# Automating the Calibration of Two Piston Gage Pressure Balances

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## Abstract

We propose a method to automate the data-gathering process in the cross-float calibration of one piston gage pressure balance against a second piston gage pressure balance. Rather than adjusting small "trim" masses until the balances generate equal pressure, the two balances are brought into only approximate pressure equilibrium. They are then sequentially connected to a pressure transducer, through use of constant volume valves, to measure the differential pressure. We present results of the method on a 200 MPa hydraulic system. We explore limitations to the method, including the allowable pressure difference between the two pressure balances, the sequencing procedure of the pressure balances, and the characteristics of the pressure transducer.

## 1. Introduction

The determination of the effective area of a piston gage pressure balance is most often accomplished by calibration against another piston gage pressure balance of known effective area. In the "cross-float calibration technique," the pressure balances are connected to a common pressure line. The masses are adjusted on one or both pressure balances until each balance generates the same pressure. Summing the forces on the reference piston along with its known area gives the common pressure, and the forces on the test piston with the common pressure yields the test balance piston area [1].

Critical to the cross-float calibration is a means for determining if the balances are generating the same pressure. Two common methods are the differential pressure cell (DP cell) and the fall rate [2]. In the DP cell method, an electronic differential pressure cell separates the fluid lines of the two balances. The cell indicates which balance is generating a higher pressure, allowing adjustment of the masses on the balances. For proper operation the cell must be "zeroed" at each operating pressure by applying the same pressure to each side and adjusting the output to read zero. Care must be taken to limit the pressure difference on a DP cell to prevent damaging the diaphragm separating the fluids. In the fall rate method, a valve is closed in the pressure line connecting the two balances, and the rates at which the rotating pistons fall in the cylinders are measured. This natural fall rate is due to the leakage of the pressure fluid through the piston-cylinder gap. Then the valve between the balances is opened, and the fall rates are re-measured. If they are different than the natural fall rates, the masses are adjusted. Proximity indicators are used to measure the piston height vs. time.

Both the DP cell and fall rate method require operator interaction to find the pressure balance point. Until the relative pressure difference is less than $10^{-4}$ to $10^{-5}$, the methods only indicate the direction of the imbalance, not the magnitude. The operator must make repeated measurements of the pressure equilibrium and then adjust the masses on the balances. The speed of finding the pressure equilibrium and its ultimate resolution depend on the skill and experience of the operator. The operator must decide whether the equilibrium is "good enough," and go on to the next pressure setting. Once the balance point is found, the operator must record the trim masses in a logbook and transcribe the values to a data reduction program or spreadsheet.

In this paper we propose an alternative method in the cross-float calibration of two pressure balances. This method does not require that the balances generate equal pressure, only that they are "close enough" so that the pressure difference measured between them contributes minimal uncertainty to the effective area of the test balance. The data-taking can be automated, and operator judgment is not required to assess the equilibrium point or the uncertainty with the equilibrium. We evaluate the method using a prototype system with 200 MPa pressure balances.

## 2. Proposed Method: Transducer Assisted Crossfloat

In the calibration method that we refer to as Transducer Assisted Crossfloat (TAC), the two pressure balances are connected sequentially to a high precision pressure transducer. The pressure difference between the balances (at the location of the transducer) is determined by the difference in output of the transducer. A schematic of the hardware required for the method is shown in Fig. 1, where pressure balance TEST is calibrated against pressure balance REF. Constant volume valves, designed at NIST by Markus [3], connect the balances to the transducer with zero displacement of fluid. The constant volume valves are air operated and can be used up to 420 MPa. Variable volume valves provide for raising the pistons to compensate for piston-cylinder gap leakage and for relative flow of the pressure fluid between the balances. We are currently evaluating the system for hydraulic pressure balances operating up to 280 MPa. The method should also be applicable to pneumatic pressure balances and lower pressure hydraulic balances.

A critical element of the TAC method is the pressure transducer. Ideally, this transducer should provide an expanded uncertainty in the pressure difference between the two balances of a few parts in $10^6$ of the measured pressure. This would make its uncertainty comparable to the resolution of the pressure equilibrium in the fall rate or DP cell methods. In our prototype system, we are using an absolute pressure transducer made by Paroscientific Corporation[1]. This transducer uses a vibrating quartz crystal incorporated in a Bourdon tube as the sensing element. The frequency of oscillation depends on the pressure-induced tension in the tube. A

[1] Certain commercial equipment, instruments, materials, or software are identified in this paper to foster understanding. Such identification does not imply recommendation or endorsement by NIST, nor does it imply that the materials or equipment are necessarily the best available for the purpose.

programmable on-board frequency counter in the transducer housing provides adjustable integration times and pressure resolution. A second quartz crystal compensates for temperature variations. With an integration time of 3.3 s, the transducer resolution is 56 Pa.

