Pressure Coefficients of Air Dielectric Capacitors

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Abstract: Normally, precision nitrogen dielectric or fused silica dielectric standard capacitors are used to transfer capacitance values from national metrology institutes such as the National Institute of Standards and Technology (NIST). Recently, NIST has adopted a measurement technique for extrapolating 1 kHz values of capacitance to 10 MHz. The technique employs the use of a specific design of air dielectric capacitance standard that is subject to differences in value depending on the atmospheric air pressure. This paper describes a series of measurements used to determine and correct for pressure differences of these air dielectric capacitors. The uncorrected values are observed to be in error by approximately 90 ppm for an elevation difference of about 1500 m between NIST and Sandia National Laboratories. This paper will address the pressure correction, how the user can correct for differences and how uncertainties are obtained. A more accurate transfer of capacitance results from the application of the correction.

1.0 Introduction

The Primary Standards Laboratory at Sandia National Laboratories has obtained calibrations of four terminal-pair (4TP) air dielectric capacitors at frequencies from 1 kHz to 10 MHz from NIST as a special test service. In determining the values above 1 kHz, NIST employs a method based on the 4TP admittance model defined by Cutkosky [1] and wideband one-port measurements made using a network analyzer to characterize the frequency response behavior of inductive and resistive components of the 4TP capacitor. The high frequency measurements are extrapolated back to 1 kHz using a method developed by Suzuki, *et al.* [2,3] that allows the capacitance and dissipation factor to be predicted in the 1 kHz to 10 MHz frequency range. The NIST implementation of the method is discussed in several recent papers [4, 5, 6].

One difficulty in the use of air dielectric capacitors is the dependence of the permittivity of air on atmospheric pressure. It is necessary to account for differences in atmospheric pressure as part of the uncertainty analysis of the capacitance measurement or to correct for these differences if

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they are known. Additionally, the permittivity is dependent of relative humidity and temperature to a lesser degree. It is important to know the influence of these effects on permittivity not only at the time of calibration, but also at the time of use so that differences in environmental conditions can adequately be accounted for.

In this paper, the dependencies of the permittivity of air on pressure, relative humidity, and temperature are established through the use of the Clausius-Mossotti relation [7] and this relationship is verified through comparison with measurements on a set of four commercial 4TP capacitance standards (HP/Agilent 16380A capacitor set) [8]. The frequency extension calibration technique discussed is specific to the Agilent 16380A capacitor set which includes the Agilent 16381A (1 pF), 16382A (10 pF), 16383A (100 pF), and 16384A (1000 pF) capacitors. The models developed for the pressure dependence of the permittivity of air are more generally applicable.

2.0 Model of the Permittivity (Dielectric Constant) of Gases

Table 1 gives the permittivity, ε (often called the dielectric constant), of some common gases at a temperature of 20 °C and pressure of one atmosphere (101.325 kPa).

Molecular Form	Gas	ε
----	Air (dry, CO_2 free)	1.000536
N_2	Nitrogen	1.000548
H_2O	Saturated Water Vapor	1.00022

Table 1. Permittivity of a few common gases [11].

The density dependence of the permittivity is given by the Clausius-Mossotti equation

$$\frac{\varepsilon-1}{\varepsilon+2} = \rho_m \left(\frac{4\pi N\alpha}{3} + \frac{4\pi N\mu^2}{9kT} \right) \tag{1}$$

where ρ_m is the molar density, N is Avogadro's number, k is the Boltzmann constant, T is the temperature, α is the molecular polarizability, and μ is the permanent dipole moment[*]. Therefore, in regions where the gas can be considered to be ideal, using the relation $k = R/N$, the ideal gas law, and the approximation $\varepsilon + 2 \approx 3$, Equation 1 becomes

$$\varepsilon \approx 1 + \frac{4\pi}{kT}\left(\alpha + \frac{\mu^2}{3kT} \right)p . \tag{2}$$

where, p is the gas pressure (R is the molar gas constant).

Without the approximation, the permittivity is given by

$$\varepsilon = \frac{2A+1}{1-A} \tag{3}$$

where

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[*] Values of physical constants used throughout this paper are from Reference 9.

$$A = \frac{4\pi}{3kT}\left(\alpha + \frac{\mu^2}{3kT}\right)p \,. \qquad\qquad (4)$$

Here, equations 3 and 4 are determined algebraically from the solution of Equation 1.

From Equation 2, under isothermal conditions, a plot of the permittivity as a function of gas pressure has a slope equal to $\frac{4\pi\alpha}{kT}$ provided that the gas is nonpolar ($\mu = 0$). Therefore, a plot of the normalized value of capacitance as a function of gas pressure should be approximately linearly dependent on pressure, and standard air capacitors at elevations above sea level will have measured capacitance values that are smaller than those obtained at sea level due to the pressure dependence of the permittivity.

