A Weighted Ensemble Method of Evaluating Unexpected Drift in DC Voltage Standards

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1. Abstract

In our previous 1998 NCSL paper "Improved Uncertainty in 10 Volt DC Standard Cells Using an Ensemble and Modified Uncertainty Predictions", we looked at improved uncertainty for Zener DC Voltage Standards using an ensemble with weighted averaging. Control charting techniques were also demonstrated as a method of tracking individual DC Voltage Standards. This paper is a follow up to the original paper. It presents a method for determining if a particular DC voltage standard is deviating from its expected voltage. This approach compares the individual DC voltage standards' own projected curve fit to the value predicted by a weighted ensemble of the remaining voltage standards. A linear unbiased minimum variance estimate is used for this comparison. The uncertainty for the ensemble comparison is at least as good if not better than the individual DC standard's own regression uncertainty; therefore, this is a valuable independent way to determine if a particular DC voltage standard has drifted away from its expected regression prediction. Data is provided in this paper for a group of DC voltage standards using this comparison method.

2. Introduction

American Airlines uses Zener DC Voltage Standards (DCVS) for the purpose of calibrating high accuracy multifunction calibrators and digital multimeters. This paper is the second part of a paper, which was originally written for two reasons. The first reason was to characterize the accuracy of our DC voltage standards, and the second was to develop an ensemble from these standards that would have a much lower uncertainty than that of the individual DC standards. The original paper did not adequately address the issue of control chart tracking for an individual DCVS. This paper demonstrates a reliable and independent method of verifying that a DCVS is following its expected regression prediction.

3. Regression Estimates of Zener DC Voltage Standards

Our first paper gives a detailed approach to the curve fitting of a DCVS [1]. The voltage drift rate of a zener standard is quite predictable. A good curve fit of this drift rate can yield a very low uncertainty for the standard. Table 1 illustrates how low the uncertainty is for the four DC standards we have been tracking since 1997. Two of these standards follow a linear regression quite well and the other two correlate well to a logarithmic function. A graph of DC Voltage Standard A along with its upper and lower confidence limits is shown in Figure 1.

Table 1-Deviation and Uncertainty of Fluke 732A DC Standards

Voltage Standard	Type of Curve Fit	Deviation from 10VDC (PPM)	Uncertainty as of 3/1/2001 (PPM)
A	Linear (y=mx+b)	0.96	.14
B	Linear (y=mx+b)	4.67	.18
C	Log (y=m ln(x-k)+b)	9.07	.17
D	Log (y=m ln(x-k)+b)	4.66	.15

