

GNU MP

The GNU Multiple Precision Arithmetic Library
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1 Introduction to MP

GNU MP is a portable library for arbitrary precision integer and rational number arithmetic.¹ It aims to provide the fastest possible arithmetic for all applications that need more than two words of integer precision.

Most often, applications tend to use just a few words of precision; but some applications may need thousands of words. GNU MP is designed to give good performance for both kinds of applications, by choosing algorithms based on the sizes of the operands.

There are five groups of functions in the MP library:

1. Functions for signed integer arithmetic, with names beginning with `mpz_`.
2. Functions for rational number arithmetic, with names beginning with `mpq_`.
3. Functions compatible with Berkeley MP, such as `itom`, `madd`, and `mult`.
4. Fast low-level functions that operate on natural numbers. These are used by the functions in the preceding groups, and you can also call them directly from very time-critical user programs. These functions' names begin with `mpn_`.
5. Miscellaneous functions.

As a general rule, all MP functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator. (The BSD MP compatibility functions disobey this rule, having the output argument(s) last.) Multi-precision numbers, whether output or input, are always passed as addresses to the declared type.

1.1 Nomenclature and Data Types

In this manual, *integer* means a multiple precision integer, as used in the MP package. The C data type for such integers is `MP_INT`. For example:

```
MP_INT sum;

struct foo { MP_INT x, y; };

MP_INT vec[20];
```

Rational number means a multiple precision fraction. The C data type for these fractions is `MP_RAT`. For example:

```
MP_RAT quotient;
```

A *limb* means the part of a multi-precision number that fits in a single word. (We chose this word because a limb of the human body is analogous to a digit, only larger, and containing several digits.) Normally a limb contains 32 bits.

1.2 Thanks

I would like to thank Gunnar Sjoedin and Hans Riesel for their help with mathematical problems, Richard Stallman for his help with design issues and for revising this manual, Brian Beuning and Doug Lea for their testing of various versions of the library, and Joachim Hollman for his many valuable suggestions.

¹ The limit of the precision is set by the available memory in your computer.

Special thanks to Brian Beuning, he has shaken out many bugs from early versions of the code!

John Amanatides of York University in Canada contributed the function `mpz_probab_prime_p`.

2 Initialization

Before you can use a variable or object of type `MP_INT` or `MP_RAT`, you must initialize it. This fills in the components that point to dynamically allocated space for the limbs of the number.

When you are finished using the object, you should clear out the object. This frees the dynamic space that it points to, so the space can be used again.

Once you have initialized the object, you need not be concerned about allocating additional space. The functions in the MP package automatically allocate additional space when the object does not already have enough space. They do not, however, reduce the space in use when a smaller number is stored in the object. Most of the time, this policy is best, since it avoids frequent re-allocation. If you want to reduce the space in an object to the minimum needed, you can do `_mpz_realloc (&object, mpz_size (&object))`.

The functions to initialize numbers are `mpz_init` (for `MP_INT`) and `mpq_init` (for `MP_RAT`).

`mpz_init` allocates space for the limbs, and stores a pointer to that space in the `MP_INT` object. It also stores the value 0 in the object.

In the same manner, `mpq_init` allocates space for the numerator and denominator limbs, and stores pointers to these spaces in the `MP_RAT` object.

To clear out a number object, use `mpz_clear` and `mpq_clear`, respectively.

Here is an example of use:

```
{
    MP_INT temp;
    mpz_init (&temp);

    ... store and read values in temp zero or more times ...

    mpz_clear (&temp);
}
```

You might be tempted to copy an integer from one object to another like this:

```
MP_INT x, y;

x = y;
```

Although valid C, **this is an error**. Rather than copying the integer value from `y` to `x` it will make the two variables share storage. Subsequent assignments to one variable would change the other mysteriously. And if you were to clear out both variables subsequently, you would confuse `malloc` and cause your program to crash.

To copy the value properly, you must use the function `mpz_set`. (see Section 3.1.1 [Assigning Integers], page 8)

3 Integer Functions

This chapter describes the MP functions for performing integer arithmetic.

The integer functions use arguments and values of type pointer-to-MP_INT (see Section 1.1 [Nomenclature], page 3). The type MP_INT is a structure, but applications should not refer directly to its components. Include the header `gmp.h` to get the definition of MP_INT.

