ChaosPlot

by Jason Regier

What is ChaosPlot?

ChaosPlot is a simple program which plots the chaotic behavior of a damped, driven anharmonic oscillator. This program, intended to be used for educational purposes, allows the user to change parameters in the differential equation describing the oscillator and to modify various plotting parameters.

How Does It Work?

ChaosPlot evaluates the following differential equation:

$$\frac{d^2x}{dt^2} = F_0 \sin(\omega t) - kx - qx^3 - b\frac{dx}{dt}$$

for various values of the parameters F_0, ω, k, q, and b and the initial conditions x_0 and v_0. F_0 is the amplitude of the driving force and ω controls its frequency. k is the spring constant for the ideal spring portion of our spring potential. q represents the amplitude of the anharmonic term of the spring potential which makes our spring vary from one which follows Hooke's law. b is the amplitude of the drag term, which depends on the velocity of the spring's motion.

To plot this complex equation, ChaosPlot splits the second order differential equation into two first order differential equations and solves them simultaneously using a fourth-order Runge-Kutta algorithm:

$$\frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = f(x,t) = F_0 \sin(\omega t) - kx - qx^3 - bv$$

The results of this algorithm are passed to a plotting routine which plots the points on either the x vs. t, v vs. t, or v vs. x axes. The input parameters, including equation constants, scale of the plot, step size, and time length, can be controlled from the **Change Plot Parameters** menu item under the **File** menu. To change between each of the 3 plots listed above, one need merely select the desired plot from the **Plot** menu. As usual, to change the printing parameters, select **Page Setup…** from the **File** menu. To Print the plot currently displayed on screen, select **Print…** from the **File** menu. The current plot parameters will be printed below the current plot. To quit the program, click in the close box in the upper left of the Plot window or select **Quit** from the **File** menu.

Notes...

Here are some physical interpretations of various input equation parameters:

$F_0 = b = q = 0, k > 0$	Simple harmonic oscillator
$F_0 = k = q = 0, b > 0$	Nonoscillatory damped motion
$F_0 = q = 0$, k and $b > 0$	Damped simple harmonic oscillator
$F_0 = 0, k < 0$, and $q, b > 0$	Damped anharmonic oscillator

$q = 0$ and $F_0, \omega, k, b > 0$	Damped, driven harmonic oscillator
$k < 0$ and $F_0, \omega, q, b > 0$	Damped, driven, anharmonic oscillator

Here are some input parameters that you might find interesting:

Fo	ω	k	q	b	x(0)	v(0)	H	xmin	xmax	vmin	vmax	tmax
0.23	1	-1	1	0.1	1	0	0.1	-3	3	-3	3	150
1	0.05	-1	1	0.1	2	0	0.1	-3	3	-3	3	139
1	0.05	-1	1	0.5	2	0	0.1	-3	3	-3	3	250
0.4	1.5	0.5	0.35	0.05	1	0	0.01	-3	3	-3	3	50
0.4	1.5	0.5	0.35	0.1	1	0	0.1	-3	3	-3	3	150

Remember, using excessively small step sizes will lead to a very accurate solution of the differential equation, but can take a VERY long time to execute and to print.

You might notice some small wobbles in the plots produced by ChaosPlot. These tiny wobbles are most likely not real, but are rather a result of connecting successive data points. The finite step size in the Runge-Kutta algorithm means that curves are not truly smooth; they are formed by connecting successive data points with straight lines. I have found that a smaller step size does not necessarily remove the wobbles. In fact, sometimes the wobbles may be more noticeable because of severe overplotting using small increments. If you really care whether a wobble is truly there or is a figment of the algorithm, change the plot parameters to zoom in on that portion of the plot.

Legalese...

I hope you have fun with ChaosPlot and learn a little about the fundamentals of chaos! Experiment with this program and send me some interesting parameter settings. If you think ChaosPlot is worth keeping, please send me $5-$10 as a reimbursement for the time I spent working on this project. I can be reached at:

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