Improvements In The Determination Of Effective Area Through A Piston-Cylinder Pressure Calibration Chain

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Abstract
Since the mid-1980s, DH Instruments has maintained a pressure calibration chain made up of multiple, interconnected piston-cylinders both to assure the coherence of pressure measurements over the range of less than10 kPa to 500 MPa and as a vehicle to maintain traceability to national metrology institutes (NMIs).

In 2001, a three year calibration chain maintenance cycle was completed. The results reflect changes made to the calibration chain that reduce the uncertainty in effective area over the covered range and thus the uncertainty in DHI's accredited scope of pressure.

This paper summarizes the principles of the calibration chain, examines the improvements made over the past few years, and exposes the benefits realized. These benefits include lower uncertainty in effective area and pressure and better agreement in inter-laboratory comparisons with various standards laboratories around the world.

1. The Challenges Of Traceability in Pressure

In establishing traceability in fluid pressure at elevated ranges, there is a significant challenge compared to many other measurands in the International System of Units (SI) [1]. Pressure (Pa) is intensive[1] and has no sources of defined reference points by intrinsic standards.

When examining traceability in quantities (or measurands) such as length (m), mass (kg), current (A), amount of substance (mol) and mass flow (kg/s), these measurands have the advantage that larger amounts can be obtained by the addition of known smaller amounts. For example, to calibrate a 2 kg/s mass flow device, an option is to use two 1 kg/s standards in parallel. If a 10 kg mass needs to be determined, two 5 kg masses may be used. To measure a 1 meter device, two 0.5 meter standards in series can be used. However it is not possible to use two 200 kPa standards to calibrate one 400 kPa test instrument.

From this aspect temperature (K) can be put in the same category as pressure. Yet temperature has the support of the International Temperature Scale of 1990 (ITS90) with intrinsic reference points well dispersed over a very wide range. Research has taken place to this end, and is currently being pursued at NIST, but at this point pressure has no intrinsic reference points to support traceability. It is completely dependent upon derived standards, specifically the manometer and the piston gauge. Though the manometer has performed well in support of lower pressures, a practical limitation of the height of the liquid column keeps it from being a useful instrument for traceability in ranges higher than about 350 kPa. For a pressure laboratory with daily applications reaching 700 MPa (100 000 psi) the manometer can only support the first 0.05 % of the range.

[1] Defined as non-extensive, i.e. unable to be extended through additive techniques.

The answer to the traceability problem described above for many years has been the piston gauge. The piston gauge defines pressures when a piston is floating by the simplified equation:

$$pressure = \frac{m \cdot g}{A_{(t,p)}}$$

where:

- m is true mass loaded on the piston-cylinder assembly
- g is the acceleration of gravity at the location of the piston gauge
- $A_{(t, p)}$ is the effective area of the piston-cylinder assembly at temperature (t) and pressure (p)

Since the amount of mass that can be loaded on a piston gauge has practical limits and gravity is constant, the only method of measuring pressure high enough to meet industry needs is by decreasing the area of the piston-cylinder.

It would seem that the problem of traceability at higher pressures could be solved by simply manufacturing piston-cylinders with smaller areas. But as piston-cylinder diameters get smaller it becomes impracticable to dimensionally determine the area. The relative uncertainty becomes significantly larger as the area gets smaller. If the uncertainty in the determination of area by dimensional measurement of a 50 mm diameter piston-cylinder is ±5 ppm, with the same dimensional measurement capability, the uncertainty in a 1.5 mm diameter piston-cylinder area is ±166 ppm. Uncertainty in the actual location of the effective area with the piston-cylinder gap results in much higher uncertainty in effective area as the gap/area ratio decreases. In addition, dimensional measurements yield effective area values at null pressure but, at higher pressures, the change in effective area with pressure becomes significant and there is no experimental determination of the deformation of the piston-cylinder at higher pressures.