3.1 Initializing Integer Objects

Most of the functions for integer arithmetic assume that the output is stored in an object already initialized. For example, `mpz_add` stores the result of addition (see Section 3.3 [Integer Arithmetic], page 10). Thus, you must initialize the object before storing the first value in it. You can do this separately by calling the function `mpz_init`.

void `mpz_init` (MP_INT **integer*) [Function]
 Initialize *integer* with limb space and set the initial numeric value to 0. Each variable should normally only be initialized once, or at least cleared out (using `mpz_clear`) between each initialization.

Here is an example of using `mpz_init`:

```
{
    MP_INT integ;
    mpz_init (&integ);
    ...
    mpz_add (&integ, ...);
    ...
    mpz_sub (&integ, ...);

    /* Unless you are now exiting the program, do ... */
    mpz_clear (&integ);
}
```

As you can see, you can store new values any number of times, once an object is initialized.

void `mpz_clear` (MP_INT **integer*) [Function]
 Free the limb space occupied by *integer*. Make sure to call this function for all MP_INT variables when you are done with them.

void * `_mpz_realloc` (MP_INT **integer*, mp_size *new_alloc*) [Function]
 Change the limb space allocation to *new_alloc* limbs. This function is not normally called from user code, but it can be used to give memory back to the heap, or to increase the space of a variable to avoid repeated automatic re-allocation.

void `mpz_array_init` (MP_INT *integer_array*[], size_t *array_size*, mp_size *fixed_num_limbs*) [Function]
 Allocate **fixed** limb space for all *array_size* integers in *integer_array*. The fixed allocation for each integer in the array is *fixed_num_limbs*. This function is useful for decreasing the working set for some algorithms that use large integer arrays. If the

fixed space will be insufficient for storing the result of a subsequent calculation, the result is unpredictable.

There is no way to de-allocate the storage allocated by this function. Don't call `mpz_clear`!

3.1.1 Integer Assignment Functions

These functions assign new values to already initialized integers (see Section 3.1 [Initializing Integers], page 7).

`void mpz_set (MP_INT *dest_integer, MP_INT *src_integer)` [Function]
Assign *dest_integer* from *src_integer*.

`void mpz_set_ui (MP_INT *integer, unsigned long int initial_value)` [Function]
Set the value of *integer* from *initial_value*.

`void mpz_set_si (MP_INT *integer, signed long int initial_value)` [Function]
Set the value of *integer* from *initial_value*.

`int mpz_set_str (MP_INT *integer, char *initial_value, int base)` [Function]

Set the value of *integer* from *initial_value*, a '\0'-terminated C string in base *base*. White space is allowed in the string, and is simply ignored. The base may vary from 2 to 36. If *base* is 0, the actual base is determined from the leading characters: if the first two characters are '0x' or '0X', hexadecimal is assumed, otherwise if the first character is '0', octal is assumed, otherwise decimal is assumed.

This function returns 0 if the entire string up to the '\0' is a valid number in base *base*. Otherwise it returns -1.

3.1.2 Combined Initialization and Assignment Functions

For your convenience, MP provides a parallel series of initialize-and-set arithmetic functions which initialize the output and then store the value there. These functions' names have the form `mpz_init_set...`

Here is an example of using one:

```
{
    MP_INT integ;
    mpz_init_set_str (&integ, "3141592653589793238462643383279502884", 10);
    ...
    mpz_sub (&integ, ...);

    mpz_clear (&integ);
}
```

Once the integer has been initialized by any of the `mpz_init_set...` functions, it can be used as the source or destination operand for the ordinary integer functions. Don't use an initialize-and-set function on a variable already initialized!

`void mpz_init_set (MP_INT *dest_integer, MP_INT
 *src_integer)` [Function]

Initialize *dest_integer* with limb space and set the initial numeric value from *src_integer*.

`void mpz_init_set_ui (MP_INT *dest_integer, unsigned long
 int src_ulong)` [Function]

Initialize *dest_integer* with limb space and set the initial numeric value from *src_ulong*.

`void mpz_init_set_si (MP_INT *dest_integer, signed long
 int src_slong)` [Function]

Initialize *dest_integer* with limb space and set the initial numeric value from *src_slong*.

`int mpz_init_set_str (MP_INT *dest_integer, char
 *src_cstring, int base)` [Function]

Initialize *dest_integer* with limb space and set the initial numeric value from *src_cstring*, a '\0'-terminated C string in base *base*. The base may vary from 2 to 36. There may be white space in the string.

If the string is a correct base *base* number, the function returns 0; if an error occurs it returns -1. *dest_integer* is initialized even if an error occurs. (I.e., you have to call `mpz_clear` for it.)

