Adapting the ISO GUM for a Practical Approach to Uncertainty Calculation of Complex Numbers

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Abstract

The ISO Guide to the Expression of Uncertainty in Measurement (GUM) was published in 1993. This document describes how to express measurement uncertainty, and offers some examples for actual uncertainty calculations. However, there is no example of uncertainty propagation for complex numbers such as electrical impedance.

At CPEM 1992*, our Agilent Technologies Japan Measurement Standards Center presented the theory of impedance calibration, and the paper was later published by IEEE I&M in 1993. Since 1992, our standards center has been calibrating four-terminal-pair (4TP) AC resistance standards at high frequencies up to 13 MHz, based on the 4TP capacitance standard. The uncertainty was calculated by the computer simulation method.

The authors have now re-calculated the uncertainty in 2001, using sensitivity coefficients in accordance with the ISO GUM. This calculation uses a novel new approach to sensitivity coefficients of complex numbers, so as to apply the ISO GUM, instead of the former simulation method. The paper concludes with a presentation of an example of uncertainty calculation for complex numbers, such as electrical impedance.

*CPEM : Conference on Precision Electromagnetic Measurements

Introduction

Agilent Technologies Japan Measurement Standards Center has been calibrating *R* (resistance) and *X* (reactance) or *B* (susceptance) of four-terminal-pair (4TP) AC resistance standards at high frequencies up to 13 MHz, in accordance with the method given in [1] since 1992. The uncertainty of this calibration was calculated by computer simulation because the mathematical model is complicated and the ISO GUM had not yet been published. We then determined to extend the uncertainty calculation in accordance with the GUM. However, we had to consider how to deal with complex numbers so as to apply it consistent with the intent of the GUM.

The objective of this paper is to suggest a rigorous method to deal with complex numbers when calculating the uncertainty in accordance with the ISO GUM for 4TP AC resistance calibration.

The following two key techniques lead to an easy calculation method for the uncertainty.

1. Adoption of the cross ratio for the mathematical model
   This helps to derive partial derivatives for calculating sensitivity coefficients
2. Use of complex numbers for a sensitivity coefficient
   This enables us to realize uncertainty for the major part of the capacitance effect for the major part of resistance.

These techniques help us to calculate the magnitude of the calibration uncertainty.

Using this method, we confirmed that uncertainty of complex numbers such as electrical impedance (R, X) can be realistically calculated in accordance with the ISO GUM instead of computer simulation method.

Summarized calibration method for 4TP AC resistance

The paper given in [1] describes 4TP AC resistance calibration theory for frequencies up to about 10 MHz. This section describes the calibration method briefly. This calibration uses a commercial 4TP LCR meter and the LCR meter is considered as an ideal LCR meter with error adapter. See Figure 1. The error adapter is a mathematical tool for expressing the reciprocal and linear errors of the 4TP calibration system. The theory applies error-correction equations to this calibration as the well-known OPEN/SHORT/LOAD technique for a one-port impedance meter at high frequencies.

