Linear Programming Examples
AS-EASY-AS v1.6 for Win95/NT

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AS-EASY-AS and LINEAR PROGRAMMING

A number of people have contacted us, since the release of AS-EASY-AS v5.7, telling us how impressed they are with the new features, in particular the powerful linear programming option, but they don't know exactly how or when to use it.

Unfortunately, linear programming can be a fairly complex topic, and we don't have universal answers on when/how to use it. In response to all those comments, however, we've put together these very detailed examples of linear programming situations and solutions, hoping that they will give some more insight to this unique, powerful AS-EASY-AS feature.

Statement of problem (1)

A designer of expensive leather jackets, created two new jacket designs for the new season, a long one and a short one. Each short leather jacket requires 1 labor-hour from the cutting department and 3 labor-hours from the sewing department. Each long leather jacket requires 2 labor hours from the cutting department and 4 labor-hours from the sewing department. This designer is sharing cutting and sewing services with other designers, and as such, there are only 32 labor-hours per week available in the cutting department and 84 labor-hours per week available in the sewing department for him. In addition, because of the limited appeal of long leather jackets, the distributor cannot take any more that 12 long leather jackets per week. If the designer makes \$50 profit on each short jacket and \$80 on each long one, how many jackets of each type should he have manufactured per week in order to maximize his profit?

Objective Function

If we let, X1 be the number of short, and X2 be the number of long leather jackets produced per week, so that the designer's profit is maximized, then the profit, per week, would be,

P = 50*X1 + 80*X2

Looking at this equation from a mathematical standpoint, it looks like the profit (P) can be made as large as we like by simply producing more and more leather jackets. But, here is where the various real-life limitations come in. Any manufacturer, no matter how small or how large, has manufacturing limits imposed by available materials, available manpower, demand, available plant capacity, etc. These limits are usually referred to as constraints.

Here is what these constraints look like in our situation (again, assuming X1 short jackets and X2 long jackets are produced).

Cutting department constraint

```
Sewing time       Sewing time    Equal    Maximum Labor
for short    +    for long       to or    hours
jackets           jackets        Less     available in
per week          per week       than     Assembly dept.
-------------------------------------------------------
   1*X1      +      2*X2          =<           32
```

Sewing department constraint

```
Sewing time       Sewing time   Equal   Maximum Labor
for short    +    for long      to or   hours
jackets           jackets       Less    available in
per week          per week      than    Assembly dept.
-----------------------------------------------------
    3*X1     +       4*X2        =<          84
```

Demand constraint
As stated in the problem, the distributor cannot take any more than 12 long leather jackets per week, so

X2 =< 12

Non-Negative Constraints
Many times we overlook these constraints, and luckily, in most situations they have no bearing, but they are very important and we should always include them if we want to guarantee the correct solution every time.

X1 >= 0
X2 >= 0

Mathematical Model
Now, let us group all our data and equations together, in preparation to enter them into AS-EASY-AS.

We want to maximize: **P = 50*X1 + 80*X2** (Objective function)

Subject to the constraints:

```
  X1  +2*X2  =<   32
3*X1  +4*X2  =<   84
         X2  =<   12
  X1         >=    0
         X2  >=    0
```

AS-EASY-AS Model
A portion of the AS-EASY-AS screen, showing the entry for this problem, is shown below (Note that all instructions given further down are based on the assumption that the data entry in AS-EASY-AS corresponds to the cells shown below).

	A	B	C	D
1	*1*	*2*	*LE*	*32*
2	*3*	*4*	*LE*	*84*
3	*0*	*1*	*LE*	*12*
4	*1*	*0*	*GE*	*0*
5	*0*	*1*	*GE*	*0*
6	*50*	*80*	*EQ*	*MAX*
7				

After you've entered the data in your AS-EASY-AS worksheet, we are ready to solve this problem.

1. Select Array, Linear
2. Specify A1..D7 as the Input Range
3. Specify B9 as the Output Range
4. Click on OK to finish the operation

The program will solve the problem for you and the screen should look like the one shown below.

	A	B	C	D
1	*1*	*2*	*LE*	*32*
2	*3*	*4*	*LE*	*84*
3	*0*	*1*	*LE*	*12*
4	*1*	*0*	*GE*	*0*
5	*0*	*1*	*GE*	*0*
6	*50*	*80*	*EQ*	*MAX*
7				
8				
9	*MAX*	*1480*		
10	*X1*	*20*		
11	*X2*	*6*		

The results in cells A9..B11 indicate that this was a maximization problem. The maximum value of the objective function is 1480, and the corresponding X1 and X2 values are 20 and 6. Or, in terms of our real life problem, given the constraints we have, the designer can maximize his weekly profit ($1,480), by having 20 short and 6 long jackets manufactured per week.