3.2 Conversion Functions

`unsigned long int mpz_get_ui (MP_INT *src_integer)` [Function]

Return the least significant limb from *src_integer*. This function together with `mpz_div_2exp(..., src_integer, CHAR_BIT*sizeof(unsigned long int))` can be used to extract the limbs of an integer efficiently.

`signed long int mpz_get_si (MP_INT *src_integer)` [Function]

If *src_integer* fits into a `signed long int` return the value of *src_integer*. Otherwise return the least significant bits of *src_integer*, with the same sign as *src_integer*.

`char * mpz_get_str (char *string, int base, MP_INT
 *integer)` [Function]

Convert *integer* to a '\0'-terminated C string in *string*, using base *base*. The base may vary from 2 to 36. If *string* is NULL, space for the string is allocated using the default allocation function.

If *string* is not NULL, it should point to a block of storage enough large for the result. To find out the right amount of space to provide for *string*, use `mpz_sizeinbase(integer, base) + 2`. The "+ 2" is for a possible minus sign, and for the terminating null character. (see Chapter 7 [Miscellaneous Functions], page 21).

This function returns a pointer to the result string.

3.3 Integer Arithmetic Functions

`void mpz_add (MP_INT *sum, MP_INT *addend1, MP_INT *addend2)` [Function]

`void mpz_add_ui (MP_INT *sum, MP_INT *addend1, unsigned long int addend2)` [Function]
Set *sum* to *addend1* + *addend2*.

`void mpz_sub (MP_INT *difference, MP_INT *minuend, MP_INT *subtrahend)` [Function]

`void mpz_sub_ui (MP_INT *difference, MP_INT *minuend, unsigned long int subtrahend)` [Function]
Set *difference* to *minuend* – *subtrahend*.

`void mpz_mul (MP_INT *product, MP_INT *multiplier, MP_INT *multiplicand)` [Function]

`void mpz_mul_ui (MP_INT *product, MP_INT *multiplier, unsigned long int multiplicand)` [Function]
Set *product* to *multiplier* times *multiplicand*.

Division is undefined if the divisor is zero, and passing a zero divisor to the divide or modulo functions, as well passing a zero mod argument to the powm functions, will make these functions intentionally divide by zero. This gives the user the possibility to handle arithmetic exceptions in these functions in the same manner as other arithmetic exceptions.

`void mpz_div (MP_INT *quotient, MP_INT *dividend, MP_INT *divisor)` [Function]

`void mpz_div_ui (MP_INT *quotient, MP_INT *dividend, unsigned long int divisor)` [Function]
Set *quotient* to *dividend* / *divisor*. The quotient is rounded towards 0.

`void mpz_mod (MP_INT *remainder, MP_INT *dividend, MP_INT *divisor)` [Function]

`void mpz_mod_ui (MP_INT *remainder, MP_INT *dividend, unsigned long int divisor)` [Function]
Divide *dividend* and *divisor* and put the remainder in *remainder*. The remainder has the same sign as the dividend, and its absolute value is less than the absolute value of the divisor.

`void mpz_divmod (MP_INT *quotient, MP_INT *remainder, MP_INT *dividend, MP_INT *divisor)` [Function]

`void mpz_divmod_ui (MP_INT *quotient, MP_INT *remainder, MP_INT *dividend, unsigned long int divisor)` [Function]
Divide *dividend* and *divisor* and put the quotient in *quotient* and the remainder in *remainder*. The quotient is rounded towards 0. The remainder has the same sign as the dividend, and its absolute value is less than the absolute value of the divisor.

If *quotient* and *remainder* are the same variable, the results are not defined.

`void mpz_mdiv (MP_INT *quotient, MP_INT *dividend, MP_INT *divisor)` [Function]

`void mpz_mdiv_ui (MP_INT *quotient, MP_INT *dividend, unsigned long int divisor)` [Function]

Set *quotient* to *dividend* / *divisor*. The quotient is rounded towards $-\infty$.

`void mpz_mmod (MP_INT *remainder, MP_INT *dividend, MP_INT *divisor)` [Function]

`unsigned long int mpz_mmod_ui (MP_INT *remainder, MP_INT *dividend, unsigned long int divisor)` [Function]

Divide *dividend* and *divisor* and put the remainder in *remainder*. The remainder is always positive, and its value is less than the value of the divisor.

For `mpz_mmod_ui` the remainder is returned, and if *remainder* is not NULL, also stored there.

`void mpz_mddivmod (MP_INT *quotient, MP_INT *remainder, MP_INT *dividend, MP_INT *divisor)` [Function]

`unsigned long int mpz_mddivmod_ui (MP_INT *quotient, MP_INT *remainder, MP_INT *dividend, unsigned long int divisor)` [Function]

Divide *dividend* and *divisor* and put the quotient in *quotient* and the remainder in *remainder*. The quotient is rounded towards $-\infty$. The remainder is always positive, and its value is less than the value of the divisor.