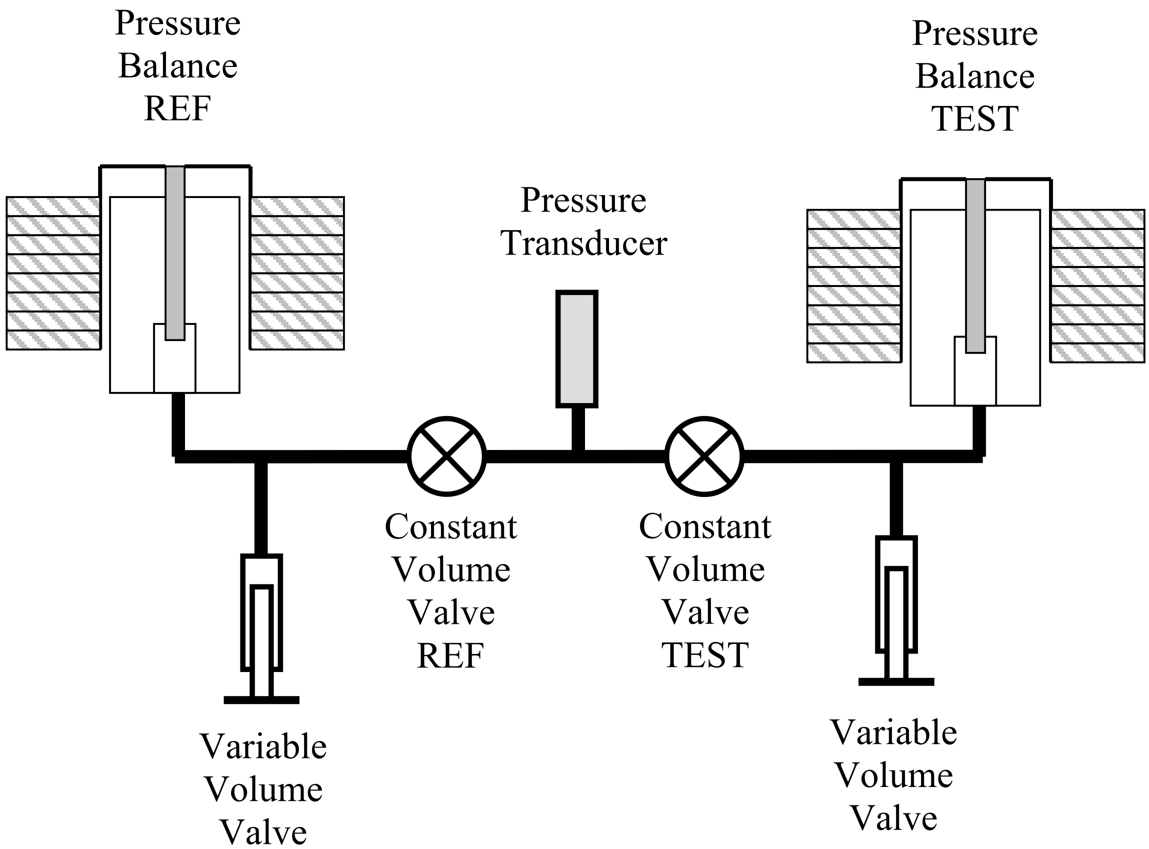


Figure 1. Schematic of hardware set-up for Transducer Assisted Crossfloat.

An important performance characteristic of the present transducer, which influences the proposed technique, is drift in its output with time for constant applied pressure. We have measured output drift rates of +0.05 Pa/s to –0.92 Pa/s, with the drift rates predominantly negative (for positive pressure steps). Drift rates tend to be higher (in magnitude) immediately after applying a large pressure change, with the drift rate approaching zero over several hours as the pressure is held constant. Over a period of 10 to 20 minutes the drift rate usually varies by less than 0.1 Pa/s. Since the transducer cannot be connected to each pressure balance at the same instant in time, the experimental protocol must compensate for the output drift.

## 3. Baseline Measurement Protocol

The measurement protocol is summarized in Table 1 [4]. One pressure balance is referred to as REF and the other is referred to as TEST. We make five measurements of the pressure difference between TEST and REF, which allows an estimate of Type A uncertainty and requires a reasonable amount of time to perform the experiment. Later, we will look at the effect of reducing the number of difference measurements. To estimate the reading of REF at the time of the reading of TEST, we space the measurements on the balances equally in time, and average the readings on REF before and after TEST. We make six readings of the pressure on the REF balance and five readings on the TEST balance. We sample the pressure transducer six times (with one temperature reading) at each connection to REF or TEST. The group of six readings is used to evaluate the stability of the transducer and pressure balances. With a transducer integration time of 3.3 s, the six pressure readings and one temperature reading require about 23 s. Starting at time 0, we sample the pressure at REF six times; at 30 s the valves are switched and piston positions are adjusted (if necessary); at 60 s no further adjustments are made; at 90 s we sample the pressure at TEST; and so on. REF is measured at 0, 3, 6, 9, 12, and 15 minutes; TEST is measured at 1 ½, 4 ½, 7 ½, 10 ½, and 13 ½ minutes. The entire measurement cycle requires 15 ½ minutes. The computer automatically samples the pressure transducer at the required times and intervals; at present the valves are switched manually.