A much better choice than dimensionally measuring all piston-cylinder sizes is to dimensionally characterize a large piston-cylinder and transfer the dimensionally determined value to the smaller effective areas by a comparison method called a crossfloat. The technique that is the most successful at transferring effective areas with lower uncertainties is the ratio based crossfloat.

The ratio based crossfloat determines a ratio between the effective areas of two piston-cylinders. The process exploits the excellent repeatability of the piston gauges while using exchanging techniques that eliminate many systematic uncertainties. Hence effective areas determined this manner have uncertainties that are only slightly larger than the uncertainty in effective area of the reference piston-cylinder's effective area. The ratio based crossfloat is the method used to characterize the piston-cylinder pressure calibration chain.

2. The Piston-Cylinder Pressure Calibration Chain

DHI's piston-cylinder calibration chain was introduced to the measurement community in a 1989 paper [2]. That paper describes the ratio based crossfloat and the calibration chain in detail. For the sake of continuity in this paper the principal concept of the calibration chain is described in this section. Since the ratio based crossfloat determines a ratio between the effective areas of two piston-cylinders, continuity of the relationships can be realized with groups of piston-cylinders. Figure 1 is a diagram representing the possible relationships in a group of four piston-cylinders. The group is comprised of two levels of piston-cylinders. A and B are the same nominal effective area as are C and D, i.e. they have a nominal ratio of 1:1. A and B are different from C and D by a nominal ratio of 2:1, 5:2 or 5:1.

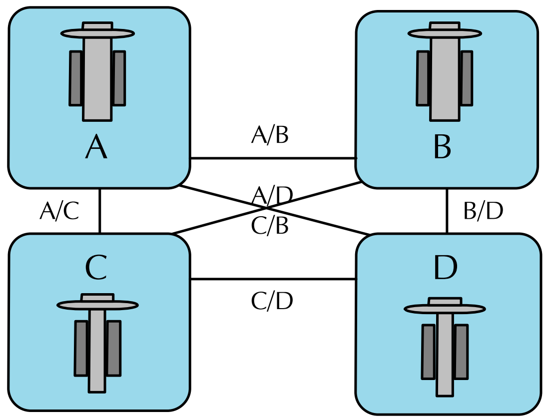

Figure 1. Definition of the relationships in a calibration chain loop.

Each line in Figure 1 is a measured ratio between the two piston-cylinder effective areas. There are four different combinations of these relationships that should be equal to the first ratio A/B. These are:

1. $A/B = A/D \times \frac{1}{B/D}$
3. $A/B = A/C \times C/D \times \frac{1}{B/D}$
2. $A/B = A/C \times \frac{1}{C/B}$
4. $A/B = A/D \times \frac{1}{C/D} \times \frac{1}{B/C}$

The difference between the A/B direct ratio and the combined ratios are errors that are used to evaluate the crossfloats performed and are eventually used in the uncertainty analysis of the calibration chain. The final relationship of all four piston-cylinders is called a loop.

After a loop is determined the C and D piston-cylinders become A and B for the next loop to build relationships with the next pair of two piston-cylinders with smaller effective areas. The continuation of this creates a chain of comparisons from largest to the smallest effective areas, and lowest to highest pressure respectively.

Once all loops are completed the effective area value of the large, dimensionally measured piston-cylinder is transferred mathematically up the chain to the smallest using the experimentally determined ratios. The propagation of uncertainty is low due too the excellent repeatability of the piston gauges and the effectiveness of the ratio based crossfloat technique used to build the relationships.

3. History of the Piston-Cylinder Pressure Calibration Chain

The original calibration chain was characterized in 1983. It consisted of three sets of two piston-cylinders that covered a hydraulic pressure range from 700 kPa (100 psi) to 350 MPa (50 000 psi). The traceability was obtained through two external piston-cylinders whose effective areas were determined by crossfloat by LNE (France). This calibration chain was re-characterized in 1985 and 1987.

The piston-cylinders mass to pressure coefficients were psi/kg and the external piston-cylinders were MPa/kg (SI) which made the nominal ratios areas not a whole number. Thus the connection between the calibration chain piston-cylinders and external piston-cylinders was more difficult than if all the ratios of the areas were a nominal value.