3.0 Pressure Coefficient Measurements on 16380A Series Capacitors

Capacitance measurements of air capacitors in N_2 gas should be most easily modeled since the gas is of a single constituent, the gas is nonpolar, and the value of the polarizability is known precisely ($\alpha = 1.7403 \times 10^{-30}$ m^3). These measurements can be utilized to support the underlying principles and validate the value of the polarizability given for N_2; in addition, the measurements provide an assurance that other unknown effects are not of significance.

A pressure chamber, previously developed to establish pressure coefficients of zener voltage reference standards was used to make the measurements. Measurements using the system with N_2 gas were conducted over a broad pressure range. The system was pumped for several hours to achieve a low base pressure and to insure that the system was relatively free of water vapor. Capacitance measurements were conducted in a 3-terminal (3T) configuration at 1 kHz using an Andeen-Hagerling 2500A capacitance bridge with an expanded uncertainty of ± 5 ppm (k=2). Connections were made from the bridge to the capacitor under test by connecting the high lead of the bridge to the HPOT connection on the capacitor, connecting the low lead of the bridge to the LPOT connection, and connecting the bridge guard to the case of the capacitor using bnc connectors, RG-58 cables and vacuum feed-through connections to the pressure system. The leads and feed-through connectors do not contribute significant uncertainty to the measurement, in part due to the relatively low values of capacitance measured and in part due to the 3T configuration. The pressure in the chamber was increased by regulating the flow of N_2 gas into the system using a Druck DPI-500 pressure controller. The readout of the controller was used to measure the pressure with an assigned uncertainty of ± 0.15% (k=2). The overall uncertainty in the measurement at the time of test is estimated to be ± 10 ppm (k=2) when the standard deviations of the data, the uncertainty in pressure, and the uncertainty in the capacitance measurement are considered.

3.1 Results of Measurements with N_2 Gas

Results of the measurements for the four capacitors are shown in Figure 1. Extrapolation of the measurements is used to establish C_0, the value of the capacitor under vacuum, which is then used to normalize the results. The normalization allows the measurements to be displayed as C/C_0 which is equivalent to a direct measurement of the permittivity ε. The results of

calculations using the Claussius-Mossoti relation are also shown (tabulated values of the calculations are shown later in Table 3). The value of the polarizability for N_2 is from Reference 10. Agreement between the measured and predicted curves is very good and it is therefore assumed that the physical construction of the capacitors is such that it is suitable to use the Claussius-Mossoti relation for analysis.
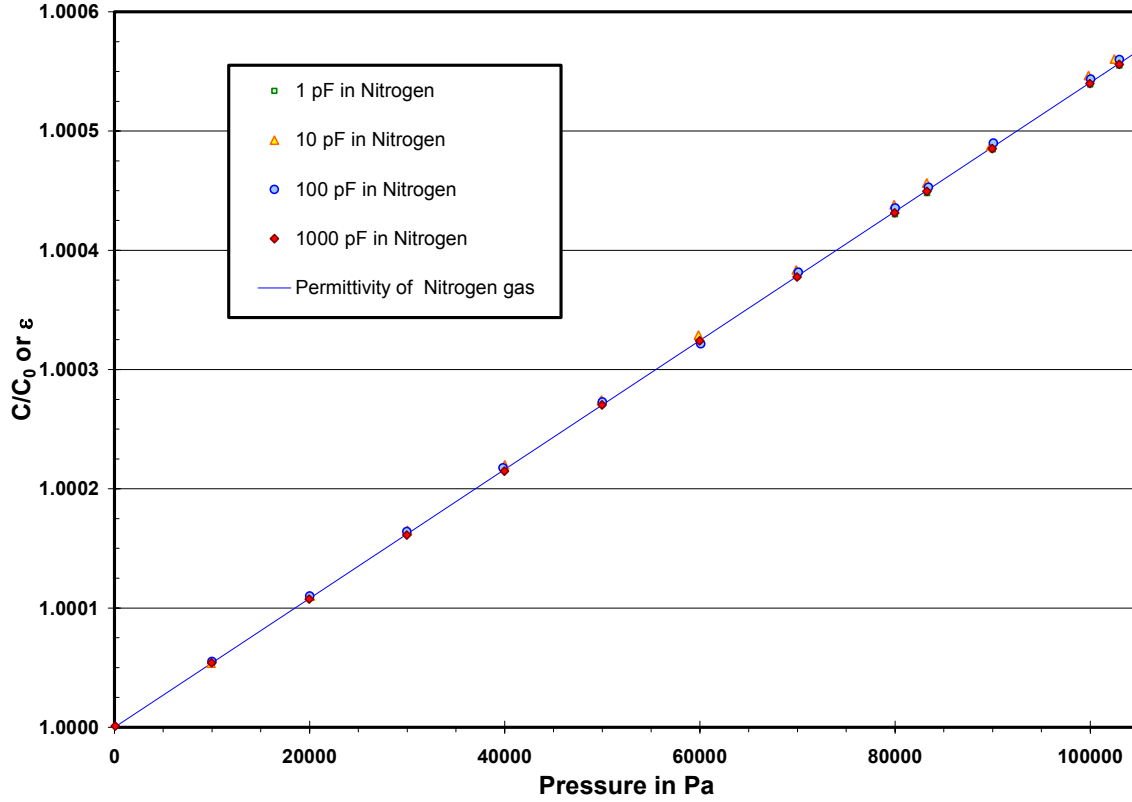