<u>Observations</u>

In order to provide a better understanding into the powerful process of linear programming, here we present some common questions that might come up.

Q1. Well, since we know that long leather jackets make significantly more profit for the designer, why not manufacture just long jackets and skip the short ones altogether?

A1. Because for starters, according to the stated constraints, the distributor can only take 12 long jackets a week. so the maximum profit that could be achieved would be 12 * 80 = 960 which is lower than that produced by our solution, i.e., it's not the 'maximum'.

Q2. If Q1 is true, then why not produce the max 12 long jackets and then use the remaining resources for short jackets? It might generate more profits!

A2. Production of 12 long jackets would use up 12 * 2 = 24 hours of the 32 available in the cutting department. There would only be 8 labor-hours left, which would result in a maximum of 8 short jackets produced. This would result in an additional profit of 8 * 50 = 400. Added to the 960 profit realized by the 12 long jackets (see A1), it would give us a total of 1360, which is still less than the amount given by the optimized solution!

Q3. The cutting time required for short jackets is 1 labor-hour, and that for long jackets is 2 labor-hours, i.e., twice as long. However, in terms of profit, long jackets produce only about 60% more (less than a factor of 2). Wouldn't it make sense to produce ONLY short jackets and maximize the profit, especially where there is no distributor limit as in Q1 above?

A3. The cutting dept. could produce a maximum of

```
32 labor hours per week
-----------------------  =  32 short jackets in a week
1 labor-hour per jacket
```

whereas the sewing dept could complete a maximum of

```
84 labor hours per week
-----------------------  =  28 short jackets in a week
3 labor-hour per jacket
```

The 28 short jackets (maximum possible per week), would generate a profit of $1250, which is less than the profit generated by the combination suggested by the AS-EASY-AS solution.

Statement of problem (2)

A cattle rancher uses three types of cattle food, Type(1), Type(2), and Type(3). The cost per pound of each is: Type(1)=$1.5, Type(2)=$3.5, and Type(3)=$2.0. The rancher wants to meet published nutritional minimum daily requirements (MDR), in milligrams of Vitamins A, B and C per animal. The MDR and nutritional content for the three types of cattle food, in mg/lb, is shown in the table below.

Vitamin	MDR	Type(1)	Type(2)	Type(3)
A	120	8	2	20
B	180	9	11	5
C	100	1	10	20

Because of protein content, however, an animal cannot eat more than 15 lbs of Type(1), 10 lbs of Type(2) and 5 lbs of Type(3) of cattle food per day. How many pounds of each type of cattle food should the rancher purchase per day in order to minimize his cost, and still meet the MDR?

Objective Function

If we let X1, X2 and X3 be the number of pounds of each type of food the rancher buys per day, for each animal, then the total cost per day (the function that needs to be minimized), is:

P = 1.5*X1 + 3.5*X2 + 2.0*X3

Looking at this equation from a mathematical standpoint, it looks like the expense can be continuously minimized by simply reducing the pounds of each type of cattle food purchased, or by purchasing only Type(1) feed which is the least expensive one (shown as X1 in the above equation). But, here is where the various real-life limitations come in. There are minimum nutritional requirements, minimum feed per day per animal, etc. These limits are usually referred to as constraints.

Here is what these constraints look like in our situation. (Note that all the quantities below are per day, per animal).

Vitamin A Nutritional Constraints

```
 Vitamin A      Vitamin A      Vitamin A          Minimum
 obtained   +   obtained   +   obtained    At     Vitamin A
 from food      from food      from food   Least  required
 Type(1)        Type(2)        Type(3)            per day
---------------------------------------------------------
    8*X1    +      2*X2    +      20*X3     >=      120
```

Vitamin B Nutritional Constraints

```
 Vitamin B      Vitamin B      Vitamin B          Minimum
 obtained   +   obtained   +   obtained    At     Vitamin B
 from food      from food      from food   Least  required
 Type(1)        Type(2)        Type(3)            per day
---------------------------------------------------------
    9*X1    +      11*X2   +       5*X3     >=      180
```

Vitamin C Nutritional Constraints

```
 Vitamin C      Vitamin C      Vitamin C            Minimum
 obtained   +   obtained   +   obtained     At      Vitamin C
 from food      from food      from food    Least   required
 Type(1)        Type(2)        Type(3)              per day
-----------------------------------------------------------
    1*X1    +     10*X2    +     20*X3       >=       100
```

