An Automatic System for Calibration of Voltage Ratio Devices

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Abstract

An automatic system is implemented for calibration of voltage ratio devices using a simple self-calibrating method at rated voltages from 2 V: 1 V to 1000 V: 1 V with standard measurement uncertainty of 2.5×10^{-7}. The system applies a fixed test voltage from a stable voltage source to the device under test (DUT). With the aid of a programmable low thermal scanner, it then uses a precision digital multimeter to measure sequentially the voltage drops across individual sections of the DUT. The DUT's voltage ratios are determined based on the measurement results of volt-drops by a set of addition and multiplication operations.

The system software is written in Visual Basic. An user-friendly graphical user interface is developed for test parameter setting, instrument control (including scanner switching), data acquisition, result calculation and real-time event monitoring. For oil immersed DUTs, the system uses a simple control module to control the oil temperature in order to minimise measurement uncertainties.

1. Introduction

Voltage ratio devices are essential equipment for precision electrical measurements. With a voltage ratio device, an unknown voltage can be scaled down to a convenient value that can be calibrated by a reference voltage standard. However, calibration of a multi-section volt ratio box (e.g. Guildline 9700PL) is a lengthy and tedious process, which is also prone to errors. A simple automatic system is implemented for calibration of voltage ratio boxes (VRBs) at considerably reduced turn around times and with significantly less operator intervention required.

2 The System

2.1 The System Hardware

The calibration system, as shown in Figure 1, comprises a stable voltage source (Wavetek 4808), a programmable low thermal scanner (Dataproof 320A Opt3H), a precision digital multimeter (Wavetek 1281) and the device under test (DUT). Test voltage is applied from the Wavetek 4808 to the DUT. The Wavetek 1281 measures the voltage drops across individual sections of the DUT sequentially via the Dataproof 320A Opt3H.

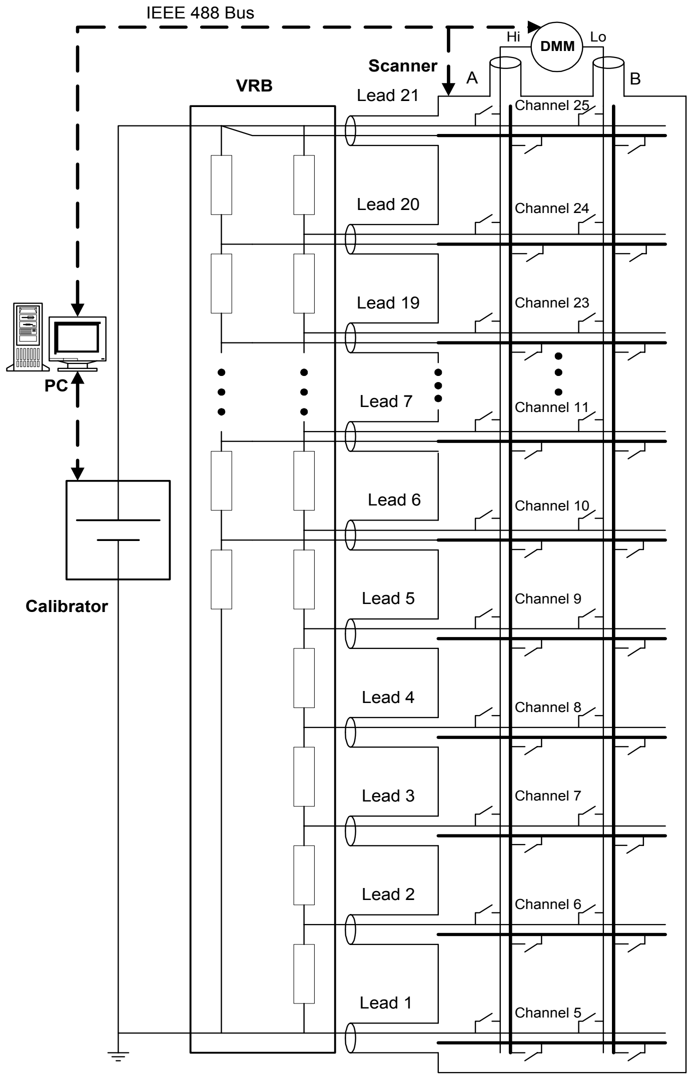


Figure 1. The Schematic Diagram of the System

For oil immersed DUTs (e.g. Guildline 9700PL), the DUT's oil temperature is kept within 20 mK of the target temperature by a simple electronic circuit so that measurement uncertainty due to temperature effect is minimized.

2.2 The System Software

The calibration system uses a simple self-calibrating method to determine the VRB's voltage ratios. The DUT's lowest section is taken as the reference section. The volt-drop measured across the reference section is compared with those volt-drops measured across the subsequent sections. Using a set of mathematical equations, the system calculates the DUT's voltage ratios based on the measured volt-drops. (Refer to Appendix A for derivation of the equations for DUT's voltage ratios from the measured volt-drops.)

The system software is written in Visual Basic. It controls the process of volt-drop measurement and voltage ratio calculation. Test parameter settings, instrument control, data acquisition and data analysis are performed automatically. A graphical user interface (GUI) is implemented for real-time event monitoring.