Figure 1. Measured values of C/C_0 of the 16380A capacitor set in a high purity N_2 gas environment as a function of gas pressure at approximately 23 °C. Values equate directly to the permittivity of N_2 gas. Uncertainty bars are indiscernible on the plot scale, but the uncertainty in the measurement values is estimated to be ±10 ppm (k=2).

3.2 Calculations for Dry Air

In order to extend the model of Section 2 for dry air (humidity is a further complication that will be considered in the next section), it is necessary to consider the gas constituents that make up air, their polarizabilities, and where applicable, their permanent dipole moments. A reasonably accurate model can be developed by combining these properties according to their partial pressures. This ignores the small influence of the molecules upon each other, but this is justified by the relatively large spacing between molecules in a dilute gas as in Reference 7. When combined in this manner, Equation 4 becomes

$$A = \frac{4\pi}{3kT} \sum_i \left(\alpha_i + \frac{\mu_i^2}{3kT} \right) p_i . \tag{5}$$

where the i denotes summation over each constituent of air and p_i is now the partial pressure of the gas constituent. The partial pressures can be obtained by assuming the model of the US Standard Atmosphere, 1976 [11] which gives the fractional volumes and molecular polarizabilities are as given in Table 2. The fractional volumes are assumed to be independent of pressure.

	M_i (kg/kmol)	F_i (dimensionless)	α (m^3)	μ
N_2	28.0134	0.78084	17.403×10^{-31}	0
O_2	31.9988	0.209476	15.812×10^{-31}	0
Ar	39.948	0.00934	16.4×10^{-31}	0
CO_2	44.00995	0.000314	29.110×10^{-31}	0
Ne	20.183	0.00001818	3.9560×10^{-31}	0
He	4.0026	0.00000524	2.0496×10^{-31}	0
Kr	83.8	0.00000114	24.844×10^{-31}	0
Xe	131.3	0.000000087	40.440×10^{-31}	0
CH_4	16.04303	0.000002	25.930×10^{-31}	0
H_2	2.01594	0.0000005	8.0450×10^{-31}	0

Table 2. Gas constituents, molar densities, fractional volumes, molecular polarizabilities, and permanent dipole moments for the US Standard Atmosphere, 1976. Values of the molecular polarizabilities and permanent dipole moments are from Reference 10.

Using these values together with Equation 3 and Equation 5, the computed permittivity for air and nitrogen is given in Table 3. The results show that the permittivity of dry air is about 11 ppm smaller than that of N_2 gas alone. A separate computation using the same formulae for CO_2 free dry air gives a value of 1.0005372 which is 0.8 ppm higher than the reference value of 1.0005364 given in Reference 10 and Table 1. This provides some confirmation of the model.

	p (Pa)	ε_{AIR}	ε_{N2}
	0	1.0000000	1.0000000
	10000	1.0000530	1.0000541
	20000	1.0001060	1.0001081
	30000	1.0001589	1.0001622
	40000	1.0002119	1.0002163
	50000	1.0002649	1.0002703
	60000	1.0003179	1.0003244
	70000	1.0003709	1.0003785
	80000	1.0004239	1.0004325
	90000	1.0004769	1.0004866
	100000	1.0005299	1.0005407
1 atm =	101325	1.0005369	1.0005479
	110000	1.0005829	1.0005948

Table 3. Computations of the permittivity of dry air and N_2 gas.

