Chapter 6

Solving equations

Contents

Introduction

This chapter describes how to use the HP 49G to solve equations, or systems of equations, to get a numeric solution. To solve equations for symbolic solutions, see chapter 5, “Working with expressions.”

This chapter also includes information on how to use the financial solver to calculate details on amounts of money borrowed or lent.

About solving equations

You can use the HP 49G numeric solver to solve four types of equations. You select the type of equation that you want to solve from the numeric solver choose list.

To access the numeric solver choose list, press ↱ NUM.SLV.

The following options are available from the choose list:

- Solve equation

  Use this option to solve an equation for an unknown variable. For example, you can use the this option to solve the following equation for x:

$$4\sin(x) + 5\cos(x)\ln(x^2 + 3) = 0$$

- Solve polynomial equation

  Use this option to find the roots of a polynomial equation. For example, you can use this option to find the roots of the following polynomial:

$$5x^3 + 4x^2 - 3x + 2$$

- Solve linear systems

  Use this option to solve a system of linear equations, that is, a set of simultaneous equations. For example, you can use this option to solve the following simultaneous equations to find the value of x and y.

$$3x + 2y = 5$$
$$2x - 8y = 7$$

- Solve differential equation

  Use this option to solve first order differential equations, that is an equation that contains a derivative. For example, the following first-order differential equation describes the rate of radioactive decay:

$$\frac{dN}{dT} = -KN$$

Solving an equation

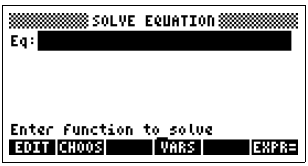
In solving an equation, the HP 49G uses any existing values it has stored for the variables in the equation. These may be variables that you have created, or variables that the calculator has used in, or generated from, previous calculations. Before you solve an equation, if necessary use File Manager to delete any variables corresponding to the ones that your equation contains.

Example

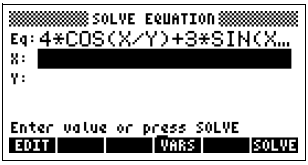
This example demonstrates how to solve an equation in x and y for x, when $y = 2$. The equation is:

$$4\cos\left(\frac{x}{y}\right) + 3\sin(\pi x) = \sqrt{2}$$

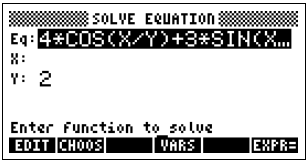
1. Open the numeric solver choose list and select SOLVE EQUATION from the menu. The Solve Equation input form is displayed.
   ↱ NUM.SLV ENTER



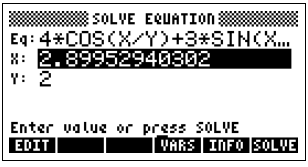
2. Enter the equation to solve and place it in the EQ field.
   4 X COS x ÷ ALPHA Y ▶ + 3 X SIN
   ↰ π X x ▶ ↱ = √x 2 ENTER



3. The variables that the equation contains are displayed. Enter the known value for y in the Y: field.
   ▼ ▼ 2 OK



4. Select the X: field and press SOLVE to solve the equation for x. The solution appears in the X: field.
   ▲ SOLVE



To speed up the solve process, enter an estimate of the value for the variable that you want to find.

Interpreting results

After it solves an equation, the numeric solver returns information relating to the solution process.

To display the solution information, press INFO. If the numeric solver found a solution to the equation, it displays one of the following three messages:

Zero	The numeric solver was able to solve the equation within the limits of its accuracy.
Sign Reversal	The numeric solver found two points where the value of the equation has opposite signs, but it cannot find a point in between where the value is 0. Possible causes are that the two points are less than one in the 12th decimal place apart, or the equation is not real-valued between the sign reversal points.
Extremum	This represents one of the following conditions: • The numeric solver found a point where the value of the equation approximates a local minimum or maximum. The point may or may not represent a root. • The numeric solver stopped searching at either the largest or smallest number in the calculator's range.

If the numeric solver did not find a solution, it displays one of the following explanatory messages:

Bad Guess(es)	One or more of the initial guesses lie outside the domain of the equation.
Constant?	The value of the equation is the same at every point that was sampled.

