A New Design of the Piston-Cylinder Assembly for Advanced Metrological Characteristics

Speaker/Author: S.Y. Woo,
Mass & Force Group, Korea Research Institute of Standards and Science
P.O. Box 102, Yusong, Taejon, Republic of Korea
Phone: +82-42-868-5118
e-mail: sywoo@kriss.re.kr

Authors: Y.J. Lee, I.M. Choi, B.S. Kim, J.T. Chung*
Mass & Force Group, Korea Research Institute of Standards and Science
* R&D Division, Korea Gas Corporation
* Il-Dong, Ansan, KyungGi-Do, , Republic of Korea

Abstract
Oil piston gauges are well-known standard instruments for measuring high pressure with high accuracy and low uncertainty. Most of them adopt simple or free deformation design for piston-cylinder assembly in order to utilize its ease of characterization. Pressure is measured when a known gravitational force applied to a piston is in equilibrium with the upward force generated by the hydraulic pressure applied to the effective area of a piston-cylinder assembly. In general, the dependence of effective area upon applied pressure, for a simple cylinder design, is characterized as a linear function of pressure. This coefficient has been considered as a constant for this design. However, through the finite element analysis coupled with the numerical iterative method, we have found that the hydraulic pressure exerting on the floor of cylinder surrounded by O-ring may deform the cylinder, causing non-linearity of the effective area, i.e. the change of the coefficient. This may hinder precise measurement of pressure. By the appropriate changes of the cylinder configuration, this non-linear effect could be reduced significantly.

1. Introduction

The primary standards used for high-pressure measurements are pressure balances, frequently called as piston gauges. Pressure balance consists of the piston with the accurately known diameter, the corresponding cylinder and weights. The downward force generated by gravity from weights put on the piston is balanced with the force on the base of piston generated by the oil pressure applied to the medium. The generated pressure is determined by the force exerted on the piston divided by the effective area, which is determined by the piston-cylinder assembly, where the effective area is approximately the average of the areas of the piston and the cylinder. In principle, the increasing pressure causes the piston and the cylinder to deform, making it difficult to determine the effective area of the equipment (that is, the pressure) accurately. Accordingly, the determination of the changes of the piston and the cylinder with the changing pressure is crucial to the precise measurement of pressure [1]. In general, the effective area in simple piston-cylinder assembly where the cylinder and the piston can freely deform may be expressed as the following [2],

$$A_e(p) = A_0(1 + \lambda p) \tag{1}$$

where $A_e(p)$ is the effective area at the pressure p, A_0 is the area under reference temperature and atmospheric pressure, and λ is the pressure distortion coefficient representing the change of effective area according to the pressure. The pressure distortion coefficient usually varies depending on the material, shape of the piston and cylinder and the exerted pressure. The accurate determination of this value is the greatest concern in the high-pressure measurement.

Most of the high pressure balances commonly used in the standard laboratories today are in a simple piston-cylinder configuration as shown in Fig. 1 where the piston and the cylinder can deform freely without external pressure. On the floor of the cylinder, O-ring is used for the purpose of sealing. When pressure is measured, the oil-pressure exerts on and deforms the cylinder floor surrounded by the O-ring, causing the non-linearity in the change of effective area [3]. Such non-linearity, if caused by the deformation of the cylinder floor, can be minimized by changing the configuration of the cylinder appropriately. This paper presents how to reduce the non-linearity in the change of effective area caused by O-ring through the numerical method with FEM (Finite Element Method).

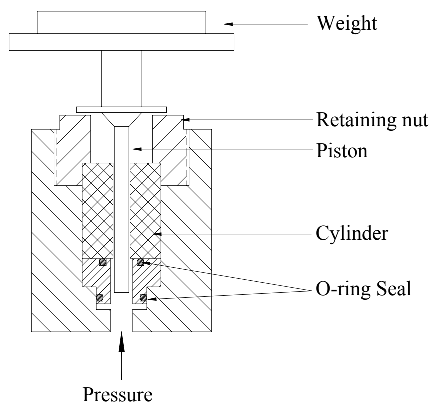


Fig. 1. Schematic sketch of the simple type piston-cylinder assembly under study.