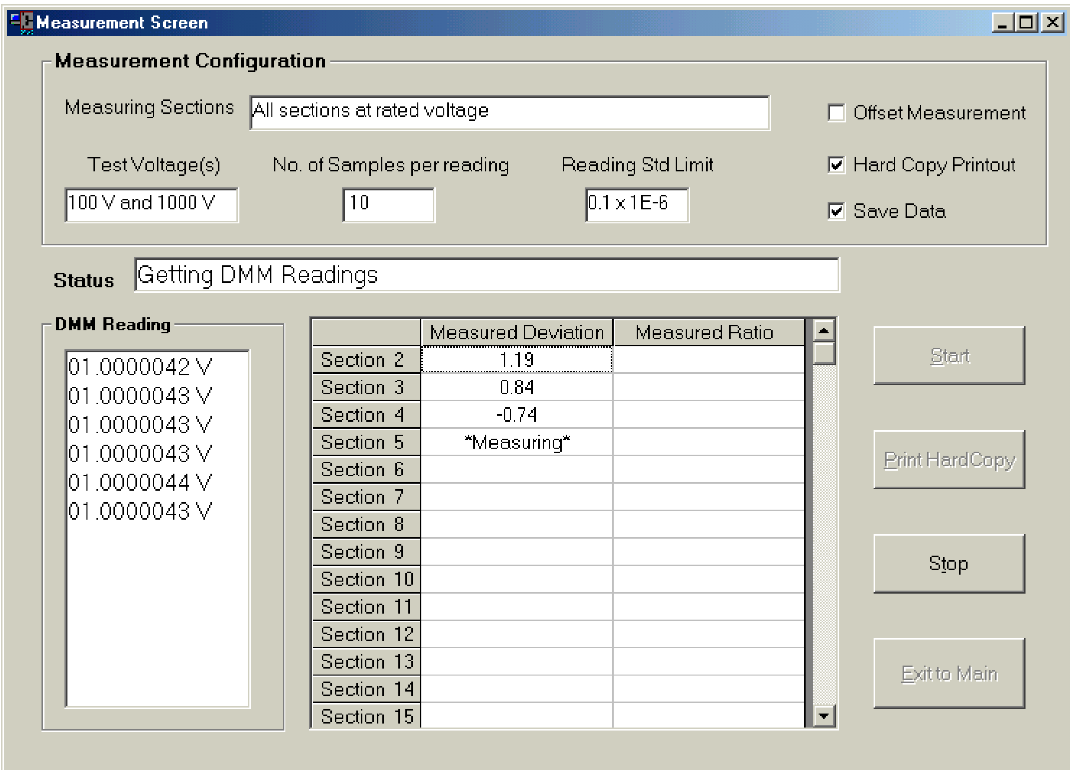


Figure 2. A GUI screen: real-time monitoring of volt-drop measurement

Standards and Calibration Laboratory
DC Laboratory
AutoVRBCal Version 2.0

Measurement Date : 12 Nov 2001 Starting Time : 12:11:04

Test Config.: All sections at rated voltage
Test Voltage : At Rated Voltage
No. of Samples : 10 Reading Standard Deviation : 0.1 X1E-6

Raw Data:

Sections		Measured Voltage (V)	
01-02-01	1.00000133	1.00000257	1.00000163
02-03-02	1.00000270	1.00000358	1.00000267
03-04-03	1.00000363	1.00000287	1.00000366
04-05-04	1.00000288	0.99998434	1.00000296
05-06-05	4.99999430	5.00001400	4.99999410
06-07-06	10.00000920	10.00002410	10.00001070
07-08-07	10.00002410	10.00001430	10.00002420
08-09-08	10.00001470	10.00002470	10.00001470
09-10-09	10.00002570	10.00001440	10.00002520
10-11-10	49.93815900	49.93826200	49.93815700
11-12-11	99.55352600	99.55357000	99.55345200
12-13-12	99.55351700	99.55345800	99.55346700
13-14-13	99.55339800	99.55333300	99.55336800
14-15-14	99.55330700	99.55329400	99.55329200
15-16-15	99.55327500	99.55335400	99.55327100
16-17-16	99.55335900	99.55333600	99.55335300
17-18-17	99.55333700	99.55328100	99.55332400
18-19-18	99.55328100	99.55321700	99.55327800
19-20-19	99.55321100	99.55328600	99.55320500

Calculated Results:

Section No.	Deviation	Ratio (Ref:1.5 V)	Ratio (Ref:15 V)
Section 2 (2V):	1.09	0.54	
Section 3 (3V):	0.90	1.02	
Section 4 (4V):	-0.78	1.07	
Section 5 (5V):	-18.58	-2.62	
Section 6 (10V):	3.96	-0.64	0.00
Section 7 (20V):	1.41	0.07	0.71
Section 8 (30V):	-0.99	-0.02	0.61
Section 9 (40V):	1.00	0.18	0.82
Section 10 (50V):	-1.10	0.08	0.72
Section 11 (100V):	2.08	1.12	1.76
Section 12 (200V):	0.81	1.53	2.17
Section 13 (300V):	-0.34	1.55	2.19
Section 14 (400V):	-0.50	1.44	2.07
Section 15 (500V):	-0.06	1.36	1.99
Section 16 (600V):	0.81	1.44	2.07
Section 17 (700V):	-0.20	1.47	2.10
Section 18 (800V):	-0.50	1.43	2.07
Section 19 (900V):	-0.62	1.33	1.97
Section 20 (1000V):	0.78	1.33	1.96

Comments : Testing of version 2.0

Operator Name : Y C Chau STD(2)

Figure 3. Sample Print-out for Calculation of a Guildline 9700PL VRB

Notes:-

(1): "Deviation" stands for the voltdrop difference between any two consecutive sections within the VRB (in $\times 10^{-6}$). (Refer to Appendix A for details.)

(2): "Ratio (Ref. 1.5 V)" stands for volt ratio w.r.t. the "1.5 V-to-0 V" section.

(3): "Ratio (Ref. 15 V)" stands for volt ratio w.r.t. the "15 V-to-0 V" section.

3. Measurement Uncertainty

The VRB can be seen as a chain of resistors, identified as unique sections, and arranged in many groups. (Refer to Appendix A for detailed descriptions and derivation of equations for VRB voltage ratios.)