Solving polynomial equations

Polynomial equations are of the form:

$$ax^n + bx^{n-1} + \ldots + cx^2 + dx + e = 0$$

For example, the following equation is a third order polynomial:

$$5x^3 + 4x^2 - 3x + 2 = 0$$

You can use the HP 49G to:

- find the roots of a polynomial
- find the coefficients of a polynomial, given a set of roots.

To solve a polynomial, you express it as a vector of its coefficients. For example, consider the previous example:

$$5x^3 + 4x^2 - 3x + 2 = 0$$

In vector form, this can be expressed as follows:

$$[5\ 4\ -3\ 2]$$

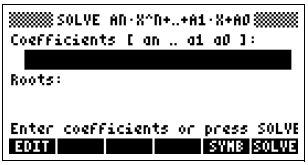
Note that if a polynomial does not include a term for a particular power, you need to include a 0 in the vector to represent the term. For example:

Equation	**Vector**
$3x^2 + 5$	$[3\ 0\ 5]$
$5x^3 - 2x$	$[5\ 0\ -2\ 0]$

Example

This example describes how to find the roots of the polynomial equation $5x^3 + 4x^2 - 3x + 2 = 0$.

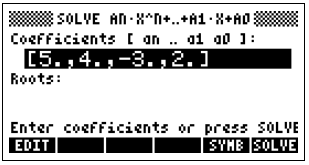
1. Open the numeric solver and select SOLVE POLY to display the Coefficients input form.



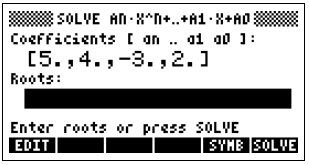
2. With the cursor in the Coefficients field, press EDIT. Matrix Writer opens, ready for use.

3. In Matrix Writer, enter the coefficients on the top row of the matrix. See chapter 8, "Vectors, lists, arrays, and matrices" for information on using Matrix Writer.

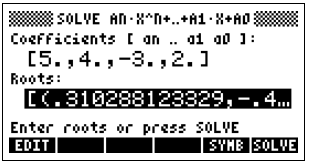
4. Press ENTER to place the values on the input form.



5. Press ▼ to place the cursor in the Roots field.

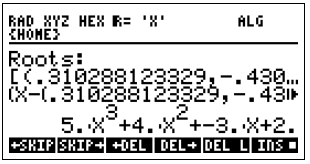


6. Press SOLVE. The HP 49G solves the equation and places the roots, in vector form, in the Roots field.



7. To copy the equation or the roots to the command line, move to the field to copy and press SYMB.

   SYMB ▲ SYMB ENTER



Finding a polynomial from a set of roots

To find a polynomial equation that corresponds to a set of roots, use the following procedure:

1. Open the numeric solver and select SOLVE POLY to display the Coefficients input form.
2. Press ▼ to place the cursor in the Roots field, and press EDIT. Matrix Writer opens, ready for use.
3. In Matrix Writer, enter the roots on the top row of the matrix, and press ENTER to place the values, in vector form, on the input form.
4. Press ▲ to place the cursor in the Coefficients field.
5. Press SOLVE. The numeric solver solves the equation and places the coefficients, in vector form, in the Coefficients field.

Solving linear systems

A linear system is a set of linear equations where there is more than one independent variable. For example, the following is a system where there are two linear equations and two independent variables.

$$3x + 2y = 5$$
$$2x - 8y = 7$$

There are three types of linear systems:

- **Exactly determined systems** are systems where there is the same number of equations as there are independent variables. The HP 49G can solve these systems to the limits of its accuracy.
- **Over-determined systems** are systems where there is a greater number of equations than there are independent variables. Usually there is no exact solution to these systems. The HP 49G returns the least-squares solution.
- **Under-determined systems** are systems where there is a smaller number of equations than the number of independent variables. Usually there is an infinite number of solutions to these systems. The HP 49G returns the solution with the minimum Euclidean norm.

Representing a system as matrices

To solve a linear system, you represent the system in matrix form within the numeric solver. In order to represent the system in matrix form, you need to transpose the equations to a form where the independent variables are to the left of the = sign, and the constant is on the right. For example, consider the following set of equations:

$$2x - 8y + 7 = 2$$
$$3x + 2y - 1 = 14$$

Before you attempt to solve this system, manipulate the equations to the following form:

$$2x - 8y = -5$$
$$3x + 2y = 15$$

These equations can be represented as a set of three matrices:

- a matrix that contains the variable coefficients

$$\begin{bmatrix} 2 & -8 \\ 3 & 2 \end{bmatrix}$$

- a matrix that contains the constants

$$\begin{bmatrix} -5 \\ 15 \end{bmatrix}$$

- a matrix that contains the variables to solve for.