Total Animal Feed Constraints

```
  Total Type(1)
  Food an animal      Less Than    15
  can eat per day
--------------------------------------
        X1               =<        15


  Total Type(2)
  Food an animal     Less Than     10
  can eat per day
--------------------------------------
        X2               =<        10


  Total Type(3)
  Food an animal     Less Than     5
  can eat per day
------------------------------------
        X3               =<        5
```

Non-Negative Constraints

Many times we overlook these constraints, and luckily, in most situations they have no bearing, but they are very important and we should always include them if we want to guarantee the correct solution every time.

X1 >= 0
X2 >= 0
X3 >= 0

Mathematical Model
Now, let us group all our data and equations together, in preparation to enter them into AS-EASY-AS.

Minimize: **P = 1.5*X1 + 3.5*X2 + 2*X3** (Objective function)
Subject to the constraints:

```
8*X1    +2*X2   +20*X3  >=  120
9*X1   +11*X2    +5*X3  >=  180
  X1   +10*X2   +20*X3  >=  100
  X1                    =<   15
          X2            =<   10
                    X3  =<    5
  X1                    >=    0
          X2            >=    0
                    X3  >=    0
```

AS-EASY-AS Model
A portion of the AS-EASY-AS screen, showing the entry for this problem, is shown below (Note that all instructions given further down are based on the assumption that the data entry in AS-EASY-AS corresponds to the cells shown below).

	A	**B**	**C**	**D**	**E**
1	*8*	*2*	*20*	*GE*	*120*
2	*9*	*11*	*5*	*GE*	*180*
3	*1*	*10*	*20*	*GE*	*100*
4	*1*	*0*	*0*	*LE*	*15*
5	*0*	*1*	*0*	*LE*	*10*
6	*0*	*0*	*1*	*LE*	*5*
7	*1*	*0*	*0*	*GE*	*0*
8	*0*	*1*	*0*	*GE*	*0*
9	*0*	*0*	*1*	*GE*	*0*
10	*1.5*	*3.5*	*2.0*	*EQ*	*MIN*

After you've entered the data in your AS-EASY-AS worksheet, we are ready to solve this problem.

1. Select Array, Linear
2. Specify A1..E10 as the Input Range
3. Specify A12 as the Output Range
4. Click on OK to finish the operation

The program will solve the problem for you and the screen should look like the one shown below.

	A	**B**	**C**	**D**	**E**
1	*8*	*2*	*20*	*GE*	*120*
2	*9*	*11*	*5*	*GE*	*180*
3	*1*	*10*	*20*	*GE*	*100*
4	*1*	*0*	*0*	*LE*	*15*
5	*0*	*1*	*0*	*LE*	*10*

6	*0*	*0*	*1*	*LE*	*5*
7	*1*	*0*	*0*	*GE*	*0*
8	*0*	*1*	*0*	*GE*	*0*
9	*0*	*0*	*1*	*GE*	*0*
10	*1.5*	*3.5*	*2.0*	*EQ*	*MIN*
11					
12	*MIN*	*38*			
13	*X1*	*15*			
14	*X2*	*2.8*			
15	*X3*	*2.9*			

The results in cells A12..B15 indicate that this was a minimization problem. The minimum value of the objective function is 38, and the corresponding X1, X2 and X3 values are 15, 2.8, and 2.9. Or, in terms of our real life problem, given the constraints we have, the rancher can minimize his cost to $38 per day per animal, and still meet all his nutritional requirements by buying 15 lbs of Type(1), 2.8 lbs of Type(2), and 2.9 lbs of Type(3) cattle food.

Statement of problem (3)

A nutritionist recommends to a colleague the following minimum daily requirements of vitamin B and vitamin C:

400 units of vitamin B
800 units of vitamin C

The local pharmacy supplies two different vitamin tablets, brand Y and brand Z. Vitamin tablet Y contains 75 units of vitamin B and 100 units of vitamin C and costs $0.05. Vitamin tablet Z contains 50 units of vitamin B and 200 units of vitamin C and costs $0.04. How many vitamin tablets of each brand should be consumed to satisfy the daily requirements at a minimal cost?