3.1 <u>Uncertainty for the voltage ratios that involve one group only</u>

According to Equation (A1) in Appendix A, within a single group (say, the i^{th} group), the voltage ratio for a chain of n sections with respect to the group's base section is:

$$R_{(i)} = \frac{V_{(i,1)} + V_{(i,2)} + \ldots + V_{(i,n)}}{V_{(i,1)}}$$

$$= 1 + \frac{1}{V_{(i,1)}} \left(\sum_{j=2}^{n} V_{(i,j)} \right)$$

If the shunting effect due to the system DMM (see Appendix B for details) is taken into consideration, a correction factor C_L should be applied and $R_{(i)}$ would become:

$$R_{(i)} = \left[1 + \frac{1}{V_{(i,1)}} \left(\sum_{j=2}^{n} V_{(i,j)} \right) \right] \left(1 + C_L \right)$$

The measurement uncertainty for $R_{(i)}$ is:

$$u^2\left(R_{(i)}\right) = \left[\frac{\partial R_{(i)}}{\partial V_{(i,1)}} u\left(V_{(i,1)}\right) \right]^2 + \sum_{j=2}^{n} \left[\frac{\partial R_{(i)}}{\partial V_{(i,j)}} u\left(V_{(i,j)}\right) \right]^2 + \left[\frac{\partial R_{(i)}}{\partial C_L} u\left(C_L\right) \right]^2$$

where

$$\frac{\partial R_{(i)}}{\partial V_{(i,1)}} = -\frac{1}{V_{(i,1)}^2} \sum_{j=2}^{n} V_{(i,j)} - \frac{C_L}{V_{(i,1)}^2} \sum_{j=2}^{n} V_{(i,j)} = c_1$$

$$\frac{\partial R_{(i)}}{\partial V_j} = \frac{1}{V_{(i,1)}} + \frac{C_L}{V_{(i,1)}} = c_2$$

$$\frac{\partial R_{(i)}}{\partial C_L} = 1 + \frac{1}{V_{(i,1)}} \sum_{j=2}^{n} V_{(i,j)} = c_3$$

For operational convenience, C_L is not corrected (i.e. $C_L = 0$), but taken as an uncertainty contribution to the ratio. It follows that:

$$c_1 = -\frac{1}{V_{(i,1)}^2}\sum_{j=2}^{n} V_{(i,j)}$$

$$c_2 = \frac{1}{V_{(i,1)}}$$

$$c_3 = 1 + \frac{1}{V_{(i,1)}}\sum_{j=2}^{n} V_{(i,j)} = R_{(i)}$$

Hence, the uncertainty equation for $R_{(i)}$ becomes:

$$u^2\left(R_{(i)}\right) = \left(n^2 - n\right)\left(\frac{u(V)}{V}\right)^2 + R_{(i)}{}^2 u^2\left(C_L\right)$$

As $V_{(i,1)} \approx V_{(i,2)} \approx \ldots \approx V_{(i,j)}$ and $u(V_{(i,1)}) \approx u(V_{(i,2)}) \approx \ldots \approx u(V_{(i,j)})$

Therefore, the relative measurement uncertainty for $R_{(i)}$ is:

$$\frac{u\left(R_{(i)}\right)}{R_{(i)}} = \sqrt{\left(\sqrt{1-\frac{1}{n}} \times \frac{u(V)}{V}\right)^2 + u^2\left(C_L\right)}$$

According to the above equation, $u(R_{(i)})/R_{(i)}$ is equal to the root sum square (RSS) of $u(V)/V$ and $u(C_L)$, whose sensitivity coefficients are $\sqrt{1-1/n}$ and 1 respectively. In the case of Guildline 9700PL VRBs, the uncertainty equations are summarized in Table 1 and the values for $\sqrt{1-1/n}$ are tabulated in Table 2.

Table 1: $u(R_{(i)})/R_{(i)}$ for a Guildline 9700PL VRB

Group	Voltage Ratios Available	Total Sections	Relative Measurement Uncertainty, $u(R_{(i)})/R_{(i)}$
1st	1V:1V to 5V:1V	5	$\sqrt{\left(0.89 \times u(V)/V\right)^2 + u^2\left(C_L\right)}$
2nd	5V:5V to 10V:5V	2	$\sqrt{\left(0.71 \times u(V)/V\right)^2 + u^2\left(C_L\right)}$
3rd	10V:10V to 50V:10V	5	$\sqrt{\left(0.89 \times u(V)/V\right)^2 + u^2\left(C_L\right)}$
4th	50V:50V to 100V:50V	2	$\sqrt{\left(0.71 \times u(V)/V\right)^2 + u^2\left(C_L\right)}$
5th	100V:100V to 1000V:100V	10	$\sqrt{\left(0.95 \times u(V)/V\right)^2 + u^2\left(C_L\right)}$

Table 2: Values of $\sqrt{1-1/n}$ for a Guildline 9700PL VRB

Group					n	$\sqrt{1-1/n}$
1 V to 5 V	5 V to 10V	10 V to 50 V	50 V to 100 V	100 V to 1000 V		
1 V : 1 V	5 V : 5 V	10 V : 10 V	50 V : 50 V	100 V : 100 V	1	0
2 V : 1 V	10 V : 5 V	20 V : 10 V	100 V : 50 V	200 V : 100 V	2	0.71
3 V : 1 V		30 V : 10 V		300 V : 100 V	3	0.82
4 V : 1 V		40 V : 10 V		400 V : 100 V	4	0.87
5 V : 1 V		50 V : 10V		500 V : 100 V	5	0.89
				600 V : 100 V	6	0.91
				700 V : 100 V	7	0.93
				800 V : 100 V	8	0.94
				900 V : 100 V	9	0.94
				1000 V : 100 V	10	0.95

3.2 Uncertainty for the voltage ratios that involve more than one group

According to Equation (A1) in Appendix A, the voltage ratio, R, for a VRB that involve m groups with respect to the VRB's reference section (i.e. $M_{(i,j)}$ w.r.t. $S_{(1,1)}$) is:

$$R = R_{(1)} \times R_{(2)} \times \ldots \times R_{(m)}$$

$$= \prod_{i=1}^{m} R_{(i)}$$

where $R_{(i)}$ is the voltage ratio of an individual group w.r.t. its base section.

