Chapter 8

Vectors, lists, arrays, and matrices

Contents

Introduction

The HP 49G provides numerous tools for creating, manipulating, and analyzing vectors, lists, arrays, and matrices. Vectors of any dimension can be created, commands can be made to operate over multiple elements in a list, and matrices can be used to specify statistical data and to solve systems of linear equations.

Vectors

A vector is a way of representing quantities that are measured both by magnitude and by direction. An example is velocity.

Most often you will work with 2- and 3-dimensional vectors, although vectors of any dimension are possible. The HP 49G enables you to specify, and work with, vectors of any dimension.

A 2-dimensional vector can be described in rectangular notation $[x, y]$— or in polar notation—$[r, \theta]$. For 3-dimensional vectors, you can use rectangular notation—$[x, y, z]$— cylindrical notation—$[r, \theta, z]$—or spherical notation—$[r, \theta, \phi]$. All these notations are available with the HP 49G.

Creating vectors

You should first decide on the notation you want to use.

Selecting vector notation

The currently set notation is indicated by the coordinates annunciator: XYZ indicates rectangular notation, R∠Z indicates cylindrical notation, and R∠∠ indicates spherical notation. You need to choose a new notation if the currently set notation is not the one you want to use. (Note: you must choose *polar* notation if you are creating a 3-D cylindrical vector.)

1. Press (MODE) to display the Calculator Modes input form.
2. Set the Coordinate System field to the notation that you want. (See page 2-11 for information on changing the fields on an input form.)
3. Press OK to set the notation you chose.

Selecting an angle unit

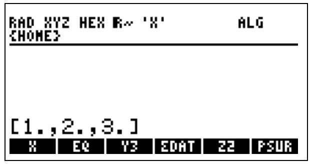
The currently set angle unit is indicated by the angle annunciator: DEG indicates degrees, RAD indicates radians—the default setting—and GRD indicates gradians. If you intend to use a vector notation that requires an angle measurement and the current angle unit is not what you want, reset the unit before entering your vector. See “Changing a mode” on page 2-18 for instructions on resetting the angle unit.

Entering vectors

You enter a vector by specifying its components between square brackets:

1. Press ↰ [ ]
2. Enter the first component.

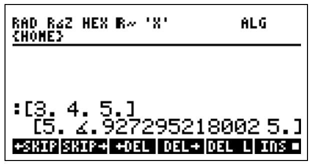
   If you are entering a real vector rather than a symbolic vector, immediately follow each component with a decimal point (as in the example below).
3. Enter each subsequent component. Separate rectangular components with a comma but precede an angular component with the angle sign: ∡. (The angle sign can be entered by pressing ALPHA ↱ 6.)



4. Press ENTER.

You can also enter a vector by creating a single-row matrix with Matrix Writer. This is explained later in this chapter.

If the coordinates notation does not match your entry, the HP 49G converts your entry to that notation. In the example at the right, the second argument—4.—is converted to an angle measurement because the coordinates notation is set to polar (indicated by the polar annunciator). Note that symbolic vectors—including those with integer elements—are not converted to the angle measurements.



Vector mathematics

Two vectors can be added and subtracted on the HP 49G just as real numbers are added and subtracted. For instance, to add two vectors, enter the first vector, press +, enter the second vector and press ENTER.

You can also multiply and divide a vector by a scalar.

The HP 49G also provides a number of special commands for working with vectors. Three of these commands—absolute magnitude, dot product, and cross product—are described in detail in the next three sections.

Absolute magnitude

The *absolute magnitude* of a vector—also known as the *scalar magnitude*—is the square root of the sum of the squares of the value of each element.

To calculate the absolute magnitude of [1 2 4]:

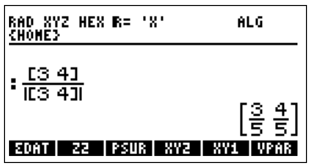
1. Press [⬑] [ABS].
2. Press [⬑] [[ ]].
3. Enter 1 [⬏] [,] 2 [⬏] [,] 4
4. Press [ENTER].

The result is $\sqrt{21}$.

An example where you need to calculate the absolute magnitude is in finding the unit vector. The unit vector is found by dividing the given vector by its magnitude:

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

Suppose you want to find the unit vector of [3 4]:



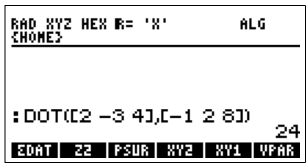
1. Press [⬑] [[ ]] 3 [⬏] [,] 4 [▶] to enter the numerator.
2. Press [÷].
3. Press [⬑] [ABS].
4. Press [⬑] [[ ]] 3 [⬏] [,] 4 to complete the denominator.
5. Press [ENTER] to obtain the unit vector, which in this case is: $\left[\frac{3}{5}, \frac{4}{5}\right]$

Dot product

The *dot product* of two vectors of equal dimensions is the sum of the products of each corresponding pair of elements. The dot product is also known as the *inner* or *scalar product.*

