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Reference Book

This document describes the rules for all the calendars implemented in InterCal, along with some background and historical information. Between the times versions 1.3 and 1.4 of InterCal were released, in September 1997, an excellent book describing the rules for all the calendars implemented by InterCal (plus several others) was published. I strongly recommend that anyone interested enough to read "Calendar System Facts" also read the new book. It is <u>Calendrical Calculations</u> by Nachum Dershowitz and Edward M. Reingold of the Department of Computer Science of the University of Illinois at Urbana-Champaign. The publisher is Cambridge University Press. The ISBN numbers are 0-521-56413-1 (hardback) and 0-521-56474-3 (paperback). The book has a web page whose URL is

http://emr.cs.uiuc.edu/home/reingold/calendar-book/index.html

I am indebted to Dershowitz and Reingold for providing me with the rules for the Mayan and Chinese calendars in advance of publication. (Actually, the rules for many of the calendars, including the Mayan but not the Chinese, had been published by them previously in articles in the computer science literature.)

"Calendar System Facts" occasionally refers to <u>Calendrical Calculations</u> rather than repeat long and complex rules. But the historical and background information in the two sources is different. And <u>Calendrical Calculations</u> is written in a more formal, more mathematical, and more computer-science-oriented style than "Calendar System Facts". Some readers will prefer the way Dershowitz and Reingold explain the rules, while other may prefer the style herein.

Basic Astronomical Data

The length of the tropical year = 365.242199 days in 1900. At the present time it is slowly getting shorter. The tropical year is the mean time between vernal (spring) equinoxes.

The length of the synodic month = 29.530588853 days. At the present time it is also getting shorter very slowly, but tidal effects will cause it to grow steadily longer in the long run. The synodic month is the mean time between new moons.

Julian Day Numbers

Astronomers keep a running count of days which began several thousand years in the past. This running day count is independent of all calendar systems, and can therefore be used as a convenient way to transform dates from one system to another. It also is a convenient way to define the starting points (first day of the first month of the year 1) of calendar systems. By convention, these days begin at noon. They are called Julian Days, which is unfortunate because they must not be confused with the Julian Calendar system. Thus calendar days, which in the Gregorian and related systems begin at midnight, usually have their Julian Day numbers equal to some integer plus 0.5. Given the Julian Day of any calendar date, the day of the week can easily be computed simply by dividing by seven and examining the remainder.

The zero point of the Julian Day system occurred at noon Greenwich Mean Time (GMT) on January 1, 4713 B.C. in the Julian Calendar, or November 24, 4714 B.C. in the Gregorian calendar. When astronomers use the Julian day, the fractional part is meaningful and all Julian Day numbers are assumed to refer to GMT (more technically, to a quantity called Universal Time which is very close to GMT). For calendrical purposes, differences of time zones are generally ignored—that is, at each location a given date starts at midnight local time (Gregorian and its relatives, Mayan, Chinese, Elliott Super) or sunset local time (Jewish and Islamic).

General Calendrical Rules ("Linear" Calendars)

Most calendars are based on the common assumption that time proceeds linearly. From any moment one can count days backwards or forwards infinitely far (in theory). As a result, there is a one-to-one correspondence between calendar dates and a straight count of the days (such as the one provided by the Julian Day number). Most calendars supported by InterCal fall into this category. Exceptions are the Mayan and Chinese, in which the year number continually cycles over a relatively narrow range. (However, the Mayan did count **days** linearly—see the discussion of the Long Count in the Mayan Calendar section.)

1) The infinite range of years is divided into two *eras*, each of which is sometimes given a common abbreviation (e.g., B.C. and A.D.)

2) There is never a year 0. Positive-numbered years are in one era; negative-numbered years are in the other. The abbreviation for the era tells the sign of the year. Actual plus and minus signs are normally not used. (This rule is due to the fact that modern calendars are all evolved from earlier ones which were developed before the concepts of zero and negative integers were well understood.) <u>Calendrical Calculations</u> does not follow this convention for most calendars. Instead, it allows a year 0, ignores eras, and uses negative numbers

for years before zero. This is all done for mathematical convenience. As long as the rules are clearly stated and consistently followed, either method works fine.

3) The sequence of weekdays (Sunday…Saturday) is assumed (by InterCal) to extend indefinitely into both the past and future without interruption. Babylonians developed the seven-day week. The emperor Constantine officially adopted it for use in the Roman Empire in the fourth century, although it had been in common use for a very long time before that. The rules of most calendars are completely independent of this ever-repeating seven-day cycle. The Jewish calendar is an exception. Its rules are carefully designed to prevent certain holy days from falling on certain days of the week.

4) Calendars are nothing more than day-naming systems. Months and years are merely convenient larger groupings of days. Usually, calendars do not depend on finer divisions of time (hours, minutes, etc.) in their rules. Once again, the Jewish calendar is an exception. The time of day of new moon nearest the start of a year can affect the date of Rosh Hashonah.