3.3 Consideration of Humidity

Having just considered the case of N_2 alone, then the case of dry air, consideration must be made of the content of water vapor in the air and its effect on the permittivity. Given the temperature, the saturation vapor pressure of water can easily be determined from tabulated data such as that given in Reference 12 (for completeness, the values used here are listed in Appendix A). If the relative humidity is also known, through measurement, then the partial pressure of water vapor in air is given simply by the relation

$$PP(T) = RH \bullet SVP(T). \tag{5}$$

where, *PP(T)* is the partial pressure of water vapor in the air, *RH* is the relative humidity in fractional percentage, *SVP(T)* is the saturation vapor pressure of water vapor in air at temperature *T*.

With the partial pressure of the water vapor known and the atmospheric pressure measured, the constituents of the air less the partial pressure or water vapor is given by the standard atmosphere model of the previous section. The contribution to the permittivity of the water vapor can be computed in the same manner as before (Equation 3 and 5) with the values of the polarizability and permanent dipole moment taken from reference data. Unlike the case of the major constituent gases of air, water has a permanent dipole moment that is given as $\mu = 1.8545$ D (Debye units) and the polarizability is 14.5×10^{-31} m^3 [10]. Calculations of the predicted permittivity values are as shown in Figure 2 along with measured values.
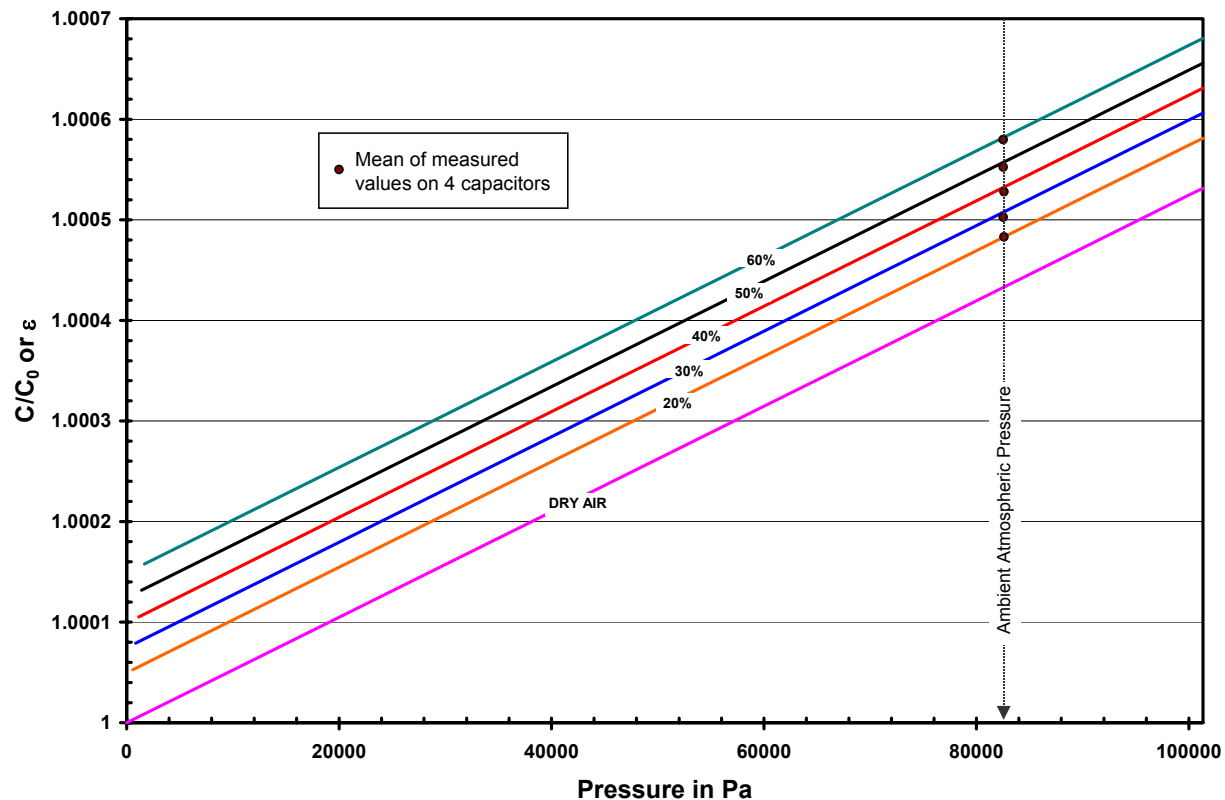

Figure 2. Permittivity of air vs. pressure with relative humidity as a parameter along with measured values.

