14.231	0.00	-2.440	14.237	-3.560	14.231
3	1	-0.50	-2.440	24.048	-3.560	7.209	0.50	-2.440	7.233	-3.560	24.047
3	1	-1.00	-2.440	35.468	-3.560	3.074	1.00	-2.440	3.159	-3.560	35.468
3	1	-1.50	-2.440	46.664	-3.560	1.088	1.50	-2.440	1.354	-3.560	46.633
3	1	-2.00	-2.440	56.024	-3.560	0.316	2.00	-2.440	1.065	-3.560	56.024

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
2	4	0.000	0	1.4851	0	0.8006	0.000	0	1.4851	0	0.8006
2	4	-0.125	0	1.500	0	0.871	0.125	0	1.500	0	0.871
2	4	-0.250	0	1.546	0	1.055	0.250	0	1.546	0	1.055
2	4	-0.375	0	1.623	0	1.295	0.375	0	1.623	0	1.295
2	4	-0.500	0	1.729	0	1.525	0.500	0	1.729	0	1.525
2	4	0.000	1.840	3.951	2.155	3.946	0.000	1.840	3.951	2.155	3.946
2	4	-0.125	1.840	3.366	2.155	6.880	0.125	1.840	4.596	2.155	1.960
2	4	-0.250	1.840	2.842	2.155	10.630	0.250	1.840	5.301	2.155	0.835
2	4	-0.375	1.840	2.377	2.155	14.892	0.375	1.840	6.065	2.155	0.292
2	4	-0.500	1.840	1.970	2.155	19.348	0.500	1.840	6.883	2.155	0.085
2	4	0.000	-1.840	3.951	-2.155	3.946	0.000	-1.840	3.951	-2.155	3.946
2	4	-0.125	-1.840	4.596	-2.155	1.960	0.125	-1.840	3.366	-2.155	6.880
2	4	-0.250	-1.840	5.301	-2.155	0.835	0.250	-1.840	2.842	-2.155	10.630
2	4	-0.375	-1.840	6.065	-2.155	0.292	0.375	-1.840	2.377	-2.155	14.892
2	4	-0.500	-1.840	6.883	-2.155	0.085	0.500	-1.840	1.970	-2.155	19.348

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
2	3	0	0	2.203	0	0.975	0	0	2.203	0	0.975
2	3	-1/6	0	2.247	0	1.052	1/6	0	2.247	0	1.052
2	3	-1/3	0	2.377	0	1.250	1/3	0	2.377	0	1.250
2	3	-1/2	0	2.593	0	1.500	1/2	0	2.593	0	1.500
2	3	-2/3	0	2.893	0	1.736	2/3	0	2.893	0	1.736
2	3	0	1.785	5.238	2.205	5.228	0	1.785	5.238	2.205	5.228
2	3	-1/6	1.785	4.214	2.205	9.095	1/6	1.785	6.391	2.205	2.600
2	3	-1/3	1.785	3.356	2.205	13.999	1/3	1.785	7.682	2.205	1.097
2	3	-1/2	1.785	2.622	2.205	19.521	1/2	1.785	9.101	2.205	0.386
2	3	-2/3	1.785	2.016	2.205	25.158	2/3	1.785	10.641	2.205	0.113
2	3	0	-1.785	5.238	-2.205	5.228	0	-1.785	5.238	-2.205	5.228
2	3	-1/6	-1.785	6.391	-2.205	2.600	1/6	-1.785	4.214	-2.205	9.095
2	3	-1/3	-1.785	7.682	-2.205	1.097	1/3	-1.785	3.356	-2.205	13.999
2	3	-1/2	-1.785	9.101	-2.205	0.386	1/2	-1.785	2.622	-2.205	19.521
2	3	-2/3	-1.785	10.641	-2.205	0.113	2/3	-1.785	2.016	-2.205	25.158

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
2	2	0.00	0	4.053	0	1.239	0.00	0	4.053	0	1.239
2	2	-0.25	0	4.253	0	1.317	0.25	0	4.253	0	1.317
2	2	-0.50	0	4.852	0	1.516	0.50	0	4.852	0	1.516
2	2	-0.75	0	5.843	0	1.754	0.75	0	5.843	0	1.754
2	2	-1.00	0	7.219	0	1.960	1.00	0	7.219	0	1.960
2	2	0.00	1.680	7.746	2.305	7.705	0.00	1.680	7.746	2.305	7.705
2	2	-0.25	1.680	5.613	2.305	13.329	0.25	1.680	10.338	2.305	3.845
2	2	-0.50	1.680	3.940	2.305	20.320	0.50	1.680	13.354	2.305	1.625
2	2	-0.75	1.680	2.697	2.305	27.962	0.75	1.680	16.727	2.305	0.573
2	2	-1.00	1.680	1.834	2.305	35.379	1.00	1.680	20.368	2.305	0.167
2	2	0.00	-1.680	7.746	-2.305	7.705	0.00	-1.680	7.746	-2.305	7.705
2	2	-0.25	-1.680	10.338	-2.305	3.845	0.25	-1.680	5.613	-2.305	13.329
2	2	-0.50	-1.680	13.354	-2.305	1.625	0.50	-1.680	3.940	-2.305	20.320
2	2	-0.75	-1.680	16.727	-2.305	0.573	0.75	-1.680	2.697	-2.305	27.962
2	2	-1.00	-1.680	20.368	-2.305	0.167	1.00	-1.680	1.834	-2.305	35.379

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
2	1	0.00	0	12.836	0	1.656	0.00	0	12.836	0	1.656
2	1	-0.50	0	15.465	0	1.718	0.50	0	15.465	0	1.718
2	1	-1.00	0	22.987	0	1.867	1.00	0	22.987	0	1.867
2	1	-1.50	0	34.327	0	2.027	1.50	0	34.327	0	2.027
2	1	-2.00	0	47.816	0	2.132	2.00	0	47.816	0	2.132
2	1	0.00	1.280	15.072	2.560	14.231	0.00	1.280	15.071	2.560	14.231
2	1	-0.50	1.280	9.480	2.560	24.045	0.50	1.280	24.450	2.560	7.210
2	1	-1.00	1.280	8.269	2.560	35.454	1.00	1.280	36.123	2.560	3.074
2	1	-1.50	1.280	11.355	2.560	46.589	1.50	1.280	48.038	2.560	1.088
2	1	-2.00	1.280	18.325	2.560	55.708	2.00	1.280	58.355	2.560	0.316
2	1	-0.885	1.280	8.157							
2	1	0.00	-1.280	15.071	-2.560	14.231	0.00	-1.280	15.072	-2.560	14.231
2	1	-0.50	-1.280	24.450	-2.560	7.210	0.50	-1.280	9.480	-2.560	24.045
2	1	-1.00	-1.280	36.123	-2.560	3.074	1.00	-1.280	8.269	-2.560	35.454
2	1	-1.50	-1.280	48.038	-2.560	1.088	1.50	-1.280	11.355	-2.560	46.589
2	1	-2.00	-1.280	58.355	-2.560	0.316	2.00	-1.280	18.325	-2.560	55.708
2	1						0.885	-1.280	8.157		