5) Calendars differ according to the significance given to the larger groupings (such as months or years). Although not all calendars fit into the following groups, all linear calendars implemented by InterCal **do**.

   a) Lunar calendars—in lunar calendars the goal is to have the date within each month be a good indicator of the phase of the moon. Most lunar calendars have their months start and end as close to new moon as feasible. In such calendars, the first of the month is near new moon, the eighth near first quarter, the sixteenth near full moon, and the twenty-third near third quarter. Thus it is desirable to have the average length of a month be equal to the synodic month. Years are just convenient larger groupings of months, and bear no particular relation to the motion of the earth-moon system around the sun.

   b) Solar calendars—in solar calendars the goal is to have the date (month and day) be a good indicator of the season of the year. Thus it is desirable for the average length of a year to be equal to the tropical year. Months are just convenient groupings of days which bear no fixed relation to the phase of the moon.

   c) Luni-solar calendars—in luni-solar calendars the object is to be both a lunar and a solar calendar at the same time. Naturally, this leads to more complex calendrical rules. The tropical year is nowhere close to an integer multiple of the synodic month. The first priority in luni-solar calendars usually is to keep accurate with respect to the moon. Extra months are added every so often to achieve reasonable accuracy with respect to the sun (when averaged over several years).

6) Lunar, Solar, and Luni-Solar calendars differ according to whether their rules are **defined** in terms of the **actual** sun and/or moon, or **defined** in terms of **mathematical formulas** which are meant to approximate the sun and moon. I call the former type *physical* and the latter type *mathematical*. A **physical** calendar cannot actually be implemented with a finite computer algorithm. One must make approximations which mean that the error, as you go farther into the past or future, cannot be quantified. The motions of the sun and moon are not known to infinite precision, and their orbits are not perfectly stable or computable over very long time frames. A **mathematical** calendar can be implemented with perfect accuracy (as far as its rules are concerned) on a computer, but of course over very long time spans it will become a poorer and poorer approximation to the sun and moon which it is supposed to track.

Calendar systems evolve over time. Rules are changed or standardized, and occasionally a new system replaces an older system. InterCal does not attempt to follow the various rules changes in any of its implemented calendars. In fact, for most ancient calendars there were few fixed rules and those that may have existed are largely unknown to modern scholars. InterCal simply extends the rules of each calendar in its present form indefinitely into both the past and the future. Thus, InterCal does not attempt historical accuracy, only mathematical accuracy. **The only exceptions are the "Western Historical Calendar", which is just a graft of the Gregorian Calendar onto the Julian, and the French Revolutionary, in which the original and modified rules have been hybridized (details are in the appropriate sections of this document).**

"Cyclic" Calendars

Calendars are not all linear. For example, in many Mesoamerican cultures (Mayan, Aztec, etc.) an underlying assumption was that the universe is periodically destroyed and re-created. Therefore, Mesoamerican calendars often consist of repeating cycles of days (or day numbers) and months (or day names) without any linearly increasing component (such as a year). If one could determine the date in the Gregorian calendar corresponding to a particular date in one of the cycles, then it would be possible to convert a Julian Day number into the cyclic calendar date unambiguously. **But the reverse is not true.** Given a date in the cyclic calendar, it is only possible to determine an infinite **set** of Julian Days which correspond to it. The days in the set are all separated by the number of days in the calendar cycle.

InterCal supports two cyclic calendars—the Mayan and the Chinese. Actually there are several different Mayan calendars which the Maya themselves frequently used in conjunction with each other. InterCal supports three of these. Additional aspects of the Mayan calendar are discussed later in this document. Also, the Chinese calendar can be treated as a linear calendar by counting its 60-year cycles. InterCal does this. The InterCal user interface for cyclic calendars looks different from the interface for linear calendars. The Mayan interface is radically different; the Chinese interface is only slightly modified from the standard. These differences are discussed in detail in the Users' Guide.

Julian Calendar

Using the classification scheme above, the Julian Calendar is a mathematical solar calendar. It was instituted by Julius Caesar on January 1 (Julian) in the year which is now known as 45 B.C. Of course at first the epoch of the Julian calendar (the first day of the first year) was not the same as it is now. It was a modification and improvement over an older calendar in use in Rome at the time. In order to get the calendar back in synchronism with the sun, Caesar lengthened the year 46 B.C. by several months. In his new system, January, March, May, July, September, and November had 31 days. April, June, August, October, and December had 30 days. February normally had 29, but had 30 in leap years. Augustus Caesar took a day from February and added it to August about 30 B.C. Actually, he also renamed the month, which had not been called August before. Additional rule changes took place during the next few decades. Also, the rules were not always correctly followed. The calendar reached its final form, with the month lengths being what we are used to, in 8 A.D. The Roman year at one time began in March, not January. The adoption of January as the first month occurred around 153 B.C. However, England was reckoning the start of the year at March 25 at the time it switched from the Julian calendar to the Gregorian (see below). So it is not quite correct to say that the Julian calendar was finalized in 8 A.D. There were still a few rule variations which varied from time to time and place to place. The Julian calendar was the official calendar of the Roman Empire, and later the entire Christian world, until its replacement by the Gregorian calendar.

There have been different numbering schemes for the days of February in leap years. One common method, adopted by the InterCal program, was to insert an unnumbered day (simply called the bisextile day) between February 24 and February 25.