2. Analysis Theory

If the radial distance between the piston and the cylinder is fixed, and parallel to the piston axis, the effective area may be expressed as the following [2],

$$A_o = \pi r_0^2 (1 + \frac{h_0}{r_0}) \tag{2}$$

where r_0 is the radius of the piston, and h_0 is the distance between the piston and the cylinder. Since the configurations of the piston and cylinder, before the pressure is exerted, are approximately parallel, A_o in equation (1) can be obtained from equation (2). However, when pressure is exerted on the piston-cylinder assembly, it complicates the configurations. The equation to obtain the effective area for the piston-cylinder assembly in complicated configuration may be expressed as the following [2],

$$A_e = \pi r_0^2 \left(1 + \frac{h_0}{r_0} + \frac{(u_0 + U_0)}{r_0} + \frac{1}{r_0 p_1} \int_0^l p(z) \frac{d(u+U)}{dz} dz\right) \tag{3}$$

where u_0 and U_0 represent the radial distortions of the piston and the cylinder at the lower part of the cylinder under pressure p_1 referenced to the initial state without the exerted pressure. l is the engagement length of the piston and the cylinder representing the overlap between the piston and the cylinder. $u(z)$ and $U(z)$ represent the radial distortions with respect to the bottom of the engagement length for the piston and cylinder respectively. Also, $p(z)$ is the pressure at any point along the engagement length referenced to the pressure at the top of the engagement length. Accordingly, from the equation (1), the coefficient of pressure distortion at the pressure p may be expressed as the following.

$$\lambda = \frac{1}{\mathrm{p}}\left(\frac{\mathrm{A_e}}{\mathrm{A_0}} - 1\right) \tag{4}$$

Therefore, one can see that in order to calculate the pressure distortion coefficient, one should obtain $p(z)$, the pressure distribution between the piston and the cylinder stated in equation (3), and $u(z)$ and $U(z)$, which are the distortions of the piston and the cylinder. Considering the flow of the viscous fluid flowing along the engagement length between the piston and the cylinder, mass flow quantity Q_m is expressed by Navier-Stokes equation as the following [4],

$$Q_m = -\frac{\pi r_0 h^3(z)\rho(p)}{6\eta(p)} \frac{dp}{dz} \tag{5}$$

where $\rho(p)$ is the density (in kg m^{-3}) at the pressure p, and $\eta(p)$ is the viscosity (in Pa s) at the pressure p. The fluid used for this analysis is di-2-ethyl-hexyl-sebacate. The density and the viscosity of the oil at 20 ℃ are known according to the following equations [4,5].

$$\rho = 912.67 + 0.7521p - 1.6448 \times 10^{-3} p^2 + 1.45625 \times 10^{-6} p^3 \tag{6}$$

$$\eta = 0.02155(1 + 0.00190036p)^{8.81} \tag{7}$$

The pressure at any point along the engagement length changes from the measured pressure to the atmospheric pressure, accompanying the deformation of the piston and the cylinder. Accordingly, the distance between the piston and the cylinder can no longer be parallel flow path, resulting in the change of $h(z)$, which is applied to equation (5). As a result, the pressure distribution $p(z)$ as well as the $h(z)$ should be re-established. One of the solutions for such complicated problem is the numerical method using FEM [4].