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1.5	4	0.000	0	3.247	0	2.047	0.000	0	3.247	0	2.047
1.5	4	-0.125	0	3.278	0	2.239	0.125	0	3.278	0	2.239
1.5	4	-0.250	0	3.370	0	2.750	0.250	0	3.370	0	2.750
1.5	4	-0.375	0	3.523	0	3.427	0.375	0	3.523	0	3.427
1.5	4	-0.500	0	3.735	0	4.119	0.500	0	3.735	0	4.119
1.5	4	0.000	1.250	4.073	1.640	3.964	0.000	1.250	4.073	1.640	3.964
1.5	4	-0.125	1.250	3.516	1.640	6.883	0.125	1.250	4.694	1.640	1.990
1.5	4	-0.250	1.250	3.023	1.640	10.613	0.250	1.250	5.381	1.640	0.870
1.5	4	-0.375	1.250	2.593	1.640	14.851	0.375	1.250	6.129	1.640	0.346
1.5	4	-0.500	1.250	2.225	1.640	19.305	0.500	1.250	6.936	1.640	0.149
1.5	4	0.000	-1.250	4.073	-1.640	3.964	0.000	-1.250	4.073	-1.640	3.964
1.5	4	-0.125	-1.250	4.694	-1.640	1.990	0.125	-1.250	3.516	-1.640	6.883
1.5	4	-0.250	-1.250	5.381	-1.640	0.870	0.250	-1.250	3.023	-1.640	10.613
1.5	4	-0.375	-1.250	6.129	-1.640	0.346	0.375	-1.250	2.593	-1.640	14.851
1.5	4	-0.500	-1.250	6.936	-1.640	0.149	0.500	-1.250	2.225	-1.640	19.305

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1.5	3	0	0	4.645	0	2.534	0	0	4.645	0	2.534
1.5	3	-1/6	0	4.727	0	2.750	1/6	0	4.727	0	2.750
1.5	3	-1/3	0	4.972	0	3.318	1/3	0	4.972	0	3.318
1.5	3	-1/2	0	5.376	0	4.049	1/2	0	5.376	0	4.049
1.5	3	-2/3	0	5.933	0	4.763	2/3	0	5.933	0	4.763
1.5	3	0	1.160	5.435	1.695	5.245	0	1.160	5.435	1.695	5.245
1.5	3	-1/6	1.160	4.532	1.695	9.101	1/6	1.160	6.559	1.695	2.629
1.5	3	-1/3	1.160	3.753	1.695	13.991	1/3	1.160	7.802	1.695	1.138
1.5	3	-1/2	1.160	3.011	1.695	19.523	1/2	1.160	9.190	1.695	0.434
1.5	3	-2/3	1.160	2.645	1.695	25.111	2/3	1.160	10.709	1.695	0.170
1.5	3	0	-1.160	5.435	-1.695	5.245	0	-1.160	5.435	-1.695	5.245
1.5	3	-1/6	-1.160	6.559	-1.695	2.629	1/6	-1.160	4.532	-1.695	9.101
1.5	3	-1/3	-1.160	7.802	-1.695	1.138	1/3	-1.160	3.753	-1.695	13.991
1.5	3	-1/2	-1.160	9.190	-1.695	0.434	1/2	-1.160	3.011	-1.695	19.523
1.5	3	-2/3	-1.160	10.709	-1.695	0.170	2/3	-1.160	2.645	-1.695	25.111

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1.5	2	0.00	0	7.908	0	3.298	0.00	0	7.908	0	3.298
1.5	2	-0.25	0	8.240	0	3.532	0.25	0	8.240	0	3.532
1.5	2	-0.50	0	9.225	0	4.132	0.50	0	9.225	0	4.132
1.5	2	-0.75	0	10.830	0	4.866	0.75	0	10.830	0	4.866
1.5	2	-1.00	0	13.007	0	5.527	1.00	0	13.007	0	5.527
1.5	2	0.00	0.935	8.404	1.795	7.721	0.00	0.935	8.404	1.795	7.721
1.5	2	-0.25	0.935	6.701	1.795	13.330	0.25	0.935	10.684	1.795	3.871
1.5	2	-0.50	0.935	5.598	1.795	20.324	0.50	0.935	13.493	1.795	1.658
1.5	2	-0.75	0.935	5.039	1.795	27.914	0.75	0.935	16.786	1.795	0.553
1.5	2	-1.00	0.935	5.052	1.795	35.379	1.00	0.935	20.434	1.795	0.210
1.5	2	-0.885	0.935	4.967							
1.5	2	0.00	-0.935	8.404	-1.795	7.721	0.00	-0.935	8.404	-1.795	7.721
1.5	2	-0.25	-0.935	10.684	-1.795	3.871	0.25	-0.935	6.701	-1.795	13.330
1.5	2	-0.50	-0.935	13.493	-1.795	1.658	0.50	-0.935	5.598	-1.795	20.324
1.5	2	-0.75	-0.935	16.786	-1.795	0.553	0.75	-0.935	5.039	-1.795	27.914
1.5	2	-1.00	-0.935	20.434	-1.795	0.210	1.00	-0.935	5.052	-1.795	35.379
1.5	2						0.885	-0.935	4.967		