The rules of the Julian calendar as implemented by InterCal are:

1) The negative year era is labeled B.C. (Before Christ). The positive year era is labeled A.D. (Anno Domini, Latin for Year of the Lord).

2) Months are named (in order) January, February, March, April, May, June, July, August, September, October, November, December. Years start in January.

3) The Julian Day of midnight at the start of January 1, 1 A.D. Julian (a Saturday) is 1721423.5 (this is the *epoch* of the Julian calendar).

4) January, March, May, July, August, October, and December have 31 days.

5) April, June, September, and November have 30 days.

6) February has 28 days in normal years and 29 days in leap years.

7) The leap year cycle is four years long. Leap years occur every fourth year, as follows:

   …9 B.C., 5 B.C., 1 B.C., 4 A.D., 8 A.D., 12 A.D.… (Note that I do not simply say "years evenly divisible by 4" because that simplification of the rule only works for positive years. Given an unambiguous definition of the cycles, a more precise way of stating the rule would be: "Years **whose position in their cycle** is divisible by 4 are leap years.")

8) The extra day in leap years is added between February 24 and February 25, is not numbered, and is referred to as "bisextile day". **This rule was not standardized—other possibilities were used in various places and times. But this rule was common and is the one implemented by InterCal.**

With a leap year cycle of four years containing 4(365) + 1 days, the average year is 365.25 days, for an error of 0.0078 days per year, or about one day every 128 years.

Gregorian Calendar

The Gregorian Calendar is a slight modification of the Julian. So it is also a mathematical solar calendar. The differences between the Gregorian and Julian calendars involve the leap year rule, the epoch, and the numbering system of days in February of leap years.

The rule differences between Gregorian and Julian:

1) The Julian Day of midnight at the start of January 1, 1 A.D. Gregorian (a Monday) is 1721425.5 (two days later than the epoch of the Julian calendar).

2) The extra day in February of leap years is simply added at the end, becoming February 29.

3) Leap year cycles are 400 years long. One cycle began in 1 A.D. and ended in 400 A.D. The cycle preceding that one began in 400 B.C. and ended in 1 B.C. Within any particular 400-year cycle, years **whose position in the cycle** is divisible by 4 but not by 100 **AND** the final (four-hundredth) year are leap years. As examples, 401 B.C., 1 B.C., and 2000 A.D. are leap years, but 501 B.C. and 1900 A.D. are not.

The Gregorian calendar was instituted by Pope Gregory XIII in 1582 A.D. It had been noticed that the seasons were moving earlier and earlier compared to given calendar dates. Equivalently, any fixed calendar date was occurring later and later in the year as defined by the seasons. It was clear that if the trend were allowed to continue, Easter (which is supposed to be a spring time feast) would move into summer, and then autumn, etc. The Pope dropped ten days so that the vernal equinox would occur close to March 21. That was the date it had in the fourth century A.D. at the time of the Council of Nicaea (325 A.D.). That Council had developed the rule (which later became standard) used to compute the date of Easter. The rule assumes that the vernal equinox is always on March 21. By decree of the Pope, October 4 1582 was immediately followed by October 15 1582. The dates October 5 through October 14 1582 were simply dropped. They did not exist. (This caused rioting in several cities. People thought their lives were being shortened by 10 days.) The leap year rule was changed to try to keep the calendar in synchronism with the seasons. Finally, the rule for Easter was modified to keep it consistent with the new leap year rule and the new epoch of the calendar.

Of course, the Pope is Catholic. So his decree was universally ignored **except** in Roman Catholic countries and their colonies. However, religious beliefs could not alter the fact that the seasons were noticeably sliding. Gradually, Protestant countries adopted the Gregorian calendar. The dates varied from country to country. The later a country made the change, the more days had to be dropped. As European influence spread over the globe, most non-Christian countries adopted the Gregorian calendar, at least for civil use, as a matter of economic convenience. The last to make the change were, in general, Eastern Orthodox Christians. Today this calendar is the civil calendar nearly everywhere (but exceptions, especially in the Middle East, do still exist). Sample conversion dates: England—September 2 1752 was followed by September 14 1752; Japan in 1873; Turkey 1908; Greece converted in 1923. When England switched, it also began reckoning the start of a year as January 1.

The average length of a year in the Gregorian calendar is (400(365)+100-3)/400 = exactly 365.2425 days. This is in error (in the same direction as the error for the Julian Calendar) by about 0.0003 days per year, or one day in 3322 years.

A slight change to the Gregorian Calendar has been proposed which would improve the accuracy considerably. Under the modified rules, the leap year cycle would be 4000 years long. Years *whose position in the cycle* is divisible by 4 but not by 100 **AND** years whose position in the cycle is divisible by 400 but not by 4000 would be leap years. Thus 1900 A.D., 2100 A.D., and 101 B.C. would **not** be leap years (as with the present system). 401 B.C., 2000 A.D., and 3200 A.D. **would** be leap years (also the same as in the present system), but 1 B.C. and 4000 A.D. would **not** be leap years (different from the present system). To my knowledge no country has bothered to adopt this suggestion. Of course it will make no practical difference until the year 4000 A.D. This change, if adopted, would also force a modification in the rule for calculating the date of Easter. InterCal **does not** use the modified rules, but sticks to the traditional Gregorian

Calendar rules. However, the modified French Revolutionary Calendar uses this very same rule.

