

# Pressure coefficients of two kinds of double wall-type 1-ohm resistors

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## Abstract

Pressure coefficients of five Leeds & Northup (L&N) made double wall-type 1-ohm resistors (model 4210) and five Yokogawa Electric Works (YEW) made double wall-type 1-ohm resistors (model 2781) were measured by being pressurized and depressurized in a container. Resistances of the L&N made resistors were almost linearly dependent on the pressure in the range of from 810 hPa to 1110 hPa. Their pressure coefficients were measured to be within from +0.7 n $\Omega$ /hPa to +2.2 n $\Omega$ /hPa. Resistances of the YEW made resistors were almost independent of the pressure. The L&N made resistors are used as standards of resistance in Japan Electric Meters Inspection Corporation.

## 1. Resistance standard and resistor

Since 1990, quantum Hall resistance has been used as a reference for resistance standard on an international basis [1]. Before 1990, resistance of a resistor had been measured by the method using a cross capacitor [2]. However, the measurement of resistance based on a cross capacitance is often accompanied by complicated operations, difficult data analysis and time-consuming measuring procedure. Actual measurement was performed once in several years or scores of years, and multiple resistors were placed under bank control during this period. In this manner, resistance standard was maintained in most cases. A 1-ohm resistor was used as a reference in many cases, and a high-performance 1-ohm resistor was developed. These 1-ohm resistors are currently used as an intermediary resistor for international comparison of resistance standards or for supply in Japan.

The resistor is available in many types depending on the structure. The following describes the open type and sealed type resistors in particular. Since measurement current is flown for measurement of resistance, temperature is changed by Joule heat. Copper-manganese-nickel alloy or nickel-chromium alloy was used as resistor wires. Temperature coefficient is smaller than those of other metals, several ppm/K for copper-manganese-nickel alloy and 0.1 to several ppm/K nickel-chromium alloy.

Since uncertainty in resistance measurement is 0.01 ppm or less at present, influence of heat generation upon temperature change cannot be ignored.

The open type resistor can be defined as a resistor whose resistance wire makes a direct contact with the outside air or oil of an oil bath. The resistor of this type has high heat radiation efficiency but the resistance wire may be changed by oxidation and annual change is relatively large. To reduce annual change, the container is sealed and is filled with dry nitrogen or argon. This is called a sealed resistor. In the sealed resistor, the heat release efficiency is determined by the ratio between the volume inside the resistor and surface area. To increase the surface area, a resistance wire is placed between two cylinders having different diameters and covers are placed on the top and bottom. This is called a double-walled resistor. The cylinder with a greater diameter is referred to as an outer wall, while that with a smaller diameter is called an inner wall.

The first double-walled resistor ever developed is a Thomas type resistor [3] and [4]. It was put on the market under the name of "4210 1-ohm resistor" by Leeds & Northup (L&N).

The annual change of the 4210 1-ohm resistor was generally very small. In some excellent cases, it is as small as  $0.1 \mu \Omega$  or thereabout at the annual rate. This is often used even now. Regret to say, however, L&N has discontinued production and sales of all resistors including Model 4210. The 4210 1-ohm resistor has a resistance wire wound directly on the inner wall and bonded with insulating varnish. The outside of the inner wall is in direct contact with outside air or oil, so the heat releasing efficiency is considered to be relatively good, on the one hand. On the other hand, the inner or outer walls may be subjected to deformation if there is a change in the pressure of the external world, and dependency on pressure is exhibited, as has been indicated so far [5].

Model 2781 resistor by Yokogawa is the only double-walled resistor produced and marketed at present, to the best knowledge of the present author. The Model 2781 resistor has the resistance wire wound on the Bakelite-made spool in such a way that neither the resistance wire nor the frame directly contact the inner or outer wall, so the pressure dependency must be small. However, there has been no report on accurate measurement or analysis of this point. On the other hand, Bakelite has a low thermal conductivity, so heat release efficiency may not be so high.

We have measured the pressure dependency of five L&N-made Model 4210 1-ohm resistors and five Yokogawa-made Model 2781 1-ohmm resistors, using a special-purpose pressure vessel to change the pressure applied to resistors. We measure the resistance using Guideline-made Model 9975 Resistance Bridge.

The resistance of Model 4210 resistor changes exhibits almost linear change within the range from 810 hPa to 1110 hPa. The minimum pressure coefficient was  $+0.7 \text{ n}\Omega/\text{hPa}$  and the maximum pressure coefficient was  $+2.2 \text{ n}\Omega/\text{hPa}$ . We also used the same range to measure the resistance of Model 2781 resistor, and could not find out a change in resistance.