Table 1. Experiment Protocol for Five Difference Measurements

| **Elapsed Time (min:sec)** | **Operation** | **REF CVV Condition** | **TEST CVV Condition** | **Trans. Average** | **Trans. Diff.** |
|---|---|---|---|---|---|
| 0:00 | Start, Measure REF | Open | Closed | $I_{REF,1}$ | |
| 0:30 | Switch CVV, adjust TEST | Closed | Open | | |
| 1:30 | Measure TEST | Closed | Open | $I_{TEST,1}$ | $\Delta I_1$ |
| 2:00 | Switch CVV, adjust REF | Open | Closed | | |
| 3:00 | Measure REF | Open | Closed | $I_{REF,2}$ | |
| 3:30 | Switch CVV, adjust TEST | Closed | Open | | |
| 4:30 | Measure TEST | Closed | Open | $I_{TEST,2}$ | $\Delta I_2$ |
| 5:00 | Switch CVV, adjust REF | Open | Closed | | |
| 6:00 | Measure REF | Open | Closed | $I_{REF,3}$ | |
| 6:30 | Switch CVV, adjust TEST | Closed | Open | | |
| 7:30 | Measure TEST | Closed | Open | $I_{TEST,3}$ | $\Delta I_3$ |
| 8:00 | Switch CVV, adjust REF | Open | Closed | | |
| 9:00 | Measure REF | Open | Closed | $I_{REF,4}$ | |
| 9:30 | Switch CVV, adjust TEST | Closed | Open | | |
| 10:30 | Measure TEST | Closed | Open | $I_{TEST,4}$ | $\Delta I_4$ |
| 11:00 | Switch CVV, adjust REF | Open | Closed | | |
| 12:00 | Measure REF | Open | Closed | $I_{REF,5}$ | |
| 12:30 | Switch CVV, adjust TEST | Closed | Open | | |
| 13:30 | Measure TEST | Closed | Open | $I_{TEST,5}$ | $\Delta I_5$ |
| 14:00 | Switch CVV, adjust REF | Open | Closed | | |
| 15:00 | Measure REF | Open | Closed | $I_{REF,6}$ | |
| 15:30 | End | | | | |

The output of the transducer during a typical measurement cycle is shown in Fig. 2. The REF and TEST pressure balances each contain 2.5 mm diameter, free deformation type piston cylinders. The piston and cylinders are made of tungsten carbide, with full-scale pressures of

200 MPa when loaded with 100 kg of mass. The TEST and REF balances for this and all experiments mentioned in the paper were PG479 and PG49, respectively. (PGxx is the NIST label for a specific pressure balance). The pressure transmitting fluid was diethylhexyl-sebacate. In the figure, the nominal pressure on each balance was 100 MPa. The fall rate method was used to find the trim masses for pressure equilibrium, then an additional 0.5 g (10 ppm of the total mass) was added to the TEST balance prior to the measurement cycle. The difference in transducer output between TEST and REF was also about 10 ppm of the nominal output, as expected. The scatter of the six readings at each connection to REF or TEST could be due to instability in the pressure generated by the balance, the transducer resolution (56 Pa), or both.

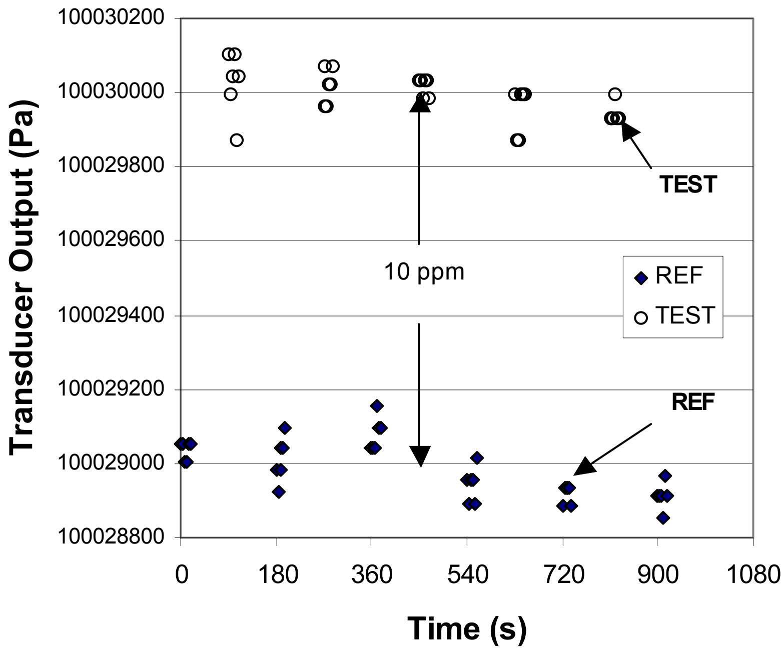


Figure 2. Transducer output vs. time during TAC protocol showing readings from REF and TEST balances. 100 MPa nominal pressure, TEST balance with 0.5 g mass (10 ppm) above pressure equilibrium.

## 4. Experiments Conducted and Data Analysis

To evaluate the transducer assisted crossfloat (TAC) method, we used it to calibrate PG479 against PG49 at 20 MPa intervals from 20 MPa to 200 MPa. The pressure balances were also calibrated using the fall rate method for comparison. For the TAC method, the pressure balances were operated with a pressure difference between. We ultimately want to avoid the mass adjustment process, so we need to find how large a pressure difference can be tolerated with TAC. For our experiments, we found the approximate pressure equilibrium through the fall rate method, and added (or subtracted) additional *trim mass* to one of the balances. The amount of trim mass was adjusted so that the relative pressure difference (pressure difference divided by the absolute pressure) was at fixed values for the various pressure operating points. For all pressures, TAC calibrations were performed at relative differential pressures of $+10^{-4}$ and $-10^{-4}$. At 100 MPa and 200 MPa, relative differential pressures of 0, $+10^{-5}$, $+/-10^{-3}$, and $+10^{-2}$ were also tested. Both pressure balances were instrumented with proximity sensors to determine piston height, and thermometers to measure piston temperature. Piston rotation rate was measured, as was barometric pressure, humidity, and ambient temperature for the buoyancy correction.