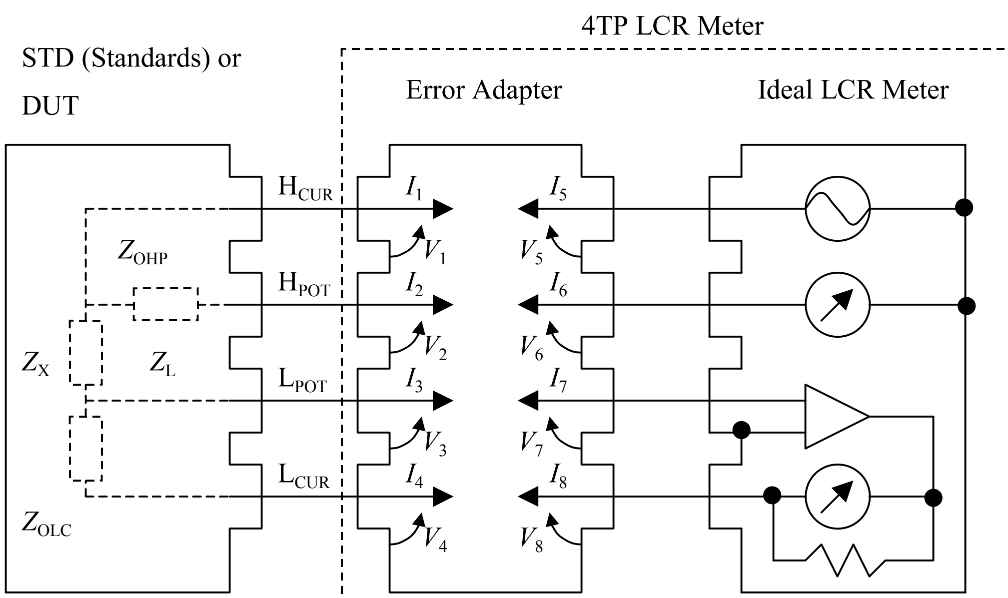

Figure 1. Concept of 4TP AC resistance calibration theory

The basic equation is expressed as follows:

$$Z_{DUT} = \frac{Z_{MDUT} - Z_{Msht}}{Z_{Mopn} - Z_{MDUT}} \frac{Z_{Mopn} - Z_{Mstd}}{Z_{MSTD} - Z_{Msht}} Z_{STD} \tag{1}$$

where, Z_{DUT} : Calculated impedance value of DUT 4TP resistance
Z_{MDUT} : Measured impedance value of the DUT 4TP resistance
Z_{Msht} : Measured impedance value of 4TP SHORT
Z_{Mopn} : Measured impedance value of 4TP OPEN
Z_{MSTD} : Measured impedance value of STD 4TP capacitance
Z_{STD} : Calibrated impedance value of the STD 4TP capacitance
It is assumed that Z_{sht}=0 and $Z_{opn}= \infty$.

This method uses an ideal 4TP open termination ($Z_{opn}= \infty$), an ideal 4TP short termination (Z_{sht}=0) and a 4TP air dielectric capacitor as the LOAD standard (Z_{STD}). It then transforms from capacitance value to resistance value using right angle of the LCR meter. See Figure 2. The capacitor is calibrated by using the method given in [2].

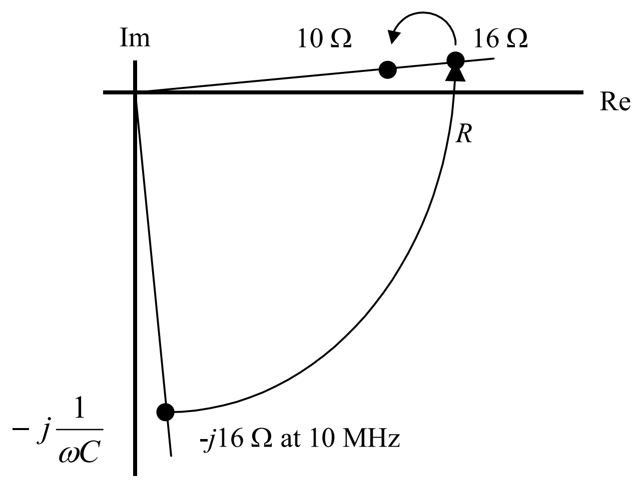

Figure 2. Transform from Capacitance to Resistance

Our standards center has been calibrating resistors ranging from 10 Ω to 100 kΩ impedance. In this paper, we will demonstrate a method to calculate uncertainty of 10 Ω at 1 MHz and 10 MHz. We selected a 1000 pF air capacitor as the standard for 10 Ω because the impedance values of the 1000 pF capacitor at 1 MHz and 10 MHz are 160 Ω and 16 Ω, respectively.

Uncertainty Calculation

Figure 2 shows that the uncertainty of the capacitance (Imaginary part) affects the uncertainty of the resistance (Real part) geometrically. The following process demonstrates it mathematically, using complex numbers for sensitivity coefficients.

#1. Mathematical model

Equation (1) is a mathematical model, based on the assumption that Z_{sht}=0 and Z_{opn}= ∞. If each device has a specific value, then equation (1) becomes

$$Z_{DUT} = \frac{Z_{sht}Z_{opn}(Z_{MDUT}-Z_{MSTD})(Z_{Msht}-Z_{Mopn})+Z_{opn}Z_{STD}(Z_{MDUT}-Z_{Msht})(Z_{Mopn}-Z_{MSTD})-Z_{STD}Z_{sht}(Z_{MDUT}-Z_{Mopn})(Z_{Msht}-Z_{MSTD})}{-Z_{sht}(Z_{MDUT}-Z_{Msht})(Z_{Mopn}-Z_{MSTD})+Z_{opn}(Z_{MDUT}-Z_{Mopn})(Z_{Msht}-Z_{MSTD})-Z_{STD}(Z_{MDUT}-Z_{MSTD})(Z_{Msht}-Z_{Mopn})} \tag{1a}$$