The pressure vessel and resistance measuring method will be discussed in Section 2. The result of measuring pressure dependency will be reported in Section 3. The uncertainty of pressure coefficient measurement will be discussed in Section 4.

## **2. Measurement method**

### **2.1 Pressure vessel**

Since pressure changes in conformity to temperature, an oil bath is normally used in order to keep the resistor temperature constant wherever possible. One of the methods is to use a deep oil bath to change the position of the oil bath, thereby changing the pressure applied to the resistor [5]. This method maintains the temperature constant and allows easy change of the pressure applied to the resistor. It permits pressurization but not depressurization. In Japan, a resistor may be used in the highland such as in Nagano, and this makes it necessary to study the impact of depressurization. When a resistor is carried by air, the pressure in the cabin is reduced to 800 hPa or less. It is also necessary to make sure that the irreversible change does not occur to resistance when pressure is reduced to this level (that there is no history left behind).

This time we have used an aluminum-made special-purpose pressure vessel, which allows both pressurization and depressurization. A resistor was placed in the vessel, which was almost filled with paraffin oil. To change the pressure applied to the resistor, air pressure was applied or reduced by a pump. To keep the temperature constant wherever possible, the entire vessel was placed in an oil bath (Model 9723CR by Guideline). In the oil bath, temperature distribution and temperature fluctuation are of the order of several mK. When a pressure vessel is used, air will go in or go out when pressure is applied or reduce. This will cause the temperature to be changed by compression and expansion, with the result that temperature stability may be reduced. Impact of compression and expansion will be discussed in Section 4.

To ensure a gradual change in pressure, we have used a cylinder and piston type pump. To keep the required pressure constant for a long time, we closed the valve to maintain the pressure vessel sealed when pressure was measured. Sayama's pressure gauge (Model 204 by Setra Systems used as a sensor) was employed to measure the pressure.

Resolution for measurement was 0.1 hPa. The normal time for measurement was 30 minutes and pressure fluctuation during this time was 0.3 hPa or thereabout. The impact of pressure fluctuation will be discussed in Section 4.

## **2.2 Resistance measurement**

In the common step of resistance measurement, the resistance ratio between the resistor with its value known and the resistor to be tested is measured. Uncertainty in resistance measurement is determined by the stability of the resistance of the resistor used as a standard and the uncertainty in the measurement of resistance ratio. The purpose of the present measurement, however, is to check changes in resistance with respect to pressure change. So it is sufficient if resistance of the standard resistor does not change. Factors of uncertainty in the measurement of resistance ratio can be divided into two types; variation for each measurement and reproducibility despite repeated measurements as exemplified by offset. The factor of reproducibility can be considered as constant even when the pressure is changed. So variations for each measurement can be considered as a factor of uncertainty in the present measurement.

Giving consideration to superb maneuverability, we used a Guideline-made Model 9975 resistor bridge for this measurement. Model 9975 Resistor Bridge for this measurement is designed in such a way that the variation in measurement is minimized in 10: 1 measurement. So we used the 10-ohm resistor (Model 2781 10-ohm resistor by Yokogawa) for standard resistance, since the resistance of this resistor had been known as stable as a result of long-time experience of measurement. Measurement current to be fed to the 1-ohm resistor was 3mA. When the resistance of the 1-ohm resistor is measured under this condition using the 10-ohm resistor as a standard, the impact of the variation is normally 10 n $\Omega$ . The impact of the uncertain in resistance measurement will be discussed in Section 4.

## **3. Result of measuring the pressure coefficient**

### **3.1 Model 4210 1-ohm resistors by Leeds & Northrup**

We used five Model 4210 1-ohm resistors by Leeds & Northrup to measure the pressure dependency in our present study. Their serial numbers are 1693133, 1711390, 1711391, 1724591 and 1724595. Fig. 1 shows the resistance dependency on pressure. It shows the resistance change with respect to pressure change where the resistance at 1010 hPa is used as a reference. The serial numbers 1693133 and 1724595 are within the range from 910 to 1110 hPa, while the 177390, 1711391 and 1724591 are within the range from 810 to 1110 hPa.

For all five resistors, the changes of resistance with respect to pressure were substantially linear, and were reproducible. Table 1 shows the pressure coefficient obtained by linear approximation. The temperature during the measurement was 25 °C. Table 2 shows the temperature coefficient at 25 °C.