Western Historical

This “calendar” is not a true calendar at all. It is merely a concatenation of the Julian and Gregorian calendars, with the “stitch” point being October 4, 1582 (Julian Day number 2299159.5). Up to and including that day, the Julian Calendar rules are used. After that day, the Gregorian Calendar rules are used. Thus this calendar accurately represents historical dates (in the countries which adopted the Gregorian calendar as soon as it was decreed) from the stabilization of the Julian calendar forward. For Christian countries which waited until later to adopt the Gregorian Calendar, there is a period from October 15 1582 until their date of switching during which the Western Historical calendar is not historically accurate. Of course for non-Christian countries, the Julian Calendar was never used so the Western Historical calendar makes less sense.

Jewish Calendar

The Jewish calendar is a mathematical luni-solar calendar. It evolved slowly over many, many centuries. During the Babylonian captivity the calendar took on several features of the Babylonian calendar, including the changing of most month names to their Babylonian equivalents. But long after the captivity the calendar continued to be improved. Many sources (most recently Sky and Telescope Magazine, September 1994 issue, “Hayyim Selig Slonimski”, pages 93–95) state that it reached its present form in 359 A.D., during the time of Patriarch Hillel II. But according to the Encyclopedia Judaica it did not reach its modern form until the tenth century A.D. Hillel II took a very important step, however. He made public the rules of the Jewish calendar, which until then had been a closely guarded secret known only to a few Jews and no Gentiles. The Encyclopedia Judaica speculates that Hillel II standardized the leap year rule (Rule #5 below). The more subtle features embodied in Rule 7f below apparently took a few hundred more years to become standardized.

The starting point of the Jewish calendar was intended by its developers to correspond to the creation of the world. Therefore, its zero point is quite far in the past and predates any written history. Not surprisingly, no era label for negative years ever developed. Negative years have not been needed. An era label for positive years exists, at least for use by scholars if not by the ordinary folks, but is rarely seen. It is A.M. (Anno Mundi, Latin for Year of the World). But InterCal needs era labels since it allows both positive and negative years. So I have invented (without intent to offend) the abbreviation B.W. (Before the World) and use A.M. even though it is not common practice.

The Jewish calendar month names and most terms used in its definition are Hebrew words. I know of no **standard** transliteration scheme for converting Hebrew into English. Therefore the English spellings of Jewish months, calendrical terms, and the names of religious holidays (such as Rosh Hashonah) are somewhat arbitrary. I have used the spellings found in the Jewish Calendar program (freeware written by Frank Yellin of Redwood City California). I wish to thank Mr. Yellin for making his source code available on the Internet. In spite of reading the article on the Jewish calendar in the Encyclopedia Judaica and in spite of pestering several Jewish friends, I was unable to develop a computer algorithm for this calendar until I examined Yellin's source. (This was before <u>Calendrical Calculations</u> was published.) The "rules" below are equivalent to the rules in <u>Calendrical Calculations</u>. My contributions in this area were to provide a faster implementation, remove Yellin's limits of dates corresponding to 1 A.D. through 2999 A.D., and extend the rules into negative years. I heartily recommend Yellin's program to anyone interested in the Jewish calendar. It has much more extensive labeling of Jewish religious festivals, holy days, etc. than InterCal.

The rules of the Jewish calendar, when stated (as below) in algorithmic form for use in a computer program, sound complicated and arbitrary. The rationale for the rules is explained in the Encyclopedia Judaica. To put them in the simplest terms—the 19-year leap year cycle with its implied cycle of months is followed exactly, and months start on or near days having a new moon. Most of the complexity is introduced because it is desired to prevent certain holy days (especially Yom Kippur) from falling on certain days of the week.

My rendering of the rules of the Jewish Calendar:

1) The months of a normal year are named (in order) Tishrei, Cheshvan, Kislev, Tevet, Shvat, Adar, Nissan, Iyar, Sivan, Tamuz, Ab, Elul. There are 12 months in normal years.

2) The months of a leap year are named (in order) Tishrei, Cheshvan, Kislev, Tevet, Shvat, Adar I, Adar II, Nissan, Iyar, Sivan, Tamuz, Ab, Elul. There are 13 months in leap years.

3) Month lengths are as follows: Tishrei, Shvat, Adar I, Nissan, Sivan, and Ab have 30 days; Tevet, Adar, Adar II, Iyar, Tamuz, and Elul have 29 days; Cheshvan and Kislev can each have either 29 or 30 days—their variable lengths are related to the special rules for determining the start of each year (1 Tishrei = Rosh Hashonah). The precise rule for their lengths can be found below (Rule #8).

4) The extra month added in leap years is Adar I. Adar and Adar II are actually the same month. Festivals which occur in Adar in normal years are observed in Adar II in leap years.