For `mpz_mddivmod_ui` the remainder is small enough to fit in an `unsigned long int`, and is therefore returned. If *remainder* is not NULL, the remainder is also stored there.

If *quotient* and *remainder* are the same variable, the results are not defined.

`void mpz_sqrt (MP_INT *root, MP_INT *operand)` [Function]

Set *root* to the square root of *operand*. The result is rounded towards zero.

`void mpz_sqrtrem (MP_INT *root, MP_INT *remainder, MP_INT *operand)` [Function]

Set *root* to the square root of *operand*, as with `mpz_sqrt`. Set *remainder* to *operand* – *root*², (i.e. zero if *operand* is a perfect square).

If *root* and *remainder* are the same variable, the results are not defined.

`int mpz_perfect_square_p (MP_INT *square)` [Function]

Return non-zero if *square* is perfect, i.e. if the square root of *square* is integral. Return zero otherwise.

`int mpz_probab_prime_p (MP_INT *n, int reps)` [Function]

An implementation of the probabilistic primality test found in Knuth's *Seminumerical Algorithms* book. If the function `mpz_probab_prime_p(n, reps)` returns 0 then *n* is not prime. If it returns 1, then *n* is 'probably' prime. The probability of a false positive is $(1/4)^{reps}$, where *reps* is the number of internal passes of the probabilistic algorithm. Knuth indicates that 25 passes are reasonable.

void mpz_powm (MP_INT *res, MP_INT *base, MP_INT *exp, [Function]
MP_INT *mod)

void mpz_powm_ui (MP_INT *res, MP_INT *base, unsigned long [Function]
int exp, MP_INT *mod)
Set *res* to (*base* raised to *exp*) modulo *mod*. If *exp* is negative, the result is undefined.

void mpz_pow_ui (MP_INT *res, MP_INT *base, unsigned long [Function]
int exp)
Set *res* to *base* raised to *exp*.

void mpz_fac_ui (MP_INT *res, unsigned long int n) [Function]
Set *res* $n!$, the factorial of *n*.

void mpz_gcd (MP_INT *res, MP_INT *operand1, MP_INT [Function]
*operand2)
Set *res* to the greatest common divisor of *operand1* and *operand2*.

void mpz_gcdext (MP_INT *g, MP_INT *s, MP_INT *t, MP_INT [Function]
*a, MP_INT *b)
Compute *g*, *s*, and *t*, such that $as + bt = g = \text{gcd}(a, b)$. If *t* is NULL, that argument is not computed.

void mpz_neg (MP_INT *negated_operand, MP_INT *operand) [Function]
Set *negated_operand* to $-operand$.

void mpz_abs (MP_INT *positive_operand, MP_INT [Function]
*signed_operand)
Set *positive_operand* to the absolute value of *signed_operand*.

int mpz_cmp (MP_INT *operand1, MP_INT *operand2) [Function]

int mpz_cmp_ui (MP_INT *operand1, unsigned long int [Function]
operand2)

int mpz_cmp_si (MP_INT *operand1, signed long int operand2) [Function]
Compare *operand1* and *operand2*. Return a positive value if *operand1* > *operand2*, zero if *operand1* = *operand2*, and a negative value if *operand1* < *operand2*.

void mpz_mul_2exp (MP_INT *product, MP_INT *multiplier, [Function]
unsigned long int exponent_of_2)
Set *product* to *multiplier* times 2 raised to *exponent_of_2*. This operation can also be defined as a left shift, *exponent_of_2* steps.

void mpz_div_2exp (MP_INT *quotient, MP_INT *dividend, [Function]
unsigned long int exponent_of_2)
Set *quotient* to *dividend* divided by 2 raised to *exponent_of_2*. This operation can also be defined as a right shift, *exponent_of_2* steps, but unlike the >> operator in C, the result is rounded towards 0.

`void mpz_mod_2exp (MP_INT *remainder, MP_INT *dividend, [Function]
 unsigned long int exponent_of_2)`

Set *remainder* to *dividend* mod (2 raised to *exponent_of_2*). The sign of *remainder* will have the same sign as *dividend*.

This operation can also be defined as a masking of the *exponent_of_2* least significant bits.

3.4 Logical Functions

`void mpz_and (MP_INT *conjunction, MP_INT *operand1, [Function]
 MP_INT *operand2)`

Set *conjunction* to *operand1* logical-and *operand2*.