In 1988 the calibration chain was replaced with a more modern calibration chain with piston-cylinders that have SI mass to pressure coefficients that covered a hydraulic pressure range of 200 kPa to 500 MPa. The traceable connections to the two external SI piston-cylinders were maintained but for the first time the effective areas originated from a 35 mm tungsten carbide piston-cylinder, S/N 116, characterized by both dimensional measurements and crossfloats in national metrology institutes [3]. This reduced uncertainties and increased confidence in the effective areas that had originally only been determined by crossfloat with another NMI. This calibration chain was re-characterized on a two year interval in 1990 and 1992, then extended to a three year interval and re-characterized in 1995 and 1998.

The basis of the traceability for the calibration chain evolved throughout the 1990s. By 1995 controlled clearance[2] 50 mm piston-cylinders had been developed in order to reach lower uncertainties on effective area from dimensional characterization. Three of these piston-cylinders were used in a Cooperative Research and Development Agreement (CRADA) with NIST (US)[4]. One of the three was used as the starting point of the calibration chain. In 1998 traceability was again obtained through the 35 mm piston-cylinder 116, but also included a 2.5 mm diameter piston-cylinder that was characterized up to 200 MPa and provided the calibration chain with a source of verification for effective area and elastic deformation of the piston-cylinders at high pressure.

In addition to advantages of traceability through large diameter piston-cylinders, support of the low pressure gas piston-cylinders in the range of 10 to 700 kPa became crucial. In 1998 the calibration chain grew to include two sets of large diameter piston-cylinders. These included two 35 and two 16 mm diameter tungsten carbide, gas operated, gas lubricated piston-cylinders. The benefits experienced with the addition of these piston-cylinders included better support of customer's piston gauges in these ranges and an easier link from the 35 mm reference piston-cylinder 116 to the 7.9 mm diameter hydraulic piston-cylinders.

Even with all the changes in the piston-cylinder pressure calibration chain, the observed stability of the effective areas was very acceptable. Table 1 shows the differences in piston-cylinder effective areas that were observed from 1988 to 1998. These differences should not be mistaken as being only the stability of the piston-cylinders because the differences are significantly influenced from differences in the values used for traceability and errors, however small, in the technique used to transfer the effective areas.

In lieu of some disagreement between the areas determined between 1988 and 1990, the differences of the piston-cylinder effective areas were within the uncertainty of each range. Note that Table 1 also shows the change from 1990 to 1998, which seems to be a better representation of the stability of tungsten carbide piston-cylinders. The largest change was observed with the highest range of piston-cylinders. This particular range was the only range with a cylinder made of tungsten carbide and a piston made of steel. This was necessary because the diameter is only 1.6 mm and if it were made of tungsten carbide the probability of the piston breaking would be too high. All other pistons and cylinders were constructed of tungsten carbide.

[2] Though by design these piston gauges are controlled clearance, they are specifically used as free deformation in the calibration chain.

Table 1. Differences in effective areas determined in the calibration chain from 1988 to 1998.

S/N	RANGE [MPa]	Difference from 1988 - 1990 [ppm]	Differences from 1990 - 1992 [ppm]	Differences from 1992 - 1995 [ppm]	Differences from 1995 - 1998 [ppm]	Differences from 1988 - 1998 [ppm]	Differences from 1990 - 1998 [ppm]
512	0.2 to 20	11.4	-0.6	2.4	-3.0	**10.2**	**-1.2**
20	0.2 to 20	4.5	0.4	-2.9	-1.5	**0.5**	**-3.9**
22	0.5 to 50	6.1	-1.5	1.0	-0.9	**4.7**	**-1.4**
23	0.5 to 50	24.0	-0.5	4.6	-5.9	**22.2**	**-1.8**
24	1 to 100	27.5	-4.1	1.0	2.9	**27.3**	**-0.2**
25	1 to 100	24.5	-1.0	0.0	6.0	**29.5**	**5.0**
26	2 to 200	7.7	24.9	-4.3	6.1	**34.5**	**26.8**
512	2 to 200	n/a	n/a	-7.3	8.9	**N/A**	**N/A**
2594	5 to 500	43.3	15.8	6.6	42.3	**108.1**	**64.8**
2690	5 to 500	51.0	13.3	10.2	27.8	**102.3**	**51.3**