The measurements shown in Figure 2 were taken by placing the capacitors in a Thunder Scientific Model 2500 two-temperature, two-pressure humidity chamber. Pressure is not controllable in this system, so the pressure is at the ambient pressure of approximately 82.530 kPa. Actual pressure and temperature were recorded in the test and as before, the capacitance was measured using an Andeen-Hagerling 2500A capacitance bridge. Values of capacitance were recorded after allowing the chamber to stabilize at a value of 20% RH. The humidity setting was then changed in increments of 10%. Tests were terminated after reaching 60% RH. The data were again normalized to C_0, the vacuum level value of capacitance, to allow a direct measure of the permittivity. The mean of the four capacitance values is shown in Figure 2 along with the predicted characteristic curves. The standard deviation in the data across all four capacitors was the highest at 60% RH ($\sigma = 3\times10^{-6}$).

The capacitors were next placed into the pressure chamber described previously in Section 3.0. The system was pumped for several hours to achieve a low base pressure and ambient air was introduced through the DPI-500 pressure controller. The ambient air being introduced had a humidity level of approximately 38%. Upon introduction into the chamber, the gas is expanded such that the humidity level in the chamber is lower than ambient until the chamber pressure is at the ambient atmospheric pressure. Above ambient pressure, the humidity level will be higher than the ambient humidity. As a result, it is necessary to record the relative humidity from inside the chamber. This was accomplished with a Rotronic Hygrolog-D psychrometer.
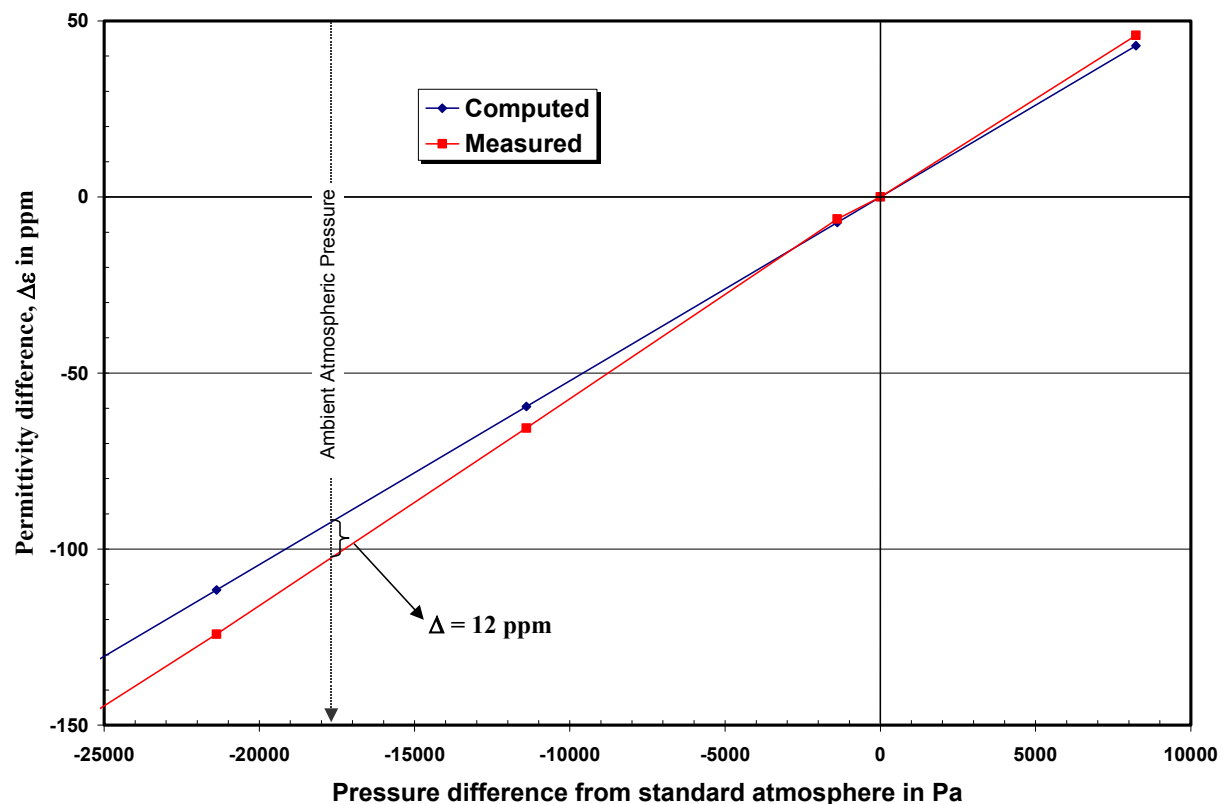

Figure 3. Permittivity as a function of pressure in air.