5) The leap year cycle is 19 years long. One cycle started with the year 1 A.M. and ended with the year 19 A.M. Another cycle started with the year 19 B.W. and ended with the year 1 B.W. Within any 19-year cycle, leap years occur in the third, sixth, eighth, eleventh, fourteenth, seventeenth, and nineteenth years. All other years in the cycle are normal years.

6) The Jewish calendar day begins at sunset, not midnight. InterCal follows the lead of the Encyclopedia Judaica and other sources in assigning weekdays to calendar dates as follows: If a calendar day begins at sunset on a Friday and ends at sunset on a Saturday, that date is displayed as Saturday, and so forth for other days. But note that the times of day used in Rule #7 below are measured from 6:00 p.m.

7) The Julian Day of 1 Tishrei (Rosh Hashonah) of any year Y is determined as follows:

   a) Given any non-zero integer Y, count the months elapsed from the start of the year 1 A.M. to the start of the year Y (using rules 1, 2, and 5 above). Since negative years are allowed, this number of months could be negative.

   b) Multiply that number of months by the number of "parts" in a synodic month. "Parts" of a day are equal to exactly 3 1/3 seconds. There are 25920 parts per day. By **definition** (this is what ensures that the Jewish calendar is mathematical, not physical) a synodic month in the Jewish calendar contains exactly 765433 parts.

   c) Add 5604 to the result of step b. (This is the time of day, in parts, at which <u>by definition</u> new moon occurred on 1 Tishrei of the year 1 A.M.)

   d) Determine the number of **days** elapsed from the new moon associated with 1 Tishrei 1 A.M. to the new moon associated with 1 Tishrei in the year Y. That is, using integer division, divide the result from step c by 25920. Keep the remainder handy for use in later rules. That remainder is the time of day, in parts, of the new moon nearest the start of the year in question. (Integer division and modular arithmetic with negative numbers require care. Truncation must be towards minus infinity, and the remainder must always be non-negative. The lack of a year 0 must also be handled.)

   e) The Julian Day at midnight <u>during</u> 1 Tishrei 1 A.M. (a Monday) is 347997.5. Add 347997.5 to the **integer** part of the result of step d. Determine the day of the week of this new moon.

   f) Tentatively, the Julian Day at midnight during 1 Tishrei (Rosh Hashonah) of the year in question will be the just-calculated answer to step e. It represents the Julian Day at midnight of the day containing the new moon nearest the start of the year Y.

If necessary, adjust the date of Rosh Hashonah by one or two days using the following rules:

i) Determine (using rule 5) whether or not year Y is a leap year. Also determine whether or not the year **before** the year Y was a leap year.

ii) If the time of day of new moon is noon or later (the remainder in step d equals or exceeds 19440 parts) **OR**

iii) If the year Y is **not** a leap year and if the new moon is on a Tuesday and its time of day (remainder in step d) is greater than or equal to 9924 parts (9 hours plus 204 parts, or shortly after 3:00 a.m.) **OR**

iv) If the **preceding** year **is** a leap year and the new moon is on a Monday and its time of day is greater than or equal to 16789 parts (15 hours plus 589 parts, or a few minutes after 9:00 a.m.) **THEN**

Make the tentative day of Rosh Hashonah be the day **after** the day of new moon—that is, the new tentative Julian Day of Rosh Hashonah equals the answer to step e plus 1.

v) Set the actual Rosh Hashonah of the year in question to the tentative day (after possible adjustment using rules f.ii through f.iv) **unless** it would be a Sunday, Wednesday, or Friday. In those cases, make the actual Rosh Hashonah one day **after** the tentative day.

To summarize, the Julian Day at midnight during Rosh Hashonah is the answer to step 7e plus 0 or 1 or 2, depending on whether or not any of the conditions in steps 7.f.ii through 7.f.iv are true and whether or not the condition in step 7.f.v is true.

8) Determine the lengths of Cheshvan and Kislev as follows:

a) Subtract the Julian Day of Rosh Hashonah of the year Y from the Julian Day of Rosh Hashonah of the year **following** the year Y. Simply reapply Rule #7 in order to do this.

b) In spite of the apparent complexity of the rules, there are only six possible answers to step 8a. (Actually, the purpose of rules 7.f.ii through 7.f.v is to force these lengths to be the only ones possible.) The possible year lengths are: 353, 354, 355, 383, 384, and 385. Years of length 353 and length 383 are called “defective”. Years of length 354 and 384 are called “regular”. Years of length 355 and 385 are called “complete”. In defective years, both Cheshvan and Kislev have 29 days. In regular years, Cheshvan has 29 days and Kislev has 30. In complete years, both Cheshvan and Kislev have 30 days.

The average length of a Jewish month is 765433 parts, or 29.53059414… days for an error of 0.000005287 days per month. In 235 months (the number of months in each 19-year cycle) this error has built up to 0.00124 days for an average error of 0.0000654 days per year. Thus the Jewish calendar develops an error of one day with respect to the moon in approximately 15300 years (over four times as accurate with respect to the moon as the Gregorian calendar is with respect to the sun). This is incredibly accurate for a calendar which is over one thousand years old in its present form! (Its month is slightly too long, so new moons will gradually slip backwards through the month as the millennia go by.)