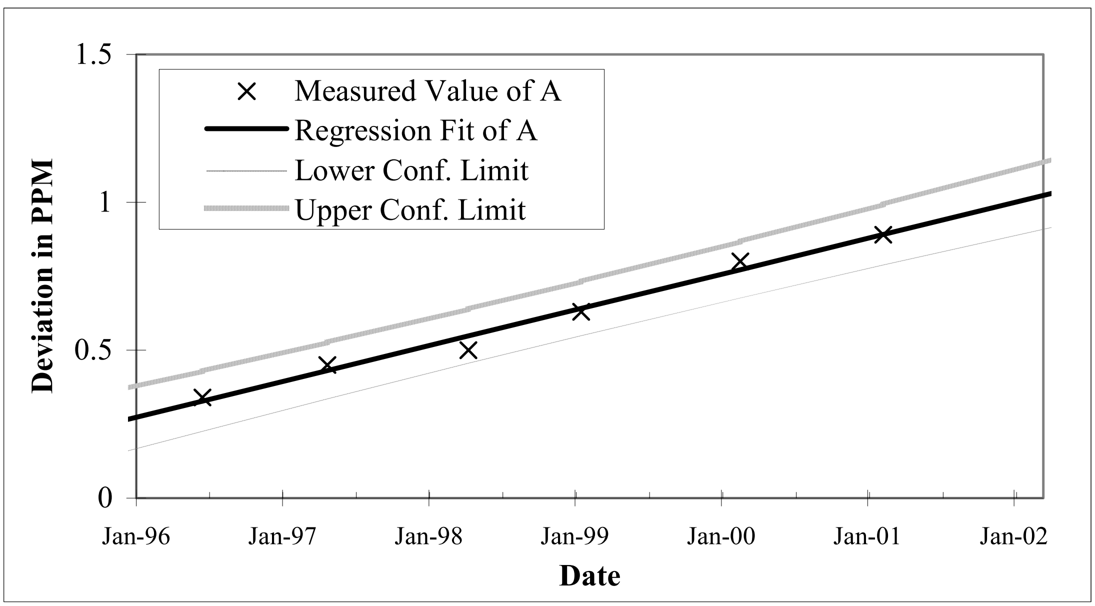

Figure 1. Regression Fit and Confidence Limits for Voltage Standard A.

4. Ensemble Comparison Method

The primary focus of this paper is to show the value of creating an ensemble measurement for a particular voltage standard. The uncertainty of this measurement can be equal to if not less than the uncertainty of the individual DCVS regression line. More importantly, this group mean estimate becomes a valuable tool when it is graphically compared to the regression line of the DCVS.

Voltage tracking of a group of DC voltage standards is typically accomplished by making differential voltage measurements between the various standards. A Low level differential voltage measurement is quite accurate when performed with an instrument such as a nanovolt meter that has a very low level voltage range. This results in uncertainties that are significantly less than the regression uncertainty. Differential voltage data combined with regression information provides an independent estimate of a DC voltage standard's value.

Four DC Voltage Standards have been tracked daily with differential voltage measurements using an HP34420A and an automated scanner. A block diagram of this voltage tracking system is given in Figure 2.

Taking DCVS 'A' as an example, an ensemble estimate 'V_{Aest}', is given by

$$(1a)\quad V_{Aest} = w_b(V_{Breg} + V_{ab}) + w_c(V_{Creg} + V_{ac}) + w_d(V_{Dreg} + V_{ad})$$

Equations for the voltage of other standards can easily be derived from 1a. They are

$$(1b)\quad V_{Best} = w_a(V_{Areg} - V_{ab}) + w_c(V_{Creg} + V_{bc}) + w_d(V_{Dreg} + V_{bd})$$

$$(1c)\quad V_{Cest} = w_a(V_{Areg} - V_{ac}) + w_b(V_{Breg} - V_{bc}) + w_d(V_{Dreg} + V_{cd})$$

$$(1d)\quad V_{Dest} = w_a(V_{Areg} - V_{ad}) + w_b(V_{Breg} - V_{bd}) + w_c(V_{Creg} - V_{cd})$$

where 'w_a', 'w_b', 'w_c', *and* 'w_d' are the weighting coefficients for the other three standards whose regression estimates are 'V_{Areg}', 'V_{Breg}', 'V_{Creg}'and 'V_{Dreg}', and 'V_{ab}', 'V_{ac}', and 'V_{ad}' are the six differential voltage measurements (eg. 'V_{ab}'='V_a'-'V_b').