The measurement uncertainty for R is:

$$u^2(R) = \sum_{i=1}^{m} \left[\frac{\partial R}{\partial R_{(i)}} u\left(R_{(i)}\right) \right]^2$$

where

$$\frac{\partial R}{\partial R_{(i)}} = \prod_{x=1}^{m} R_{(x)} \Bigg|_{x \neq i} \qquad (\text{e.g.} \frac{\partial R}{\partial R_{(1)}} = R_{(2)} \times R_{(3)} \times \ldots \times R_{(m)})$$

Therefore, the relative measurement uncertainty for R is:

$$\frac{u(R)}{R} = \sqrt{\sum_{i=1}^{m}\left[\frac{u(R_{(i)})}{R_{(i)}}\right]^2}$$

Hence, the uncertainty for the voltage ratio (i.e. $u(R)/R$) is the RSS of the uncertainties for all the groups involved (i.e. $u(R_{(i)})/R_{(i)}$).

3.3 Analysis of Measurement Uncertainty

3.3.1 Uncertainty in the mean of the voltage ratios, $u(R_{mean})/R_{mean}$

This is estimated from the standard deviation of the mean of the results obtained from repeated measurements. Typical value for this uncertainty contribution is $\pm 0.1\times10^{-6}$, assuming rectangular distribution and infinite degrees of freedom.

3.3.2 Uncertainty due to the measured voltdrop, $u(V)/V$

Voltdrops at the VRB terminals are measured by a 7½-digit precision DMM. Assuming rectangular distribution and infinite degrees of freedom, the estimated uncertainties due to effects such as meter resolution and thermal emf are:

Voltdrop (V)	Uncertainty ($\times10^{-6}$)
1 V	0.087
5 V	0.13
10 V	0.065
50 V	0.13
100 V and above	0.065

3.3.3 Uncertainty due to DMM shunting, $u(C_L)$

With reference to Appendix B, errors due to DMM shunting are within $\pm 0.09\times10^{-6}$ with an uncertainty of $\pm 0.01\times10^{-6}$. This shunting error is not corrected but accounted as an uncertainty contribution. The estimated value for this uncertainty contribution is $\pm 0.1\times10^{-6}$, assuming rectangular distribution and infinite degrees of freedom.

3.4 Uncertainty Budgets

2 V : 1 V to 5 V : 1 V

Source of Uncertainty	Type	$\lvert c_i u(x_i) \rvert$ ($\times 10^{-6}$)	P.d.*	Divisor, y	$\lvert c_i u(x_i) \rvert / y$ ($\times 10^{-6}$)	v_i or v_{eff}
Due to $u(R_{mean})/R_{mean}$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Due to $u(V)/V$	B	0.89×0.087	Normal	1	0.077	∞
Due to $u(C_L)$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Comb. Std. Uncertainty			Normal		0.112	∞

10 V : 1 V

Source of Uncertainty	Type	$\lvert c_i u(x_i) \rvert$ ($\times 10^{-6}$)	P.d.*	Divisor, y	$\lvert c_i u(x_i) \rvert / y$ ($\times 10^{-6}$)	v_i or v_{eff}
u_c of the ratio 5 V : 1 V	B	0.112	Normal	1	0.112	∞
Due to $u(R_{mean})/R_{mean}$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Due to $u(V)/V$	B	0.71×0.13	Normal	1	0.092	∞
Due to $u(C_L)$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Comb. Std. Uncertainty			Normal		0.167	∞

20 V : 1 V to 50 V : 1 V

Source of Uncertainty	Type	$\lvert c_i u(x_i) \rvert$ ($\times 10^{-6}$)	P.d.*	Divisor, y	$\lvert c_i u(x_i) \rvert / y$ ($\times 10^{-6}$)	v_i or v_{eff}
u_c of the ratio 10 V : 1 V	B	0.167	Normal	1	0.167	∞
Due to $u(R_{mean})/R_{mean}$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Due to $u(V)/V$	B	0.89×0.065	Normal	1	0.058	∞
Due to $u(C_L)$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Comb. Std. Uncertainty			Normal		0.195	∞

Note *: P.d. stands for Probability Distribution and Rect. stands for Rectangular Distribution.

100 V : 1 V

Source of Uncertainty	Type	$\lvert c_i u(x_i) \rvert$ ($\times 10^{-6}$)	P.d.*	Divisor, y	$\lvert c_i u(x_i) \rvert /y$ ($\times 10^{-6}$)	ν_i or ν_{eff}
u_c of the ratio 50 V : 1 V	B	0.195	Normal	1	0.195	∞
Due to $u(R_{mean})/R_{mean}$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Due to $u(V)/V$	B	0.71×0.13	Normal	1	0.092	∞
Due to $u(C_L)$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Comb. Std. Uncertainty			Normal		0.230	∞

200 V : 1 V to 1 000 V : 1 V

Source of Uncertainty	Type	$\lvert c_i u(x_i) \rvert$ ($\times 10^{-6}$)	P.d.*	Divisor, y	$\lvert c_i u(x_i) \rvert /y$ ($\times 10^{-6}$)	ν_i or ν_{eff}
u_c of the ratio 100 V : 1 V	B	0.23	Normal	1	0.23	∞
Due to $u(R_{mean})/R_{mean}$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Due to $u(V)/V$	B	0.95×0.065	Normal	1	0.062	∞
Due to $u(C_L)$	B	0.1	Rect.	$\sqrt{3}$	0.058	∞
Comb. Std. Uncertainty			Normal		0.252	∞

4 Conclusion

A simple automatic calibration system for VRBs is implemented. This system can perform full calibration for multi-section VRBs at combined standard uncertainty of 2.5×10^{-7}. For oil immersed VRBs, this system maintains the oil temperature within 0.02 °C of the target temperature so that uncertainty due to temperature effect is minimized.

5 References

[1] NAMAS Information Sheet B4153, "In-House Checking of D.C. Resistive Volt Ratio Boxes", Edition 1, by NAMAS Executive, January 1989.

[2] Guide to the Expression of Uncertainty in Measurement. Corrected and reprinted in 1995. Published by ISO.