To find the dot product of [2 –3 4] and [–1 2 8]:

1. Press ↰ MTH to select the MATH menu.
2. Press OK or ENTER to select the VECTOR menu.
3. Press ▼ to highlight the DOT command and press OK or ENTER.
4. Press ↰ [ ] to enter a pair of square brackets to enclose the first vector.
5. Enter 2 ↱ , 3 +/- ↱ , 4.
6. Press ▶ to move your cursor outside the square brackets, thereby indicating that the first vector is complete.
7. Press ↱ , to indicate the end of the first argument.
8. Press ↰ [ ] to enter a pair of square brackets to enclose your second vector.
9. Enter 1 +/- ↱ , 2 ↱ , 8.
10. Press ENTER to return the dot product of the two vectors, in this case, 24.

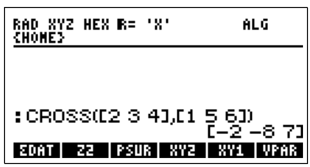


Cross product

For two vectors [*a b c*] and [*d e f*], the cross product is [*(bf – ce) (cd – af) (ae – bd)*]. The cross product of two vectors is also known as the *vector product* or *outer product.*

To find the cross product of [2 3 4] and [1 5 6]:

1. Press ↰ MTH to select the MATH menu.
2. Press OK or ENTER to select the VECTOR menu.
3. Press ▼ twice to highlight the CROSS command and press OK or ENTER.
4. Enter the two vectors, separating them with a comma.
5. Press ENTER to return the cross product of the two vectors, in this case, [–2 –8 7].



Lists

A list is a collection of any number of objects. The objects can be of any type—numbers, character strings, and so on—and objects of different types can appear in the one list. A list is represented on the HP 49G by a pair of braces ({}) surrounding a collection of objects.

Creating a list

1. Press ↰ {}

   This indicates that you want to create a list.
2. Enter the elements you want to include in the list, separating each with a comma (that is, ↱ ,).
3. Press ENTER.

Working with lists

Single-argument commands

Commands that require only one argument can be applied to each element in a list in a single operation.

For example, to find the square root of 5, 6, and 7:

1. Press √x.
2. Press↰ {} 5 ↱ , 6 ↱ , 7.
3. Press ENTER.

   The positive square root of each of the three numbers in the list is returned, with the three results displayed in a list.

Multiple-argument commands

Commands that require more than one argument can be applied to each element in a list in a single operation.

For example, suppose you want to calculate how many samples of 4 objects can be made from populations of 5, 6, and 7 objects.

1. Press ↰MTH to select the MATH menu.
2. Use the arrow keys to highlight the PROBABILITY menu.
3. Press OK or ENTER.
4. Press OK or ENTER to select the COMB command.

5. Press ↰ { } 5 ↱ , 6 ↱ , 7.
6. Press ▶

   The cursor is now outside the list of the populations to be sampled.
7. Press ↱ , 4
8. Press ENTER.

   The answers are returned in a list: {5 15 35}.

Note that the command was applied to two parameters—the list of the various populations, and the size of the sample—each separated by a comma within a pair of braces.

Arrays and matrices

An *array* is any rectangular arrangement of objects. An array of numbers—real or complex—is usually referred to as a *matrix*.

You can create arrays from many types of objects: real numbers, complex numbers, symbolic expressions, character strings, programs and so on. You could, for example, create a database—such as a list of contacts and their telephone numbers—as a type of array.

Creating arrays

Using Matrix Writer

You create an array with a special tool called *Matrix Writer*. You open Matrix Writer by pressing ↰ MTRW.

When you open the Matrix Writer, the display becomes a table, with the rows and columns numbered like a spreadsheet. The size of the array is indicated by the figures in the top left-hand corner of the table. (The size is 0 × 0 at the start, but this increases as you enter objects into the array.) The row–column coordinates of the currently active cell are shown at the bottom left-hand corner of the display.

To create an array using Matrix Writer:

1. Press ↰ MTRW to open Matrix Writer.
2. Enter the object that is to appear in the first cell of the array.

   This object appears on the command line as you type it.

3. Press ENTER to move the object from the command line to the first cell of the array.

   The active cell now becomes 1–2 (that is, the cell at row 1 and column 2).

4. Enter the remaining objects that are to go into row 1 of the array, pressing ENTER after each to move it from the command line to the next available cell.

5. When you have typed the last object in the first row of the array, press ▼ to move to the second row of the array and then press ◄ until cell 2–1 becomes the active cell (that is, the first cell in the second row).

6. Enter the objects that are to appear in the second row of your array, pressing ENTER after each object.

   Note that the cursor now automatically moves to the first cell of the next row after you enter an object in the last column of a row.

   If you need to add more objects to a row you have already created, use the arrow keys to position your cursor in the appropriate blank cell and enter a new object. See "Quickly moving through an array" below to learn how to quickly move through large arrays.

7. When you have entered all the objects that will comprise your array, press ENTER.

   Matrix Writer closes and the array you have created appears on the default display screen.