`void mpz_ior (MP_INT *disjunction, MP_INT *operand1, [Function]
 MP_INT *operand2)`

Set *disjunction* to *operand1* inclusive-or *operand2*.

`void mpz_xor (MP_INT *disjunction, MP_INT *operand1, [Function]
 MP_INT *operand2)`

Set *disjunction* to *operand1* exclusive-or *operand2*.

This function is missing in the current release.

`void mpz_com (MP_INT *complemented_operand, MP_INT [Function]
 *operand)`

Set *complemented_operand* to the one's complement of *operand*.

3.5 Input and Output Functions

Functions that perform input from a standard I/O stream, and functions for output conversion.

`void mpz_inp_raw (MP_INT *integer, FILE *stream) [Function]`
Input from standard I/O stream *stream* in the format written by `mpz_out_raw`, and put the result in *integer*.

`void mpz_inp_str (MP_INT *integer, FILE *stream, int base) [Function]`
Input a string in base *base* from standard I/O stream *stream*, and put the read integer in *integer*. The base may vary from 2 to 36. If *base* is 0, the actual base is determined from the leading characters: if the first two characters are '0x' or '0X', hexadecimal is assumed, otherwise if the first character is '0', octal is assumed, otherwise decimal is assumed.

`void mpz_out_raw (FILE *stream, MP_INT *integer) [Function]`
Output *integer* on standard I/O stream *stream*, in raw binary format. The integer is written in a portable format, with 4 bytes of size information, and that many bytes of limbs. Both the size and the limbs are written in decreasing significance order.

`void mpz_out_str (FILE *stream, int base, MP_INT *integer) [Function]`
Output *integer* on standard I/O stream *stream*, as a string of digits in base *base*. The base may vary from 2 to 36.

4 Rational Number Functions

All rational arithmetic functions canonicalize the result, so that the denominator and the numerator have no common factors. Zero has the unique representation 0/1.

The set of functions is quite small. Maybe it will be extended in a future release.

- `void mpq_init (MP_RAT *dest_rational)` [Function]
 Initialize *dest_rational* with limb space and set the initial numeric value to 0/1. Each variable should normally only be initialized once, or at least cleared out (using the function `mpq_clear`) between each initialization.
- `void mpq_clear (MP_RAT *rational_number)` [Function]
 Free the limb space occupied by *rational_number*. Make sure to call this function for all MP_RAT variables when you are done with them.
- `void mpq_set (MP_RAT *dest_rational, MP_RAT *src_rational)` [Function]
 Assign *dest_rational* from *src_rational*.
- `void mpq_set_ui (MP_RAT *rational_number, unsigned long int numerator, unsigned long int denominator)` [Function]
 Set the value of *rational_number* to *numerator*/*denominator*. If *numerator* and *denominator* have common factors, they are divided out before *rational_number* is assigned.
- `void mpq_set_si (MP_RAT *rational_number, signed long int numerator, unsigned long int denominator)` [Function]
 Like `mpq_set_ui`, but *numerator* is signed.
- `void mpq_add (MP_RAT *sum, MP_RAT *addend1, MP_RAT *addend2)` [Function]
 Set *sum* to *addend1* + *addend2*.
- `void mpq_sub (MP_RAT *difference, MP_RAT *minuend, MP_RAT *subtrahend)` [Function]
 Set *difference* to *minuend* – *subtrahend*.
- `void mpq_mul (MP_RAT *product, MP_RAT *multiplicator, MP_RAT *multiplicand)` [Function]
 Set *product* to *multiplicator* * *multiplicand*
- `void mpq_div (MP_RAT *quotient, MP_RAT *dividend, MP_RAT *divisor)` [Function]
 Set *quotient* to *dividend* / *divisor*.
- `void mpq_neg (MP_RAT *negated_operand, MP_RAT *operand)` [Function]
 Set *negated_operand* to –*operand*.
- `int mpq_cmp (MP_RAT *operand1, MP_RAT *operand2)` [Function]
 Compare *operand1* and *operand2*. Return a positive value if *operand1* > *operand2*, zero if *operand1* = *operand2*, and a negative value if *operand1* < *operand2*.

`void mpq_inv (MP_RAT *inverted_number, MP_RAT *number)` [Function]
 Invert *number* by swapping the numerator and denominator. If the new denominator becomes zero, this routine will divide by zero.

`void mpq_set_num (MP_RAT *rational_number, MP_INT *numerator)` [Function]
 Make *numerator* become the numerator of *rational_number* by copying.