The measured differences in permittivity from those measured at 1 standard atmosphere were computed from the data and compared with the computed values using the Clausius-Mossotti model. These results simulate the corrections that are to be applied to the capacitance values when the capacitors are used in an environment where the atmospheric pressure differs from that at the time of calibration. The results are shown in Figure 3. For the mean pressure difference between Sandia National Laboratories (at an altitude of 1653 m) and that of NIST, Gaithersburg (at an approximate altitude of 150 m), the calculations are within approximately 12 ppm of the measured differences.

4.0 Uncertainty of the Capacitor Frequency Characteristic Prediction (CFCP) Method

Using a circuit model developed by Yonekura and Wakasugi [10] for the 16380A capacitor set, NIST has established the uncertainty of the extrapolations through a series of circuit simulations where the capacitance values of the equivalent circuit model, shown in Figure 4, are obtained by 1 kHz measurements with a commercial capacitance bridge and variations in the values are input to the circuit simulation routine in order to determine the Type A and Type B components due to these reference values [6].
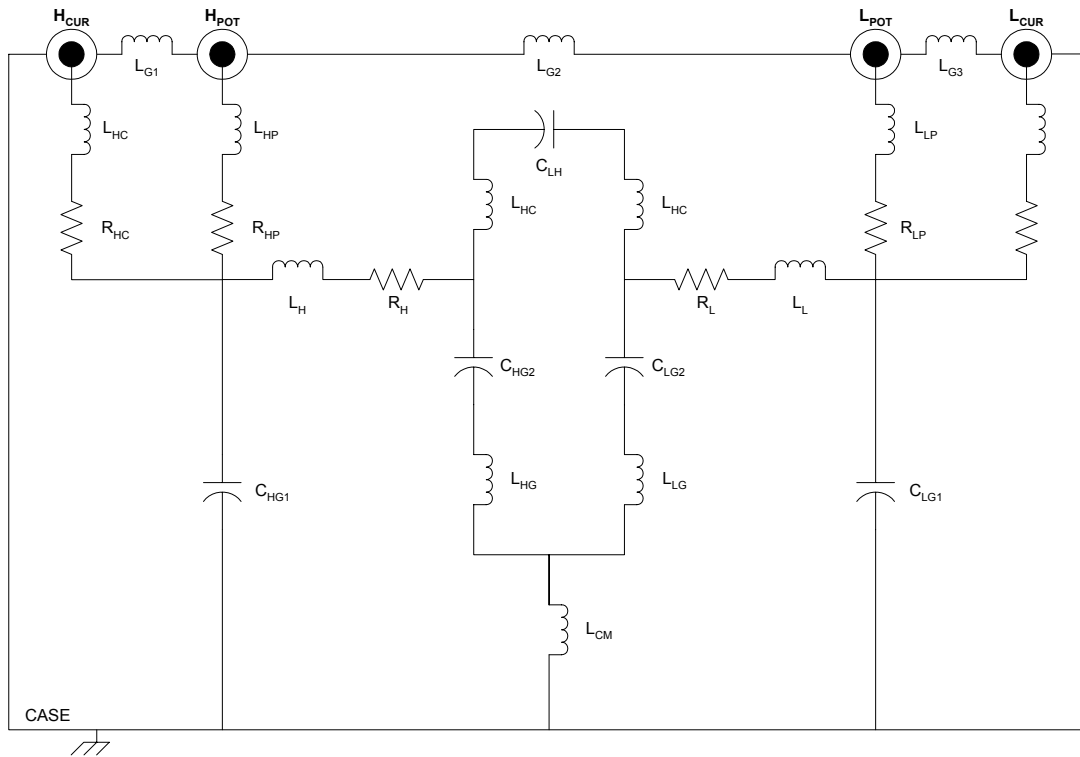

Figure 4. Yonekura and Wakasugi equivalent circuit [7].