On average, there are (235)29.5305941 days in each 19-year cycle or 365.246822 days per year. This is too long by 0.004623 days per year, or an error of one day every 216 years. This is slightly more accurate than the Julian calendar, but considerably less accurate than the Gregorian. Since the years are too long, actual astronomical events seem to slip backwards. Conversely, the calendar dates of events become later and later compared to the seasons with which they are supposed to be associated. As an example, in the present century Rosh Hashonah typically happens in September, with occasional forays into early October. 11000 years from now, given the present rules, Rosh Hashonah will have slipped nearly two months. It will occur from about 24 October through about 20 November.

Islamic Calendar

The Islamic calendar is a physical lunar calendar. That implies that any computer algorithm only approximates the true calendar. Therefore, going far into the past or the future is even less meaningful than for the mathematical calendars. Nevertheless, InterCal forges ahead!

In the true Islamic calendar, months start at the first official sighting of the new moon. Since it takes one to two days for a new moon to be visible after astronomical new moon. Islamic months are offset by a day or two from astronomical new moon. Note that this calendar is purely lunar, and makes no attempt at all to track the sun. Therefore the months cycle quickly through the seasons—it only takes 33 years for Ramadan (as an example) to cycle from a spring month all around the year and back to spring again.

The formulas used by InterCal (described below) are approximations to the Islamic calendar. I believe this system is in common use throughout most of the Islamic world for planning purposes.

The rules of the mathematical approximation to the Islamic calendar as implemented by InterCal are:

1) The negative year era is labeled B.H. (Before the Hegira). The positive year era is labeled A.H. (Anno Hegira, Year of the Hegira).

2) Months are named (in order) Muharram, Safar, Rabi‘ al-Awwal, Rabi‘ al-Akhir, Jumada’ al-Ula, Jumada’ al-Akhirah, Rajab, Sha’baan, Ramadan, Shawwal, Dhul-Qi’dah, Dhul-Hijjah. Years start in Muharram.

3) Days begin at sunset. The convention used for displaying days is the same as that used for the Jewish calendar, whose days also start at sunset. The Julian Day of midnight **during** Muharram 1, 1 A.H. Islamic (a Friday) is 1948439.5.

4) Muharram, Rabi‘ al-Awwal, Jumada’ al-Ula, Rajab, Ramadan, and Dhul-Qi’dah have 30 days.

5) Safar, Rabi‘ al-Akhir, Jumada’ al-Akhirah, Sha’baan, and Shawwal have 29 days.

6) Dhul-Hijjah has 29 days in normal years and 30 days in leap years.

7) The leap year cycle is thirty years long. One cycle began in the year 1 A.H. and ended in 30 A.H. Year #1 in the preceding cycle was 30 B.H. and year #30 in that cycle was 1 B.H. Leap years occur in years whose position in their cycle is 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, or 29.

With a leap year cycle of thirty years containing 354(30) + 11 days and 360 months, the average month is 29.53055556 days, for an error of 0.00003329 days per month, or approximately one day every 2500 years. The months are slightly too short. This **approximation** to the Islamic calendar has about the same error with respect to the moon that the Gregorian calendar has with respect to the sun. Remember that the official Islamic calendar is based on observations of the real moon, and so by definition has no perceptible error. Because the approximation’s months are too short, the first of each month computed by InterCal will come earlier and earlier compared to the actual start of the month as the millennia go by.

Elliott Super Calendar

The Elliott Super Calendar was invented as a toy by the author of InterCal. It is a mathematical luni-solar calendar. The eras are imaginatively named B.Z. and A.Z. for Before Zero and After Zero.

Just for fun, I decided to find a common multiple of the tropical year and the synodic month which was considerably closer than the 19-year (235-month) Metonic cycle discovered by the Babylonians and used in several other calendars (including the Jewish). I wanted accuracy, but did not want ridiculously long cycles of tens or hundreds of thousands of years. Although such long cycles might produce even greater accuracy, they would be unnecessarily complicated and not worthwhile in view of the long-term instability of the sun and moon's motions. I found a very nice match with a 1689-year cycle containing 20890 months. This is only about four times as long as the Gregorian cycle, yet produces considerably greater accuracy for the sun and throws in the moon as well!

The Elliott Super Calendar uses astronomical new moon in its calculations. That is, it uses as definition of new moon the time at which the earth, moon, and sun are most nearly aligned in a straight line. The Jewish calendar evolved from a system in which actual sightings of a new crescent moon began each month. And the Islamic calendar continues to use such sightings for its official definition. Since a new crescent takes one or two days to become visible after astronomical new moon, the starts of months in the Jewish and Islamic calendars tend to be offset from those in the Elliott calendar by one or two days.

Taking a lesson from the Caesars, I named the first month after myself. Then I bested them by naming all the other months after my wife, children, parents, sisters, and my wife's parents and siblings.

For the zero point, I tried to find a place not too far away from the zero point of the Gregorian calendar. I also had two other criteria. For some year near the zero point I wanted new moon to occur very close to midnight at the start of the first day of the year. I also wanted the average date of the vernal equinox to be the first day of the year. (This rule was often used in ancient calendars, including the Babylonian and the Roman systems which pre-dated the Julian.) These considerations led to the following rules.