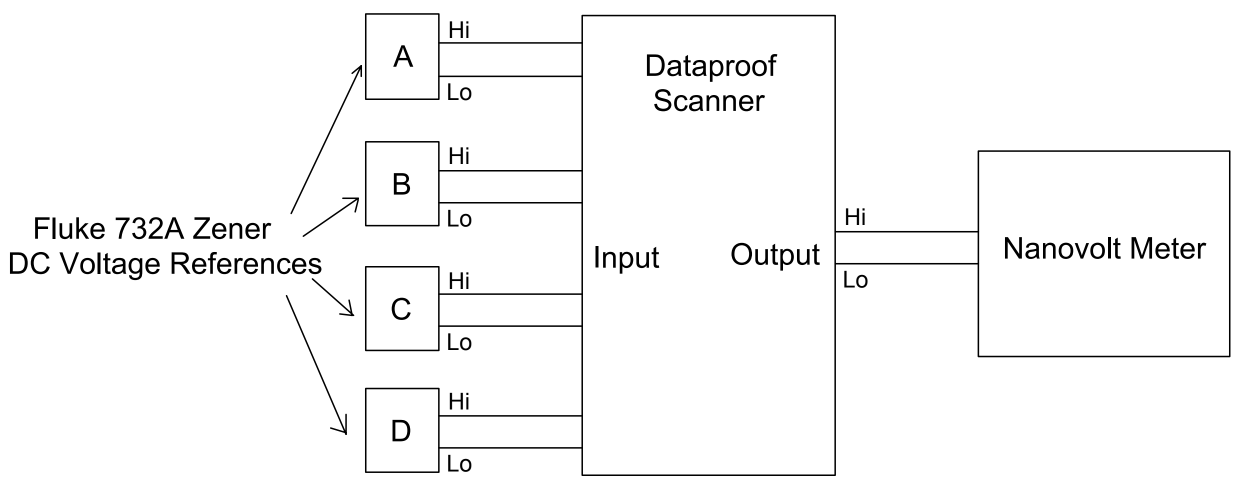

Figure 2. Scanner Differential Voltage Measurement System.

We have selected a weighting method known as the 'linear unbiased minimum variance estimate' (LUMV) for averaging the individual standards [2]. This method is guaranteed to be the 'minimum variance' estimate. The LUMV gives more weighting to a standard with less uncertainty and, conversely, a standard with higher uncertainty has less influence on the group average. The weighting coefficients using the LUMV method are given by equations 2a, 2b, and 2c.

(2a) $w_b = \left(\dfrac{\dfrac{1}{U_B^2}}{\dfrac{1}{U_B^2} + \dfrac{1}{U_C^2} + \dfrac{1}{U_D^2}} \right)$, (2b) $w_c = \left(\dfrac{\dfrac{1}{U_C^2}}{\dfrac{1}{U_B^2} + \dfrac{1}{U_C^2} + \dfrac{1}{U_D^2}} \right)$, (2c) $w_d = \left(\dfrac{\dfrac{1}{U_D^2}}{\dfrac{1}{U_B^2} + \dfrac{1}{U_C^2} + \dfrac{1}{U_D^2}} \right)$

where 'U_A', 'U_B', and 'U_C' are uncertainties for the other three standards being used to provide the estimates for 'A'.

We can also calculate uncertainty for the ensemble from Mandel [3]. The uncertainty (which is the confidence limit for a given confidence level) of a regression line is given by

(3) $$U_{reg} = \sqrt{t_c^2 \hat{V}(\delta)} \left[\frac{1}{n} + \frac{1}{N} + \frac{N(x - \bar{x})}{u} + U_{cal}^2 \right]^{1/2}$$

where N is the number of points used in the regression.
n is the number of replicates.
$\bar{x}$ is the mean of the set of 'x' data.
$u = N \sum x_i^2 - \left(\sum x_i \right)^2$
$t_c^2 \hat{V}(\delta)$ is the critical value of the 'student t' distribution for N-2 degrees of freedom for a given level of significance (which is '1-confidence coefficient').

An estimate of the uncertainty of a voltage standard as derived from the 'V_{ab}' differential measurements is a combination of the uncertainty of the regression line of 'B' and the uncertainty of the measurement of Vab. This total uncertainty is

(4) $$U_{A_{ab}} = \sqrt{U_{Breg}^2 + U_{M_{ab}}^2}$$

where $U_{A_{ab}}$ is the total uncertainty of the standard A estimate derived from standard B.
U_{Breg} is the uncertainty of the regression line for standard B (from equation 3).
$U_{M_{ab}}$ is the uncertainty of the 'V_{ab}' differential voltage measurement.

Finally, the uncertainties for the ensemble estimate of voltage standards are given by

(5a) $$U_{Aens} = w_b U_{ab} + w_c U_{ac} + w_d U_{ad}$$

(5b) $$U_{Bens} = w_a U_{ab} + w_c U_{bc} + w_d U_{bd}$$

(5c) $$U_{Cens} = w_a U_{ac} + w_b U_{bc} + w_d U_{cd}$$

(5d) $U_{Dens} = w_a U_{ad} + w_b U_{bd} + w_c U_{cd}$

In summary, a linear unbiased minimum variance ensemble estimate can be calculated for voltage standards 'A', 'B', 'C', and 'D' from equations 1a-d. The corresponding uncertainties are also represented by equations 5a-d.