4. The 2001 Piston-Cylinder Pressure Calibration Chain

The calibration chain continues to evolve in response to new developments and goals to improve uncertainties on effective area. For the 2001 calibration chain, changes were made relative to 1998. These changes helped to improve the final uncertainties in effective area and helped to increase confidence in the areas determined.

All of the hydraulic piston-cylinders were converted to a new mounting system design in which all components relevant to the elastic deformation of the piston-cylinder are designed into a dedicated piston-cylinder module (Type 7000). Relative to the previous system (Type 5000), this reduced inconsistencies in the load of the cylinder retaining nut and other influences of the deformation of the piston-cylinder thus lowering the uncertainty contributed by elastic deformation for both theoretical and measured values and improving repeatability.

The 2.5 mm diameter (2 to 200 MPa range) and the 1.6 mm diameter (5 to 500 MPa/kg range) piston-cylinders were replaced with new tungsten carbide pistons and cylinders. This was to ensure that the performance of the highest ranges in the calibration chain was the best available and to eliminate the steel piston used for the 1.6 mm diameter size. With the new modules the piston is protected in operation and not expose during installation and removal so there is much less chance of breaking the 1.6 mm tungsten pistons.

Added to the calibration chain was a level of 50 mm diameter piston-cylinders above the 35 mm piston-cylinders. This created a 1:1 link with a 50 mm diameter primary piston-cylinder that was characterized by the NIST dimensional group.

Figure 2 shows the structure of the 2001 piston-cylinder pressure calibration chain. The connections between boxes (piston-cylinders) are ratios determined by at least two ratio based crossfloats that are averaged. When possible the piston-cylinder's platforms are swapped between crossfloats to average out platform based systematic errors such as temperature measurement or leveling.