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1.5	1	0.00	0	20.111	0	4.588	0.00	0	20.111	0	4.588
1.5	1	-0.50	0	23.256	0	4.778	0.50	0	23.256	0	4.778
1.5	1	-1.00	0	31.894	0	5.217	1.00	0	31.894	0	5.217
1.5	1	-1.50	0	43.943	0	5.610	1.50	0	43.943	0	5.610
1.5	1	-2.00	0	56.789	0	5.667	2.00	0	56.789	0	5.667
1.5	1	0.00	0	20.111	2.060	14.225	0.00	0	20.111	2.060	14.225
1.5	1	-0.50	0	23.256	2.060	23.977	0.50	0	23.256	2.060	7.218
1.5	1	-1.00	0	31.894	2.060	35.154	1.00	0	31.894	2.060	3.088
1.5	1	-1.50	0	43.943	2.060	45.576	1.50	0	43.943	2.060	1.244
1.5	1	-2.00	0	56.789	2.060	52.950	2.00	0	56.789	2.060	0.332
1.5	1	0.00	0	20.111	-2.060	14.225	0.00	0	20.111	-2.060	14.225
1.5	1	-0.50	0	23.256	-2.060	7.218	0.50	0	23.256	-2.060	23.977
1.5	1	-1.00	0	31.894	-2.060	3.088	1.00	0	31.894	-2.060	35.154
1.5	1	-1.50	0	43.943	-2.060	1.244	1.50	0	43.943	-2.060	45.576
1.5	1	-2.00	0	56.789	-2.060	0.332	2.00	0	56.789	-2.060	52.950

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1	4	0.000	0	5.558	0	4.091	0.000	0	5.558	0	4.091
1	4	-0.125	0	5.606	0	4.503	0.125	0	5.606	0	4.503
1	4	-0.250	0	5.752	0	5.611	0.250	0	5.752	0	5.611
1	4	-0.375	0	5.992	0	7.109	0.375	0	5.992	0	7.109
1	4	-0.500	0	6.324	0	8.688	0.500	0	6.324	0	8.688
1	4	0.000	0	5.558	0.880	4.314	0.000	0	5.558	0.880	4.314
1	4	-0.125	0	5.606	0.880	6.843	0.125	0	5.606	0.880	2.751
1	4	-0.250	0	5.752	0.880	10.185	0.250	0	5.752	0.880	2.047
1	4	-0.375	0	5.992	0.880	14.003	0.375	0	5.992	0.880	1.915
1	4	-0.500	0	6.324	0.880	17.945	0.500	0	6.324	0.880	2.058
1	4						0.355			0.880	1.911
1	4	0.000	0	5.558	-0.880	4.314	0.000	0	5.558	-0.880	4.314
1	4	-0.125	0	5.606	-0.880	2.751	0.125	0	5.606	-0.880	6.843
1	4	-0.250	0	5.752	-0.880	2.047	0.250	0	5.752	-0.880	10.185
1	4	-0.375	0	5.992	-0.880	1.915	0.375	0	5.992	-0.880	14.003
1	4	-0.500	0	6.324	-0.880	2.058	0.500	0	6.324	-0.880	17.945
1	4	-0.355			-0.880	1.911					

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1	3	0	0	7.700	0	5.153	0	0	7.700	0	5.153
1	3	-1/6	0	7.822	0	5.634	1/6	0	7.822	0	5.634
1	3	-1/3	0	8.185	0	6.914	1/3	0	8.185	0	6.914
1	3	-1/2	0	8.782	0	8.602	1/2	0	8.782	0	8.602
1	3	-2/3	0	9.600	0	10.310	2/3	0	9.600	0	10.310
1	3	0	0	7.700	0.990	5.609	0	0	7.700	0.990	5.609
1	3	-1/6	0	7.822	0.990	9.084	1/6	0	7.822	0.990	3.371
1	3	-1/3	0	8.185	0.990	13.572	1/3	0	8.185	0.990	2.247
1	3	-1/2	0	8.782	0.990	18.602	1/2	0	8.782	0.990	1.875
1	3	-2/3	0	9.600	0.990	23.680	2/3	0	9.600	0.990	1.875
1	3						0.575			0.990	1.848
1	3	0	0	7.700	-0.990	5.609	0	0	7.700	-0.990	5.609
1	3	-1/6	0	7.822	-0.990	3.371	1/6	0	7.822	-0.990	9.084
1	3	-1/3	0	8.185	-0.990	2.247	1/3	0	8.185	-0.990	13.572
1	3	-1/2	0	8.782	-0.990	1.875	1/2	0	8.782	-0.990	18.602
1	3	-2/3	0	9.600	-0.990	1.875	2/3	0	9.600	-0.990	23.680
1	3	-0.575			-0.990	1.848					



$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1	2	0.00	0	12.280	0	6.902	0.00	0	12.280	0	6.902
1	2	-0.25	0	12.721	0	7.455	0.25	0	12.721	0	7.455
1	2	-0.50	0	14.018	0	8.887	0.50	0	14.018	0	8.887
1	2	-0.75	0	16.105	0	10.677	0.75	0	16.105	0	10.677
1	2	-1.00	0	18.878	0	12.303	1.00	0	18.878	0	12.303
1	2	0.00	0	12.280	1.165	8.061	0.00	0	12.280	1.165	8.061
1	2	-0.25	0	12.721	1.165	13.345	0.25	0	12.721	1.165	4.506
1	2	-0.50	0	14.018	1.165	19.921	0.50	0	14.018	1.165	2.568
1	2	-0.75	0	16.105	1.165	27.053	0.75	0	16.105	1.165	1.724
1	2	-1.00	0	18.878	1.165	33.736	1.00	0	18.878	1.165	1.483
1	2	0.00	0	12.280	-1.165	8.061	0.00	0	12.280	-1.165	8.061
1	2	-0.25	0	12.721	-1.165	4.506	0.25	0	12.721	-1.165	13.345
1	2	-0.50	0	14.018	-1.165	2.568	0.50	0	14.018	-1.165	19.921
1	2	-0.75	0	16.105	-1.165	1.724	0.75	0	16.105	-1.165	27.053
1	2	-1.00	0	18.878	-1.165	1.483	1.00	0	18.878	-1.165	33.736