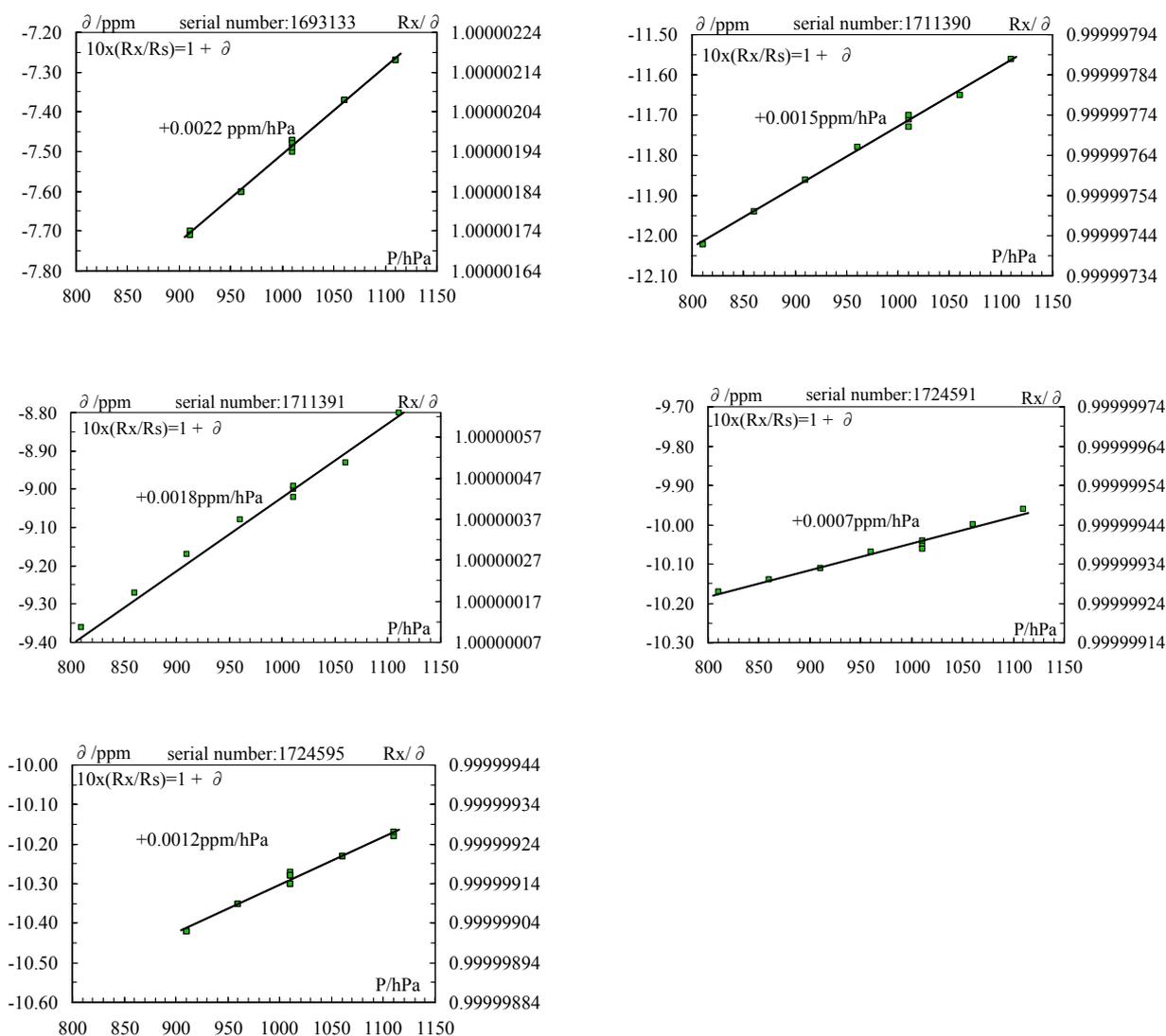


Fig. 1. Pressure dependence of the Leeds & Northrup made 1-ohm resistors of model 4210. Vertical axes show the resistance change from that measured at 1010 hPa. Pressure was changed as following procedure when measuring the resistors of serial numbers of 1693133 and 1724595: 1010 hPa  $\rightarrow$  960 hPa  $\rightarrow$  910 hPa  $\rightarrow$  1010 hPa  $\rightarrow$  1060 hPa  $\rightarrow$  1110 hPa  $\rightarrow$  1010 hPa  $\rightarrow$  1010 hPa  $\rightarrow$  960 hPa  $\rightarrow$  910 hPa  $\rightarrow$  1010 hPa  $\rightarrow$  1060 hPa  $\rightarrow$  1110 hPa  $\rightarrow$  1010 hPa. Resistance change was measured after 30 minutes for stabilizing the temperature. Pressure was changed as follows when measuring the resistors of serial numbers of 1711390, 1711391, 1724591: 1010 hPa  $\rightarrow$  810 hPa  $\rightarrow$  860 hPa  $\rightarrow$  910 hPa  $\rightarrow$  960 hPa  $\rightarrow$  1010 hPa  $\rightarrow$  1110 hPa  $\rightarrow$  1060 hPa  $\rightarrow$  1010 hPa. Resistance change was measured after 10 minutes for stabilizing the temperature.