4.1 Area Calculation in TAC Method

The equation for pressure on the TEST balance, $P_{TEST}$, at its reference level is:

$$P_{TEST} = P_{REF} + \Delta\rho gh + \Delta P_{PT} \quad , \tag{1}$$

where $P_{REF}$ is the pressure generated by the REF balance; $\Delta P_{PT}$ is the pressure difference measured by the pressure transducer; $\Delta\rho = \rho_f - \rho_a$ is the fluid density minus the air density; and $h$ is the height difference between the balance reference levels (positive if REF is higher than TEST). Using the definition of pressure for the balances as the forces divided by the effective area, we find:

$$\frac{\sum M_{TEST}\left(1-\rho_a/\rho_m\right)g+\gamma C_{TEST}}{A_{E,TEST}\left(1+\left(\alpha_p+\alpha_c\right)\left(T_{TEST}-23\right)\right)} = \frac{\sum M_{REF}\left(1-\rho_a/\rho_m\right)g+\gamma C_{REF}}{A_{E,REF}\left(1+\left(\alpha_p+\alpha_c\right)\left(T_{REF}-23\right)\right)} + \Delta\rho gh + \Delta P_{PT} \ . \tag{2}$$

The terms in the numerator for the REF and TEST balances are the buoyancy corrected force due to the masses and the surface tension; the term in the denominator is the effective area at 23 ºC ($A_E$) times the thermal expansion correction. $C$ is the piston circumference, $\gamma$ is the fluid surface tension, $\alpha_p$ and $\alpha_c$ are the piston and cylinder thermal expansion coefficients, $T_{REF}$ is the temperature of the REF piston-cylinder, and $T_{TEST}$ is the temperature of the TEST piston-cylinder. The area of the REF balance is known from its calibration equation:

$$A_{E,REF} = A_{0,REF}(1 + b_{REF} P_{REF}) \ . \tag{3}$$

$A_{0,REF}$ and $b_{REF}$ are calibration constants. A simple model of the pressure transducer is that the pressure is linear with its indicated output:

$$P = f \cdot I + P_0 \quad , \tag{4}$$

where $I$ is the indicated output, $P_0$ is the pressure when $I = 0$, and $f$ is a scaling factor. Since the transducer measures absolute pressure, when used to measure gage pressure, the atmospheric pressure must be subtracted. The differential pressure is then:

$$\Delta P_{PT} = f \cdot \Delta I \quad . \tag{5}$$

Changes in the atmospheric pressure are insignificant over the time required for a difference measurement. Each indicated difference is calculated as:

$$\Delta I_j = I_{TEST,j} - (I_{REF,j} + I_{REF,j+1})/2 \quad . \tag{6}$$

This removes the systematic effect of the drift in the output of the transducer. The difference used in eq. 5 is the mean of the five $\Delta I_j$ readings. Each $I_{TEST,j}$ and $I_{REF,j}$ is the average of the six transducer readings sampled sequentially.

Eq. (2) with the masses, balance temperatures, air density, and $\Delta P_{PT}$ from the pressure transducer will yield $A_{E,TEST,TAC}$ , where the subscript TAC indicates the area was obtained using the TAC method. With $\Delta P_{PT} = 0$, eq. (2) gives the area from the fall rate method, $A_{E,TEST,FR}$ .

### 4.2 Calibration of Pressure Transducer

When used with the factory calibration, $f = 1.000000$. We have calibrated the transducer against two internal NIST pressure balance standards, about four months apart. In both calibrations, masses were loaded on the pressure balance with the pressure line connected to the transducer, and the pressure was allowed to stabilize for a minimum of 30 minutes prior to taking the data. The values of $f$ determined by the two calibrations were 0.999963 and 0.999977, with standard uncertainties ($u(f)$) on the fit of $28\text{x}10^{-6}$ and $30\text{x}10^{-6}$. However, the proposed TAC method requires a transducer reading within 60 seconds or less of an applied pressure change. We know that the transducer output drifts over time. Hence a more appropriate calibration is to measure the change in output of the transducer after a known change in pressure at about the elapsed time used in the TAC. We performed this calibration using the timing sequence of TAC (shown in Table 1), with the transducer connected only to PG49. Differential mass was added and removed from PG49 every 90 seconds. This calibration was performed at a nominal pressure of 100 MPa with differential masses of 0.5 g, 1.0 g, 2.0 g, 5.0 g, 10.0 g, 20.0 g, 50.0 g, 100.0 g, 200.0 g, and 500.0 g; and at 200 MPa with differential masses of 10.0 g and 100.0 g. This calibration gave $f = 0.999624$, and is the value used in the data analysis. $u(f)$ for this calibration method was $55\text{x}10^{-6}$.

### 4.3 Uncertainty Analysis

#### 4.3.1 Uncertainty in TAC Method

Using the methods described in Taylor and Kuyatt [5], the uncertainty[2] in the pressure difference is given by:

$$u(\Delta P_{PT}) = \left[\{\Delta I \cdot u(f)\}^2 + \{f \cdot u(\Delta I)\}^2\right]^{1/2} \quad , \tag{7}$$

where $u(f)$ and $u(\Delta I)$ are the uncertainties in the transducer scaling factor and the indicated pressure difference, respectively. The effect of the relative uncertainty of the pressure difference on the relative uncertainty of the effective area is $u(\Delta P_{PT})/P_{TEST}$. $u(\Delta I)$ is a Type A uncertainty, evaluated by considering the stability of the transducer readings on pressure balances REF and TEST, and the repeatability of the five difference readings.

$$u(\Delta I) = \left[\{u(I_{REF})\}^2 + \{u(I_{TEST})\}^2 + \{u(\Delta I_{TEST-REF})\}^2\right]^{1/2} \quad . \tag{8}$$

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[2] Uncertainty refers to standard uncertainty unless noted otherwise.