Using the cross ratio K, the mathematical model (1a) is transformed to equation (2).

$$Z_{DUT} = \frac{Z_{sht}(Z_{opn}-Z_{STD})-KZ_{opn}(Z_{sht}-Z_{STD})}{(Z_{opn}-Z_{STD})-K(Z_{sht}-Z_{STD})} \tag{2}$$

$$\text{where, } K = \frac{Z_{MDUT}-Z_{Msht}}{Z_{MDUT}-Z_{Mopn}}\frac{Z_{Mstd}-Z_{Mopn}}{Z_{Mstd}-Z_{Msht}}$$

For a sensitivity coefficient of OPEN unit, equation (2) is transformed to (2a) using Y_{opn} instead of Z_{opn} because Z_{opn} = ∞.

$$Z_{DUT} = \frac{Z_{sht}(1-Z_{STD}Y_{opn})-K(Z_{sht}-Z_{STD})}{(1-Z_{STD}Y_{opn})-KY_{opn}(Z_{sht}-Z_{STD})} \tag{2a}$$

From equation (2) and (2a), the sensitivity coefficients are calculated by partial derivatives.

#2. Sensitivity coefficient

The uncertainty is expressed using Y_{opn} instead of Z_{opn} because Z_{opn}= ∞.

$$u^2(Z_{DUT}) = c_{STD}^2 u^2(Z_{STD}) + c_{sht}^2 u^2(Z_{sht}) + c_{opn}^2 u^2(Y_{opn}) \tag{3}$$

From partial derivatives of equation (2) and (2a), each sensitivity coefficient is expressed by

$$\frac{\partial Z_{DUT}}{\partial Z_{STD}} = \frac{(Z_{DUT}-Z_{opn})(Z_{DUT}-Z_{sht})}{(Z_{STD}-Z_{opn})(Z_{STD}-Z_{sht})} = c_{STD} \tag{4}$$

$$\frac{\partial Z_{DUT}}{\partial Z_{sht}} = \frac{(Z_{DUT}-Z_{STD})(Z_{DUT}-Z_{opn})}{(Z_{sht}-Z_{STD})(Z_{sht}-Z_{opn})} = c_{sht} \tag{5}$$

$$\frac{\partial Z_{DUT}}{\partial Y_{opn}} = \frac{(Z_{DUT}-Z_{sht})(Z_{DUT}-Z_{STD})}{(-Y_{opn}Z_{sht}+1)(Y_{opn}Z_{STD}-1)} = c_{opn}\,. \tag{6}$$

Substituting Z_{sht}=0, Z_{opn}=∞ and Y_{opn}=0 in equation (4), (5) and (6), then

$$c_{STD} = \frac{(Z_{DUT}-Z_{opn})(Z_{DUT}-Z_{sht})}{(Z_{STD}-Z_{opn})(Z_{STD}-Z_{sht})} = \frac{Z_{DUT}}{Z_{STD}} \tag{7}$$

$$c_{sht} = \frac{(Z_{DUT}-Z_{STD})(Z_{DUT}-Z_{opn})}{(Z_{sht}-Z_{STD})(Z_{sht}-Z_{opn})} = 1-\frac{Z_{DUT}}{Z_{STD}} \tag{8}$$

$$c_{\mathrm{opn}} = \frac{(Z_{\mathrm{DUT}} - Z_{\mathrm{sht}})(Z_{\mathrm{DUT}} - Z_{\mathrm{STD}})}{(-Y_{\mathrm{opn}} Z_{\mathrm{sht}} + 1)(Y_{\mathrm{opn}} Z_{\mathrm{STD}} - 1)} = Z_{\mathrm{DUT}}(Z_{\mathrm{STD}} - Z_{\mathrm{DUT}}). \qquad (9)$$

The sensitivity coefficients are calculated using typical values of Z_{STD} and Z_{DUT}. The capacitance calibration certificate gives C (capacitance) and D (dissipation factor) values, after which their values are transformed to the impedance expression ($R+jX$) using the equation (10). The equation (1) gives R and X values of the DUT. Table 1 shows the typical values and the transformed values.