3. Piston-Cylinder Assembly

The equipment used for the study is the piston-cylinder assembly used in the oil-operated pressure balance model DH5306 manufactured by Desgranges et Huot in France, with the nominal area of 2 mm^2 and the measurement range of 5-500 MPa. The piston is made of hard steel (nominal diameter 1.58 mm, Young's Modulus 2×10^{11} Pa, and Poisson ratio 0.3). The cylinder is made of tungsten carbide (outer radius 17 mm, inner radius 1.58 mm, height 27 mm, Young's Modulus 6.2×10^{11} Pa, and Poisson ratio 0.218)[5]. The engagement length representing the overlap between the piston and the cylinder is 27 mm, same as the length of the cylinder. The diameter of the piston is (1.5796 ± 0.00005) mm, and the average gap between the piston and the cylinder h_0 is $(0.61\pm0.1)\,\mu$m. The average diameter of the O-ring (OR106) used for sealing under the cylinder is 8.0 mm.

In order to analyze the piston-cylinder assembly with FEM, a geometrical model as shown in Fig.2 was established. And the table below represents the coordinates of keypoints of the geometrical model. Generally axis symmetrical, piston and cylinder can be modeled in 2D. In the figure, the pressure p_1 is exerted on the straight line 5-6, 10-11 and 11-12, and the inter-distance pressure $p(z)$ on the straight line 6-1 and 12-7. The straight line 2-3 is restrained to the

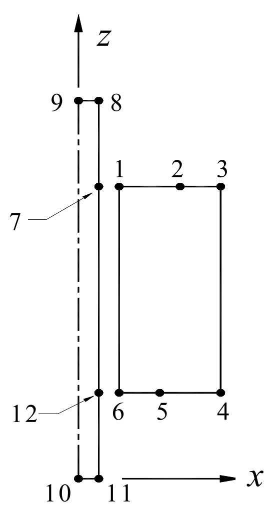


Fig. 2. Model and keypoints of the piston-cylinder assembly used in finite element analysis.

Keypoint	x (mm)	z (mm)
1	0.79041	36
2	5.9	36
3	8.5	36
4	8.5	9
5	4.0	9
6	0.79041	9
7	0.78980	36
8	0.78980	44
9	0	44
10	0	0
11	0.78980	0
12	0.78980	9

z direction by the retaining nut, and 8-9 to the z direction by the deadweights on the piston. Also, the straight line 9-10 is restrained to the x direction by symmetry. We configured the element net with the triangle elements with a two hundredth of the engagement length between the piston and the cylinder as a side, and made the net coarse toward the outer radius of the cylinder to create about total 4000 nodes, so as to control the sufficient memory maintained when calculating the deformation qualities.

4. Results of the Analysis

Fig. 3 represents the pressure distortion coefficients at 100, 200, 300, 400 and 500 MPa. As shown in the figure, the pressure distortion coefficient with O-ring seal is not a constant but gradually decreasing with pressure. It is generally known that the pressure distortion coefficient is constant in the simple piston-cylinder assembly regardless of the variation of pressure. But it is only applied to the special case when no upward force is generated by the O-ring seal. When there is no upward force by the O-ring seal, the pressure distortion coefficients are 0.748x10^{-6} MPa^{-1} at 10 MPa, 0.755x10^{-6} MPa^{-1} at 500 MPa, all of which correspond to the calculated value of 0.746x10^{-6} MPa^{-1} in the simple theory[6] within 1 %. Considering that the uncertainty of the pressure distortion coefficient is 5-10 %, one can see that the deviation from the calculated value by the simple theory is insignificant. However, when the O-ring is used under the cylinder, the pressure distortion coefficient varies, 10.4 % at 10 MPa, 2.1 % at 500 MPa, showing the tendency that the relatively high pressure results in the smaller increase of the pressure distortion coefficient. This non-linear property is caused by the O-ring: the pressure on the engagement length is added to the pressure under the cylinder surrounded by the O-ring. This is a universal phenomenon occurring in the piston-cylinder assembly with this configuration. Therefore, simply considering the pressure distortion coefficient as a constant will lead to an error.