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

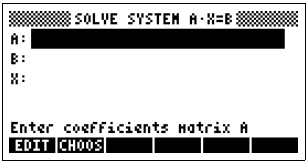
When you solve this system, you specify the first two matrices and the answers are returned in the third matrix.

Example

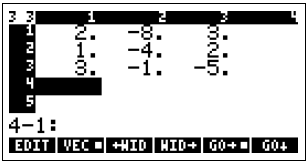
To solve the following linear system:

$$\begin{aligned} 2x - 8y + 3z &= -5 \\ x - 4y + 2z &= 3 \\ 3x - y - 5z &= 4 \end{aligned}$$

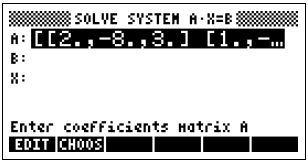
1. Open the numeric solver and select SOLVE LIN SYS to display the Solve System input form.
   (↱) (NUM.SLV) 4 (ENTER)



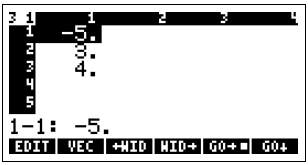
2. Make sure that the cursor is in the A: field and press EDIT. Matrix Writer opens, ready for use. Create a coefficients matrix.
   EDIT …



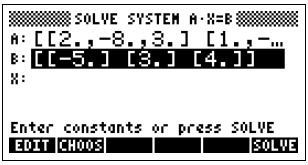
3. Press (ENTER) to return to the Solve System input form. The matrix that you created appears in the A: field.
   (ENTER)



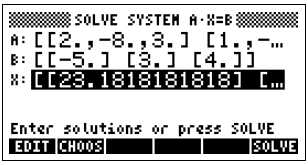
4. Place the cursor in the B: field and press EDIT to display Matrix Writer. Create a constants matrix.
   (▼) EDIT …



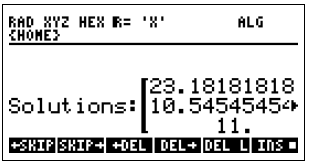
5. Press (ENTER) to return to the Solve System input form. The matrix that you created appears in the B: field.
   (ENTER)



6. Press (▼) to place the cursor in the X: field and press SOLVE. The numeric solver solves the linear system and writes the answers to a matrix. The results matrix is displayed in the X: field.
   (▼) SOLVE



7. Return to the default screen. The results matrix is written to the history.
   ENTER



Solving differential equations

This section explains how to use the numeric solver to solve differential equations.

1. Open the numeric solver and select SOLVE DIFF EQ to display the Solve Equation input form.
2. Use the method described in "Solving an equation" on page 6-3 to specify the equation.
3. Use the arrow keys to navigate to the fields and press EDIT. The default settings are for an equation where x is a function of y. The fields are as follows:

F:	Holds the equation to be solved.
INDEP:	Specifies the independent variable. This defaults to x.
INIT:	Contains the initial value of the independent variable.
FINAL:	Contains the independent variable's final value.
SOLN:	Specifies the solution variable. This defaults to y.
INIT:	Contains the solution variable's initial value.
FINAL:	Displays the solution variable's final value when the equation is solved. You cannot edit this value.
TOL:	Contains the acceptable level of absolute error. This defaults to 0.0001.
STEP:	Contains the initial step size to be used as the solver attempts to find a solution.
STIFF	Check this field when the solve process does not work, or is taking a long time. It displays additional fields where you can enter partial derivative information in order to obtain a stiff solution. See the *Advanced User's Guide* for more information.

4. Press SOLVE. The calculator solves the equation. If the process takes a long time, or does not produce a solution:
   a. Press CANCEL to stop the process.
   b. Re-edit the input form to check the STIFF option.
   c. Enter partial derivative information before trying again.

Using the financial solver

Use the financial solver to perform time-value-of-money calculations, and to calculate amortization amounts that relate to these calculations.

- Time-value-of-money calculations relate to a borrowed amount of money that is to be repaid, at a fixed compounding interest rate, over a period of time.
- Amortization is the way the repayments are divided between the principal amount and interest on the loan.

