Chapter 5

Working with expressions

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Introduction

This chapter explains how to use the main features of the HP 49G's computer algebra system to work with expressions.

It describes how to:

- factorize, expand, and simplify expressions
- use the calculus functions to perform differentiations and integrations
- perform a differentiation in step-by-step mode.

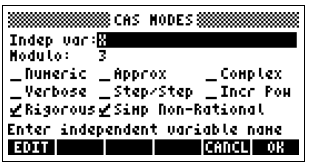
You can use either Equation Writer or the command line to create and edit expressions. The examples in this chapter do not explain how to create the expressions. See chapter 2, "Basic operation", and chapter 3, "Creating and editing expressions", for information on how to do this.

Configuring the CAS

The HP 49G has powerful symbolic capabilities. It can return numeric or symbolic solutions to problems involving algebraic objects. You use the CAS Modes input form to set the way that the HP 49G deals with calculations, and how it performs symbolic manipulation.

To configure the CAS:

1. Press (MODE) to display the Calculator Modes input form, then press CAS. The CAS Modes input form is displayed.



2. Use the arrow keys to navigate around the options.
   - For those options that require a value, enter the value and press OK to apply it to the option.
   - For the other options, select or de-select them by pressing CHK.
3. When you have configured the CAS modes, press OK twice to return to the default screen.

On the CAS Modes input form, two key settings determine how the HP 49G handles symbolic solutions. These are the Numeric option and the Approx option.

Numeric option

When the Numeric option is set, the calculator returns only numeric solutions to calculations. Otherwise, the calculator returns symbolic solutions.

With the numeric option set, any variables that a calculation contains must exist in the current path, and contain a numeric value. If this is not the case, the calculator returns an error. Any constants with an approximate numeric value, such as π, are evaluated and the approximate value substituted into the calculation.

For example, consider the calculation `SIN(2*π*X + 3*π*X)`

- If the Numeric option is set, and a numeric value for X is stored in the current path, the calculator returns the approximate answer, accurate to 12 decimal places. That is, it substitutes the numeric value for X and the approximate value for π into the equation.
- If the Numeric option is set and there is no numeric value for X stored in the current path, the calculator returns an error.
- If the Numeric option is clear, and no value for *X* is stored in the current path, the calculator simplifies the expression and returns `SIN(5.X.π)`.

Approx option

The Approx option setting determines how the HP 49G expresses answers to calculations:

- When the Approx option is clear, the HP 49G finds exact solutions to calculations where possible. That is, it expresses the solution as an irreducible expression containing exact terms.
- When the Approx option is set, the HP 49G expresses solutions to 12-digit accuracy.

Note that you can change the setting of the Approx option by using the CAS Modes input form or by pressing simultaneously ↱ and ENTER.

- If the calculator is in exact mode, pressing ↱ ENTER changes to approximate mode.
- If the calculator is in approximate mode, pressing ↱ ENTER changes the setting to exact mode.

For example, consider the expression $\sin(x^3 + 2x)$.

If the value for variable *X* is defined as $\sqrt{2}$ in the current path:

- When the Approx option is set, evaluating the expression returns –.586176193022
- When Approximate mode is clear, evaluating the expression returns SIN(4 $\sqrt{2}$).

Other options

In addition to the above options, the CAS Modes input form is used to set the following parameters:

- **Independent variable**

  The independent variable is the default variable that the calculator uses for operations such as differentiation and integration. The default value is X.

- **Modulo**

  Enter the modulus that you want to use in modulo arithmetic operations. The default value is 3.

- **Complex**

  Check this option if you want to work with complex numbers. If you do not need to use complex numbers, de-select this option. Some transformations behave differently in complex mode.

  If complex mode is not set and a calculation returns only a complex solution for most calculations, the calculator prompts for a switch to complex mode.

- **Verbose**

  Check this option to display messages that indicate progress as the calculator performs the operations that you specify.

- **Step/step**

  Check this option to perform linear algebra operations, integrations, and differentiations in step-by-step mode. See “Differentiating an expression step-by-step” on page 5-19.

- **Incr Pow**

  Check this option to display polynomial expressions with the terms in increasing power order, for example $x + x^2 + x^3$. The default is to display polynomials with the terms in decreasing order.