Changing the cylinder configuration with two methods as shown in Fig. 4 can minimize such non-linear property. Design I is to make a hole (H in radial direction, V in axial direction) a little bit larger than the diameter of the cylinder at the gate of the engagement length to reduce the effect from the deformation of the cylinder floor. And design II is to process the gate of engagement length so that it can gradient (H in radial direction, V in axial direction). Fig. 5 shows the variation of the pressure distortion coefficient with H=0.1 mm and axial depths 0.5, 1.0,

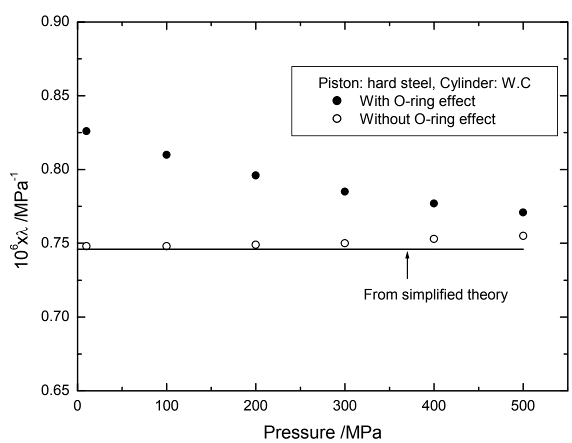
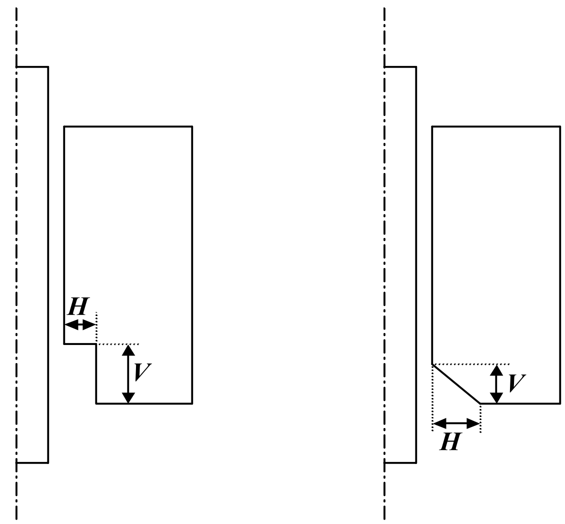


Fig. 3. Variation of the pressure distortion coefficients with and without O-ring effect.

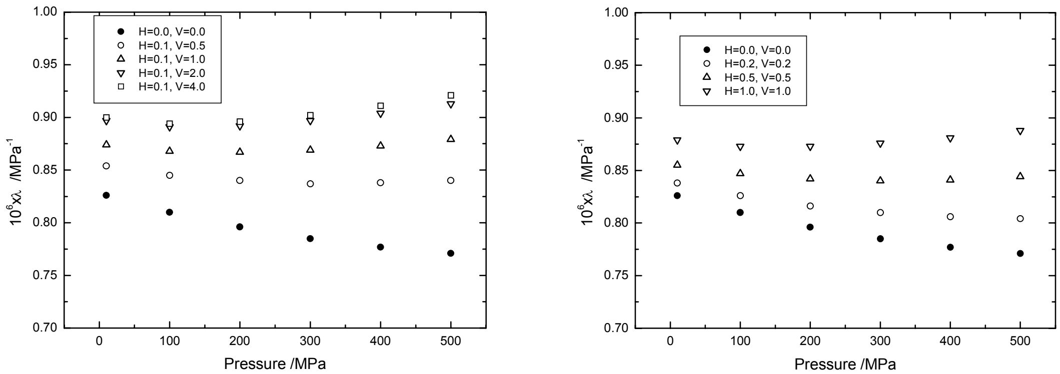


(a) Design I (b) Design II

Fig. 4. Two new designs of the piston-cylinder assembly suggested by authors.