`void mpq_set_den (MP_RAT *rational_number, MP_INT *denominator)` [Function]
 Make *denominator* become the denominator of *rational_number* by copying. If *denominator* < 0 the denominator of *rational_number* is set to the absolute value of *denominator*, and the sign of the numerator of *rational_number* is changed.

`void mpq_get_num (MP_INT *numerator, MP_RAT *rational_number)` [Function]
 Copy the numerator of *rational_number* to the integer *numerator*, to prepare for integer operations on the numerator.

`void mpq_get_den (MP_INT *denominator, MP_RAT *rational_number)` [Function]
 Copy the denominator of *rational_number* to the integer *denominator*, to prepare for integer operations on the denominator.

5 Low-level Functions

The next release of the GNU MP library (2.0) will include changes to some `mpn` functions. Programs that use these functions according to the descriptions below will therefore not work with the next release.

The low-level function layer is designed to be as fast as possible, **not** to provide a coherent calling interface. The different functions have similar interfaces, but there are variations that might be confusing. These functions do as little as possible apart from the real multiple precision computation, so that no time is spent on things that not all callers need.

A source operand is specified by a pointer to the least significant limb and a limb count. A destination operand is specified by just a pointer. It is the responsibility of the caller to ensure that the destination has enough space for storing the result.

With this way of specifying source operands, it is possible to perform computations on subranges of an argument, and store the result into a subrange of a destination.

All these functions require that the operands are normalized in the sense that the most significant limb must be non-zero. (A future release of might drop this requirement.)

The low-level layer is the base for the implementation of the `mpz_` and `mpq_` layers.

The code below adds the number beginning at `src1_ptr` and the number beginning at `src2_ptr` and writes the sum at `dest_ptr`. A constraint for `mpn_add` is that `src1_size` must not be smaller than `src2_size`.

```
mpn_add (dest_ptr, src1_ptr, src1_size, src2_ptr, src2_size)
```

In the description below, a source operand is identified by the pointer to the least significant limb, and the limb count in braces.

```
mp_size mpn_add (mp_ptr dest_ptr, mp_srcptr src1_ptr, [Function]
                 mp_size src1_size, mp_srcptr src2_ptr, mp_size src2_size)
```

Add $\{src1_ptr, src1_size\}$ and $\{src2_ptr, src2_size\}$, and write the `src1_size` least significant limbs of the result to `dest_ptr`. Carry-out, either 0 or 1, is returned.

This function requires that `src1_size` is greater than or equal to `src2_size`.

```
mp_size mpn_sub (mp_ptr dest_ptr, mp_srcptr src1_ptr, [Function]
                 mp_size src1_size, mp_srcptr src2_ptr, mp_size src2_size)
```

Subtract $\{src2_ptr, src2_size\}$ from $\{src1_ptr, src1_size\}$, and write the result to `dest_ptr`.

Return 1 if the minuend < the subtrahend. Otherwise, return the negative difference between the number of words in the result and the minuend. I.e. return 0 if the result has `src1_size` words, -1 if it has `src1_size - 1` words, etc.

This function requires that `src1_size` is greater than or equal to `src2_size`.

```
mp_size mpn_mul (mp_ptr dest_ptr, mp_srcptr src1_ptr, [Function]
                 mp_size src1_size, mp_srcptr src2_ptr, mp_size src2_size)
```

Multiply $\{src1_ptr, src1_size\}$ and $\{src2_ptr, src2_size\}$, and write the result to `dest_ptr`. The exact size of the result is returned.

The destination has to have space for `src1_size + src1_size` limbs, even if the result might be one limb smaller.

This function requires that *src1_size* is greater than or equal to *src2_size*. The destination must be distinct from either input operands.

`mp_size mpn_div (mp_ptr dest_ptr, mp_ptr src1_ptr, mp_size [Function]
src1_size, mp_srcptr src2_ptr, mp_size src2_size)`

Divide $\{src1_ptr, src1_size\}$ by $\{src2_ptr, src2_size\}$, and write the quotient to *dest_ptr*, and the remainder to *src1_ptr*.

Return 0 if the quotient size is at most $(src1_size - src2_size)$, and 1 if the quotient size is at most $(src1_size - src2_size + 1)$. The caller has to check the most significant limb to find out the exact size.

The most significant bit of the most significant limb of the divisor has to be set.

This function requires that *src1_size* is greater than or equal to *src2_size*. The quotient, pointed to by *dest_ptr*, must be distinct from either input operands.