[3] "Measurement Terminology and Concepts", IEE Lecture Notes, by Ron Walker.

Appendix A

The Voltage Ratios of a Volt Ratio Box

A.1 Structural Layout of a Volt Ratio Box

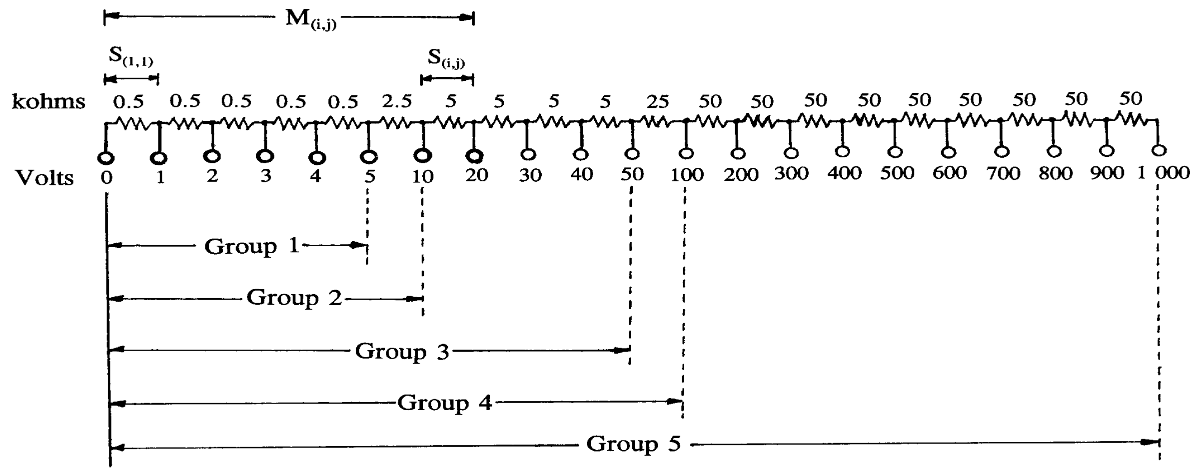


Figure A1: A Typical Volt Ratio Box

The VRB shown in Figure A1 can be understood as a tapped chain of resistors and arranged in 5 groups. Group 1 has 5 sections with $S_{(1,1)}$ as the base section. Within the group, all sections have the same nominal resistance. Likewise, Group 2 has 2 sections with $S_{(2,1)}$ as the base section. It can be seen that, the entire Group 1 is in fact the base section of Group 2. The arrangements for other groups are similar. Section $S_{(1,1)}$ is taken as the Reference Section for the entire VRB.

A.2 The VRB's Voltage Ratios Expressed in terms of Measured Volt-drops

When a fixed voltage is applied across the VRB's 1000 V and 0 V terminals, the voltage ratio for $M_{(i,j)}$ and $S_{(1,1)}$ (i.e. $R_{(i,j)}$) can be seen as the voltdrop ratio for $M_{(i,j)}$ and $S_{(1,1)}$. That is:

$$R_{(i,j)} = \frac{v(M_{(i,j)})}{v_{(1,1)}} = \frac{\sum_{k=1}^{j} v_{(i,k)}}{v_{(i,1)}} \times \frac{\sum_{k=1}^{J_{(i-1)}} v_{(i-1,k)}}{v_{(i-1,1)}} \times \frac{\sum_{k=1}^{J_{(i-2)}} v_{(i-2,k)}}{v_{(i-2,1)}} \times \cdots \times \frac{\sum_{k=1}^{J_{(1)}} v_{(1,k)}}{v_{(1,1)}} \qquad \text{... Equation (A1)}$$

where

i is the group number;

j is the section number (within a particular group);

$J_{(i)}$ is the total number of sections in the i^{th} group;

$S_{(i,j)}$ is a resistor section (i.e. the j^{th} section in i^{th} group);

$M_{(i,j)}$ is the resistance chain from sections $S_{(1,1)}$ to $S_{(i,j)}$;

$v_{(i,j)}$ is the voltage drop across the section $S_{(i,j)}$;

$v(M_{(i,j)})$ is the voltage drop across the resistance chain $M_{(i,j)}$; and

$R_{(i,j)}$ is the voltage ratio between $M_{(i,j)}$ and $S_{(1,1)}$.

Equation (A1) can be rewritten as:

$$R_{(i,j)} = \frac{V_{(i)}(j + \sum_{k=1}^{j} d_{(i,k)})}{V_{(i)}(1 + d_{(i,1)})} \times \frac{V_{(i-1)}(J_{(i-1)} + \sum_{k=1}^{J_{(i-1)}} d_{(i-1,k)})}{V_{(i-1)}(1 + d_{(i-1,1)})} \times \cdots \times \frac{V_{(1)}(J_{(1)} + \sum_{k=1}^{J_{(1)}} d_{(1,k)})}{V_{(1)}(1 + d_{(1,1)})}$$

... Equation (A2)

where $V_{(i)}$ is the nominal volt-drop across the section $S_{(i,j)}$;
$d_{(i,j)}$ is the relative deviation of voltdrop for the section $S_{(i,j)}$
(i.e. $d_{(i,j)} = (V_{(i)} - v_{(i,j)})/\ V_{(i)}$).

Equation (A2) can then be simplified as:

$$R_{(i,j)} = j\left(1 + \frac{\sum_{k=1}^{j}(d_{(i,k)} - d_{(i,1)})}{j}\right) \times J_{(i-1)}\left(1 + \frac{\sum_{k=1}^{J_{(i-1)}}(d_{(i-1,k)} - d_{(i-1,1)})}{J_{(i-1)}}\right) \times ... \times J_{(1)}\left(1 + \frac{\sum_{k=1}^{J_{(1)}}(d_{(1,k)} - d_{(1,1)})}{J_{(1)}}\right)$$

... Equation (A3)

Let D be the voltdrop difference between any two consecutive sections.
(i.e. $D_{(\mathrm{i,k})} = d_{(\mathrm{i,k})} - d_{(\mathrm{i,k-1})}$)

$$R_{(i,j)} = j\left(1 + \frac{\sum_{k=1}^{j}\left(\sum_{x=1}^{k} D_{(i,x)}\right)}{j}\right) \times J_{(i-1)}\left(1 + \frac{\sum_{k=1}^{J_{(i-1)}}\left(\sum_{x=1}^{k} D_{(i-1,x)}\right)}{J_{(i-1)}}\right) \times \cdots \times J_{(1)}\left(1 + \frac{\sum_{k=1}^{J_{(1)}}\left(\sum_{x=1}^{k} D_{(1,x)}\right)}{J_{(1)}}\right)$$

... Equation (A4)

Let n be the deviation factor for a particular section.