$$R = \frac{D}{2\pi f C}, \quad X = -\frac{1}{2\pi f C} \qquad (10)$$

Table 1. Typical values of Z_{STD} and Z_{DUT}

	Z_{STD}				Z_{DUT}	
	C	D	R	X	R	X
1 MHz	1000.16 pF	0.00003	0.004774 Ω	-159.129 Ω	10.010 Ω	0.162 Ω
10 MHz	1026.6 pF	0.00091	0.014108 Ω	-15.5031 Ω	10.029 Ω	1.574 Ω

Each sensitivity coefficient is calculated using equations (7), (8), (9) and the values in table 1. An example of calculating c_{STD} at 1 MHz as follows:

$$c_{\mathrm{STD}} = \frac{Z_{\mathrm{DUT}}}{Z_{\mathrm{STD}}} = \frac{10.010 + j0.162}{0.004774 - j159.129} = -0.001 + j0.0629 \qquad (11)$$

The calculation results of all sensitivity coefficients are shown in Table 2.

Table 2. Sensitivity coefficients

Sensitivity Coefficient	1 MHz		10 MHz	
	Re	Im	Re	Im
c_{STD}	-0.001	$j0.0629$	-0.101	$j0.647$
c_{sht}	1.001	$-j0.0629$	1.101	$-j0.647$
c_{opn}	-74.35 Ω^2	$-j1596\ \Omega^2$	-73.56 Ω^2	$-j187\ \Omega^2$

Table 2 shows the sensitivity coefficients of Im (imaginary part) are more influential than them of Re (real part) regarding the capacitance standard and the open. In case of the short (c_{sht}), it is to the contrary.

#3 Example of each standard uncertainty, $u(Z_{\mathrm{STD}})$

The $u(Z_{\mathrm{sht}})$ and $u(Y_{\mathrm{opn}})$ evaluation process is not described here because they are not important for the objective of this paper. Only the calibration uncertainty $u(Z_{\mathrm{STD\_cal}})$ of $u(Z_{\mathrm{STD}})$ is shown here.
The calibration certificate gives us the expanded uncertainty for capacitance C and dissipation factor D, and states that it was obtained using a coverage factor of k=2. The standard uncertainty is U/k. For example, the 1000 pF standard uncertainties (k=1) are given in Table 3.

Table 3. 1000 pF calibration uncertainty, $u(Z_{\mathrm{STD\_cal}})$

$u(Z_{STD\_cal})$	1 MHz	10 MHz
C	32 ppm	0.095 %
D	0.000015	0.00029

The C and D expressions of $u(Z_{STD\_cal})$ are transformed to an impedance expression ($R+jX$). Since the value of D is sufficiently small compared to the C value, the standard uncertainties of C and D are assumed to be independent. That means the change of C (ΔC) affects the X value and the change of D (ΔD) affects the R value. This validity is shown by the following calculation example at 1 MHz.

From the equation (10),

$$\frac{\partial X}{\partial C} = \frac{1}{2\pi f C^2}, \quad \frac{\partial R}{\partial C} = \frac{-D}{2\pi f C^2}, \quad \frac{\partial R}{\partial D} = \frac{1}{2\pi f C} \qquad (12)$$

Then, each contribution is shown by

$$\Delta X = \frac{\partial X}{\partial C}\Delta C = \frac{1}{2\pi f C^2}\Delta C, \; \Delta R = \frac{\partial R}{\partial C}\Delta C + \frac{\partial R}{\partial D}\Delta D \qquad (13)$$

Table 1 gives a C value of 1000.16 pF, and Table 3 gives the ΔC value of 32 ppm. Then, ΔX is

$$\Delta X = \frac{1000.16\times10^{-12}\times32\times10^{-6}}{2\pi\times1\times10^{6}\times\left(1000.16\times10^{-12}\right)^2} = 0.005092 \;\; \Omega. \qquad (14)$$

$$\frac{\partial R}{\partial C}\Delta C = \frac{\Delta C}{2\pi f C^2}\left(-D\right) = 0.005092\times0.00003 = 1.53\times10^{-7} \qquad (15)$$

This term can be neglected. Then, ΔR is

$$\Delta R = \frac{\partial R}{\partial C}\Delta C + \frac{\partial R}{\partial D}\Delta D \cong \frac{\partial R}{\partial D}\Delta D = \frac{1}{2\pi f C}\Delta D. \qquad (16)$$

Table 3 gives a ΔD value of 0.000015. Therefore,

$$\Delta R \cong \frac{0.000015}{2\pi\times1\times10^{6}\times1000.16\times10^{-12}} = 0.002387 \;\; \Omega \qquad (17)$$

The transformed uncertainties (k=1) are shown in Table 4.