At 1 kHz, the effect on the total impedance of the various inductances and resistances is small and, as mentioned previously, reference values for the capacitances can be obtained by measurement with a capacitance bridge. Three capacitance values are obtained, C_{LH}, C_{HG}, and C_{LG} as shown in the simplified circuit model of Figure 5 (where $C_{HG} = C_{HG1} + C_{HG2}$ and $C_{LG} = C_{LG1} + C_{LG2}$).
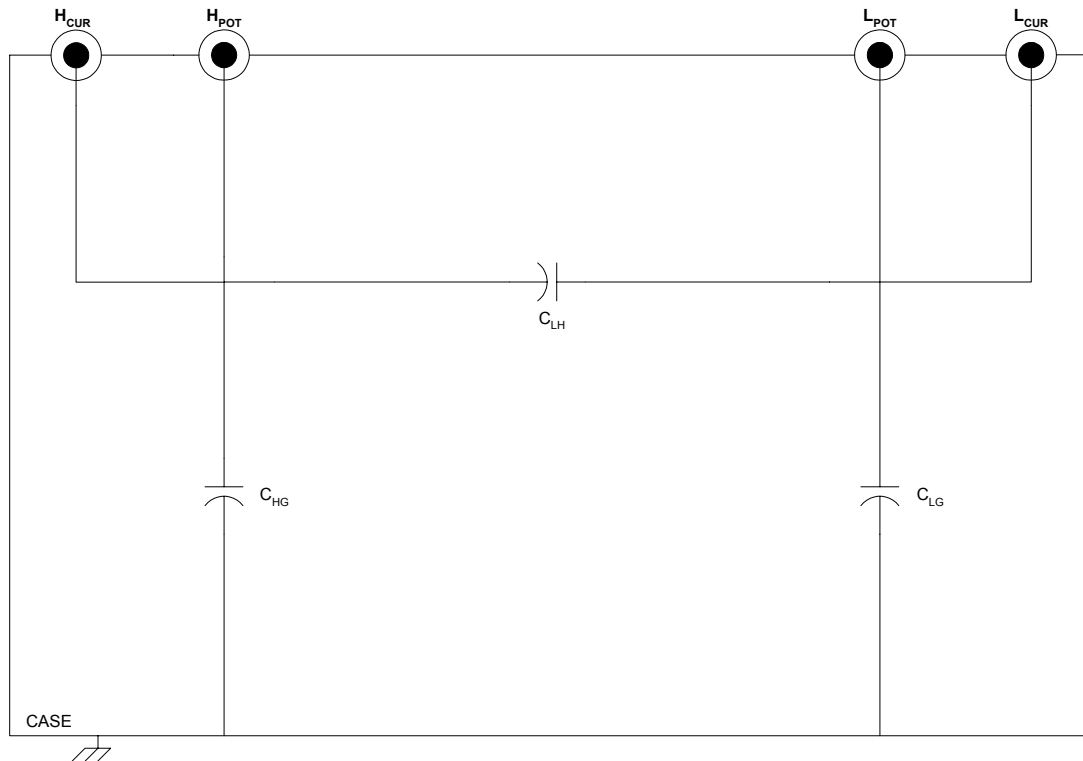
Figure 5. A simplified equivalent circuit valid for low frequency (1 kHz) measurements.

The Type-B uncertainty component in these 1 kHz reference measurements was conservatively estimated by NIST to be 10 ppm, and these variations, when input into the circuit simulation routine produce uncertainty components at 1 MHz and 10 MHz as given in Table 4.

	A [ppm]	*B* [ppm]	*U (k=2)* [ppm]
1 pF			
1 MHz	5	10	700
10 MHz	5	10	1400
10 pF			
1 MHz	4	10	24
10 MHz	4	31	1100
100 pF			
1 MHz	7	10	26
10 MHz	7	10	1100
1000 pF			
1 MHz	5	10	25
10 MHz	5	36	1100

Table 4. Type A and Type B components of the 1 kHz capacitance reference measurement uncertainty and expanded uncertainty of the CFCP method for a 4TP capacitor set as reported by NIST [4].

The uncertainty components due to these reference measurements (as determined by NIST) is small relative to the overall uncertainty, however, upon measurement of these reference values at Sandia, it was noted that the reference values differ by about 90 ppm because of the variation in the permittivity of air with atmospheric air pressure noted in the previous section. The altitude difference between the laboratories is 1500 m resulting in a mean atmospheric pressure difference of 17.75 kPa. In order to obtain uncertainties of the same order of magnitude as those evaluated by NIST, it is necessary to apply the corrections developed to the reference capacitance values. An example of the calculation is shown below.

Sample Calculation of the Correction:
HP 16384A, nominal 1000 pF capacitor
Measured value of C_{LH} at NIST: 1000.258 ± 0.01 pF
Estimated ambient at NIST: ($p = 101.325 \pm 4$ kPa, $T = 23 \pm 1$ °C, $RH = 35 \pm 10$ %)
Computed permittivity: $\varepsilon_{NIST} = 1.0006185$
Ambient at Sandia: ($p = 83.166 \pm .08$ kPa, $T = 23 \pm 1$ °C, $RH = 38 \pm 5$ %)
Computed permittivity: $\varepsilon_{SNL} = 1.0005307$
Uncertainty in corrected value: $U = \pm 35$ ppm ($k = 2$)
Calculated value of C_{LH} at Sandia: 1000.170 ± 0.035 pF
Measured value of C_{LH} at Sandia: 1000.168 ± 0.005 pF

The uncertainty of the correction is computed by considering the uncertainty of the measurement at NIST, the estimated uncertainties in the pressure, temperature, and humidity at both laboratories, and the sensitivity of the permittivity to each of these values.