5. Measurement Data

The plot shown in figure 3 compares the regression line for voltage standard 'A' to the ensemble measurements calculated from equation 1a. This good correlation demonstrates that the ensemble measurement provides an excellent independent verification of the 10-volt DC standard. Figures 3, 4, and 5 show similar plots of the other three voltage standards as derived from equations 1b, 1c, and 1d respectively. Extremely good correlation is evident from these graphs; the worst case deviations appear to be no more than about .25 PPM.

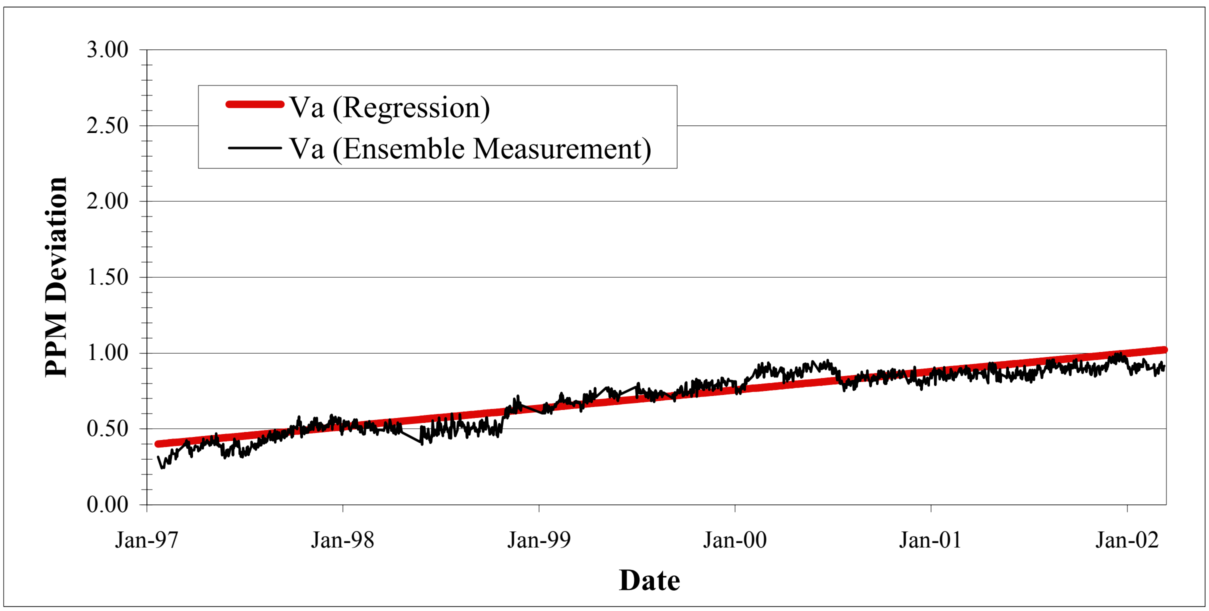

Figure 3. Control Chart for Voltage Standard A.

Figure 6 shows the same comparisons, but with an added feature. Maximum allowable PPM deviation lines have been plotted on the graph too. This becomes a graphical tool to let the metrologist know whether or not the standard has drifted unexpectedly. If so, then standard should be corrected with a new curve fit. The maximum drift allowed for the standard should be determined by the accuracy requirement, which, in turn, depends on what equipment the standard is calibrating.