The financial solver allows you to calculate any one of the parameters for a time-value-of-money transaction. You can perform modelling by entering all parameters except one, and calculating the value for the missing one.

Time-value-of-money calculation parameters

The financial solver uses the following parameters:

N	The total number of compounding interest periods and payments. A compounding interest period is the period after which the amount of interest that the loan has accrued is added to the principal. The financial solver assumes that this period corresponds to the payment period.
I%YR	The annual interest rate, expressed as a nominal percentage.
PV	The value of the loan at the beginning of the first period.
PMT	The periodic payment amount, or the repayment amount that is to be made in each period.
FV	The value of the loan at the end of the N^{th} period. For example, if you were calculating details of a complete loan repayment, this value would be 0.
BEG/END	Whether the payment is made at the beginning or end of the payment period.

Time-value-of-money calculations

1. Press ↰ (FINANCE) to open the financial solver.

   The Time Value of Money input form is displayed.

2. Depending on the value that you want to calculate, enter values into the fields.
   - To enter a value in a field, place the cursor in the field, enter the value and press (ENTER). The value appears in the highlighted field.
   - To edit an existing value, place the cursor in the field and press EDIT. Edit the value on the command line and press (ENTER).
   - To specify whether payments are made at the beginning or the end of the payment period, place the highlight in the Beg/End field and press CHOOS. (The Beg/End field is immediately below the P/Yr field. It displays either BEG or END.) Select the value you want from the list.

3. Use the arrow keys to move the cursor to the field for the value to be determined and press SOLVE.

   The financial solver solves the calculation and the computed value appears in the field.

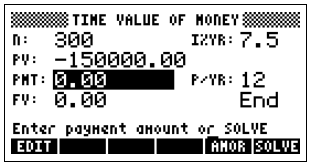
4. Press (CANCEL) to return to the default screen. The value that you computed is displayed in the history.

For example, to calculate the monthly payments on a $150,000 mortgage over a 25 year period at an interest rate of 7.5%:

1. Press ↰ (FINANCE) to open the financial solver.

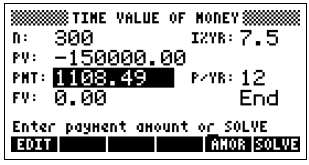
   The Time Value of Money input form is displayed.

2. Enter the values into the relevant fields. Note that the number of payments is 300, or 25 times 12.



3. Use the arrow keys to place the cursor in the PMT field, and press SOLVE. The monthly amount appears in the PMT field.

   SOLVE



Amortizing the calculation

After you have performed a time-value-of-money calculation, you can amortize the results, that is, calculate the amount of principal and the amount of interest that you pay over a period.

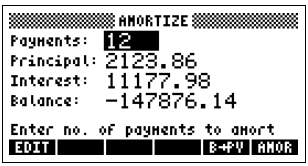
The starting value for the amortization calculations, that is the point from which the payment and interest details is calculated, is the initial value (stored in the PV field) in the Time Value of Money input form. To amortize payments from the previous example, perform the following:

1. Enter details for the time-value-of-money calculation, and find the monthly payment value as in the previous example.
2. Press AMOR. The Amortize input form is displayed.

   AMOR

3. In the Payments field, ensure that the number of payments to amortize is set to 12, and press AMOR. The financial solver amortizes the payments and displays the results.

   AMOR



   For the first year's payments, the financial solver tells you:

   - the principal remaining after the number of payments is made
   - the interest component of the payments
   - the balance of the principal after the number of payments have been made.

Once you have amortized a batch of payments, you can set the loan balance as the starting value for amortization. This way you can amortize payments for each year to compare principal and interest details at different stages of the loan.

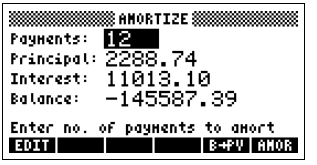
To amortize the second year of the loan's payments:

1. Press B→PV.

   The starting value is set to the value in the Balance field.

2. In the Payments field, ensure that the number of payments to amortize is set to 12, and press AMOR. The financial solver amortizes the second year's payments, and displays the details.

   AMOR



3. When you finish, press CANCEL to return to the Time Value of Money input form. Note that the current starting amount is displayed in the PV field.