- **Rigorous**
  Check this option to specify that you do not want $|x|$ terms simplified to x.
- **Simp Non-Rational**
  Check this option to specify that non-rational expressions are simplified.

Using the computer algebra system

The HP 49G computer algebra system consists of a collection of commands and functions that you apply to expressions. Commands and functions appear in the same way on the menus, but they differ in the way that they operate.

Commands have the following properties:

- They produce one result.
- They cannot be included in an algebraic object.

Functions have the following properties:

- They can produce more than one result.
- They can be a part of an algebraic expression.
- You need to evaluate functions in order to get results. That is, once you apply a function to an object, you need to press EVAL, or use the EVAL command, in order to get a result.

See the *Advanced User's Guide* or the *Pocket Guide* for details of whether an operation is a function or a command.

In this chapter, commands and functions are referred to collectively as commands.

The following is a list of the categories and how you access commands in each category. Within each category, commands are displayed in a choose list.

↱ ALG	Displays the algebraic commands, for performing operations such as factorizing or expanding.
↰ ARITH	Displays the complex, the integer, and polynomial arithmetic commands.
↰ CALC	Displays the calculus commands for performing operations such as integrating and differentiating. This category also includes commands for working with limits and series, and for dealing with Taylor polynomials.
↱ CMPLX	Displays the commands relating to complex numbers.
↰ EXP&LN	Displays the commands for manipulating exponential and logarithmic expressions.
↰ MATRICES	Displays the commands for working with matrices.
↰ S.SLV	Displays the commands you use to solve an equation symbolically.
↱ TRIG	Displays the commands for manipulating trigonometric expressions.

For example, if you want to simplify a trigonometric expression, you press ↱ TRIG to display the trigonometric commands.

You can use the computer algebra system to manipulate expressions both from the command line and from within Equation Writer.

- From the command line, you place the expression between the command's parentheses.
- Within Equation Writer, you select the expression, or part of the expression, then select a command from the menu.

Working from the command line

1. Use one of the listed key combinations to display the choose list containing the command that you want.
2. Use the arrow keys to highlight the command to use, and press ENTER to place it on the command line. The command appears with a set of parentheses after it.
3. Ensure that the cursor is between the command's parentheses and insert the expression to operate on, and any other arguments the command needs. Separate each argument with a comma (→ ,).
4. Press ENTER to apply the command.

Inserting an expression onto the command line

There are three ways to insert an expression onto the command line:

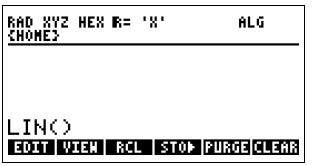
- Use the command line editor to enter the expression directly onto the command line.
- Use Equation Writer:
  a. Place the cursor where you want to insert the expression.
  a. Press EQW to open Equation Writer.
  b. Create the expression.
  c. Press ENTER to exit Equation Writer and place the expression on the command line at the cursor position.
- Retrieve an expression stored in memory or from the history. See Chapter 2, "Basic operation", and chapter 7, "Storing objects", for details.

Command line example

This example uses the LIN command to linearize a trigonometric expression, and the EXPAND command to simplify the result. The expression to linearize is:
$\sin(\pi x)$

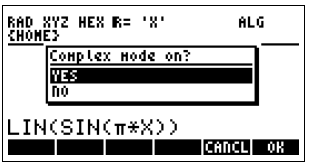
Before you start, ensure that the CAS Modes input form is set to the default setting. See "Configuring the CAS" on page 5-2 for details.