$\sigma$	UR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR	Meas. Bias	Prod. Bias	PR	Prod. Bias	CR
1	1	0.00	0	26.049	0	9.830	0.00	0	26.049	0	9.830
1	1	-0.50	0	29.027	0	10.133	0.50	0	29.027	0	10.133
1	1	-1.00	0	36.801	0	10.668	1.00	0	36.801	0	10.668
1	1	-1.50	0	46.583	0	10.642	1.50	0	46.583	0	10.642
1	1	-2.00	0	55.507	0	9.518	2.00	0	55.507	0	9.518
1	1	-1.250			0	10.772	1.250	0			10.772
1	1	0.00	0	26.049	1.510	14.120	0.00	0	26.049	1.510	14.120
1	1	-0.50	0	29.027	1.510	22.966	0.50	0	29.027	1.510	7.471
1	1	-1.00	0	36.801	1.510	32.135	1.00	0	36.801	1.510	3.463
1	1	-1.50	0	46.583	1.510	38.882	1.50	0	46.583	1.510	1.489
1	1	-2.00	0	55.507	1.510	40.920	2.00	0	55.507	1.510	0.659
1	1	0.00	0	26.049	-1.510	14.120	0.00	0	26.049	-1.510	14.120
1	1	-0.50	0	29.027	-1.510	7.471	0.50	0	29.027	-1.510	22.966
1	1	-1.00	0	36.801	-1.510	3.463	1.00	0	36.801	-1.510	32.135
1	1	-1.50	0	46.583	-1.510	1.489	1.50	0	46.583	-1.510	38.882
1	1	-2.00	0	55.507	-1.510	0.659	2.00	0	55.507	-1.510	40.920

## 5. Risk Tables (with guardbands)

Table 2 illustrates how product biases (PB) and measurement biases (MB) affect the values of guardbands for consumer risks of 0.80%, 1.6%, 2.26% and 5.00%. The biases are expressed in units of product sigma, and the risks are expressed in percent. A description of how the table is to be interpreted follows.

Column 1 contains the sigma model.

Column 2 contains the uncertainty ratio.

Column 3 contains the number of product sigma the mean of the measurement system distribution is biased relative to its certified bilateral performance specification nominal.

Column 4 contains the number of product sigma the mean of the product measurand distribution is biased relative to its bilateral performance specification nominal. This value will be zero for the first group of five rows. The non-zero product biases of the middle and last group of five rows are those that result in the greatest (worst-case) consumer risks at zero measurement bias.

Column 5 contains the indication that the tabulated consumer risks are valid for the parameter values and polarity either in columns 1, 2, 3 and 4, or 1, 2, 6 and 7.

Column 6 contains the number of product sigma the mean of the measurement system distribution is biased relative to its bilateral performance specification nominal. This value will be zero for the first group of five rows. The non-zero product biases of the middle and last group of five rows are those that result in the greatest (worst-case) consumer risks at zero measurement bias.

Column 7 contains the number of product sigma the mean of the product measurand distribution is biased relative to its bilateral performance specification nominal. This value will be zero for the first group of five rows. The non-zero product biases of the middle and last group of five rows are those that result in the greatest (worst-case) consumer risks at zero measurement bias.

Column 8 contains the guardband factor necessary to achieve a value of consumer risk of 0.80%.

Column 9 contains the guardband factor necessary to achieve a value of consumer risk of 1.60%.

Column 10 contains the guardband factor necessary to achieve a value of consumer risk of 2.26%.

Column 11 contains the guardband factor necessary to achieve a value of consumer risk of 5.00%.

1	2	3	4	5	6	7	8	9	10	11
$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
4	4	0.000	0		0.000	0	1	1	1	1
4	4	-0.250	4.155		0.250	-4.155	0.873	0.898	0.912	0.950
4	4	-0.125	-4.155		0.125	4.155	0.967	0.992	1	1