1) The negative year era is labeled B.Z. (Before Zero). The positive year era is labeled A.Z. (After Zero).

2) Months are named (in order) Denis, Jill, Nicole, Abigail, Ernest, Gladys, Gerard, Veronica, Anne, Colette, Joan, Ericka, and Anthony. Years start in Denis.

3) Normal years have 12 months (Denis through Ericka).

4) Leap years have 13 months (the month Anthony is added after Ericka).

5) Denis, Nicole, Ernest, Gerard, Anne, and Joan always have 30 days.

6) Jill, Abigail, Gladys, Veronica, Colette, and Ericka always have 29 days.

7) In leap years, the added month (Anthony) can have either 30 or 31 days. See Rules #8 and #10 below.

8) The leap year cycle is 1689 years long. One cycle began with 1 A.Z. and ended with 1689 A.Z. The cycle preceding that one began with 1689 B.Z. and ended with 1 B.Z. During each cycle there are three types of years. Normal years have 12 months. Leap Year Type 1 has the month Anthony added with Anthony having 30 days. Leap Year Type 2 has the month Anthony added with Anthony having 31 days. During each cycle there are 1067 normal years, 294 Type 1 leap years, and 328 Type 2 leap years. These leap years are distributed approximately evenly throughout the cycle.

9) The Julian Day of midnight at the start of Denis 1, 1 A.Z. Elliott (a Thursday) is 1751822.5. (This date corresponds to 25 March 84 A.D. Julian, which is 23 March 84 A.D. Gregorian.)

10) The rule for which years within each cycle are of which type is most easily understood in terms of a table. Such a table, having 1689 entries, would specify the type of each year. But such a table is too long for this document. So instead, I specify the rule for computing that table. That is exactly what InterCal does during initialization.

    a) Define two auxiliary tables, Type2 (with 328 entries) and Type 1 (with 294). For each integer N from 1 to 328, set its entry in Type2 to the nearest integer to 1689N/328 (**round**, don't truncate).

    b) For each integer M from 1 to 294, set its entry in Type1 to the nearest integer to 1689M/294 - 3. (**Round**, don't truncate.) (The purpose of the "-3" is to offset entries in Type 1 from entries in Type2.) Occasionally during this process, the computed value of the M'th entry in Type1 will equal the (M-1)'th entry. Check for this, and when it happens go to the other table (Type2). Set the M'th entry of Type1 to the average of the two values in Type2 which are closest to, **but both larger than**, the duplicated value. When averaging, **truncate** to the next lowest integer if the average is not an integer.

    c) Generate the final table, T1689, as follows:

        i) For every integer J from 1 to 1689, set T1689(J) to "normal" **unless** J can be found in Type1 or Type2.

ii) If J is in Type1, set T1689(J) to “Leap Year Type 1”.

iii) If J is in Type2, set T1689(J) to “Leap Year Type 2”. **If computed correctly, there are no duplicates between or within the auxiliary tables.**

As samples, the first few entries in Type2 are 5, 10, 15, 21, 26, 31, 36, 41, 46, 51… The first few entries in Type1 are 2, 7, 12, 18, 23, 28, 38, 43, 48, 54…

The rules above imply that there are exactly 616894 days in each 1689-year cycle. Thus the average length of a year is 365.2421551 days, so years are too short by 0.0000439 days, which amounts to an error of one day in 22780 years. This is nearly seven times as accurate as the Gregorian calendar and far more accurate than the Julian and Jewish calendars. The rules also imply that, on average, there are exactly 616894/20890 = 29.5305887984 days per month. Thus the months are too short by 0.0000000516 days. That is equivalent to an error of 0.0010779 days per 1689-year cycle or 0.000000638 days per year. Thus, with respect to the moon, the Elliott calendar builds up an error of one day every one and a half million years (approximately). This is one hundred times as accurate as the Jewish calendar, which (for the moon) is a very accurate calendar. It also far exceeds the time span over which we can claim to know the mean synodic month to the number of decimal places given at the start of this document. All we can really say is that this calendar accurately tracks the moon as well as can be done with present knowledge.

French Revolutionary Calendar

The French Revolutionary Calendar was instituted after the Revolutionaries achieved full control of France. It was the official calendar of France from late November 1793 to the last day of 1805 (Gregorian). In keeping with the passion of the Revolutionaries for rationality and naturalism, the calendar has every month the same length—30 days. The months are named for natural phenomena (mostly weather-related) such as Vintage, Hot, Rain, and Wind.

There are really two French Revolutionary calendars. The one actually used was a physical solar calendar in which years started on the day (in Paris) of the actual autumnal equinox. There was thus no fixed pattern of leap years. To determine the length of a year one had to calculate the dates of two successive autumnal equinoxes. Shortly after the calendar was instituted, the complexities of maintaining a physical calendar became apparent. A suggestion was being considered to turn the calendar into a mathematical solar calendar. It would use a leap year rule very much like the Gregorian one, but with a more accurate 4000-year leap cycle instead of a 400-year cycle. Before this calendar reform could be accomplished officially, Napoleon abolished the calendar and returned France to the Gregorian.