1. Open the EXP&LN menu and select the LIN command.
   ↰ EXP&LN ▼ ▼ ENTER

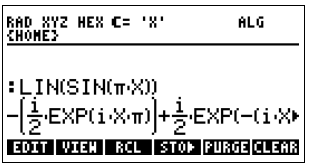


2. Enter the expression inside the command's brackets.
   SIN ↰ π X x

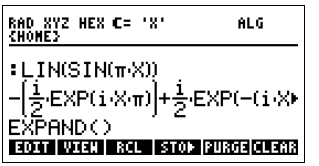
3. Apply the command to the expression.
   ENTER



4. Since the linearization returns a complex result, the calculator prompts to switch to complex mode. Accept the switch to complex mode, and linearize the expression.
   ENTER

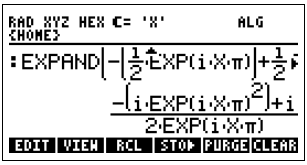


5. Open the Algebra menu and place the EXPAND command on the command line.
   ↱ ALG ENTER



6. Retrieve the result from history.
   HIST ENTER

7. Apply the command to simplify the result.
   ENTER



Working in Equation Writer

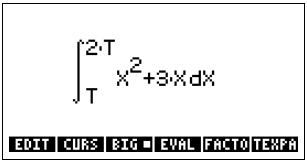
In Equation Writer, you can apply a computer algebra command to the entire expression, or you can select a part of the expression to apply a command to. See chapter 3, "Creating and editing expressions", for details on how to use Equation Writer.

In Equation Writer, you can use only those commands that require one argument, and you cannot use commands that return more than one result.

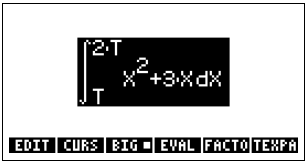
The following example demonstrates how to use Equation Writer to evaluate the integral of an expression between two indefinite limits, and factorize a part of the result. The expression to evaluate is:

$$\int_{t}^{2t} (x^2 + 3x)\,dx$$

1. Open Equation Writer and enter the expression.
   EQW . . .



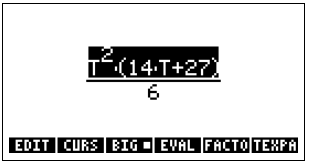
2. Select the expression.
   ▲▲



3. Evaluate the expression
   EVAL

4. Select the numerator component of the expression.
   ▲

5. Factorize the numerator.
   FACTO



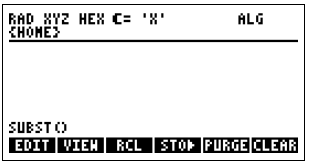
Performing substitutions

Use the SUBST command in the algebraic command list (↱ ALG) to perform substitutions.

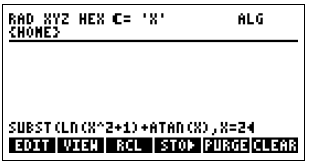
The following example substitutes the value 2 for x in

$\ln(x^2 + 1) + \text{atan}(x)$

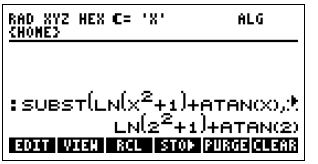
1. From the Algebra menu, select the SUBST command and place it on the command line.
   ↱ ALG 6 ENTER



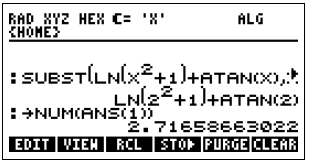
2. Use the command line editor to place the arguments between the command's parentheses.
   ↱ LN x y^x 2 + 1 ▶ + ↰ ATAN x ▶
   ↱ , x ↱ = 2



3. Apply the command.
   ENTER

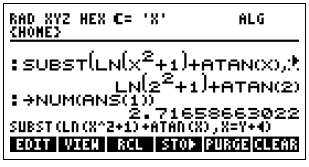


4. Because the calculator is in exact mode, the command produces the following exact result. Obtain an approximate result.
   ↱ →NUM ENTER

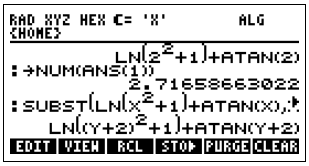


You can also use the SUBST command to substitute expressions. For example, to substitute the expression $y+2$ for x in the previous example:

1. Retrieve the original command from history and edit it to replace the substitution value.
   (HIST) (▲) . . (ENTER) . . (◀)(◀)(◀) (ALPHA) Y (+) 2



2. Apply the command.
   (ENTER)



Expanding and factorizing

The HP 49G can expand and factorize most algebraic expressions. The commands that perform these functions are in the algebraic category. If the expression that you are working on contains exponential or trigonometric functions, you may need to simplify the expression before using EXPAND or FACTOR. See "Exponential and trigonometric expressions" on page 5-14.