**Table 2. Sigma Models of 4 to 1, and Uncertainty Ratios of 4 to 1 with Guardbanding.**

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
4	4	0.000	0		0.000	0	1	1	1	1
4	4	-0.125	0		0.125	0	1	1	1	1
4	4	-0.250	0		0.250	0	1	1	1	1
4	4	-0.375	0		0.375	0	1	1	1	1
4	4	-0.500	0		0.500	0	1	1	1	1
4	4	0.000	4.155	or	0.000	-4.155	0.936	0.961	0.974	1
4	4	-0.125	4.155		0.125	-4.155	0.905	0.929	0.943	0.981
4	4	-0.250	4.155		0.250	-4.155	0.873	0.898	0.912	0.950
4	4	-0.375	4.155		0.375	-4.155	0.842	0.867	0.880	0.918
4	4	-0.500	4.155		0.500	-4.155	0.811	0.835	0.849	0.887
4	4	0.000	-4.155	or	0.000	4.155	0.936	0.961	0.974	1
4	4	-0.125	-4.155		0.125	4.155	0.967	0.992	1	1
4	4	-0.250	-4.155		0.250	4.155	0.998	1	1	1
4	4	-0.375	-4.155		0.375	4.155	1	1	1	1
4	4	-0.500	-4.155		0.500	4.155	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
4	3	0.00	0		0.00	0	1	1	1	1
4	3	-1/6	0		1/6	0	1	1	1	1
4	3	-1/3	0		1/3	0	1	1	1	1
4	3	-1/2	0		1/2	0	1	1	1	1
4	3	-2/3	0		2/3	0	1	1	1	1
4	3	0.00	4.205	or	0.00	-4.205	0.903	0.934	0.951	0.997
4	3	-1/6	4.205		1/6	-4.205	0.861	0.892	0.909	0.955
4	3	-1/3	4.205		1/3	-4.205	0.819	0.850	0.867	0.913
4	3	-1/2	4.205		1/2	-4.205	0.778	0.808	0.826	0.872
4	3	-2/3	4.205		2/3	-4.205	0.736	0.767	0.784	0.830
4	3	0.00	-4.205	or	0.00	4.205	0.903	0.934	0.951	0.997
4	3	-1/6	-4.205		1/6	4.205	0.944	0.975	0.992	1
4	3	-1/3	-4.205		1/3	4.205	0.986	1	1	1
4	3	-1/2	-4.205		1/2	4.205	1	1	1	1
4	3	-2/3	-4.205		2/3	4.205	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
4	2	0.00	0		0.00	0	1	1	1	1
4	2	-0.25	0		0.25	0	1	1	1	1
4	2	-0.50	0		0.50	0	1	1	1	1
4	2	-0.75	0		0.75	0	1	1	1	1
4	2	-1.00	0		1.00	0	1	1	1	1
4	2	0.00	4.305	or	0.00	-4.305	0.831	0.874	0.897	0.959
4	2	-0.25	4.305		0.25	-4.305	0.768	0.811	0.835	0.897
4	2	-0.50	4.305		0.50	-4.305	0.706	0.749	0.772	0.834
4	2	-0.75	4.305		0.75	-4.305	0.643	0.686	0.710	0.772
4	2	-1.00	4.305		1.00	-4.305	0.581	0.624	0.647	0.709
4	2	0.00	-4.305	or	0.00	4.305	0.831	0.874	0.897	0.959
4	2	-0.25	-4.305		0.25	4.305	0.893	0.936	0.960	1
4	2	-0.50	-4.305		0.50	4.305	0.956	0.999	1	1
4	2	-0.75	-4.305		0.75	4.305	1	1	1	1
4	2	-1.00	-4.305		1.00	4.305	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
4	1	0.00	0		0.00	0	1	1	1	1
4	1	-0.50	0		0.50	0	1	1	1	1
4	1	-1.00	0		1.00	0	1	1	1	1
4	1	-1.50	0		1.50	0	1	1	1	1
4	1	-2.00	0		2.00	0	1	1	1	1
4	1	0.00	4.560	or	0.00	-4.560	0.592	0.668	0.710	0.818
4	1	-0.50	4.560		0.50	-4.560	0.466	0.543	0.585	0.693
4	1	-1.00	4.560		1.00	-4.560	0.342	0.391	0.460	0.568
4	1	-1.50	4.560		1.50	-4.560	0.217	0.293	0.335	0.443
4	1	-2.00	4.560		2.00	-4.560	0.104	0.171	0.211	0.318
4	1	0.00	-4.560	or	0.00	4.560	0.592	0.668	0.710	0.818
4	1	-0.50	-4.560		0.50	4.560	0.717	0.793	0.835	0.943
4	1	-1.00	-4.560		1.00	4.560	0.841	0.918	0.960	1
4	1	-1.50	-4.560		1.50	4.560	0.967	1	1	1
4	1	-2.00	-4.560		2.00	4.560	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
3	4	0.000	0		0.000	0	1	1	1	1
3	4	-0.125	0		0.125	0	1	1	1	1
3	4	-0.250	0		0.250	0	1	1	1	1
3	4	-0.375	0		0.375	0	1	1	1	1
3	4	-0.500	0		0.500	0	1	1	1	1
3	4	0.000	3.155	or	0.000	-3.155	0.915	0.947	0.966	1
3	4	-0.125	3.155		0.125	-3.155	0.873	0.906	0.924	0.974
3	4	-0.250	3.155		0.250	-3.155	0.831	0.864	0.882	0.933
3	4	-0.375	3.155		0.375	-3.155	0.790	0.822	0.842	0.891
3	4	-0.500	3.155		0.500	-3.155	0.748	0.781	0.799	0.850
3	4	0.000	-3.155	or	0.000	3.155	0.915	0.947	0.966	1
3	4	-0.125	-3.155		0.125	3.155	0.956	0.989	1	1
3	4	-0.250	-3.155		0.250	3.155	0.998	1	1	1
3	4	-0.375	-3.155		0.375	3.155	1	1	1	1
3	4	-0.500	-3.155		0.500	3.155	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
3	3	0.00	0		0.00	0	1	1	1	1
3	3	-1/6	0		1/6	0	1	1	1	1
3	3	-1/3	0		1/3	0	1	1	1	1
3	3	-1/2	0		1/2	0	1	1	1	1
3	3	-2/3	0		2/3	0	1	1	1	1
3	3	0.00	3.205	or	0.00	-3.205	0.871	0.911	0.934	0.996
3	3	-1/6	3.205		1/6	-3.205	0.815	0.856	0.879	0.940
3	3	-1/3	3.205		1/3	-3.205	0.759	0.800	0.823	0.884
3	3	-1/2	3.205		1/2	-3.205	0.704	0.745	0.767	0.829
3	3	-2/3	3.205		2/3	-3.205	0.648	0.689	0.712	0.773
3	3	0.00	-3.205	or	0.00	3.205	0.871	0.911	0.934	0.996
3	3	-1/6	-3.205		1/6	3.205	0.926	0.967	0.990	1
3	3	-1/3	-3.205		1/3	3.205	0.982	1	1	1
3	3	-1/2	-3.205		1/2	3.205	1	1	1	1
3	3	-2/3	-3.205		2/3	3.205	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
3	2	0.00	0		0.00	0	1	1	1	1
3	2	-0.25	0		0.25	0	1	1	1	1
3	2	-0.50	0		0.50	0	1	1	1	1
3	2	-0.75	0		0.75	0	1	1	1	1
3	2	-1.00	0		1.00	0	1	1	1	1
3	2	0.00	3.305	or	0.00	-3.305	0.775	0.832	0.863	0.946
3	2	-0.25	3.305		0.25	-3.305	0.691	0.748	0.780	0.862
3	2	-0.50	3.305		0.50	-3.305	0.608	0.665	0.696	0.779
3	2	-0.75	3.305		0.75	-3.305	0.525	0.582	0.613	0.696
3	2	-1.00	3.305		1.00	-3.305	0.441	0.498	0.530	0.612
3	2	0.00	-3.305	or	0.00	3.305	0.775	0.832	0.863	0.946
3	2	-0.25	-3.305		0.25	3.305	0.858	0.915	0.946	1
3	2	-0.50	-3.305		0.50	3.305	0.941	0.998	1	1
3	2	-0.75	-3.305		0.75	3.305	1	1	1	1
3	2	-1.00	-3.305		1.00	3.305	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
3	1	0.00	0		0.00	0	1	1	1	1
3	1	-0.50	0		0.50	0	1	1	1	1
3	1	-1.00	0		1.00	0	1	1	1	1
3	1	-1.50	0		1.50	0	1	1	1	1
3	1	-2.00	0		2.00	0	1	1	1	1
3	1	0.00	3.560	or	0.00	-3.560	0.456	0.558	0.614	0.757
3	1	-0.50	3.560		0.50	-3.560	0.290	0.391	0.447	0.590
3	1	-1.00	3.560		1.00	-3.560	0.138	0.229	0.282	0.424
3	1	-1.50	3.560		1.50	-3.560	0.053	0.103	0.140	0.263
3	1	-2.00	3.560		2.00	-3.560	0.022	0.045	0.064	0.138
3	1	0.00	-3.560	or	0.00	3.560	0.456	0.558	0.614	0.757
3	1	-0.50	-3.560		0.50	3.560	0.622	0.725	0.780	0.924
3	1	-1.00	-3.560		1.00	3.560	0.789	0.891	0.947	1
3	1	-1.50	-3.560		1.50	3.560	0.956	1	1	1
3	1	-2.00	-3.560		2.00	3.560	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
2	4	0.000	0		0.000	0	1	1	1	1
2	4	-0.125	0		0.125	0	1	1	1	1
2	4	-0.250	0		0.250	0	0.966	1	1	1
2	4	-0.375	0		0.375	0	0.904	1	1	1
2	4	-0.500	0		0.500	0	0.841	1	1	1
2	4	0.000	2.155	or	0.000	-2.155	0.872	0.921	0.961	1
2	4	-0.125	2.155		0.125	-2.155	0.810	0.859	0.885	0.962
2	4	-0.250	2.155		0.250	-2.155	0.747	0.796	0.824	0.899
2	4	-0.375	2.155		0.375	-2.155	0.685	0.734	0.761	0.837
2	4	-0.500	2.155		0.500	-2.155	0.622	0.671	0.699	0.774
2	4	0.000	-2.155	or	0.000	2.155	0.872	0.921	0.961	1
2	4	-0.125	-2.155		0.125	2.155	0.935	0.984	1	1
2	4	-0.250	-2.155		0.250	2.155	0.997	1	1	1
2	4	-0.375	-2.155		0.375	2.155	1	1	1	1
2	4	-0.500	-2.155		0.500	2.155	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
2	3	0.00	0		0.00	0	1	1	1	1
2	3	-1/6	0		1/6	0	1	1	1	1
2	3	-1/3	0		1/3	0	0.917	1	1	1
2	3	-1/2	0		1/2	0	0.833	1	1	1
2	3	-2/3	0		2/3	0	0.750	0.958	1	1
2	3	0.00	2.205	or	0.00	-2.205	0.806	0.867	0.901	0.994
2	3	-1/6	2.205		1/6	-2.205	0.723	0.784	0.818	0.910
2	3	-1/3	2.205		1/3	-2.205	0.639	0.701	0.735	0.827
2	3	-1/2	2.205		1/2	-2.205	0.556	0.617	0.651	0.744
2	3	-2/3	2.205		2/3	-2.205	0.473	0.534	0.568	0.660
2	3	0.00	-2.205	or	0.00	2.205	0.806	0.867	0.901	0.994
2	3	-1/6	-2.205		1/6	2.205	0.889	0.951	0.985	1
2	3	-1/3	-2.205		1/3	2.205	0.973	1	1	1
2	3	-1/2	-2.205		1/2	2.205	1	1	1	1
2	3	-2/3	-2.205		2/3	2.205	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
2	2	0.00	0		0.00	0	1	1	1	1
2	2	-0.25	0		0.25	0	0.939	1	1	1
2	2	-0.50	0		0.50	0	0.814	1	1	1
2	2	-0.75	0		0.75	0	0.689	0.960	1	1
2	2	-1.00	0		1.00	0	0.563	0.833	1	1
2	2	0.00	2.305	or	0.00	-2.305	0.662	0.748	0.794	0.919
2	2	-0.25	2.305		0.25	-2.305	0.537	0.623	0.670	0.794
2	2	-0.50	2.305		0.50	-2.305	0.412	0.498	0.545	0.669
2	2	-0.75	2.305		0.75	-2.305	0.287	0.373	0.420	0.544
2	2	-1.00	2.305		1.00	-2.305	0.166	0.248	0.295	0.419
2	2	0.00	-2.305	or	0.00	2.305	0.662	0.748	0.794	0.919
2	2	-0.25	-2.305		0.25	2.305	0.787	0.842	0.920	1
2	2	-0.50	-2.305		0.50	2.305	0.912	0.937	1	1
2	2	-0.75	-2.305		0.75	2.305	1	1	1	1
2	2	-1.00	-2.305		1.00	2.305	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
2	1	0.00	0		0.00	0	0.696	0.983	1	1
2	1	-0.50	0		0.50	0	0.640	0.959	1	1
2	1	-1.00	0		1.00	0	0.485	0.876	1	1
2	1	-1.50	0		1.50	0	0.330	0.720	1	1
2	1	-2.00	0		2.00	0	0.253	0.580	1	1
2	1	0.00	2.560	or	0.00	-2.560	0.208	0.343	0.423	0.636
2	1	-0.50	2.560		0.50	-2.560	0.080	0.154	0.212	0.394
2	1	-1.00	2.560		1.00	-2.560	0.034	0.068	0.096	0.207
2	1	-1.50	2.560		1.50	-2.560	0.018	0.036	0.051	0.113
2	1	-2.00	2.560		2.00	-2.560	0.011	0.023	0.033	0.073
2	1	0.00	-2.560	or	0.00	2.560	0.208	0.343	0.423	0.636
2	1	-0.50	-2.560		0.50	2.560	0.435	0.587	0.671	0.886
2	1	-1.00	-2.560		1.00	2.560	0.684	0.837	0.920	1
2	1	-1.50	-2.560		1.50	2.560	0.934	1	1	1
2	1	-2.00	-2.560		2.00	2.560	1	1	1	1