Assuming the groups of six consecutive readings on REF and TEST are independent observations, the stability uncertainties were calculated from the standard deviation of the mean of the REF or TEST readings [5] as follows:

$$s(I_{REF,j}) = \left[ \frac{1}{n(n-1)} \sum_{k=1}^{n} (I_{REF,j,k} - \bar{I}_{REF,j})^2 \right]^{1/2} , \qquad u(I_{REF,j}) = \left[ \frac{(n-1)}{(n-3)} \right]^{1/2} s(I_{REF,j}) \ , \qquad (9)$$

$$s(I_{TEST,j}) = \left[ \frac{1}{n(n-1)} \sum_{k=1}^{n} (I_{TEST,j,k} - \bar{I}_{TEST,j})^2 \right]^{1/2} , \qquad u(I_{TEST,j}) = \left[ \frac{(n-1)}{(n-3)} \right]^{1/2} s(I_{TEST,j}) \ , \qquad (10)$$

with $n$ equal 6. $s(I_{REF,j})$ and $s(I_{TEST,j})$ are the standard deviations of the mean. Here, the Bayesian viewpoint is taken as suggested by Kacker *et al.* [6], and the $[(n\text{-}1)/(n\text{-}3)]^{1/2}$ operator accounts for the uncertainty that arises if $n$ is small (but greater than 3). The six $u(I_{REF,j})$ and the five $u(I_{TEST,j})$ uncertainties were then averaged. The repeatability of the difference readings was calculated as the standard deviation of the mean difference with the Bayesian operator, or

$$u(\Delta I_{TEST-REF}) = \left[ \frac{(n-1)}{(n-3)} \right]^{1/2} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} (\Delta I_j - \overline{\Delta I})^2 \right]^{1/2} \ . \qquad (11)$$

In this case, $n = 5$.[3]

One option for transducer uncertainty, $u(f)$, would be to use the uncertainty of the fit parameter ($55 \times 10^{-6}$) from the calibration of the transducer response vs. differential mass on PG49 at 100 MPa and 200 MPa (see sec. 4.2). However, we have no independent means of determining if the pressure balance can produce a stable pressure 30 to 60 seconds after a mass change (monitoring the *transducer output* indicates only whether that output is steady). In addition, at present we lack data for the same calibration at other nominal pressures. The scaling factor determined from the differential pressure calibration and that found in the more traditional method differed by about $350 \times 10^{-6}$. At this time, a best guess is:

$$u(f) = 400x10^{-6} \ . \qquad (12)$$

Using this value, the uncertainty term in eq. (7) due to $u(f)$ will be less than the transducer resolution (56 Pa at 3.3 s integration time) when $f\Delta I$ is less than $1.4 \times 10^5$ Pa.

4.3.2 Uncertainty in Fall Rate Method

[3] An alternative method for determining the uncertainty $u(\Delta I)$ would be as follows. Calculate $\Delta I$ without averaging the six sequential transducer readings. Find $u(\Delta I_{TEST\text{-}REF})$ from eq. 11, and use the transducer resolution (56 Pa over a rectangular distribution) for $u(I_{REF})$ and $u(I_{TEST})$. The repeatability uncertainty will be slightly higher, as it will now include variations in the transducer reading due to the pressure balance instability. The combined standard uncertainty is approximately the same. However, the alternative method cannot indicate the relative stability of the pressure balances, which is often important in monitoring system performance.

For the fall rate method, we declare that the pressures from the REF and TEST balances are equal at the same elevation in the hydraulic line. There is a Type B uncertainty associated with the resolution of the smallest trim mass that can be placed on one of the balances, when the fluid lines are connected, that will produce a measurable change in the fall rate from the natural fall rate (obtained when the balances are isolated). For the present system, this resolution is 0.05 g for pressures less that 100 MPa, and 0.1 g for pressures between 100 MPa and 200 MPa. Taking this as a rectangular distribution, the standard pressure uncertainty is 58 Pa for pressures less than 100 MPa, and 115 Pa for pressures between 100 MPa and 200 MPa.

## 5. Experimental Results

### 5.1 Results at $|\Delta P/P| = 10^{-4}$

The Transducer Assisted Crossfloat method is compared to the fall rate method in Fig. 3. Plotted is the relative difference in the effective area of the TEST balance as determined by the TAC and FR methods, for twenty-eight separate experiments, from 20 MPa to 200 MPa. Comparing the ratio of areas of the test balance isolates the uncertainties due to the crossfloat method from the uncertainty of the effective area of the REF balance. For the TAC method, the relative pressure difference, $\Delta P/P$, between the TEST and REF balances was $-10^{-4}$ and $+10^{-4}$. For pressures of