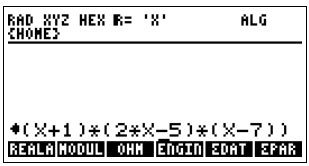
Expanding expressions

You use the EXPAND command to expand and simplify an expression. The following example expands the expression:

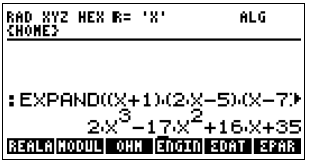
$$(x + 1)(2x - 5)(x - 7)$$

1. Open the Algebra command list and select EXPAND to place it on the command line.
   (↱) (ALG) (ENTER)

2. With the cursor between the parentheses, open Equation Writer and create the expression. When you have created it, press (ENTER) to place it on the command line between the parentheses.
   (EQW) . . . (ENTER)



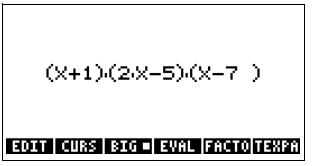
3. Press ENTER to expand the expression.
   ENTER



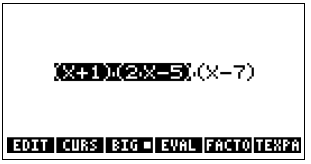
Expanding a part of the expression

To expand parts of the expression, you must work from within Equation Writer. For example, to expand the first two components of the expression in the previous example, perform the following.

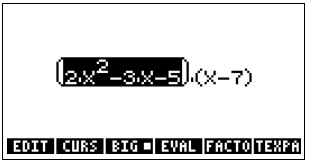
1. Open Equation Writer and create the expression.
   EQW . . .



2. Use either Cursor mode or Selection mode to select the first two component expressions only. For example, if the cursor is positioned on the first term in the expression, X, use the following keystrokes.
   ▲▲↱▶



3. Open the Algebra command list and select EXPAND to expand the selected components.
   ↱ ALG ENTER



Factorizing expressions

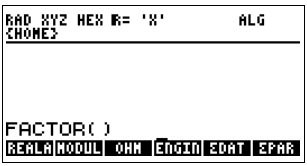
You use the FACTOR command to factorize an expression. As with EXPAND, you can factorize an entire expression, or components of the expression.

Example

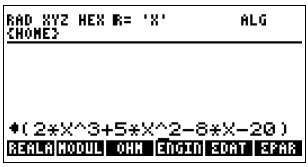
This example factorizes the following cubic polynomial expression:

$$2x^3 + 5x^2 - 8x - 20$$

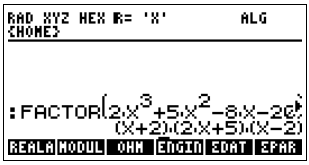
1. Open the Algebra command list and select FACTOR.
   ⓡ ALG ▼ ENTER



2. With the cursor positioned between the parentheses, open Equation Writer, create the expression, and press ENTER to place it on the command line between the parentheses.
   EQW … ENTER



3. Press ENTER to factorize the expression.
   ENTER



Exponential and trigonometric expressions

When working with expressions involving exponential and trigonometric functions, you often need to simplify them before you use FACTOR or EXPAND. The following commands, from the exponential and linearization category, simplify trigonometric and exponential expressions. At the beginning of each description, the keys you use to access the command list are displayed.

EXPLN — Exp and Lin command list—↰ EXP&LN:

Applies Euler identity. The example at the right shows the result of applying the command to sin(x).

HALFTAN — Trigonometry command list—↱ TRIG

Replaces sin(x), cos(x), and tan(x) terms with terms that use $\tan\frac{x}{2}$.

LIN — Exp and Lin command list—↰ EXP&LN

Performs Euler identities, then linearizes expressions by applying the following substitutions:

$e^x \cdot e^y \rightarrow e^{x+y}$

$(e^x)^n \rightarrow e^{nx}$

The example at the right shows the result of applying the command to sin(x).

LNCOLLECT — Exp and Lin command list—↰ EXP&LN

Simplifies an expression by collecting terms involving natural logarithms.

