

SyMan Lesson 9: Sine Curves Part II

In the previous exercise, you explored the curve of $y=\sin(x)$. In this exercise, you will continue your work with the sine function.

As you follow along with the example below, be sure to read the explanation after each step. These explanations tell you why you are doing each step, and give further helpful advice.

Step 1 **Enter and graph the function $y=\sin(x)$.**

Step 2 **Without clearing the graph display, enter and graph the function $y=\sin(2x)$. How does the second graph compare to the first?**

The period of the original $y=\sin(x)$ was 2π (the function starts to repeat itself after 2π). For $y=\sin(2x)$, the ' $2x$ ' term changes twice as fast as ' x ', so the period is only π . Obviously the coefficient of the ' x ' term controls how fast the sine curve varies— it determines the amount of horizontal "squeezing" or "stretching."

Step 3 **If the period of $y=\sin(x)$ was 2π , and the period of $y=\sin(2x)$ was π , what is the period of $y=\sin(4x)$? Use a graph to check your answer.**

period = _____

Notice that the period of $\sin(2x)$ is $\frac{1}{2}$ that of $\sin(x)$. Similarly, $\sin(3x)$ has a period that is $\frac{1}{3}$ that of $\sin(x)$; and $\sin(4x)$ has a period that is $\frac{1}{4}$ that of $\sin(x)$. (respectively, the periods are 2π , $\frac{1}{2}2\pi$, $\frac{1}{3}2\pi$, $\frac{1}{4}2\pi$ in unsimplified form)

Step 4 **Clear the graph and re-graph $y=\sin(x)$. What function will have a period that is $\frac{1}{5}$ that of $\sin(x)$? (i.e. is "squeezed" by a factor of 5) Check your answer by graphing. Also, state the exact value of the period.**

$y =$ _____ , period = _____

Step 5 **Clear the graph, re-graph $y=\sin(x)$, and then graph $y=\sin(\frac{1}{2}x)$. How are the two functions related?**

Step 6 **What equation would you enter to graph a sine curve that has a period three times as great as $\sin(x)$? (i.e. is stretched by a factor of 3) Graph your answer to check it. Also, if the period of $\sin(x)$ is 2π , what is the period of your answer?**

$y =$ _____ , period = _____

Step 7 Without graphing, describe how the graph of $y = \sin(\pi(1,5) x)$ compares to the graph of $y = \sin(x)$.

Step 8 Clear the graph, re-graph $y = \sin(x)$, and then graph $y = \sin(x) + 1$. How are the two functions related?

Step 9 What would you enter to graph a sine curve similar to $y = \sin(x)$ but shifted up 3 units? Graph your answer to check it.

$y =$ _____

Step 10 What would you enter to graph a sine curve that it is shifted down 2 units?

$y =$ _____

When you have completed Steps 1 through 10, go on to answer the following questions:

1.) Write down the equation that produces a sine curve with the specified characteristics. (stretches are implied to be horizontal stretches)

a) stretch of 2 $y =$ _____

b) stretch of 6, shift up of 2 $y =$ _____

c) stretch of 0.25, shift down of 4 $y =$ _____

d) stretch of $\pi(1, 2)$ $y =$ _____

e) stretch of $\pi(1, 10)$ shift down of 1 $y =$ _____

f) stretch of 2.5, shift up of 1.5 $y =$ _____

g) stretch of 5, shift down $\pi(1, 5)$ $y =$ _____

h) stretch of $\pi(4, 5)$ shift up $\pi(7, 3)$ $y =$ _____

i) stretch of $\pi(2, 3)$ shift down $\pi(10, 7)$ $y =$ _____

- 2.) Use the information from the previous lesson to determine the equation that produces a sine curve with the following characteristics. (stretches are horizontal)

- | | | |
|----|--|-------------|
| a) | amplitude of 2, stretch of 3 | $y =$ _____ |
| b) | stretch of 2, reflected in x-axis | $y =$ _____ |
| c) | stretch of 0.75, shift right 0.5 | $y =$ _____ |
| d) | reflected in x-axis, shift left $\pi(1, 3)$ | $y =$ _____ |
| e) | reflected in x-axis, stretch of $\pi(1, 3)$ | $y =$ _____ |
| f) | stretch of 5, shift up of 3,
reflected in x-axis | $y =$ _____ |
| g) | stretch of 5, shift down $\pi(1, 5)$
shift right 2 | $y =$ _____ |
| h) | amplitude 4, stretch of $\pi(5, 3)$
shift up $\pi(3, 4)$ | $y =$ _____ |
| i) | amplitude $\pi(1, 2)$ stretch of 3,
shift down 5, reflected in x-axis | $y =$ _____ |

- 3.) On a clear graph, graph $y = \sin(x)$, then graph $y = \sin(-x)$. Compare the graphs.

Is it possible that the second graph is a reflection of the first one in the x-axis? Is it possible that the second is a reflection in the y-axis?

To help you decide, clear the graph and then graph $y = \sin(x+0.5)$ and $y = \sin(-x+0.5)$. Once you decide if it is a reflection in the x or y axis, explain your reasoning:

- 4.) The general form of the sine function is written as $y = a \sin(bx + c) + d$, where a , b , c , d are simply constants. For example, in the case of $y = 2\sin(-3x + 4) - 5$, the constants are $a=2$, $b=-3$, $c=4$, $d=-5$.

Describe the effect that the following ranges of the constants have on the graph of the general sine function. Try choosing a value in the specified range and consider what effect such a choice would have — then generalize from your choice.

$$a > 1$$

$$0 < a < 1$$

$$-1 < a < 0$$

$$a < -1$$

$$b > 1$$

$$0 < b < 1$$

$$-1 < b < 0$$

$$b < -1$$

$$c > 0$$

$$c < 0$$

$$d > 0$$

$$d < 0$$
