

# Numeric Precision Requirements for Representation of a Location in a Global Coordinate System

The standard include file `math.h` when compiling in "non strict ascii" mode contains the following useful definitions:

<code>M_E</code>	2.7182818284590452354
<code>M_LOG2E</code>	1.4426950408889634074
<code>M_LOG10E</code>	0.43429448190325182765
<code>M_LN2</code>	0.69314718055994530942
<code>M_LN10</code>	2.30258509299404568402
<code>M_PI</code>	3.14159265358979323846
<code>M_PI_2</code>	1.57079632679489661923
<code>M_PI_4</code>	0.78539816339744830962
<code>M_1_PI</code>	0.31830988618379067154
<code>M_2_PI</code>	0.63661977236758134308
<code>M_2_SQRTPI</code>	1.12837916709551257390
<code>M_SQRT2</code>	1.41421356237309504880
<code>M_SQRT1_2</code>	0.70710678118654752440

The circumference of the Earth is approximately :

$$c_e = 24000 \text{ miles} * 1.609344e+03 \text{ meters/mile} = 38624256.000 \text{ meters}$$

From this we can see that any distance on the Earth can be expressed as a positive distance from any Origin on that surface with:

9 digits    to within 1 meter  
12 digits   to within 1 millimeter

From the above constants, we can see that on a typical modern Unix workstation we can expect up to 21 significant digits in a C language double float, which is more than sufficient for our requirements.

## Universal Coordinate Systems

There are several different ways of expressing an accurate location in a 3-space, of which three are of particular interest here: Cartesian, Spherical and World Geographic.

## Cartesian Coordinates

Cartesian coordinates are expressed as a triple of distances,  $\langle x, y, z \rangle$  where:

**X** = distance along x axis

**y** = distance along y axis

**Z** = distance along z axis

## Spherical Coordinates

Spherical coordinates are expressed as a triple with two angles in radians and one distance measurement,  $\langle r, \phi, \theta \rangle$  where:

$\phi$  = angle from z axis towards xy plane

$\theta$  = angle from x axis on xy plane

**r** = distance from center of sphere

and

$$0 \leq \phi \leq \pi$$

$$0 \leq \theta < 2\pi$$

$$r \geq 0$$

## World Geographic Coordinates

World Geographical coordinates are expressed as a triple of latitude, longitude and altitude, where altitude is a distance and is often not included and angles are expressed in degrees,  $\langle \phi_g, \theta_g, \rho_g \rangle$  where:

$\phi_g$  = angle from the Equator towards a Pole, ie *Latitude*

$\phi_{gN}$  = angle from the Equator towards the North Pole

$\phi_{gS}$  = angle from the Equator towards the South Pole

$\lambda_g$  = angle westwards from Greenwich Meridian along the equator,  
ie *Longitude*

$\rho_g$  = distance from sea level of the Earth, ie *Altitude*

and

$$0 \leq \phi_g \leq 90^\circ$$

$$0 \leq \lambda_g < 360^\circ$$

$$\rho_g \geq 0$$

## Local Coordinate Systems

Due to historical reasons and matters of convenience, surface maps are often given 2D grid reference numbers that are related to a known local benchmark in a Cartesian coordinate system that is locally semi-congruent with a World Geographic Coordinate system. The congruency falls off with distance from the specified benchmark.

Grid systems in Ireland and the UK are usually given as a 12 figure Grid Reference. This is actually two numbers that give the cartesian coordinates in meters. Distances North and East of the origin are given as positive numbers as is usual for points on a cartesian plane.

The two local systems of most immediate concern are the *Irish Grid* which covers all of Ireland and the *National Grid* covering England, Scotland and Wales. These systems are not directly compatible, although the True Irish Grid Origin is directly related to the National Grid Origin. For convenience, Irish maps are given from a False Irish Grid Origin that is translated by <-200000,-200000> relative to the True Irish Grid Reference. This makes all Irish Grid Reference numbers positive.

## Transformations between Coordinate Systems

The transformation between spherical coordinates and cartesian coordinates is:

$$x = r \sin \phi \cos \theta$$

$$y = r \sin \phi \sin \theta$$

$$z = r \cos \phi$$

World Geographic coordinates are simply related to spherical coordinates:

$$k_0 = 180/\pi \text{ degrees/radian}$$

$$r_g = r - r_e$$

$$f_{gN} = -k_0(f - \pi/2) \quad \text{for} \quad 0 \leq f < \pi/2$$

$$f_{gS} = k_0(f - \pi/2) \quad \text{for} \quad \pi/2 \leq f < \pi$$

$$q_g = k_0 q$$