2.0 and 4.0 mm in design I. The deeper the axial depth, in other words, the shorter the engagement length is, the more the pressure distortion coefficient tends to increase. The variation of the pressure distortion coefficient is smallest when V is around 0.9 mm, where the average is 0.867×10^{-6} MPa^{-1}, and the maximum difference between 10 MPa and 500 MPa is 0.011×10^{-6} MPa^{-1}, corresponding to 1.3 % of the pressure distortion coefficient. The maximum difference was reduced by five times compared with the values before design change (pressure distortion coefficient: 0.794×10^{-6} MPa^{-1}, maximum difference: 0.055×10^{-6} MPa^{-1}), indicating the significant improvement of linearity. Also, for the case where the depth V is fixed as 1.0 mm, and the radial H is increased to 1.0 mm, the pressure distortion coefficient becomes 0.888×10^{-6} MPa^{-1}. This is 1.8 % greater than when H is 0.1 mm, and the maximum difference is also increased to 0.018×10^{-6} MPa^{-1}. Therefore, it is not desirable to increase the diameter of the hole excessively. This indicates that the horizontal area needs to be minimized that causes the vertical force at the lower part of the engagement length. H was chosen to be 0.1 mm for the sake of convenient processing. To obtain more optimum H values has no meaning other than theoretical calculation.

The Fig. 5(b) represents the variation of the pressure distortion coefficient where the design II was applied but with H same as V along with the process of declination of 45 degrees. As with the design I, when the engagement length is shortened from the gradient processing depth is deepened, then the pressure distortion coefficient tends to increase. Also, if processing up to the appropriate depth, one can minimize the variation of the pressure distortion coefficient. This optimum condition is when H=V=0.6 mm and the average of the pressure distortion coefficient is 0.853×10^{-6} MPa^{-1} and the maximum difference is 0.011×10^{-6} MPa^{-1}, corresponding to approximately 1.3 % of the coefficient. These values are similar to the optimum condition of the design I. Manufacturers may apply them to production considering the convenience of the process. Also, selecting 45 degrees as the angle of gradient processing is for convenience only.



(a) Design I (b) Design II

Fig. 5. Variation of the pressure distortion coefficients obtained from the design change (piston: hard steel, cylinder: tungsten carbide).

To find the optimum gradient angle also has no meaning other than theoretical study as in the design I.

Another important investigation is to simulate the same piston-cylinder assembly with the assumption that piston is made of tungsten carbide instead of hard steel. As shown in the Fig. 6, when there is no upward hydraulic force by the O-ring seal, the pressure distortion coefficients are 0.717×10^{-6} MPa^{-1} at 10 MPa, 0.716×10^{-6} MPa^{-1} at 500 MPa, all of which correspond to the calculated value of 0.717×10^{-6} MPa^{-1} in the simply theory very well. However, when the O-ring is used under cylinder for the purpose of pressure seal, the pressure distortion coefficient changes significantly. This implies that it is very important to make an appropriate change of the cylinder configuration.

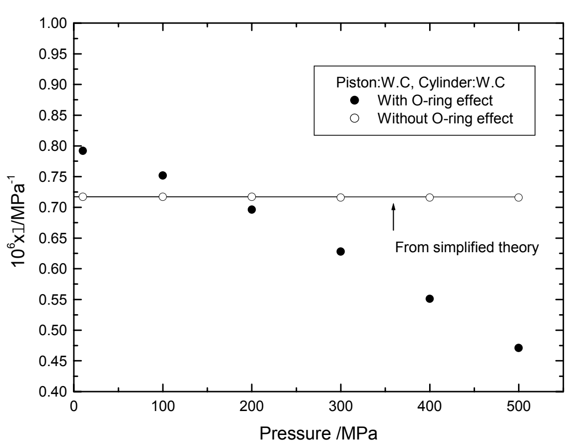
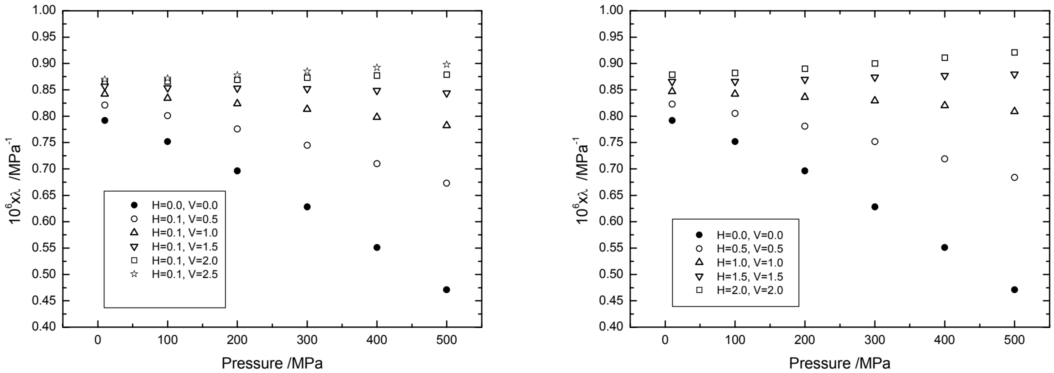