Table 4. 1000 pF calibration uncertainty, $u(Z_{STD\_cal})$

	1 MHz	10 MHz
R	0.002387 Ω	0.004496 Ω
X	0.005092 Ω	0.014333 Ω

The $u(Z_{STD\_cal})$ and the other uncertainties of $u(Z_{STD})$ are combined, and the $u(Z_{STD})$ at 1 MHz and

10 MHz are $(0.0247+j0.0158)$ Ω and $(0.0116+j0.0222)$ Ω, respectively. See uncertainty budget sheet in the Appendix for detail.

#4 Multiplication of the sensitivity coefficient and the standard uncertainty

The combined standard uncertainty is the positive square root of the combined variance. Combined variance is calculated by summing a square of each standard uncertainty multiplied by a sensitivity coefficient. In this case, if we calculate the multiplication arithmetically, a minus term appears because the sensitivity coefficient and the standard uncertainty are both complex numbers ($j^2=-1$). For example, if a standard uncertainty is denoted by $(a+jb)$ and a sensitivity coefficient is denoted by $(c+jd)$, the multiplication becomes $(\boldsymbol{ac-bd})+j(ad+bc)$. Then, the following calculation process is adopted as an additional rule. The absolute value of the each term is used. That means real part becomes $\boldsymbol{ac+|-bd|=ac+bd}$ and imaginary part becomes $ad+bc$.

For example, uncertainty of the capacitance standard at 1 MHz is $u(Z_{STD})=a+jb=(0.0247+j0.0158)$ Ω and the sensitivity coefficient is $c_{STD}=c+jd=(0.001+j0.0629)$. See table 2.

$$c_{STD}\times u(Z_{STD}) = (ac+bd)+j(ad+bc)$$
$$= ((0.0247\times0.001+0.0158\times0.0629) + j(0.0247\times0.0629+0.0158\times0.001))\ \Omega$$
$$=(0.00102+j0.00157)\ \Omega$$

From this calculation, we realized a contribution of the real and imaginary part of $u(Z_{STD})$ to the resistance uncertainty. The results of the calculation are shown in Table 5. Refer to the budget sheet for detail.

Table 5. Standard uncertainty multiplied by sensitivity coefficient

	1 MHz		10 MHz	
	R	X	R	X
$u_1(Z_{STD})$	1.02×10^{-3} Ω	1.57×10^{-3} Ω	1.55×10^{-2} Ω	9.73×10^{-3} Ω

#5 Combined standard uncertainty, Expanded uncertainty and Reporting uncertainty

The combined standard uncertainty is the positive square root of the combined variance. Table 6 shows all standard uncertainties multiplied by sensitivity coefficients for each.

Table 6. Standard uncertainty components

	1 MHz		10 MHz	
	R	X	R	X
$u_1(Z_{STD})$	1.02×10^{-3} Ω	1.57×10^{-3} Ω	1.55×10^{-2} Ω	9.73×10^{-3} Ω
$u_2(Z_{sht})$	2.99×10^{-5} Ω	2.82×10^{-5} Ω	5.30×10^{-5} Ω	5.07×10^{-5} Ω
$u_3(Y_{opn})$	4.02×10^{-5} Ω	5.19×10^{-5} Ω	7.55×10^{-6} Ω	7.46×10^{-6} Ω
$u_4(Z_{reproduc})$	8.88×10^{-4} Ω	7.40×10^{-4} Ω	1.11×10^{-3} Ω	1.26×10^{-3} Ω

A combined example of the real part at 1 MHz is shown by

$$\sqrt{\left(1.02\times10^{-3}\right)^2+\left(2.99\times10^{-5}\right)^2+\left(4.02\times10^{-5}\right)^2+\left(8.88\times10^{-4}\right)^2}=1.35\times10^{-3}\ \Omega.$$

The combined standard uncertainty, expanded uncertainty and reporting uncertainty are shown in

Table 7. Refer to the uncertainty budget sheet in detail.