`mp_limb mpn_lshift (mp_ptr dest_ptr, mp_srcptr src_ptr, [Function]
mp_size src_size, unsigned long int count)`

Shift $\{src_ptr, src_size\}$ *count* bits to the left, and write the *src_size* least significant limbs of the result to *dest_ptr*. *count* might be in the range 1 to $n - 1$, on an *n*-bit machine. The limb shifted out is returned.

Overlapping of the destination space and the source space is allowed in this function, provided *dest_ptr* \geq *src_ptr*.

`mp_size mpn_rshift (mp_ptr dest_ptr, mp_srcptr src_ptr, [Function]
mp_size src_size, unsigned long int count)`

Shift $\{src_ptr, src_size\}$ *count* bits to the right, and write the *src_size* least significant limbs of the result to *dest_ptr*. *count* might be in the range 1 to $n - 1$, on an *n*-bit machine. The size of the result is returned.

Overlapping of the destination space and the source space is allowed in this function, provided *dest_ptr* \leq *src_ptr*.

`mp_size mpn_rshiftci (mp_ptr dest_ptr, mp_srcptr src_ptr, [Function]
mp_size src_size, unsigned long int count, mp_limb inlimb)`

Like *mpn_rshift*, but use *inlimb* to feed the least significant end of the destination.

`int mpn_cmp (mp_srcptr src1_ptr, mp_srcptr src2_ptr, [Function]
mp_size size)`

Compare $\{src1_ptr, size\}$ and $\{src2_ptr, size\}$ and return a positive value if *src1* > *src2*, 0 if they are equal, and a negative value if *src1* < *src2*.

6 Berkeley MP Compatible Functions

These functions are intended to be fully compatible with the Berkeley MP library which is available on many BSD derived U*ix systems.

The original Berkeley MP library has a usage restriction: you cannot use the same variable as both source and destination in a single function call. The compatible functions in GNU MP do not share this restriction—inputs and outputs may overlap.

It is not recommended that new programs are written using these functions. Apart from the incomplete set of functions, the interface for initializing MINT objects is more error prone, and the `pow` function collides with `pow` in `libm.a`.

Include the header `mp.h` to get the definition of the necessary types and functions. If you are on a BSD derived system, make sure to include GNU `mp.h` if you are going to link the GNU `libmp.a` to you program. This means that you probably need to give the `-I<dir>` option to the compiler, where `<dir>` is the directory where you have GNU `mp.h`.

`MINT * itom (signed short int initial_value)` [Function]
Allocate an integer consisting of a MINT object and dynamic limb space. Initialize the integer to *initial_value*. Return a pointer to the MINT object.

`MINT * xtom (char *initial_value)` [Function]
Allocate an integer consisting of a MINT object and dynamic limb space. Initialize the integer from *initial_value*, a hexadecimal, '\0'-terminate C string. Return a pointer to the MINT object.

`void move (MINT *src, MINT *dest)` [Function]
Set *dest* to *src* by copying. Both variables must be previously initialized.

`void madd (MINT *src_1, MINT *src_2, MINT *destination)` [Function]
Add *src_1* and *src_2* and put the sum in *destination*.

`void msub (MINT *src_1, MINT *src_2, MINT *destination)` [Function]
Subtract *src_2* from *src_1* and put the difference in *destination*.

`void mult (MINT *src_1, MINT *src_2, MINT *destination)` [Function]
Multiply *src_1* and *src_2* and put the product in *destination*.

`void mdiv (MINT *dividend, MINT *divisor, MINT *quotient,
MINT *remainder)` [Function]

`void sdiv (MINT *dividend, signed short int divisor, MINT
*quotient, signed short int *remainder)` [Function]
Set *quotient* to *dividend* / *divisor*, and *remainder* to *dividend* mod *divisor*. The quotient is rounded towards zero; the remainder has the same sign as the dividend.

Some implementations of this function return a remainder whose sign is inverted if the divisor is negative. Such a definition makes little sense from a mathematical point of view. GNU MP might be considered incompatible with the traditional MP in this respect.