Fig. 6. Variation of the pressure distortion coefficients with and without O-ring effect with the assumption that piston is made of tungsten carbide instead of hard steel.



(a) Design I (b) Design II

Fig. 7. Variation of the pressure distortion coefficients obtained from the design change (piston: tungsten carbide, cylinder: tungsten carbide).

The Fig. 7(a) represents the variation of the pressure distortion coefficient where the design I was applied with H=0.1 mm and axial depths 0.5, 1.0, 1.5, 2.0 and 2.5 mm. The deeper the axial depth, the more the pressure distortion coefficient tends to increase. The variation of the pressure distortion coefficient is smallest when V is around 1.5 mm, where the average is 0.852×10^{-6} MPa^{-1}, and the maximum difference between 10 MPa and 500 MPa is 0.013×10^{-6} MPa^{-1}, corresponding to 1.5 % of the pressure distortion coefficient. The maximum difference was reduced by twenty times compared with the values before design change (the average of pressure distortion coefficient: 0.648×10^{-6} MPa^{-1}, maximum difference: 0.321×10^{-6} MPa^{-1}), indicating the very significant improvement of linearity.

The Fig. 7(b) shows the variation of the pressure distortion coefficient where the design II was applied with H same as V as before. The optimum condition is when H=V=1.5 mm and the average of the pressure distortion coefficient is 0.872×10^{-6} MPa^{-1} and the maximum difference is 0.014×10^{-6} MPa^{-1}, corresponding to approximately 1.6 % of the coefficient. These values are similar to the optimum condition of the design I.

5. Conclusions

Most oil-operated pressure balances commonly used are in simple piston-cylinder configuration where the piston-cylinder can deform freely at high pressures, and have O-ring chamber in the floor of the cylinder for sealing the cylinder. Then, the oil pressure exerting on the floor of cylinder surrounded by O-ring may deform the cylinder, causing non-linearity of the effective area, i.e. the change of the pressure deformation coefficient. This non-linear effect could be reduced significantly by appropriate change of the cylinder configuration. The equipment used for the analysis is the piston-cylinder assembly for 500 MPa (nominal area: 2 mm^2) used for the oil-operated pressure balance DH5306. As a result, we found that the pressure distortion coefficient calculated from the simple theory matched well with the numerical analysis ignoring the effect from O-ring. We also found that the O-ring used for sealing caused the pressure distortion coefficient to increase non-linearly. We could minimize such a non-linear property by enlarging or applying gradient process to the gate of the engagement part under the cylinder with the piston. For this piston-cylinder assembly, we could reduce the non-linearity by five times by either enlarging the hole under the cylinder by H=0.1, V=0.9, or processing with 45 degree angles with H=0.6 and V=0.6. Another simulation was carried out with the assumption that piston is made of tungsten carbide instead of hard steel. For this assembly, we could reduce the non-linearity by twenty times by either enlarging the hole under the cylinder by H=0.1, V=1.5, or processing with 45 degree angles with H=1.5 and V=1.5.

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