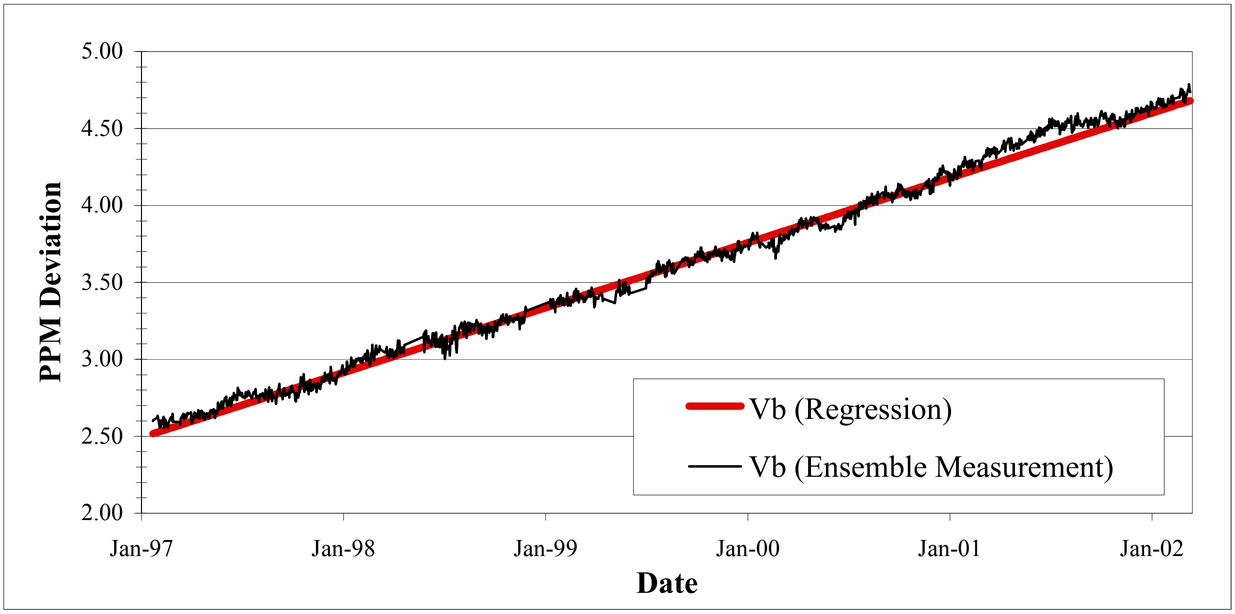

Figure 4. Control Chart for Voltage Standard B.

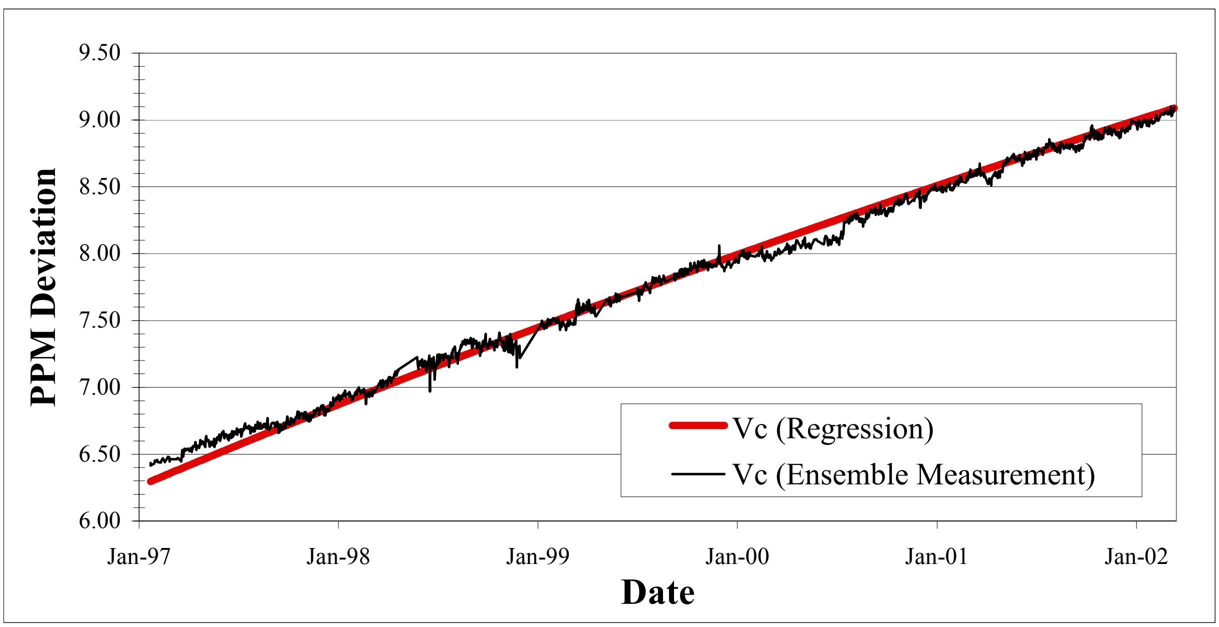

Figure 5. Control Chart for Voltage Standard C.