Objective Function

If we let X1 be the number of brand Y tablets and X2 be the number of brand Z tablets, then the total cost per day (the function that needs to be minimized), is:

P = .05*X1 + .04*X2

Looking at this equation from a mathematical standpoint, it looks like the expense can be minimized by simply purchasing brand Z tablets ($0.04). However, in real life, like in the previous examples, there are limitations. There are minimal nutritional daily requirements to be met. Such limitations limits are usually referred to as constraints.

Here is what these constraints look like in our situation:

Vitamin B Nutritional Constraints

```
 Vitamin B      Vitamin B                 Minimum units
 obtained   +   obtained      At          Vitamin B
 from           from          Least       required
 Brand Y        Brand Z                   per day
-------------------------------------------------------
    75*X1   +     50*X2        >=            400
```

Vitamin C Nutritional Constraints

```
 Vitamin C          Vitamin C                 Minimum units
```

```
 obtained       +    obtained         At       Vitamin C
 from Brand Y        from Brand Z     Least    required per day
--------------------------------------------------------------
   100*X1       +    200*X2            >=          800
```

Non-Negative Constraints

Many times we overlook these constraints. In most situations they have no bearing to the solution of the problem. However, they are very important and we should always include them if we want to guarantee the correct solution every time.

X1 >= 0
X2 >= 0

Mathematical Model

Now, let us group all our data and equations together, in preparation to enter them into AS-EASY-AS.

Minimize: **P = .05*X1 + .04*X2** (Objective function)

Subject to the constraints:

```
 75*X1       +50*X2     >=   400
100*X1      +200*X2     >=   800
    X1                 >=     0
                  X2   >=     0
```

AS-EASY-AS Model

A portion of the AS-EASY-AS screen, showing the entry for this problem, is shown below (Note that all instructions given further down are based on the assumption that the data entry in AS-EASY-AS corresponds to the cells shown below).

	A	**B**	**C**	**D**
1	*75*	*50*	*GE*	*400*
2	*100*	*200*	*GE*	*800*
3	*1*	*0*	*GE*	*0*
4	*0*	*1*	*GE*	*0*
5	*.05*	*.04*	*EQ*	*MIN*

After you've entered the data in your AS-EASY-AS worksheet, we are ready to solve this problem.

1. Select Array, Linear
2. Specify A1..D5 as the Input Range
3. Specify A7 as the Output Range
4. Click on OK to finish the operation

The program will solve the problem for you and the screen should look like the one shown below.

	A	**B**	**C**	**D**
1	*75*	*50*	*GE*	*400*
2	*100*	*200*	*GE*	*800*
3	*1*	*0*	*GE*	*0*
4	*0*	*1*	*GE*	*0*
5	*.05*	*.04*	*EQ*	*MIN*
6				
7	*MIN*	*0.28*		

8	*X1*	*4*
9	*X2*	*2*

The results in cells A7..B9 indicate that this was a minimization problem. The minimum value of the objective function is .28, and the corresponding X1 and X2 values are 4 and 2. Or, in terms of our real life problem, given the constraints we have, the individual can minimize the cost to $0.28 per day and still meet all nutritional requirements by purchasing 4 tablets of brand Y and 2 tablets of brand Z.

Statement of problem (4)

A savings & loan company has $3 million in funds to lend. Local and State laws require that at least 50% of all monies loaned for mortgages must be for first mortgages and that at least 30% of the total amount loaned must be for either first or second mortgages. Company policy requires that signature and automobile loans cannot exceed 25% of the total amount loaned and that signature loans cannot exceed 15% of the total amount loaned. How much money should be allocated to each type of loan in order to maximize the company's return?

The types of loans and annual returns for each type are given below:

Type of Loan	Annual Return
Signature	18%
First Mortgage	12%
Second Mortgage	14%
Automobile	16%

Objective Function

If we let,

X1 represent Signature loans,
X2 represent First Mortgages,
X3 represent Second Mortgages, and
X4 represent Automobile loans,

then the company's return (the function that needs to be maximized), can be expressed as:

P = 0.18*X1 + 0.12*X2 + 0.14*X3 + 0.16*X4

Looking at this equation from a mathematical standpoint, it looks like the annual return can be maximized by simply providing Signature loans. However, in real life (and as indicated in this example), we have to take into account State and Federal regulations as well as Company guidelines and limitations. These limitations will become the problems constraints.