Table 7. Combined standard uncertainty, Expanded uncertainty and Reporting uncertainty

		1 MHz	10 MHz
Combined Standard uncertainty	R	1.35×10^{-3} Ω	1.56×10^{-2} Ω
	X	1.74×10^{-3} Ω	9.81×10^{-3} Ω
Expanded uncertainty (k=2)	R	0.00271 Ω 0.0271 %	0.0311 Ω 0.311 %
	X	0.0035 Ω	0.020 Ω
Reporting uncertainty	R	0.03 %	0.4 %
	X	0.004 Ω	0.020 Ω

Conclusion

The uncertainty of impedance was calculated in accordance with the ISO GUM and the authors confirmed that the capacitance uncertainty (Imaginary part) affects resistance uncertainty (Real part) using sensitivity coefficients containing complex numbers. However, we adopted absolute values for each term in the multiplication of sensitivity coefficient and the standard uncertainty because both of them are complex numbers. This approach should be discussed in the impedance field.

We also applied the ISO GUM to uncertainty calculation of an electrical impedance (R+jX) calibration using a novel new approach. The approach uses:

1. the cross ratio for the mathematical model
2. complex numbers for a sensitivity coefficient.

Acknowledgement

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References

1. K.Suzuki, T.Aoki, and K.Yokoi, "A Calibration Method for Four-Terminal-Pair High-Frequency Resistance Standards," IEEE Trans. Instrum. Meas., vol 42, pp 379-384, Apr. 1993.

2. K.Suzuki, "A New Universal Calibration Method for 4TP Admittance Standards," IEEE Trans. Instrum. Meas., vol 40, pp 420-422, Apr. 1991.

3. ISO, "Guide to the Expression of Uncertainty in Measurement," 1995

Appendix

Agilent 42035A 10 ohm Resistance at 1 MHz

Agilent Technologies

Technical Document Chapter Number	Standard Uncertainty Component $u(x_i)$	Source of Uncertainty		Value of Standard Uncertainty $u(x_i)$				$\lvert c_i \rvert \equiv \lvert \partial f / \partial x_i \rvert$		$u_i(Z_{10}) \equiv \lvert c_i \rvert u(x_i)$				Degrees of Freedom			
7-3-3-1				Re		Im		Re	Im	Re		Im		Re	Im	Re	Im
7-3-3-1-1	$u(Z_{STD})$	Capacitance Standard		2.47E-02	ohm	1.58E-02	ohm	0.001	0.0629	1.02E-03	ohm	1.57E-03	ohm	390.9	113.2		
1)	$u(Z_{STD\_cal})$	Uncertainty of MSC	☆	2.39E-03	ohm	5.09E-03	ohm									2560	2560
2)	$u(Z_{STD\_stab})$	0.5 Year Stability	◇	4.71E-03	ohm	1.18E-02	ohm									8	8
3)	$u(Z_{STD\_rpt})$	Repeatability	☆	7.70E-04	ohm	5.95E-04	ohm									8	8
4)	$u(Z_{STD\_linear})$	4285A linearity	□	9.19E-03	ohm	9.19E-03	ohm									8	8
5)	$u(Z_{STD\_phase})$	4285A phase	□	2.23E-02	ohm	2.87E-06	ohm									8	8
6)	$u(Z_{STD\_strct})$	Hp, Lc L and R	□	4.08E-04	ohm	1.03E-03	ohm									8	8
7-3-3-1-2	$u(Z_{sht})$	Short Standard		2.83E-05	ohm	2.64E-05	ohm	1.001	0.0629	2.99E-05	ohm	2.82E-05	ohm	8	8		
1)	$u(Z_{sht\_diff})$	Difference from ideal		negl		negl										8	8
2)	$u(Z_{sht\_rpt})$	Repeatability	☆	2.83E-05	ohm	2.64E-05	ohm									8	8
7-3-3-1-3	$u(Y_{opn})$	Open Standard		3.14E-08	S	2.37E-08	S	74.35 ohm^2	1596 ohm^2	4.02E-05	ohm	5.19E-05	ohm	8	8		
1)	$u(Y_{opn\_diff})$	Difference from ideal		negl		negl										8	8
2)	$u(Y_{opn\_rpt})$	Repeatability	☆	3.14E-08	S	2.37E-08	S									8	8
7-3-3-1-4	$u(Z_{reproduc})$	Reproducibility	◇	8.88E-04	ohm	7.40E-04	ohm	1	1	8.88E-04	ohm	7.40E-04	ohm	9	9		
									$u_c^2(Z_{10}) = \Sigma u_i^2(Z_{10}) =$	1.83E-06	ohm^2	3.02E-06	ohm^2				
									$u_c(Z_{10}) =$	1.35E-03	ohm	1.74E-03	ohm				
									$\nu_{eff}(Z_{10}) =$	46.66687		104.5614					
									Coverage Factor k =	2		2					
									$k \times u_c(Z_{10}) =$	2.71E-03	ohm	3.47E-03	ohm				
	☆: Normal Distribution ◇: 50:50 normal Distribution △: Triangular Distribution □: Rectangular Distribution																
				Absolute Expanded Uncertainty						0.0028	ohm	0.0035	ohm				
				Relative Expanded Uncertainty						0.028	%						