The Matrix Writer function-key menu is explained in detail in the *Pocket Guide* and the *Advanced User's Guide*.

Using the command line

This method is suitable only for creating small arrays. For large arrays, use Matrix Writer (as described in the previous section).

1. Press ⬑ [ ] to enter array delimiters.
2. Press ⬑ [ ] to enter row delimiters.
3. Enter the row of elements, pressing ⬏ , to separate each element.
4. If you want to enter more rows, continue from step 5 below; otherwise press ENTER to create the array.
5. Press ► to move the cursor to the right of the row delimiter.
6. Repeat from step 2.

Quickly moving through an array

Key combinations are provided to help you quickly move through an array that is too large to be displayed in full:

↱ ▶| moves to the last column.

↱ |◀ moves to the first column.

↱ ▲ moves to the first row.

↱ ▼ moves to the last row.

↰ ⫼▶ moves to the next displayable set of columns.

↰ ◀⫼ moves to the previous displayable set of columns.

↱ ▲≡ moves to the previous displayable set of rows.

↱ ≡▼ moves to the next displayable set of rows.

Editing an array

1. Highlight the array in history and press EDIT.

   Matrix Writer opens with your array displayed.

> If the array you want to edit is the last object in history, you can also press ▼ to open the array in Matrix Writer.

2. Use the arrow keys to highlight the cell you want to edit.
3. Enter the new value.
4. Press (ENTER) to update the array.
5. If you want to edit other values, repeat from step 3.
6. Press (ENTER) to place the edited array in history.
7. Press (ENTER) again to save the edited array.

Matrix arithmetic

In matrix arithmetic, you need to enter one or more matrices. You can enter a matrix:

- using Matrix Writer
- by typing it on the command line
- by selecting it from history, or
- by recalling the variable name associated with it.

Adding or subtracting two matrices

1. Enter the first matrix.
2. Press (+) or (−).
3. Enter the second matrix.

   The second matrix must have the same dimensions as the first.
4. Press (ENTER).

   Each element in the second matrix is added, or subtracted, from the corresponding element in the first matrix.

Multiplying or dividing a matrix by a scalar

1. Enter the matrix.

   See page 2-4 for information on selecting a matrix from history.
2. Press (X) or (÷).
3. Enter the scalar.
4. Press (ENTER).

   Each element in the matrix is multiplied by, or divided by, the scalar.

Multiplying two matrices

Since the multiplication of matrices is not commutative, the order in which you specify the matrices is important. The number of columns in the first matrix must equal the number of rows in the second matrix.

1. Enter the first matrix.
2. Press ⓧ.
3. Enter the second matrix.
4. Press (ENTER).

The result is a matrix with the same number of rows as the first matrix and the same number of columns as the second matrix. Each element in the matrix is the product of the corresponding two elements in the original matrices.

Finding the determinant of a square matrix

1. Enter DET on the command line.
2. Press ↰ ().
3. Enter the matrix.
4. Press (ENTER).

The determinant of a matrix can be used to solve a system of linear equations. Another method is to use Gaussian elimination to generate the row-reduced echelon form of a matrix. This is discussed in the next section.

Solving a system of linear equations

A method of solving a system of linear equations is explained in chapter 6. This method uses the numeric solver. The HP 49G also has a matrix command for solving a system of linear equations. This command—RREF—uses Gaussian elimination to generate the row-reduced echelon form of an augmented matrix.

You can use the RREF command in direct mode or in step-by-step mode. (See “Setting step-by-step mode” on page 5-19 for instructions on setting step-by-step mode.) In this mode, the HP 49G performs the Gaussian elimination one step at a time. Before it performs each step, the HP 49G displays a description of the action it is about to perform. You press OK to action each step.

For example, suppose you have to solve the following system:

$$3x + 4y = 25$$

$$5x - 3y = 3$$

To solve such a system, you can:

1. Enter RREF on the command line.

   "RREF" stands for the ROW-REDUCED ECHELON FORM command.

2. Press ⬅ ( ).

3. Enter or select the augmented matrix.

   The augmented matrix is a matrix of the system's coefficients and constants (with the constants set out in the rightmost column of the matrix). In this example, the augmented matrix looks like this:

$$\begin{bmatrix} 3 & 4 & 25 \\ 5 & -3 & 3 \end{bmatrix}$$

4. Press ENTER.

   - If you are in step-by-step mode, a description of the first step in the process is displayed. Press OK to see the result of that step. Keep pressing OK until you have stepped through the entire reduction process and the row-reduced echelon form of the augmented matrix is displayed.
   - If step-by-step mode is not set, the row-reduced echelon form of the augmented matrix is immediately displayed.

   The row-reduced echelon form of the augmented matrix in our example is:

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \end{bmatrix}$$

The answer to the system of linear equations is in the rightmost column of the row-reduced echelon matrix: in our example, $x = 3$ and $y = 4$.

You can speed up the processing of large matrices by setting the LARGE MATRICES system flag (–110).