- `void msqrt (MINT *operand, MINT *root, MINT *remainder)` [Function]
 Set *root* to the square root of *operand*, as with `mpz_sqrt`. Set *remainder* to *operand* − *root*², (i.e. zero if *operand* is a perfect square).
- `void pow (MINT *base, MINT *exp, MINT *mod, MINT *dest)` [Function]
 Set *dest* to (*base* raised to *exp*) modulo *mod*.
- `void rpow (MINT *base, signed short int exp, MINT *dest)` [Function]
 Set *dest* to *base* raised to *exp*.
- `void gcd (MINT *operand1, MINT *operand2, MINT *res)` [Function]
 Set *res* to the greatest common divisor of *operand1* and *operand2*.
- `int mcmp (MINT *operand1, MINT *operand2)` [Function]
 Compare *operand1* and *operand2*. Return a positive value if *operand1* > *operand2*, zero if *operand1* = *operand2*, and a negative value if *operand1* < *operand2*.
- `void min (MINT *dest)` [Function]
 Input a decimal string from stdin, and put the read integer in *dest*. SPC and TAB are allowed in the number string, and are ignored.
- `void mout (MINT *src)` [Function]
 Output *src* to stdout, as a decimal string. Also output a newline.
- `char * mtox (MINT *operand)` [Function]
 Convert *operand* to a hexadecimal string, and return a pointer to the string. The returned string is allocated using the default memory allocation function, `malloc` by default. (See Chapter 2 [Initialization], page 5, for an explanation of the memory allocation in MP).
- `void mfree (MINT *operand)` [Function]
 De-allocate, the space used by *operand*. **This function should only be passed a value returned by `itom` or `xtom`.**

7 Miscellaneous Functions

`void mpz_random (MP_INT *random_integer, mp_size max_size)` [Function]

Generate a random integer of at most *max_size* limbs. The generated random number doesn't satisfy any particular requirements of randomness.

`void mpz_random2 (MP_INT *random_integer, mp_size max_size)` [Function]

Generate a random integer of at most *max_size* limbs, with long strings of zeros and ones in the binary representation. Useful for testing functions and algorithms, since this kind of random numbers have proven to be more likely to trigger bugs.

`size_t mpz_size (MP_INT *integer)` [Function]

Return the size of *integer* measured in number of limbs. If *integer* is zero, the returned value will be zero, if *integer* has one limb, the returned value will be one, etc. (See Section 1.1 [Nomenclature], page 3, for an explanation of the concept *limb*.)

`size_t mpz_sizeinbase (MP_INT *integer, int base)` [Function]

Return the size of *integer* measured in number of digits in base *base*. The base may vary from 2 to 36. The returned value will be exact or 1 too big. If *base* is a power of 2, the returned value will always be exact.

This function is useful in order to allocate the right amount of space before converting *integer* to a string. The right amount of allocation is normally two more than the value returned by `mpz_sizeinbase` (one extra for a minus sign and one for the terminating '\0').

7.1 Custom Allocation

By default, the initialization functions use `malloc`, `realloc`, and `free` to do their work. If `malloc` or `realloc` fails, the MP package terminates execution after a printing fatal error message on standard error.

In some applications, you may wish to allocate memory in other ways, or you may not want to have a fatal error when there is no more memory available. To accomplish this, you can specify alternative functions for allocating and de-allocating memory. Use `mp_set_memory_functions` to do this.

`mp_set_memory_functions` has three arguments, *allocate_function*, *reallocate_function*, and *deallocate_function*, in that order. If an argument is NULL, the corresponding default function is retained.

The functions you supply should fit the following declarations:

`void * allocate_function (size_t alloc_size)`

This function should return a pointer to newly allocated space with at least *alloc_size* storage units.

`void * reallocate_function (void *ptr, size_t old_size, size_t new_size)`

This function should return a pointer to newly allocated space of at least *new_size* storage units, after copying the first *old_size* storage units from *ptr*. It should also de-allocate the space at *ptr*.

You can assume that the space at *ptr* was formerly returned from *allocate_function* or *realloc_function*, for a request for *old_size* storage units.

`void deallocate_function (void *ptr, size_t size)`

De-allocate the space pointed to by *ptr*.

You can assume that the space at *ptr* was formerly returned from *allocate_function* or *realloc_function*, for a request for *size* storage units.

(A *storage unit* is the unit in which the `sizeof` operator returns the size of an object, normally an 8 bit byte.)

NOTE: call `mp_set_memory_functions` only before calling any other MP functions. Otherwise, the user-defined allocation functions may be asked to re-allocate or de-allocate something previously allocated by the default allocation functions.

8 Reporting Bugs

If you think you have found a bug in the GNU MP library, please investigate it and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

Please make sure that the bug is really in the GNU MP library.

You have to send us a test case that makes it possible for us to reproduce the bug.

You also have to explain what is wrong; if you get a crash, or if the results printed are not good and in that case, in what way.

Make sure that the bug report includes all information you would need to fix this kind of bug for someone else. Think twice.

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we won't do anything about it (aside of chiding you to send better bug reports).

Send your bug report to: `tege@gnu.ai.mit.edu`.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

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