(i.e. $n_{(i,j)} = \frac{\sum_{k=1}^{j}\left(\sum_{x=1}^{k} D_{(i,x)}\right)}{j}$; $n_{(i-1,J_{(i-1)})} = \frac{\sum_{k=1}^{J_{(i-1)}}\left(\sum_{x=1}^{k} D_{(i-1,x)}\right)}{J_{(i-1)}}$; ... ; and $n_{(1,J_{(1)})} = \frac{\sum_{k=1}^{J_{(1)}}\left(\sum_{x=1}^{k} D_{(1,x)}\right)}{J_{(1)}}$)

The VRB's voltage ratios can be expressed in terms of n as:

$$\begin{aligned} R_{(i,j)} &= j(1 + n_{(i,j)}) \times J_{(i-1)}(1 + n_{(i-1,J_{(i-1)})}) \times \cdots \times J_{(1)}(1 + n_{(1,J_{(1)})}) \\ &\cong j \times J_{(i-1)} \times J_{(i-2)} \times \cdots \times J_{(1)}\left(1 + n_{(i,j)} + n_{(i-1,J_{(i-1)})} + \cdots + n_{(1,J_{(1)})}\right) \end{aligned}$$

... Equation (A5)

It is shown that the VRB's voltage ratios can be determined by calculation based on the measured voltdrops across individual sections. Expressions of the VRB's voltage ratios and the related variables are tabulated in Table A1 below.

Table A1: Expressions for the Guildline 9700PL VRB's Sections $S_{(i,j)}$, Chains of Sections $M_{(i,j)}$, Groups of Sections, $d_{(i,j)}$, $D_{(i,j)}$, $n_{(i,j)}$ and $R_{(i,j)}$