Fund Constraints

```
 Sign.       First        Second       Auto                Total
 Loan    +   Mortg.   +   Mortg.   +   Loan       Less     Loan
 Amount      Amount       Amount       Amount     Than     Amount
--------------------------------------------------------------------
    X1   +     X2     +      X3    +     X4        <=      3000000
```

State & Federal Law Constraints

```
 First              Second                  30% of
 Mortgage    +      Mortgage      At        Total Loan
 Amount             Amount        Least     Amount
```

```
-------------------------------------------------
   X2      +      X3          >=        900000

                                                  Assures First and Second
 First          Second                            Mortgages are at least
 Mortgage   -   Mortgage    At        0           equal, to  meet "50% of
 Amount         Amount      Least                 Total Mortgage Funds
                                                  loaned must be for First Mortgages
----------------------------------------
   X2      -      X3          >=      0
```

Company Guidelines Constraints

```
 Signature      Automobile               25% of
 Loan      +    Loan           Less      Total Loan
 Amount         Amount         Than      Amount
----------------------------------------------------
   X1      +     X4             <=        750000

 Signature               15% of
 Loan         Less       Total Loan
 Amount       Than       Amount
----------------------------------
   X1          <=         450000
```

Non-Negative Constraints

Many times we overlook these constraints, and luckily, in most situations they have no bearing, but they are very important and we should always include them if we want to guarantee the correct solution every time.

X1 >= 0
X2 >= 0
X3 >= 0
X4 >= 0

Mathematical Model

Now, let us group all our data and equations together, in preparation to enter them into AS-EASY-AS.
Minimize: **P = .18*X1 + .12*X2 + .14*X3 + .16*X4** **(**Objective function)

Subject to the constraints:

```
X1   +X2   +X3     +X4   <=   3000000
      X2   +X3           >=    900000
      X2   -X3           >=         0
X1                 +X4   <=    750000
X1                       <=    450000
X1                       >=         0
      X2                 >=         0
           X3            >=         0
                    X4   >=         0
```

AS-EASY-AS Model

A portion of the AS-EASY-AS screen, showing the entry for this problem, is shown below (Note that all instructions given further down are based on the assumption that the data entry in AS-EASY-AS corresponds to the cells shown below).

	A	**B**	**C**	**D**	**E**	**F**
1	*1*	*1*	*1*	*1*	*LE*	*3000000*
2	*0*	*1*	*1*	*0*	*GE*	*900000*
3	*0*	*1*	*-1*	*0*	*GE*	*0*
4	*1*	*0*	*0*	*1*	*LE*	*750000*
5	*1*	*0*	*0*	*0*	*LE*	*450000*
6	*1*	*0*	*0*	*0*	*GE*	*0*
7	*0*	*1*	*0*	*0*	*GE*	*0*
8	*0*	*0*	*1*	*0*	*GE*	*0*
9	*0*	*0*	*0*	*1*	*GE*	*0*
10	*0.18*	*0.12*	*0.14*	*0.16*	*EQ*	*MAX*

After you've entered the data in your AS-EASY-AS worksheet, we are ready to solve this problem.

1. Select Array, Linear
2. Specify A1..F10 as the Input Range
3. Specify A12 as the Output Range
4. Click on OK to finish the operation

The program will solve the problem for you and the screen should look like the one shown below.

	A	**B**	**C**	**D**	**E**	**F**
1	*1*	*1*	*1*	*1*	*LE*	*3000000*
2	*0*	*1*	*1*	*0*	*GE*	*900000*
3	*0*	*1*	*-1*	*0*	*GE*	*0*
4	*1*	*0*	*0*	*1*	*LE*	*750000*
5	*1*	*0*	*0*	*0*	*LE*	*450000*
6	*1*	*0*	*0*	*0*	*GE*	*0*
7	*0*	*1*	*0*	*0*	*GE*	*0*
8	*0*	*0*	*1*	*0*	*GE*	*0*
9	*0*	*0*	*0*	*1*	*GE*	*0*
10	*.18*	*.12*	*.14*	*.16*	*EQ*	*MAX*
11						
12	*MAX*	*423560*				
13	*X1*	*450000*				
14	*X2*	*1125000*				
15	*X3*	*1125000*				
16	*X4*	*3000000*				

The results in cells A12..B16 indicate that this was a maximization problem. The maximum value of the objective function is 423,560, and the corresponding X1, X2, X3, and X4 values are 450,000, 1,125,000, 1,125,000, and 300,000. Or, putting it in terms that the loan company's officers would understand, given the constraints we have, the bank can maximize their returns and still meet all State/Federal law and company guidelines requirements by lending an amount equal to $450,000 for Signature loans, $1,125,000 for First mortgages, $1,125,000 for Second mortgages, and $300,000 for Automobile loans. The maximum return, under those conditions, would be: $423,559.60.