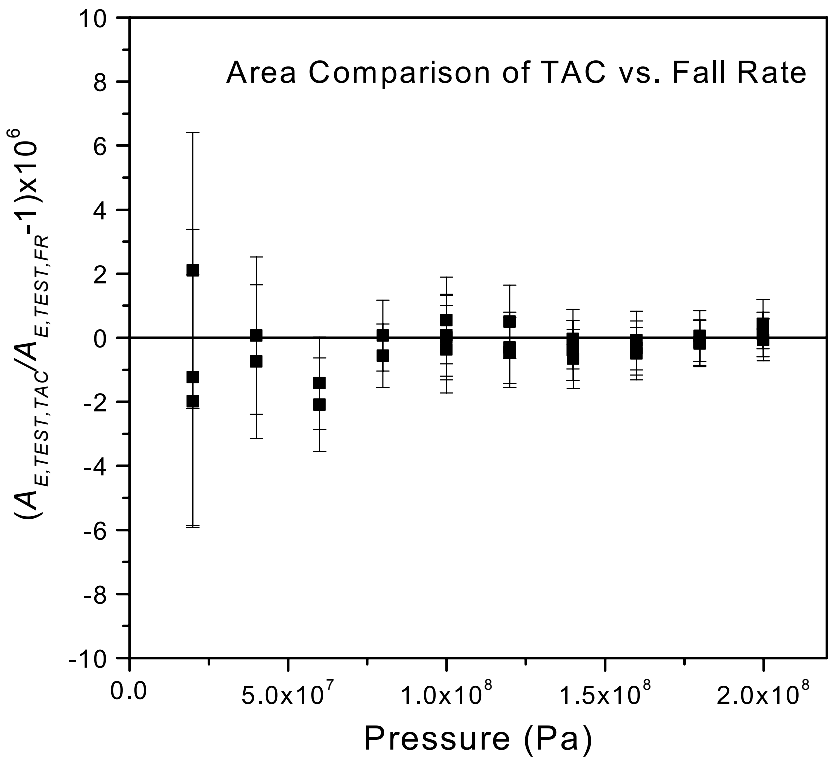


Figure 3. Difference in ratio of effective area of TEST balance using TAC method compared to fall rate method, at $\Delta P/P = +/- 10^{-4}$. TAC method also used at pressure equilibrium for 100 MPa and above. Relative combined standard uncertainty (k=1) plotted as error bars.

100 MPa and above, the TAC method was also tested at the equilibrium condition determined by the fall rate method. The vertical bars are the relative combined standard uncertainty (k=1) of the TAC method added in quadrature with the FR resolution uncertainty. The two methods gave the same results to within one standard uncertainty for all pressures except at 60 MPa. That is, only one of the twenty-eight tests fell outside the k=1 uncertainty band. If the fall rate equilibrium point had required 30 mg less mass on the TEST balance, the 60 MPa points would also have been within one standard uncertainty. The maximum difference in relative area, for all tests above 80 MPa, was $0.65 \times 10^{-6}$.

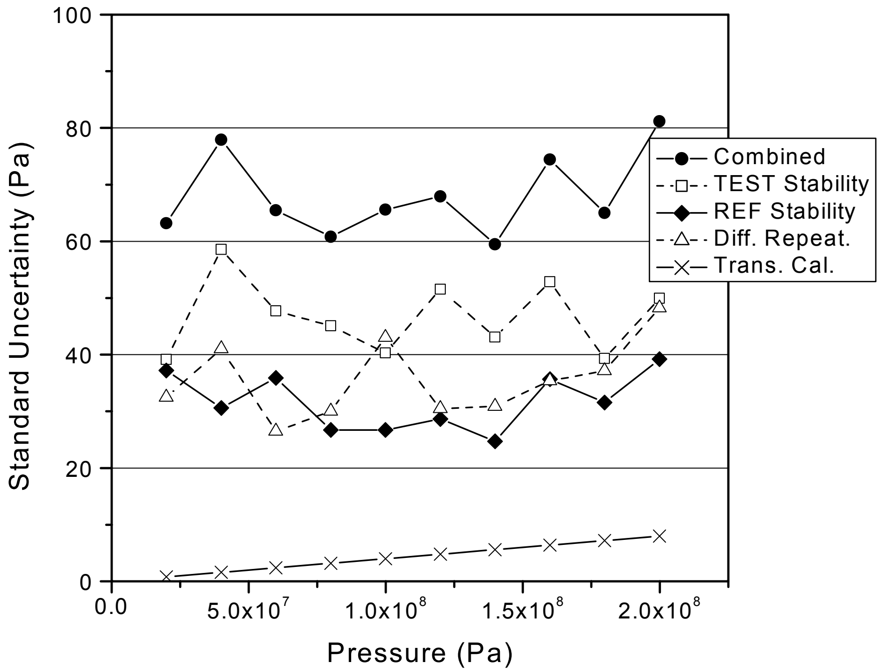


Figure 4. Standard uncertainties in pressure difference for TAC method, $\Delta P/P = +/- 10^{-4}$. Average value at each pressure shown for clarity. Range in combined standard uncertainty was 53 Pa to 102 Pa.

Component standard uncertainties for the TAC method are shown in Fig. 4 for the same experiments of Fig. 3. Uncertainties for the multiple experiments at each pressure were averaged to more easily show the trends. The various standard uncertainty components are the stability of the TEST balance, the stability of the REF balance, the repeatability of the difference measurement, and the calibration of the transducer. The largest component of standard uncertainty was the stability of the TEST balance, with the REF balance stability and the difference repeatability slightly less. The transducer calibration standard uncertainty was smallest since the pressure difference was small compared to the operating pressure. The combined standard uncertainty of the *individual* experiments varied between 53 and 102 Pa over the operating pressure, with no significant trend as the pressure was varied. (Recall we estimate the standard uncertainty of the FR method as 58 Pa for $P < 100$ MPa, and 115 Pa for $P \geq 100$ MPa). The largest relative combined standard uncertainty (k=1) occurred at 20 MPa, and was $3.6 \times 10^{-6}$. At 60 MPa and above, all relative combined standard uncertainties were less than $1.1 \times 10^{-6}$.

The TAC method can reveal relative levels of performance of the TEST and REF balances, indicated by the stability uncertainty. For an individual balance, the stability uncertainty could be due to both the transducer resolution and the balance performance. For two pressure balances sampled with the same pressure transducer, the difference in stability is likely due to the difference in balance performance rather than the difference in the pressure transducer performance. For these two pressure balances, REF appears to generate a more stable pressure than TEST.