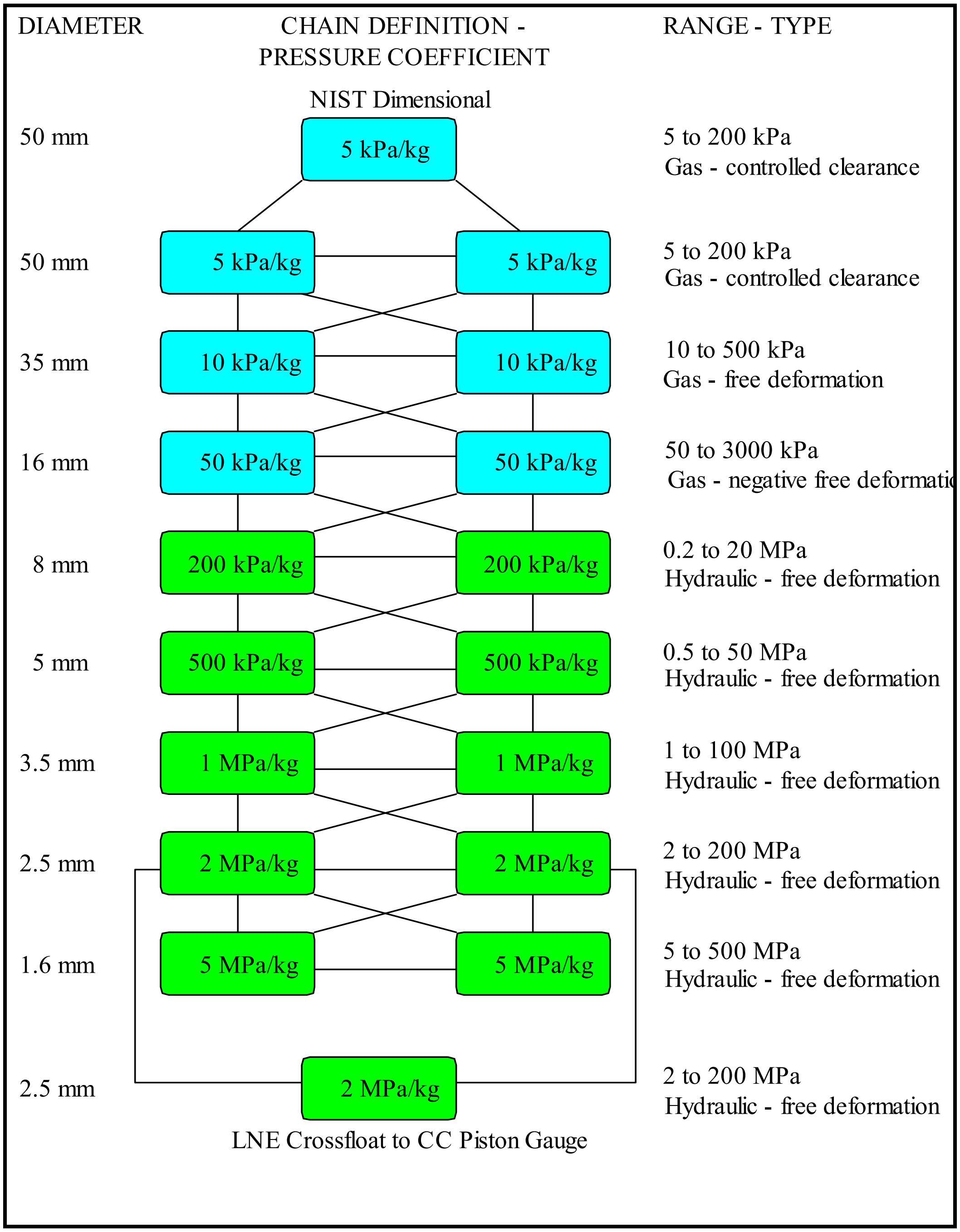

Figure 2. Structure of the 2001 piston-cylinder pressure calibration chain.

The traceability of the 2001 piston-cylinder pressure calibration chain originates from dimensional measurements made on a reference tungsten carbide 50 mm diameter piston-cylinder. When the calibration chain reference was measured dimensionally by NIST, a second 50 mm piston-cylinder to be delivered to an NMI was also characterized. A ratio based crossfloat was performed between the two to verify the coherence of the dimensional measurements made. Table 2 presents the results of the comparison and shows that the agreement is acceptable and within the dimensional uncertainty.

Table 2. Comparison of 50 mm piston-cylinders characterized dimensionally.

Ratio	Ratio of areas determined dimensionally	Ratio of areas determined by crossfloat	Dimensional uncertainty (k=2) [±ppm]	Difference [ppm]
NMI/DHI	1.0000034	1.0000008	5	-2.6

The pressure deformation coefficient reported by LNE (France) in 1998 on the 2.5 mm reference tungsten carbide piston-cylinder was used to verify the elastic deformation coefficients of the 2.5 mm piston-cylinders in the calibration chain. The results are presented in Table 3.

Table 3. Measured elastic deformation of the 2.5 mm and 1.6 mm piston-cylinders.
All values shown in scientific notation are in the units MPa^{-1}.

Calibration Chain Piston-Cylinder	2.5 mm A	2.5 mm B	Calibration Chain Piston-Cylinder	1.6 mm A	1.6 mm B
Determined from 3.5 mm A	6.32E-07	9.32E-07	Determined from 2.5 mm A	9.30E-07	8.18E-07
Determined from 3.5 mm B	6.46E-07	9.06E-07	Determined from 2.5 mm B	9.72E-07	7.57E-07
From LNE (France) 2.5 mm	6.87E-07	1.00E-06			
Average	6.39E-07	9.19E-07	Average	9.51E-07	7.87E-07
Disagreement	1.4E-08	-2.6E-08	Disagreement	4.2E-08	-6.1E-08
Disagreement	2 %	-3 %	Disagreement	5 %	-8 %
Disagreement at 200 MPa	3 ppm	-5 ppm	Disagreement at 500 MPa	21	-30
Disagreement (average - LNE) at 200 MPa	-10 ppm	-16 ppm			