Serial No.	Pressure coefficient ( n $\Omega$ / hPa )	Pressure range ( hPa )
1693133	+2.2	910-1110
1711390	+1.5	810-1110
1711391	+1.8	810-1110
1724591	+0.7	810-1110
1724595	+1.2	910-1110

Table 1. Pressure coefficients of the Leeds & Northrup made 1-ohm resistors of model 4210.

Serial No.	$\alpha_{25}$ / ( $\mu \Omega$ / $^{\circ}\text{C}$ )	$\beta$ / ( $\mu \Omega$ / $^{\circ}\text{C}^2$ )
1693133	+2.2	-0.48
1711390	+2.2	-0.50
1711391	+2.1	-0.51
1724591	+2.4	-0.50
1724595	+2.2	-0.50

Table 2. Temperature coefficients of the Leeds&Northrup made 1-ohm resistors of model 4210.

The following describes a brief estimation of the causes for the change of pressure applied to the resistance and its impact: In many cases, atmospheric pressure is not controlled in the room where calibration is performed, so atmospheric pressure is subject to change according to the weather. Table 3 shows the changes in resistance with respect to changes in atmospheric pressure at 50 hPa. As is apparent from the fact that pressure dependency can be measured by using deep oil bathy, the pressure applied to the resistor changes, depending on the position of the resistor placed in the oil bath. The specific weight of the oil used in the oil bath is about 0.8. Accordingly, if the position in the oil bath is 10 cm deeper, the pressure on the resistor changes +7.8 hPa. Table 4 shows the change in resistance with respect to this change.

Serial No.	Resistance change ( $\mu \Omega$ )
1693133	+0.110
1711390	+0.075
1711391	+0.090
1724591	+0.035
1724595	+0.060

Table 3. Resistance changes correspond to the pressure change of 50 hPa.

Serial No.	Resistance change ( $\mu \Omega$ )
1693133	+0.110
1711390	+0.075
1711391	+0.090
1724591	+0.035
1724595	+0.060

Table 4. Pressure to the resistor increases +7.8 hPa when its position in the oil bath is +10 cm deeper. Resistance changes correspond to this pressure change.

### 3.2 Yokogawa' Model 2781 1-ohm resistor

We used five Model 2781 1-ohm resistors by Yokogawa to measure the pressure dependency in our present study. We measured the changes in resistance with respect to changes in pressure within the range from 810 to 1010 hPa. For all resistors, the changes in resistance were below the uncertainty of measurement. The temperature was 23 °C during measurement.

## 4. Discussion on the uncertainty of measurement

### 4.1 Impact of uncertainty of resistance measurement and uncertainty of pressure measurement

Since changes in resistance with respect to changes in pressure are measured, offset or the like which is reproduced for every measurement does not constitute a factor of uncertainty in the measurement of pressure coefficient, as discussed earlier. Random fluctuation due to noise or detection limit of the detector constitutes uncertainty in the measurement. The following describes the pressure coefficient. Further, pressure measurement or changes in pressure during measurement also constitute a factor of uncertainty. Since the pressure coefficient is calculated in terms of linear approximation, the following will discuss the uncertainty in obtaining the pressure coefficient from two measurements as the simplest case.

Assume that  $r_A$  is the change of resistance when the pressure is changed from 1010 hPa to  $P_A$ , and  $r_B$  is the change when changed to  $P_B$ . Since other measurement conditions are the same, the change in pressure can be considered as constant ( $\delta P$ ) and uncertainty in resistance measurement as constant ( $\delta r$ ). The measured value in measurement A is ( $P_A \pm \delta P$ ,  $r_A \pm \delta r$ ), and that in measurement B is ( $P_B \pm \delta P$ ,  $r_B \pm \delta r$ ). Needless to say, the average ( $v_{ave}$ ) of pressure coefficient obtained from these two measurements can be expressed as follows:

$$v_{ave} = \frac{r_A - r_B}{P_A - P_B} \quad (1)$$

The maximum value is:

$$v_{max} = \frac{(r_A + \delta_r) - (r_B - \delta_r)}{(P_A - \delta P) - (P_B + \delta P)} \quad (2)$$