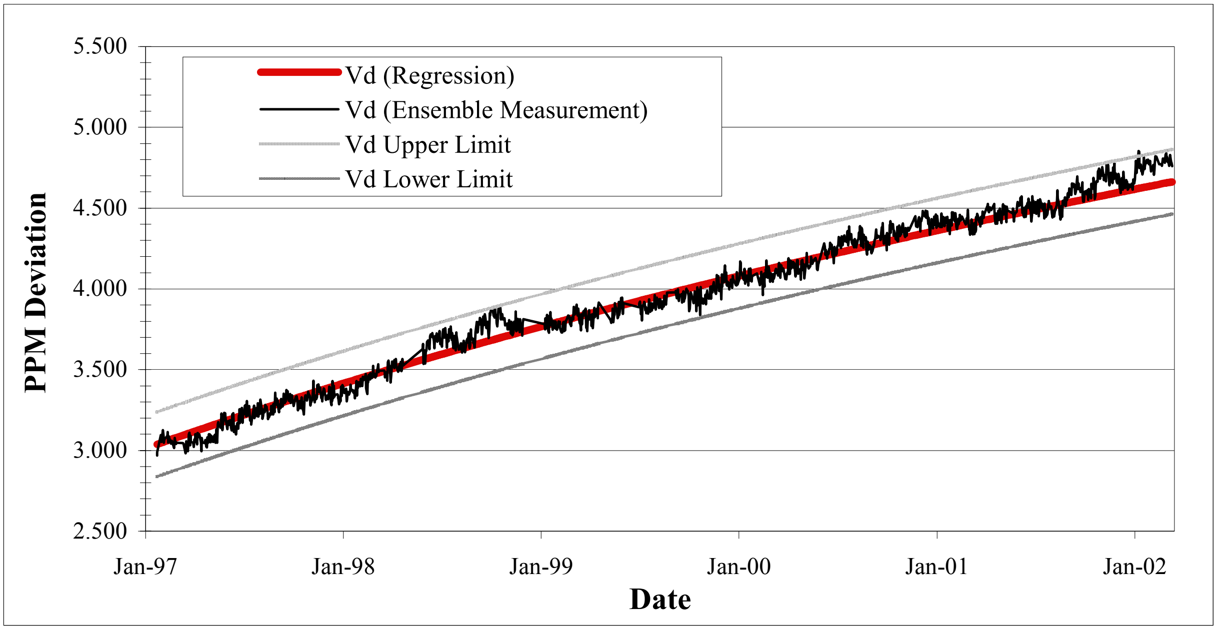

Figure 6. Control Chart for Voltage Standard D with Upper and Lower .2 PPM Limits.

6. Conclusions

Our first paper demonstrated that a group of Zener DC voltage standards could be curve fitted accurately, resulting in a low regression uncertainty for individual DC standards [4]. In this paper we have demonstrated the value of using differential voltage data, in the form of an ensemble measurement, to compare to the DC standards' own regression fit. The optimum ensemble method for our situation, the linear unbiased minimum variance method, was used. This was because it provided an estimate guaranteed to have the 'minimum variance property'. Control charts for four different standards also showed very good correlation over a long period of time, thus validating the accuracy and value of this technique.

7. References

1. G. Hilderbrand, and S. Tedder, Improved Uncertainty in 10 Volt Standard Cells Using an Ensemble and Modified Uncertainty Predictions, *Cal Lab,* vol. 6, num 1, pp. 23-26, 1999.

2. Mandel, John., Weighted Averages, Chapter 7 in *The Statistical Analysis of Experimental Data*, Dover, New York, 1984, pp132-135.

3. Ibid., p 286-288.

4. G. Hilderbrand, and S. Tedder, Improved Uncertainty in 10 Volt Standard Cells Using an Ensemble and Modified Uncertainty Predictions, *Cal Lab,* vol. 6, num 1, pp. 24, 1999.