A description of how elastic deformation coefficients are determined in the calibration chain is presented in the paper given in 1989. To summarize, the theoretical coefficients are used for ratio based crossfloats that are determined from the 50 mm to the 3.5 mm piston-cylinders, i.e. up to 100 MPa. This is done because below 100 MPa the theoretical value has a lower uncertainty than a measured value.

When performing a ratio based crossfloat the difference between the elastic deformation coefficients of the two piston-cylinders can be measured. This can be used for an evaluation of the theoretical coefficients or to trim the theoretical coefficient to create a measured value. The latter was done to the 2.5 mm and 1.6 mm piston-cylinders. Each 3.5 mm piston-cylinder was used to determine the elastic deformation coefficient of each 2.5 mm piston-cylinders and each 2.5 mm was used to determine the 1.6 mm piston-cylinders. The disagreement between these determinations are listed in units of MPa^{-1}, in percent of the theoretical value, and in ppm at the full scale pressure of the range. In addition, the 2.5 mm piston-cylinder that was measured by LNE was used to measure the A and B 2.5 mm piston-cylinders in the calibration chain. The disagreement between the average values determined for each 2.5 mm piston-cylinder in the calibration chain and the value determined from the reference 2.5 mm is shown, calculated in ppm, for the full scale pressure of that range (200 MPa). The disagreement between the elastic deformation coefficients from the calibration chain and the values transferred from LNE were within acceptable agreement, i.e. within 10%.

The final uncertainties of the effective areas in the 2001 calibration chain were reduced from previous calibration chain characterizations due to the low uncertainties returned from the dimensional measurements on the 50 mm piston-cylinder and the repeatability of the high pressure hydraulic piston-cylinders. Table 4 lists the final uncertainties of the piston-cylinder pressure calibration chain effective areas.

Table 4. Final uncertainties on effective area in the calibration chain.

Range	Diameter [mm]	Expanded @ k=2 [±ppm]
5 – 200 kPa	50	5.6
5 – 200 kPa	50	6.2
10 – 500 kPa	35	7.2
10 – 500 kPa	35	7.6
50 – 3 000 kPa	16	8.0
50 – 3 000 kPa	16	8.8
0.2 - 20 MPa	8	9.4
0.2 - 20 MPa	8	9.8
0.5 - 50 MPa	5	11
0.5 - 50 MPa	5	12
1 - 100 MPa	3.5	13
1 – 100 MPa	3.5	14
2 - 200 MPa	2.5	19
2 – 200 MPa	2.5	20
5 – 500 MPa	1.6	28
5 – 500 MPa	1.6	31

As always an uncertainty is only a prediction of a probability distribution around the value determined. To validate the results of the effective areas of the calibration chain piston-cylinders and their uncertainties, third party comparisons with reliable sources are relied upon. The following section examines the results of comparisons between effective areas determined by third party sources and with the calibration chain.

5. Intercomparisons

Because of their excellent stability and repeatability, piston gauges make very good artifacts for comparisons. In producing piston gauges for customers, we sometimes encounter situations in which clients require direct traceability to a specific national metrology institute. This requirement can be used as an opportunity to compare a piston-cylinder effective area value coming from the calibration chain to the value determined by an NMI.