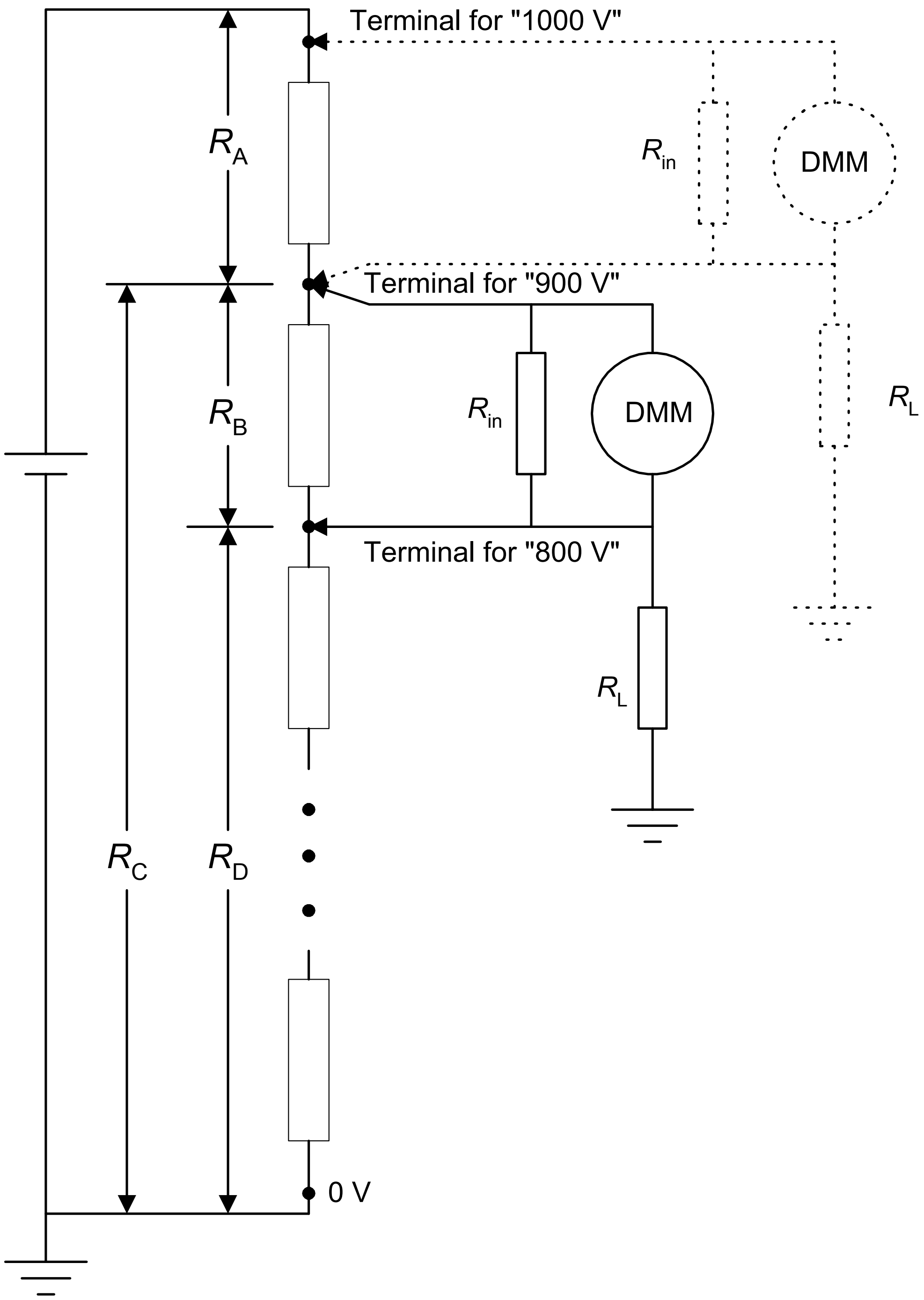
Chain, $M_{(i,j)}$ (from $S_{(1,1)}$ to $S_{(i,j)}$, inclusive)					Section No.	Section ID	Measured Voltdrop, dev. from nominal	Voltdrop Diff., between two consecutive sections	Deviation Factor, $n_{(i,j)}$	Voltage Ratio, $R_{(i,j)}$ ($M_{(i,j)}$: $S_{(1,1)}$)
1st Group (1V)	2nd Group (5 V)	3rd Group (10 V)	4th Group (50 V)	5th Group (100 V)						
$M_{(1,1)}$					01	$S_{(1,1)}$	$d_{(1,1)}$	(Ref. Section)		
$M_{(1,2)}$					02	$S_{(1,2)}$	$d_{(1,2)}$	$D_{(1,2)}=d_{(1,2)}-d_{(1,1)}$	$n_{(1,2)}=D_{(1,2)}/2$	$R_{(1,2)}=2(1+n_{(1,2)})$
$M_{(1,3)}$	$M_{(2,1)}$	$M_{(3,1)}$			03	$S_{(1,3)}$	$d_{(1,3)}$	$D_{(1,3)}=d_{(1,3)}-d_{(1,2)}$	$n_{(1,3)}=(D_{(1,3)}+2D_{(1,2)})/3$	$R_{(1,3)}=3(1+n_{(1,3)})$
$M_{(1,4)}$					04	$S_{(1,4)}$	$d_{(1,4)}$	$D_{(1,4)}=d_{(1,4)}-d_{(1,3)}$	$n_{(1,4)}=(D_{(1,4)}+2D_{(1,3)}+3D_{(1,2)})/4$	$R_{(1,4)}=4(1+n_{(1,4)})$
$M_{(1,5)}$			$M_{(4,1)}$		05	$S_{(1,5)}$	$d_{(1,5)}$	$D_{(1,5)}=d_{(1,5)}-d_{(1,4)}$	$n_{(1,5)}=(D_{(1,5)}+2D_{(1,4)}+3D_{(1,3)}+4D_{(1,2)})/5$	$R_{(1,5)}=R_{(2,1)}=5(1+n_{(1,5)})$
	$M_{(2,2)}$			$M_{(5,1)}$	06	$S_{(2,2)}$	$d_{(2,2)}$	$D_{(2,2)}=d_{(2,2)}-d_{(2,1)}$	$n_{(2,2)}=D_{(2,2)}/2$	$R_{(2,2)}=R_{(3,1)}=R_{(2,1)}\times2(1+n_{(2,2)})$
		$M_{(3,2)}$			07	$S_{(3,2)}$	$d_{(3,2)}$	$D_{(3,2)}=d_{(3,2)}-d_{(3,1)}$	$n_{(3,2)}=D_{(3,2)}/2$	$R_{(3,2)}=R_{(3,1)}\times2(1+n_{(3,2)})$
		$M_{(3,3)}$			08	$S_{(3,3)}$	$d_{(3,3)}$	$D_{(3,3)}=d_{(3,3)}-d_{(3,2)}$	$n_{(3,3)}=(D_{(3,3)}+2D_{(3,2)})/3$	$R_{(3,3)}=R_{(3,1)}\times3(1+n_{(3,3)})$
		$M_{(3,4)}$			09	$S_{(3,4)}$	$d_{(3,4)}$	$D_{(3,4)}=d_{(3,4)}-d_{(3,3)}$	$n_{(3,4)}=(D_{(3,4)}+2D_{(3,3)}+3D_{(3,2)})/4$	$R_{(3,4)}=R_{(3,1)}\times4(1+n_{(3,4)})$
		$M_{(3,5)}$			10	$S_{(3,5)}$	$d_{(3,5)}$	$D_{(3,5)}=d_{(3,5)}-d_{(3,4)}$	$n_{(3,5)}=(D_{(3,5)}+2D_{(3,4)}+3D_{(3,3)}+4D_{(3,2)})/5$	$R_{(3,5)}=R_{(4,1)}=R_{(3,1)}\times5(1+n_{(3,5)})$
			$M_{(4,2)}$		11	$S_{(4,2)}$	$d_{(4,2)}$	$D_{(4,2)}=d_{(4,2)}-d_{(4,1)}$	$n_{(4,2)}=D_{(4,2)}/2$	$R_{(4,2)}=R_{(5,1)}=R_{(4,1)}\times2(1+n_{(4,2)})$
				$M_{(5,2)}$	12	$S_{(5,2)}$	$d_{(5,2)}$	$D_{(5,2)}=d_{(5,2)}-d_{(5,1)}$	$n_{(5,2)}=D_{(5,2)}/2$	$R_{(5,2)}=R_{(5,1)}\times2(1+n_{(5,2)})$
				$M_{(5,3)}$	13	$S_{(5,3)}$	$d_{(5,3)}$	$D_{(5,3)}=d_{(5,3)}-d_{(5,2)}$	$n_{(5,3)}=(D_{(5,3)}+2D_{(5,2)})/3$	$R_{(5,3)}=R_{(5,1)}\times3(1+n_{(5,3)})$
				$M_{(5,4)}$	14	$S_{(5,4)}$	$d_{(5,4)}$	$D_{(5,4)}=d_{(5,4)}-d_{(5,3)}$	$n_{(5,4)}=(D_{(5,4)}+2D_{(5,3)}+3D_{(5,2)})/4$	$R_{(5,4)}=R_{(5,1)}\times4(1+n_{(5,4)})$
				$M_{(5,5)}$	15	$S_{(5,5)}$	$d_{(5,5)}$	$D_{(5,5)}=d_{(5,5)}-d_{(5,4)}$	$n_{(5,5)}=(D_{(5,5)}+2D_{(5,4)}+3D_{(5,3)}+4D_{(5,2)})/5$	$R_{(5,5)}=R_{(5,1)}\times5(1+n_{(5,5)})$
				$M_{(5,6)}$	16	$S_{(5,6)}$	$d_{(5,6)}$	$D_{(5,6)}=d_{(5,6)}-d_{(5,5)}$	$n_{(5,6)}=(D_{(5,6)}+2D_{(5,5)}+3D_{(5,4)}+4D_{(5,3)}+5D_{(5,2)})/6$	$R_{(5,6)}=R_{(5,1)}\times6(1+n_{(5,6)})$
				$M_{(5,7)}$	17	$S_{(5,7)}$	$d_{(5,7)}$	$D_{(5,7)}=d_{(5,7)}-d_{(5,6)}$	$n_{(5,7)}=(D_{(5,7)}+2D_{(5,6)}+3D_{(5,5)}+4D_{(5,4)}+5D_{(5,3)}+6D_{(5,2)})/7$	$R_{(5,7)}=R_{(5,1)}\times7(1+n_{(5,7)})$
				$M_{(5,8)}$	18	$S_{(5,8)}$	$d_{(5,8)}$	$D_{(5,8)}=d_{(5,8)}-d_{(5,7)}$	$n_{(5,8)}=(D_{(5,8)}+2D_{(5,7)}+3D_{(5,6)}+4D_{(5,5)}+5D_{(5,4)}+6D_{(5,3)}+7D_{(5,2)})/8$	$R_{(5,8)}=R_{(5,1)}\times8(1+n_{(5,8)})$
				$M_{(5,9)}$	19	$S_{(5,9)}$	$d_{(5,9)}$	$D_{(5,9)}=d_{(5,9)}-d_{(5,8)}$	$n_{(5,9)}=(D_{(5,9)}+2D_{(5,8)}+3D_{(5,7)}+4D_{(5,6)}+5D_{(5,5)}+6D_{(5,4)}+7D_{(5,3)}+8D_{(5,2)})/9$	$R_{(5,9)}=R_{(5,1)}\times9(1+n_{(5,9)})$
				$M_{(5,10)}$	20	$S_{(5,10)}$	$d_{(5,10)}$	$D_{(5,10)}=d_{(5,10)}-d_{(5,9)}$	$n_{(5,10)}=(D_{(5,10)}+2D_{(5,9)}+3D_{(5,8)}+4D_{(5,7)}+5D_{(5,6)}+6D_{(5,5)}+7D_{(5,4)}+8D_{(5,3)}+9D_{(5,2)})/10$	$R_{(5,10)}=R_{(5,1)}\times10(1+n_{(5,10)})$