Using  $v_{ave}$ , the  $v_{max}$  can be obtained as follows:

$$v_{max} \simeq v_{ave} \left(1 + \frac{2\delta_r}{r_A - r_B}\right) \left(1 + \frac{2\delta P}{P_A - P_B}\right) \quad (3)$$

Similarly, the minimum value ( $v_{min}$ ) can be expressed by the following equation:

$$v_{min} \simeq v_{ave} \left(1 - \frac{2\delta_r}{r_A - r_B}\right) \left(1 - \frac{2\delta P}{P_A - P_B}\right) \quad (4)$$

From these two equations, uncertainty ( $\delta v$ ) in the measurement of pressure coefficient can be estimated as follows:

$$\delta v \simeq \frac{2\delta_r}{P_A - P_B} + \frac{2(r_A - r_B) \times \delta P}{(P_A - P_B)^2} \quad (5)$$

The equation is simplified as  $\delta r \ll r_A, r_B, (r_A - r_B)$  using  $\delta P \ll P_A, P_B, (P_A - P_B)$ . Uncertainty in the measurement of pressure coefficient can be estimated at 0.1 nΩ/hPa in terms of the maximum value by substituting the measurement condition of the Leeds & Northrup's Model 4210 1-ohm resistor and the result of measuring the pressure coefficient (Table 1),  $\delta = 10$  nΩ,  $\delta P = 0.3$  hPa in the equation. The same applies to the uncertainty in the measurement of the pressure coefficient of Yokogawa's Model 2781 1-ohm resistor.

#### 4.2 Impact of expansion and compression

In the measurement conducted in the present study, the pressure of air in the pressure vessel was increased or decreased to change the pressure to be applied to the resistor. Air is about 2 cm high in the upper part of the pressure vessel, and the interior of the pressure vessel is close to a regular square having a side of 13 cm. Thus, the volume of the air is about 338 cm<sup>3</sup> (= 13 x 13 x 2). This corresponds to 0.015 moles.

Generally, gas temperature will be changed if the pressure of gas is increased or decreased in a vessel having a predetermined capacity. This temperature change causes the temperature of the vessel wall and oil inside the vessel to be changed, and temperature change is transmitted from the vessel wall to the surrounding oil in the final phase. This process of heat conduction is complicated, and it is impossible to make an accurate estimation on how the temperature distribution in the vessel changes with time.

So assuming that the vessel is thermally shut off from the surrounding, we made a simple estimation of the extent to which the temperature of internal air having changed due to heat-insulated compression and expansion changed the temperature of the interior including the vessel proper. The change of temperature is about 1 mK or less.

As discussed earlier, the temperature of the oil bath fluctuates several mK, and this makes it possible to ignore the temperature change due to air compression and expansion.

## 5. Conclusion

We measured pressure dependency of two types of double-walled 1-ohm resistors. The resistance of five Leeds & Northrup's Model 4210 1-ohm resistors underwent an almost linear change in the range from 810 hPa to 1110 hPa. The pressure coefficient was +0.7 n $\Omega$ /hPa at the minimum and +2.2 n $\Omega$ /hPa at the maximum. We also measured the resistance of five Yokogawa's Model 2781 1-ohm resistors in the same pressure range, but could not detect any change in resistance. Changes in resistance with respect to changes in pressure were shown to be reproducible, so it is possible to consider that there is no irreversible change (history) even if pressure change has occurred during the transport by air. When there is a sudden change in temperature, resistance does not get back immediately to the original level even if temperature is set back to the original level. It gets back to the original resistance only gradually, and history is said to remain.

According to the experience of the authors of the present paper, there was a sudden change in resistance value when the temperature of the Yokogawa's 1-ohm resistor was changed about 8 °C, and it returned to the original resistance slowly in one year or thereabout. When the double-walled 1-ohm resistor is transported, the temperature change will have to be made smaller than pressure change.

Uncertainty in the present measurement is 0.1 n $\Omega$ /hPa or less. A simple barometer can measure the atmospheric pressure at a resolution of 1 hPa. The uncertainty of pressure compensation is equivalent to  $1 \times 10^{-10}$  at a resistance of 0.1 n $\Omega$  or 1  $\Omega$ . It is of the order of  $10^{-9}$  even when uncertainty is the minimum in the current measurement of resistance. So uncertainty of this extent is sufficient.

## References

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