4.0 Conclusions

Pressure corrections for the permittivity of air (with consideration of air pressure, temperature, and humidity) have been established through measured data and the Clausius-Mossotti equation. The corrections compare within about 12 ppm of measured data on the Agilent/HP 16380A capacitor set. Using these corrections, it is possible to establish differences in capacitance values at ambient conditions that differ between the calibration laboratory and the user laboratory. These corrections are applied and the uncertainties determined for capacitance values measured at 1 KHz. Changes in the capacitance value at higher frequencies, up to 10 MHz, as determined using the CFCP technique, can then be applied to the reference values.

Additional consideration needs to be given to any effect on the CFCP technique itself. The argument can be advanced that these particular data are not subject to pressure dependent errors because the data is utilized in a manner that extracts only the parasitic inductive and resistive characteristics of the capacitor. The assumption is made in the method that the capacitor values in the equivalent circuit model of Figure 4 do not change with frequency [4]; the change in capacitance with frequency is apparent in nature only, being the result of the parasitic resistive and inductive elements. This assumption is not strictly valid (for example, see Reference 13). Further evaluation is necessary to determine the importance of neglecting the variation of capacitance with frequency and the effects of pressure, temperature, and humidity on the parasitic parameters of the capacitor.

5.0 Acknowledgements

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Appendix A: Saturation Vapor Pressure Values for Water Vapor in Air

Values of the saturation vapor pressure used in Section 3 are taken from Reference 15. The reference data, shown in Figure 6 were least-squares fit to a quadratic polynomial over the temperature range from 20 to 26 °C. In addition, calculations with the Clausius-Mossotti relation were compared with the tabulated reference values of the permittivity to insure that computations were being performed correctly. The calculated and the tabulated results are shown in Table 5.
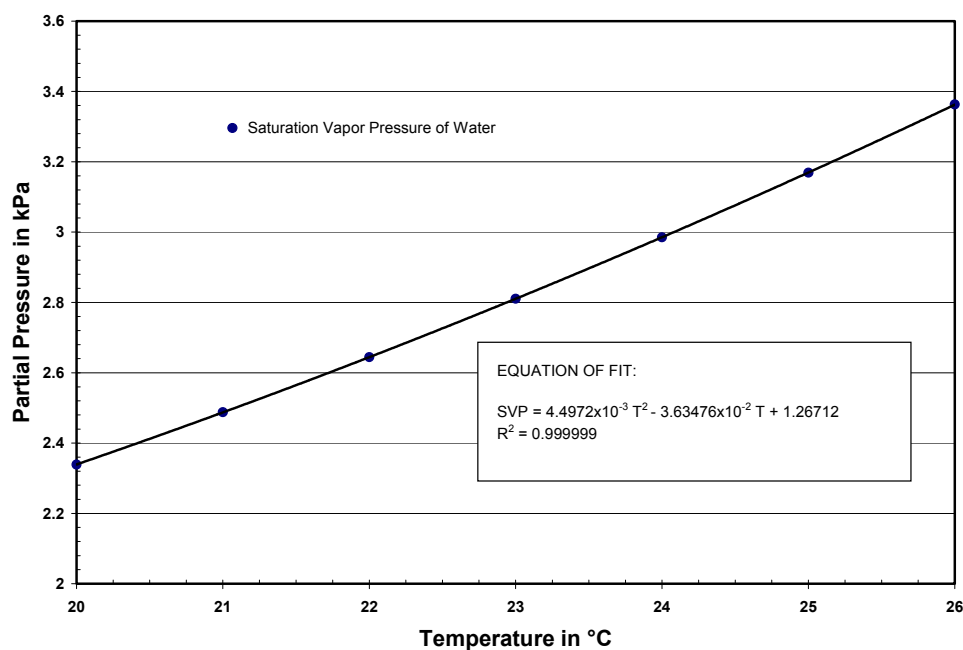

EQUATION OF FIT:

$SVP = 4.4972 \times 10^{-3}\ T^2 - 3.63476 \times 10^{-2}\ T + 1.26712$

$R^2 = 0.999999$

Figure 6. Reference values of the saturation vapor pressure of water [15].

T (K)	ε (reference)	ε (computed)
273.15	1.00007	1.00007
283.15	1.00012	1.00013
293.15	1.00022	1.00022
303.15	1.00037	1.00038
313.15	1.0006	1.0006

Table 5. Reference values of the permittivity of saturated water vapor [11] and results calculated with the Clausius-Mossotti equation.