$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1.5	4	0.000	0		0.000	0	0.966	1	1	1
1.5	4	-0.125	0		0.125	0	0.883	0.987	1	1
1.5	4	-0.250	0		0.250	0	0.800	0.904	0.973	1
1.5	4	-0.375	0		0.375	0	0.716	0.820	0.890	1
1.5	4	-0.500	0		0.500	0	0.633	0.737	0.807	1
1.5	4	0.000	1.640	or	0.000	-1.640	0.830	0.895	0.932	1
1.5	4	-0.125	1.640		0.125	-1.640	0.747	0.812	0.848	0.949
1.5	4	-0.250	1.640		0.250	-1.640	0.663	0.728	0.765	0.866
1.5	4	-0.375	1.640		0.375	-1.640	0.580	0.645	0.682	0.783
1.5	4	-0.500	1.640		0.500	-1.640	0.497	0.562	0.598	0.699
1.5	4	0.000	-1.640	or	0.000	1.640	0.830	0.895	0.932	1
1.5	4	-0.125	-1.640		0.125	1.640	0.913	0.978	1	1
1.5	4	-0.250	-1.640		0.250	1.640	0.997	1	1	1
1.5	4	-0.375	-1.640		0.375	1.640	1	1	1	1
1.5	4	-0.500	-1.640		0.500	1.640	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1.5	3	0.00	0		0.00	0	0.819	0.919	0.978	1
1.5	3	-1/6	0		1/6	0	0.784	0.894	0.962	1
1.5	3	-1/3	0		1/3	0	0.695	0.819	0.895	1
1.5	3	-1/2	0		1/2	0	0.585	0.714	0.797	1
1.5	3	-2/3	0		2/3	0	0.474	0.603	0.687	1
1.5	3	0.00	1.695	or	0.00	-1.695	0.741	0.823	0.868	0.991
1.5	3	-1/6	1.695		1/6	-1.695	0.630	0.712	0.758	0.881
1.5	3	-1/3	1.695		1/3	-1.695	0.519	0.601	0.647	0.769
1.5	3	-1/2	1.695		1/2	-1.695	0.408	0.490	0.535	0.658
1.5	3	-2/3	1.695		2/3	-1.695	0.297	0.379	0.424	0.547
1.5	3	0.00	-1.695	or	0.00	1.695	0.741	0.823	0.868	0.991
1.5	3	-1/6	-1.695		1/6	1.695	0.851	0.933	0.978	1
1.5	3	-1/3	-1.695		1/3	1.695	0.959	1	1	1
1.5	3	-1/2	-1.695		1/2	1.695	1	1	1	1
1.5	3	-2/3	-1.695		2/3	1.695	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1.5	2	0.00	0		0.00	0	0.672	0.812	0.897	1
1.5	2	-0.25	0		0.25	0	0.616	0.771	0.861	1
1.5	2	-0.50	0		0.50	0	0.477	0.650	0.757	1
1.5	2	-0.75	0		0.75	0	0.315	0.495	0.603	1
1.5	2	-1.00	0		1.00	0	0.181	0.332	0.440	0.883
1.5	2	0.00	1.795	or	0.00	-1.795	0.550	0.622	0.726	0.891
1.5	2	-0.25	1.795		0.25	-1.795	0.383	0.497	0.559	0.725
1.5	2	-0.50	1.795		0.50	-1.795	0.221	0.331	0.393	0.558
1.5	2	-0.75	1.795		0.75	-1.795	0.098	0.179	0.234	0.393
1.5	2	-1.00	1.795		1.00	-1.795	0.043	0.085	0.118	0.240
1.5	2	0.00	-1.795	or	0.00	1.795	0.550	0.622	0.726	0.891
1.5	2	-0.25	-1.795		0.25	1.795	0.715	0.829	0.891	1
1.5	2	-0.50	-1.795		0.50	1.795	0.878	0.994	1	1
1.5	2	-0.75	-1.795		0.75	1.795	1	1	1	1
1.5	2	-1.00	-1.795		1.00	1.795	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1.5	1	0.00	0		0.00	0	0.264	0.476	0.618	1
1.5	1	-0.50	0		0.50	0	0.218	0.414	0.557	1
1.5	1	-1.00	0		1.00	0	0.150	0.300	0.425	0.956
1.5	1	-1.50	0		1.50	0	0.114	0.232	0.332	0.845
1.5	1	-2.00	0		2.00	0	0.107	0.217	0.312	0.810
1.5	1	0.00	2.060	or	0.00	-2.060	0.106	0.206	0.281	0.525
1.5	1	-0.50	2.060		0.50	-2.060	0.045	0.091	0.128	0.276
1.5	1	-1.00	2.060		1.00	-2.060	0.024	0.048	0.068	0.150
1.5	1	-1.50	2.060		1.50	-2.060	0.015	0.031	0.044	0.097
1.5	1	-2.00	2.060		2.00	-2.060	0.012	0.024	0.034	0.076
1.5	1	0.00	-2.060	or	0.00	2.060	0.106	0.206	0.281	0.525
1.5	1	-0.50	-2.060		0.50	2.060	0.276	0.457	0.564	0.847
1.5	1	-1.00	-2.060		1.00	2.060	0.577	0.781	0.892	1
1.5	1	-1.50	-2.060		1.50	2.060	0.906	1	1	1
1.5	1	-2.00	-2.060		2.00	2.060	1	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1	4	0.000	0		0.000	0	0.735	0.833	0.888	1
1	4	-0.125	0		0.125	0	0.689	0.796	0.856	1
1	4	-0.250	0		0.250	0	0.582	0.697	0.765	0.964
1	4	-0.375	0		0.375	0	0.458	0.575	0.646	0.860
1	4	-0.500	0		0.500	0	0.333	0.450	0.553	0.738
1	4	0.000	0.880	or	0.000	-0.880	0.731	0.827	0.905	1
1	4	-0.125	0.880		0.125	-0.880	0.622	0.719	0.774	0.926
1	4	-0.250	0.880		0.250	-0.880	0.499	0.597	0.653	0.807
1	4	-0.375	0.880		0.375	-0.880	0.374	0.472	0.528	0.683
1	4	-0.500	0.880		0.500	-0.880	0.249	0.348	0.403	0.558
1	4	0.000	-0.880	or	0.000	0.880	0.731	0.827	0.905	1
1	4	-0.125	-0.880		0.125	0.880	0.801	0.905	0.964	1
1	4	-0.250	-0.880		0.250	0.880	0.805	0.946	1	1
1	4	-0.375	-0.880		0.375	0.880	0.727	0.938	1	1
1	4	-0.500	-0.880		0.500	0.880	0.607	0.859	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1	3	0.00	0		0.00	0	0.604	0.728	0.798	0.991
1	3	-1/6	0		1/6	0	0.541	0.676	0.752	0.962
1	3	-1/3	0		1/3	0	0.394	0.540	0.625	0.869
1	3	-1/2	0		1/2	0	0.234	0.378	0.464	0.723
1	3	-2/3	0		2/3	0	0.122	0.229	0.306	0.559
1	3	0.00	0.990	or	0.00	-0.990	0.597	0.719	0.786	0.969
1	3	-1/6	0.990		1/6	-0.990	0.446	0.568	0.637	0.821
1	3	-1/3	0.990		1/3	-0.990	0.283	0.404	0.473	0.660
1	3	-1/2	0.990		1/2	-0.990	0.141	0.244	0.310	0.494
1	3	-2/3	0.990		2/3	-0.990	0.064	0.125	0.172	0.333
1	3	0.00	-0.990	or	0.00	0.990	0.597	0.719	0.786	0.969
1	3	-1/6	-0.990		1/6	0.990	0.704	0.835	0.908	1
1	3	-1/3	-0.990		1/3	0.990	0.732	0.909	1	1
1	3	-1/2	-0.990		1/2	0.990	0.649	0.829	1	1
1	3	-2/3	-0.990		2/3	0.990	0.710	0.872	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1	2	0.00	0		0.00	0	0.332	0.503	0.643	0.867
1	2	-0.25	0		0.25	0	0.253	0.424	0.527	0.817
1	2	-0.50	0		0.50	0	0.134	0.259	0.352	0.663
1	2	-0.75	0		0.75	0	0.075	0.150	0.212	0.465
1	2	-1.00	0		1.00	0	0.051	0.102	0.145	0.329
1	2	0.00	1.165	or	0.00	-1.165	0.317	0.480	0.573	0.819
1	2	-0.25	1.165		0.25	-1.165	0.146	0.267	0.349	0.587
1	2	-0.50	1.165		0.50	-1.165	0.064	0.127	0.178	0.362
1	2	-0.75	1.165		0.75	-1.165	0.033	0.067	0.095	0.209
1	2	-1.00	1.165		1.00	-1.165	0.021	0.042	0.060	0.134
1	2	0.00	-1.165	or	0.00	1.165	0.317	0.480	0.573	0.819
1	2	-0.25	-1.165		0.25	1.165	0.500	0.679	0.778	1
1	2	-0.50	-1.165		0.50	1.165	0.602	0.835	0.956	1
1	2	-0.75	-1.165		0.75	1.165	0.584	0.962	1	1
1	2	-1.00	-1.165		1.00	1.165	0.476	1	1	1