Table 4 lists a number of comparisons recently completed in this manner. By chance, many of the comparisons were performed within a little more than one year. The table identifies:

a. The NMI with which the comparison was performed.

b. Approximate date the comparison was completed.

c. The full scale pressure of the comparison. Note that some of these have two values. In these cases the elastic deformation coefficient was determined by the NMI and the differences are shown at null pressure and the full scale pressure. Where only one pressure is shown the difference would be the same at all pressures because the NMI used the theoretical elastic deformation coefficient that was reported by DHI to determine effective area.

d. The nominal diameter of the piston-cylinder in the comparison.

e. The uncertainty in effective area reported by DHI.

f. The uncertainty in effective area reported by the NMI.

g. The difference in effective area determined from the calibration chain and the NMI.

h. The En number calculated from (e), (f) and (g).[3]

i. The theoretical difference in the effective area determined from the calibration chain and the NMI if the effective areas determined by DHI were changed by the same amount the calibration chain effective areas changed from 1998 to 2001. Since the 2.5 and 1.6 mm diameter piston-cylinders were replaced in 2001 this value is not shown.

Table 5. Results of comparisons of the calibration chain and various NMIs.

(a) NMI	(b) Date	(c) Pressure [MPa]	(d) Nominal diameter [mm]	(e) DHI UNC K=2 [ppm]	(f) NMI UNC K=2 OR 95 % [ppm]	(g) Difference [ppm]	(h) En	(i) Differences corrected for 2001 changes [ppm]
LNE (France)	Mar-99	0	35	15	10	4.2	0.23	-0.8
		0.35	35	15	11	9.0	0.49	4.0
PTB (Germany)	Dec-99	0.35	35	15	2(?)	4.0	0.27	-1.0
NIST (US)	Apr-99	0	35	15	13	4.7	0.24	-0.3
		0.35	35	15	13	6.2	0.31	1.2
LNE (France)	Mar-99	1.75	16	20	10	-1.7	-0.07	-7.7
PTB (Germany)	Dec-99	1.75	16	20	20	7.4	0.26	1.4
LNE (France)	Mar-99	7	8	25	12	22.0	0.78	15.0
NMI (Holland)	Aug-00	7	8	30	37	1.3	0.03	-5.7
		7	8	30	37	-10.0	-0.22	-17.0
		7	8	30	37	-8.5	-0.18	-16.0
LNE (France)	Jun-00	0	5	25	10	-2.6	-0.10	-9.6
		30	5	30	12	-0.5	-0.02	-7.5
		0	2.5	30	10	14.0	0.43	N/A
		120	2.5	34	14	6.1	0.17	
		0	1.6	35	10	-11.0	-0.29	N/A
		300	1.6	47	27	-25.0	-0.47	

Column (i) is strictly a theoretical value and is used to help evaluate the differences between the 1998 and 2001 calibration chains. As is mentioned in a previous section, the differences should not be mistaken for the stability of the effective areas. These differences are influenced by the method of traceability and the method of transfer. As it turned out the effective areas with the largest area, lowest uncertainty, and also the best stability, agreed much better with the NMIs determinations. If the En numbers were calculated using the corrected differences, (i), instead of the actual differences, (g), the average of the En numbers through the 5 mm level would be cut in half. This helps to build confidence in the 2001 calibration chain results.

[3] En calculated as: DHI value - NMI value / [square root($UncDHI^2 + UncNMI^2$)]

6. Conclusion

The piston-cylinder pressure calibration chain has grown from a concept in the early 1980s to a measurement standard that is depended upon by many laboratories around the world. This has been accomplished in a laboratory that has no dimensional measurement capability and attributes its success to the unprecedented qualities of the piston gauge as both a primary and transfer standard.

It is apparent from the results of the calibration chain that methods and reference standards have yet to be refined enough to measure the true stability of well manufactured tungsten carbide piston-cylinders. However it is clear that the ratio based crossfloat and the calibration chain have exceeded expectations for its 19 years of history.

Determining effective area of a 50 mm tungsten carbide piston-cylinder through dimensional measurements proved to be the right decision as a source of traceability and answered some questions regarding results that have been obtained in comparisons. At the same time this leaves room for improvement as uncertainties on the dimensional measurements should decrease as standards and techniques continue to improve. With the precision of the ratio based crossfloat the uncertainties of effective areas in the calibration chain will also certainly improve.

7. References

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