SINCOS — Trigonometry command list—↱ TRIG

Converts exponential and natural logarithmic expressions to trigonometric expressions.

TAN2SC Trigonometry command list—→ TRIG

Applies the following substitution:

$$\tan(x) \rightarrow \frac{\sin(x)}{\cos(x)}$$

TAN2SC2 Trigonometry command list—→ TRIG

Applies the following substitutions:

$$\tan(x) \rightarrow \frac{\sin(2x)}{1 + \cos(2x)}$$

$$\tan(x) \rightarrow \frac{1 - \cos(2x)}{\sin(2x)}$$

TEXPAND Trigonometry command list—→ TRIG

Expands expressions of the form Exp(nx), sin(nx), and cos(nx), where n is an integer. It applies the following substitutions:

$$e^{x+y} \rightarrow e^x e^y$$

$$\ln(xy) \rightarrow \ln(x) + \ln(y)$$

$$\sin(x + y) \rightarrow \sin(x)\cos(y) + \sin(y)\cos(x)$$

$$\cos(x + y) \rightarrow \cos(x)\cos(y) - \sin(x)\sin(y)$$

TLIN Trigonometry command list—→ TRIG

Applies the following substitution:

$$\sin(x)\sin(y) = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

TRIG Trigonometry command list—→ TRIG

Simplifies expressions by applying the following substitution:

$$\sin^2(x) + \cos^2(x) = 1$$

TRIGCOS Trigonometry command list—▷ TRIG

Applies the following substitution, and returns cosine terms if possible:

$\sin^2(x) \rightarrow 1-\cos^2(x)$

TRIGSIN Trigonometry command list—▷ TRIG

Applies the following substitution, and returns sine terms if possible:

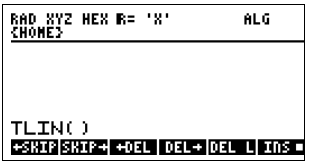
$\cos^2(x) \rightarrow 1-\sin^2(x)$

Example

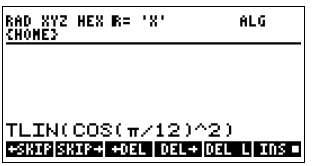
This example uses TLIN, in conjunction with EXPAND, to simplify the following trigonometric expression, and deduce its value:

$$\left(\cos\frac{\pi}{12}\right)^2$$

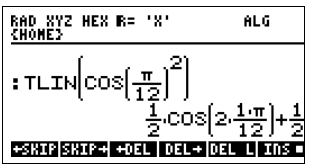
1. Open the Trig command list, and select TLIN.
   ▷ TRIG ▲ ▲ ▲ ▲ ▲ ▲ ENTER



2. With the cursor positioned between the parentheses, open Equation Writer and create the expression. Then press ENTER to place it on the command line between the parentheses.
   EQW … ENTER



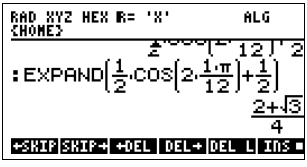
3. Press ENTER to simplify the expression.
   ENTER



4. Open the Algebra command list, and select EXPAND.
   ▷ ALG ENTER

5. Retrieve the result of step 3 from history.
(HIST) (ENTER)

6. Press (ENTER) to expand the expression.
(ENTER)



Calculus commands

Use the following commands from the Calculus choose list to differentiate and integrate expressions.

DERVX — Calculus command list—(↰) (CALC) DERIV. & INTEG

Differentiates an expression with respect to the default independent variable. The default independent variable is X.

DERIV — Calculus command list—(↰) (CALC) DERIV. & INTEG

Differentiates an expression with respect to the variable you specify as a parameter after the expression.

INTVX — Calculus command list—(↰) (CALC) DERIV. & INTEG

Integrates an expression with respect to the default independent variable.

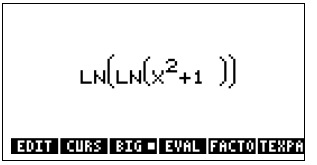
RISCH — Calculus command list—(↰) (CALC) DERIV. & INTEG

Integrates an expression with respect to the variable you specify as a parameter after the expression.