5.2 Results as $\Delta P/P$ Changes

At the operating pressures of 100 MPa and 200 MPa, the REF balance was calibrated with the TAC method at $\Delta P/P$ of $-10^{-3}$, $-10^{-4}$, 0, $+10^{-5}$, $+10^{-4}$, $+10^{-3}$, and $+10^{-2}$. The results are shown in Fig. 5a and 5b, again as a difference in area ratio between the TAC method and the FR method. Fig. 5a shows the results for $|\Delta P/P| \leq 10^{-3}$, and Fig. 5b shows all the results. For all experiments at $|\Delta P/P| \leq 10^{-3}$ (except 200 MPa and $\Delta P/P = +10^{-3}$) the area ratio agreed to $0.6 \times 10^{-6}$ or better, and the relative combined standard uncertainty was larger than the area ratio deviation. At 200 MPa and $\Delta P/P = +10^{-3}$, the area ratio deviation was $-1.1 \times 10^{-6}$ and the relative combined standard uncertainty was $0.78 \times 10^{-6}$. At 200 MPa and $\Delta P/P = +10^{-2}$, the area ratio deviated by $-5.6 \times 10^{-6}$, with a relative combined standard uncertainty of $4.0 \times 10^{-6}$. For $|\Delta P/P| \geq 10^{-3}$, the TAC method over-predicted the true pressure difference. As can be seen from eq. 2, this under-predicts the area if $P_{TEST} > P_{REF}$, and over-predicts the area if $P_{TEST} < P_{REF}$. The error was less at 100 MPa because the transducer was calibrated at that pressure.

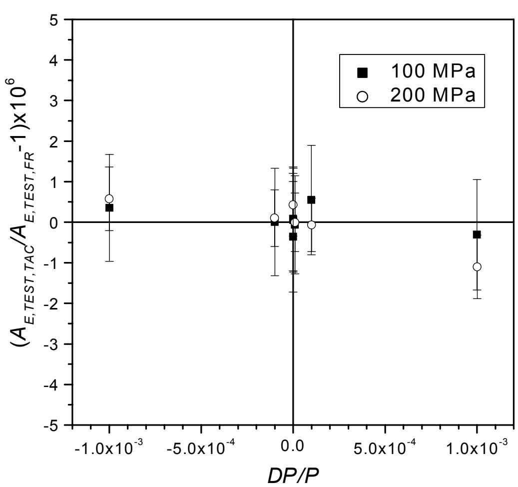


Figure 5a.

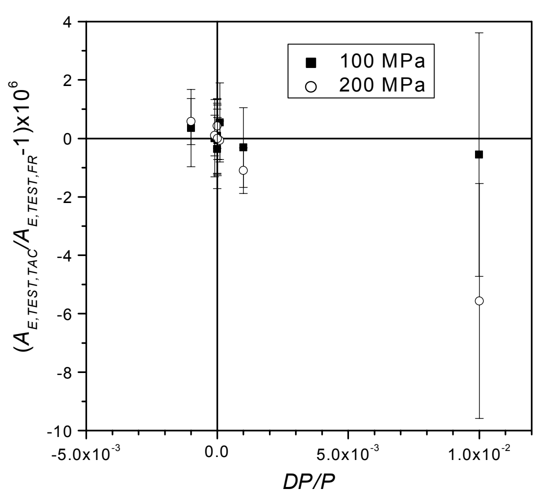


Figure 5b.

Figure 5. Difference in ratio of effective area of TEST balance using TAC method compared to fall rate method, for various $\Delta P/P$. Nominal pressures was 100 MPa and 200 MPa. Relative combined standard uncertainty (k=1) plotted as error bars.

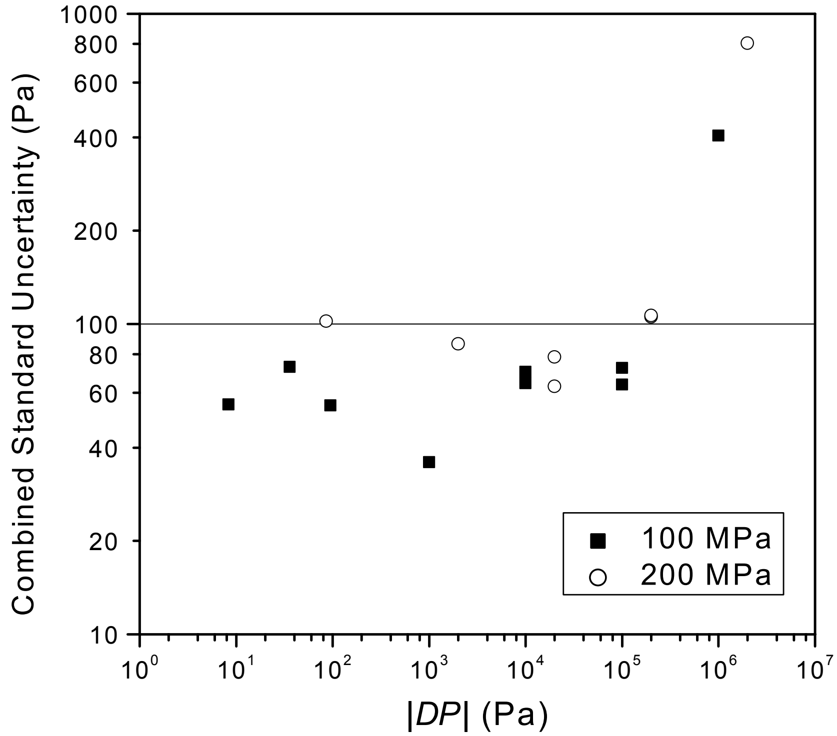


Figure 6. Combined standard uncertainty in TAC method as a function of pressure difference ($\Delta P$) between TEST and REF balance, at 100 and 200 MPa. Absolute value of $\Delta P$ plotted on log-log scale to show entire range of data.