Agilent 42035A 10 ohm Resistance at 10 MHz

Agilent Technologies

Technical Document Chapter Number	Standard Uncertainty Component $u(x_i)$	Source of Uncertainty		Value of Standard Uncertainty $u(x_i)$				$\lvert c_i \rvert \equiv \lvert \partial f / \partial x_i \rvert$		$u_i(Z_{10}) \equiv \lvert c_i \rvert u(x_i)$				Degrees of Freedom			
7-3-3-1				Re		Im		Re	Im	Re		Im		Re	Im	Re	Im
7-3-3-1-1	$u(Z_{STD})$	Capacitance Standard		1.16E-02	ohm	2.22E-02	ohm	0.101	0.647	1.55E-02	ohm	9.73E-03	ohm	16.67	3.236		
1)	$u(Z_{STD\_cal})$	Uncertainty of MSC	☆	4.50E-03	ohm	1.43E-02	ohm									2560	2560
2)	$u(Z_{STD\_stab})$	0.5 Year Stability	◇	9.64E-03	ohm	1.36E-02	ohm									8	8
3)	$u(Z_{STD\_rpt})$	Repeatability	☆	4.90E-05	ohm	3.71E-05	ohm									8	8
4)	$u(Z_{STD\_linear})$	4285A linearity	□	8.95E-04	ohm	8.95E-04	ohm									8	8
5)	$u(Z_{STD\_phase})$	4285A phase	□	4.48E-03	ohm	1.12E-06	ohm									8	8
6)	$u(Z_{STD\_strct})$	Hp, Lc L and R	□	3.98E-04	ohm	9.99E-03	ohm									8	8
7-3-3-1-2	$u(Z_{sht})$	Short Standard		3.22E-05	ohm	2.71E-05	ohm	1.101	0.647	5.30E-05	ohm	5.07E-05	ohm	8	8		
1)	$u(Z_{sht\_diff})$	Difference from ideal		negl		negl										8	8
2)	$u(Z_{sht\_rpt})$	Repeatability	☆	3.22E-05	ohm	2.71E-05	ohm									8	8
7-3-3-1-3	$u(Y_{opn})$	Open Standard		2.84E-08	S	2.92E-08	S	73.56 ohm^2	187 ohm^2	7.55E-06	ohm	7.46E-06	ohm	8	8		
1)	$u(Y_{opn\_diff})$	Difference from ideal		negl		negl										8	8
2)	$u(Y_{opn\_rpt})$	Repeatability	☆	2.84E-08	S	2.92E-08	S									8	8
7-3-3-1-4	$u(Z_{reproduc})$	Reproducibility	◇	1.11E-03	ohm	1.26E-03	ohm	1	1	1.11E-03	ohm	1.26E-03	ohm	9	9		
									$u_c^2(Z_{10}) = \Sigma u_i^2(Z_{10}) =$	2.42E-04	ohm^2	9.63E-05	ohm^2				
									$u_c(Z_{10}) =$	1.56E-02	ohm	9.81E-03	ohm				
									$\nu_{eff}(Z_{10}) =$	16.84445		3.345058					
									Coverage Factor k =	2		2					
									$k \times u_c(Z_{10}) =$	3.11E-02	ohm	1.96E-02	ohm				
	☆: Normal Distribution ◇: 50:50 normal Distribution △: Triangular Distribution □: Rectangular Distribution																
				Absolute Expanded Uncertainty						0.032	ohm	0.020	ohm				
				Relative Expanded Uncertainty						0.32	%						