Appendix B

Shunting Effect on the Measured Voltage Ratios due to the System DMM

For calibration of a Volt Ratio Box (Guildline 9700PL) similar to the one shown in Figure A1 in Appendix A, analysis of the shunting effect due to the system DMM, that would have on the measured ratios, are shown in the following example:

Example: Shunting effect on the volt-drop ratio for the VRB's 1000V and 900 V sections



Voltage applied to the VRB's "1000 V" and "0 V" terminals: 1000 V DC

VRB under test:
R_{Total} = 50 kΩ (nominal)

Resistor	Nominal Value	Actual Value (assuming 20 ppm mismatch for R_A & R_B)
R_A	50 kΩ	50 kΩ
R_B	50 kΩ	50.001 kΩ
R_C	450 kΩ	450.001 kΩ
R_D	400 kΩ	400 kΩ

System DMM (Wavetek 1281):
Measurement Range: 100 V
Input resistance, $R_{in} \geq 1 \times 10^7\ \Omega$
Leakage resistance, $R_L \geq 1 \times 10^{13}\ \Omega$

Case A: Shunting effect is neglected:

$R_A(\text{actual})/R_B(\text{actual}) = 50\ \text{k}\Omega / 50.001\ \text{k}\Omega$
$= 0.99998$

Case B: Shunting effect is considered:

(1) The voltdrop across R_A, as measured by the system DMM:

$$1000\ \text{V} \times \frac{\dfrac{R_A R_{in}}{R_A + R_{in}}}{\dfrac{R_A R_{in}}{R_A + R_{in}} + \dfrac{R_C R_L}{R_C + R_L}}$$

$$= 1000\ \text{V} \times \frac{49.751\,243\,78\ \text{k}\Omega}{49.751\,243\,78\ \text{k}\Omega + 450.000\,979\,7\ \text{k}\Omega}$$

$$= 99.551\,820\,77\ \text{V}$$

(2) The voltdrop across R_B, as measured by the system DMM:

$$1000\ \text{V} \times \frac{\dfrac{R_B R_{in}}{R_B + R_{in}}}{R_A + \dfrac{R_B R_{in}}{R_B + R_{in}} + \dfrac{R_D R_L}{R_D + R_L}}$$

$$= 1000\ \text{V} \times \frac{49.752\,233\,86\ \text{k}\Omega}{50\ \text{k}\Omega + 49.752\,233\,86\ \text{k}\Omega + 399.999\,984\ \text{k}\Omega}$$

$$= 99.553\,803\,03\ \text{V}$$

(3) The Voltage Ratio for R_A(actual)/R_B(actual):

DMM indicated voltdrop across R_A ÷ DMM indicated voltdrop across R_B

= 99.551 820 77 V ÷ 99.553 803 03 V

= 0.999980089

Error in the measured voltage ratio due to shunting effect (i.e. comparing the voltage drop of Case A to that of Case B.)**:**

$|(0.99998 - 0.999980089) \div 0.99998| \times 10^{-6} = 0.089 \times 10^{-6}$

The above method is used to estimate the effect of DMM shunting on all the measured voltage ratios. The errors due to this shunting effect are found to be within 0.09×10^{-6} with a standard uncertainty of 0.01×10^{-6}.