Combined standard uncertainties of the TAC method for the experiments of Fig. 5 are shown in Fig. 6 on a log-log plot, with $|\Delta P|$ as the ordinate. The increase in uncertainty above $|\Delta P| = 10^5$ Pa is due to the increase in the transducer calibration uncertainty, $u(f)$. Changing $\Delta P$ did not affect the other uncertainty components beyond the scatter shown in Fig. 4. A more complete calibration for $f$ at various pressures and pressure differences could yield a different value for $f$ and lower uncertainty, however the practical (and least costly) guidance to keep the uncertainties low is to restrict $|\Delta P|$ to less than $10^5$ Pa.

5.3 Uncertainty for Various Repeat Points

In the measurement protocol of Table 1, we measured the pressure difference five times at each setting. If the number of difference readings, $n$, is reduced, the time for data acquisition also decreases. According to eq. (11), the standard uncertainty $u(\Delta I_{TEST\text{-}REF})$ will increase as $n$ decreases, even if the mean and variance remain constant. $u(\Delta I_{TEST\text{-}REF})$ will be 37 % higher at $n = 4$ than $n = 5$ for the same variance. The combined standard uncertainty $u(\Delta P)$ will increase by less than 37 %, since the repeatability uncertainty will still be on the same order as the stability uncertainties. For $n = 3$ or less, the Bayesian approach cannot be used to estimate the uncertainty. For $n = 3$ and $n = 2$, we can use the frequentist approach to estimate the uncertainty [5]. Since the degrees of freedom for $n$ is low, the coverage factors must now be significantly greater than k=1 and k=2 to encompass the 68.3 % and 95.5 % confidence intervals (k = 4.5 at $n$

= 3 and 95.5 %). The repeatability uncertainty is likely to dominate the other components, and the combined standard uncertainty could reach 200 Pa or more.

## 6. Discussion

Using the baseline measurement protocol described in Section 3 and Table 1, the proposed TAC method and the traditional FR method agreed to within the combined standard uncertainty of the methods in nearly all cases, and to within the combined expanded uncertainty (k=2) for all cases. To keep the uncertainty in the method comparable to the uncertainty of the fall rate method, the pressure difference between the REF and TEST balances must be small enough so that the transducer calibration uncertainty remains small. For the case studied here, a reasonable limit was $|\Delta P| \leq 10^5$ Pa, which kept the transducer uncertainty equal to or less than other component uncertainties. The technique worked equally well if the TEST pressure was less than or greater than the REF pressure. The combined standard uncertainties for the TAC method were 53 to 112 Pa, making them on the same order as the resolution of the pressure transducer (56 Pa). The relative combined standard uncertainties were $3.6 \times 10^{-6}$ or less at 20 MPa, and $1.1 \times 10^{-6}$ or less above 50 MPa. This compares quite favorably with the relative combined standard uncertainty of the NIST Transfer Standard at 280 MPa, which is $16 \times 10^{-6}$ (k=1). If the relative uncertainties at the lower pressures are too high for a certain application, a second transducer could be used. This transducer should have a smaller full-scale range and hence a lower absolute uncertainty.

Operational considerations also limit the pressure difference. When the pressure difference between the balances becomes too large, the piston positions rise and fall excessively when the constant volume valves switch the transducer between the REF and TEST balances, and substantial adjustment is required of the variable volume valves. For this system, $|\Delta P/P| \leq 10^{-3}$ seems sufficient. For the Fall Rate and DP cell methods, much additional mass trimming would be necessary at this pressure difference.

One of the important advantages of this technique is that it offers a trade-off between Type A uncertainty of the calibration process and speed of calibration (which translates into cost). For the lowest levels of uncertainty, five or more difference measurements can be made. For a customer willing to accept higher uncertainty, four difference measurements will still provide an estimate of the Type A uncertainty using Bayesian methods, and frequentist methods can be used for three or two difference measurements. Unlike the fall rate and DP cell techniques that require operator judgment of the resolution (and uncertainty) of the pressure agreement between the REF and TEST balances, the TAC uncertainty is computed directly from the measured transducer output. Once the approximate pressures are set, the entire data acquisition sequence can proceed under the control of the computer, with the output being the computed pressure difference and its uncertainty. The only operator involvement would be adjustment of the piston positions, if necessary.

The baseline measurement protocol requires 15 ½ minutes to acquire data at one setting. Reducing the number of difference measurements from five to two will reduce the measurement time to 6 ½ minutes, although the uncertainty will increase. Further reductions in measurement time should be possible with computer-actuation of the constant volume valves, timed to occur close to the completion of the six sample points, and reduction of the settling time between piston position adjustment and data acquisition. A reasonable limit is likely to be 5 minutes.

Another optimization that we have not explored in this work is the integration time of the transducer, which affects its resolution, and the number of sample points at each setting. Reducing the integration time will allow the six sample points to be acquired faster, although the stability uncertainties may increase. Reducing the number of sample points could have the same effect.

## 7. Conclusions

We have proposed a method for calibrating two pressure balances that offers the potential for automating the data acquisition, once approximately equal pressures are set on the balances. Trial and error adjustment of small trim masses is avoided, as is operator judgment in determining pressure equilibrium between the balances. In a prototype system using two 200 MPa pressure balances, agreement between this method and the fall rate method was better than the combined standard uncertainty (k=1) in nearly all cases.

## 8. References

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