$\sigma$	UR	Meas. Bias	Prod. Bias	or	Meas. Bias	Prod. Bias	GBF CR=0.80 %	GBF CR=1.60 %	GBF CR=2.26 %	GBF CR=5.00 %
1	1	0.00	0		0.00	0	0.090	0.179	0.252	0.543
1	1	-0.25	0		0.25	0	0.087	0.175	0.246	0.533
1	1	-0.50	0		0.50	0	0.082	0.164	0.232	0.508
1	1	-0.75	0		0.75	0	0.076	0.152	0.215	0.476
1	1	-1.00	0		1.00	0	0.071	0.142	0.200	0.449
1	1	0.00	1.510	or	0.00	-1.510	0.067	0.134	0.189	0.407
1	1	-0.25	1.510		0.25	-1.510	0.047	0.096	0.136	0.298
1	1	-0.50	1.510		0.50	-1.510	0.036	0.072	0.102	0.225
1	1	-0.75	1.510		0.75	-1.510	0.028	0.057	0.080	0.178
1	1	-1.00	1.510		1.00	-1.510	0.023	0.047	0.066	0.147
1	1	0.00	-1.510	or	0.00	1.510	0.067	0.134	0.189	0.407
1	1	-0.25	-1.510		0.25	1.510	0.098	0.195	0.273	0.562
1	1	-0.50	-1.510		0.50	1.510	0.149	0.291	0.398	0.759
1	1	-0.75	-1.510		0.75	1.510	0.229	0.450	0.569	0.982
1	1	-1.00	-1.510		1.00	1.510	0.348	0.610	0.774	1