Example

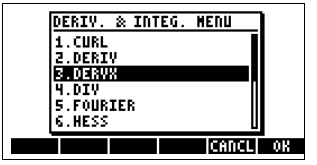
This example illustrates how to use the DERVX command to differentiate an expression and to use the EXPAND command to simplify the result.

1. Open Equation Writer and create the expression.
   EQW ▶ LN ▶ LN x y^x 2 ▶ + 1

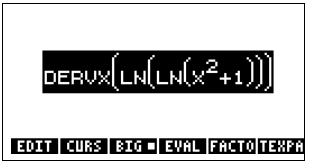


2. Select the expression.
   ▲ ▲ ▲ ▲

3. Open the Calculus command list, select DERIV & INTEG and highlight the DERVX command.
   ◀ CALC ENTER ▼ ▼

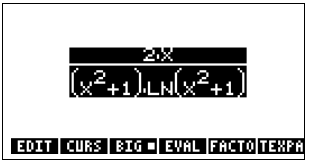


4. Apply the command to the selection. Note that as DERVX is a function, it does not differentiate the expression immediately.
   ENTER



5. Differentiate the expression.
   ▶ EVAL

6. Use the EXPAND command from the Algebra choose list to simplify the result. Note that, as EXPAND is a command, it expands the expression immediately.
   ▶ ALG ENTER



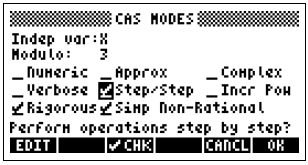
Differentiating an expression step-by-step

You can use the HP 49G to differentiate expressions in step-by-step mode. The computer algebra system displays the results of each stage of the differentiation process. The results of each step of the operation are written to History.

Setting step-by-step mode

You use the CAS Modes input form to set step-by-step mode.

1. Press (MODE).
   The Calculator Modes input form is displayed.
2. Press CAS.
   The CAS Modes input form is displayed.
3. Press ▼▼▼ to move the cursor to the Step/Step field and press CHK.
   A check mark appears next to the mode setting.
4. Press (ENTER) twice to return to the default screen.



Performing step-by-step operations

You can perform step-by-step operations from within Equation Writer.

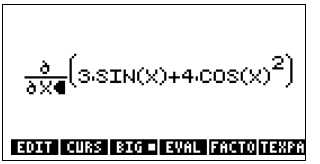
1. Use the above method to set step-by-step mode.
2. Access Equation Writer and either create or import the expression that you want to work on.
3. Use the arrow keys to select the expression.
4. Press (EVAL) to perform the first step in the operation.
   The result of the first step is displayed.
5. Press (EVAL) to perform the next step in the operation. Each time you press (EVAL), the calculator performs the next step in the process, and displays the result.

Step-by-step example

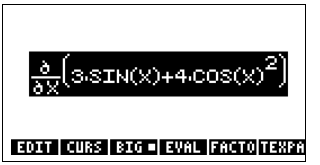
This example differentiates the following expression in step-by-step mode.

$$3\sin x + 4\cos^2 x$$

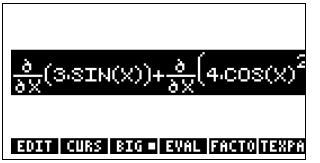
1. Use the method described in the previous section to ensure that step-by-step mode is set.
2. Press (EQW) to open Equation Writer.
3. Create the expression
   3(SIN)(x)(▶)(▶)(+)4(COS)(x)(▶)(y^x)2(▶)(▶)(▶)(↵)(∂)(x)



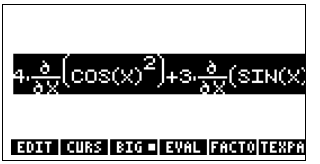
4. Select the expression.
   (▲)(▲)



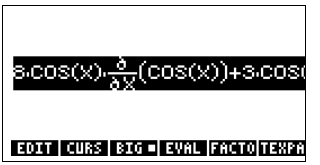
5. Evaluate the first step.
   (↵)(EVAL)



6. Evaluate the next step.
   (↵)(EVAL)



7. Evaluate the next step.
   (↵)(EVAL)



8. Evaluate the last step. This returns the final result of the derivation.
   (↵)(EVAL)