## 6. Summary

- Uncorrected and untreated product and measurement system biases can significantly increase consumer and producer risks. The magnitude of each can be as great as nearly 60% for the commonly encountered cases considered here.
- Consumer and producer risks are most significantly affected when the uncorrected and untreated product and measurement biases are of opposite polarity.
- Uncorrected and untreated product and measurement system biases can significantly decrease guardbands. For consumer risks of 0.80%, 1.60%, 2.265 and 5.00%, the guardbands, as a percentage of the performance specification are approximately 1.1%, 2.3%, 3.3% and 7.3%, respectively.

## 7. Recommendations

- Correct or statistically treat product and measurement system biases.
- Calculate consumer and producer risks based on the actual significant product and measurement system biases, if any.

## Appendix A

Two independent methods were used to calculate the consumer risk and producer risks<sup>6</sup>. “Mathcad” and a Boeing Company developed MS Excel Workbook version was also used. The mathematical basis, dimensionless form, for consumer risk is described below. Producer risk, not included here, is similarly, but not identically patterned.

The unconditional probability that the product is actually outside of the specification limits, from minus infinity to the lower specification limit, but is measured to be within the limits.

$$CR_{Minus} = \frac{1}{2\pi} \int_{-\infty}^{-L} \int_{-R(t+kL)}^{-R(t-kL)} e^{\frac{(t-u)^2 + (s-v)^2}{-2}} ds dt$$

The unconditional probability that the product is actually outside of the specification limits, from the upper specification limit to plus infinity, but is measured to be within the limits.

$$CR_{Plus} = \frac{1}{2\pi} \int_L^{\infty} \int_{-R(t+kL)}^{-R(t-kL)} e^{\frac{(t-u)^2 + (s-v)^2}{-2}} ds dt$$

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<sup>6</sup> This dimensionless form is taken from reference [2], and was verified to be accurate. The producer risk version is not given here.

$$CR_{Total} = CR_{Minus} + CR_{Plus}$$

Where: CR: Consumer risk  
 UUT: Unit under test  
 MS: Measurement system  
 L: Specification limit  
 R: Uncertainty ratio  
 t: Product distribution variable  
 k: Guardband factor  
 u: UUT bias (in UUT sigma)  
 s: Measurement system distribution variable  
 v: MS bias (in MS sigma)

Example:

For L: 2 sigma  
 R: 2.3:1  
 k: 1  
 u: +0.4 sigma  
 v: -0.7 sigma

$$CR_{Minus} = \frac{1}{2\pi} \int_{-\infty}^{-2} \int_{-2.3(t+2)}^{-2.3(t-2)} e^{\frac{(t-0.4)^2 + (s+0.7)^2}{-2}} ds dt = 0.024061$$

$$CR_{Plus} = \frac{1}{2\pi} \int_2^{\infty} \int_{-2.3(t+2)}^{-2.3(t-2)} e^{\frac{(t-0.4)^2 + (s-0.7)^2}{-2}} ds dt = 0.000883$$

$$CR_{Total} = CR_{Minus} + CR_{Plus} = 0.024061 + 0.000883 = 0.024944 \text{ or approximately } 2.49\%.$$

The reader is referred to the following references for additional perspectives on this topic.

[1] Robert G. Easterling, Mark E. Johnson, Thomas R. Bement, Christopher J. Nachtsheim, Statistical Tolerancing Based on Consumer's Risk Calculations, "Journal of Quality Technology", Vol. 23, No. 1, January 1991.

[2] David Deaver, "Maintaining Your Confidence (In a Work of Declining Test Uncertainty Ratios)", 1993, NCSL Workshop and Symposium.