Following a suggestion from Ed Reingold (one of the co-authors of Calendrical Calculations), InterCal uses a hybrid set of rules. The first few years of the calendar are handled as a special case. The original (physical) form of the calendar is used during the years 1 through 19. For all negative years, and all years +20 and beyond, the modified mathematical form is used. Thus, historians can trust that dates in the years 2 through the early part of 14 (the years during which the calendar was legally the civil calendar of France) are correctly converted. The year 20 was chosen as the boundary because it is the same in either system and its use simplifies the code. The use of the modified (mathematical) rules for the rest of the time allows high performance and a range limited only by the use of 32-bit integers to count the days. Physical calendars have limits on their range of use because celestial body positions cannot be calculated infinitely far into the past or future with any reasonable accuracy. They have poor performance because of the large number of floating-point operations required. The Chinese calendar is an excellent example of the problem.

One oddity about the French Revolutionary calendar (especially considering who invented it) was that its years are specified in Roman numerals, not Arabic. InterCal takes the liberty of ignoring this, and states years in standard Arabic numerals as for all the other calendars. No era names appear to have been defined by the Revolutionaries. The era abbreviations used by InterCal are based on non-standardized but reasonably common usage of historians specializing in the period.

My thanks to Professor David Stewart of Hillsdale College for letting me know of a need for this calendar in historical research. The rules for the calendar (both forms) are from Calendrical Calculations. Note that the French Revolutionary Calendar deliberately tried to abolish the concept of weeks. Weeks were replaced with 10-day periods called décades (3 per month). InterCal ignores this aspect of the calendar, and displays weeks as usual.

The rules of the French Revolutionary calendar as implemented by InterCal, are:

1) The negative-year era is labeled A.R., for Ancien Régime. The positive-year era is labeled R.C. for Revolutionary Calendar.

2) Months are named, in order, Vendémiaire (Vintage), Brumaire (Fog), Frimaire (Sleet), Nivôse (Snow), Pluviôse (Rain), Ventôse (Wind), Germinal (Sprouting), Floréal (Flowering), Prairial (Pasturing), Messidor (Harvest), Thermidor (Hot), and Fructidor (Fruit). Years start in Vendémiaire.

3) The Julian Day of midnight at the start of Vendémiaire 1, 1 R.C. (a Saturday) is 2375839.5, which corresponds to September 22, 1792 A.D. Gregorian. At the longitude of Paris, that day contained the autumnal equinox.

4) Every month has 30 days.

5) After the end of Fructidor there are five extra days (in normal years) called jours complémentaires. In leap years a sixth extra day was added. Technically, these extra days were not considered to belong to any month. They each had a special name:

   a) Jour de la Vertu (Virtue day);
   b) Jour du Génie (Genius);
   c) Jour du Labour (Labor);
   d) Jour de la Raison (Reason);
   e) Jour de la Récompense (Reward);
   f) Jour de la Révolution (Revolution)—leap years only.

6) For implementation convenience, InterCal treats the jours complémentaires as a very short thirteenth month, containing five days in normal years and six in leap years. InterCal does not use the special names of the days, but simply numbers them.

7) The modified calendar's leap year cycle is four thousand years long. One cycle began in 1 R.C. and will end in 4000 R.C. The preceding cycle began in 4000 A.R. and ended in 1 A.R. French Revolutionary leap years occur in years whose position in their cycle is divisible by 4 but not by 100, **except** that years whose position in their cycle are divisible by 400 but **not** by 4000 **are** leap years. For example, leap years include 20 R.C, 116 R.C., 1200 R.C., and 5 A.R. (5 A.R. has position 3996 in its cycle). Non-leap years include 25 R.C., 300 R.C. 4000 R.C., and 1 A.R. (its position in its cycle is also 4000).

8) The original calendar's leap year rule (there was no fixed cycle) was that years always began on the day (from midnight to midnight at Paris) during which the autumnal equinox occurred. In the present era, successive autumnal equinoxes defined in this way are always separated by either 365 or 366 days. Leap years were the ones with 366 days.

9) InterCal uses rule 7 for all negative years, and all positive years greater than or equal to 20. InterCal uses Rule number 8 for the positive years 1 through 19 inclusive. (As always in InterCal, there is no year 0.) Since the leap years in the range 1 to 19 under the original rules turned out to be 3, 7, 11, and 15, the net result is that the leap year sequence for the hybrid calendar is as follows: …13 A.R., 9 A.R., 5 A.R., 3 R.C., 7 R.C., 11 R.C., 15 R.C., 20 R.C., 24 R.C., …

   Note that even in the original form of the calendar, the year 20 would have been a leap year.

Mayan Calendar(s)

The rules for the Mayan calendars have only been reconstructed within the past few decades. Much information is still unknown and is probably permanently lost. After the conquest, most (but not all) of the Spanish rulers either made a conscious effort to eradicate Mesoamerican culture or at least instituted policies which were antithetical to the preservation of their culture by the natives. The situation was probably made worse by the fact that, while Mayan people were still living in the area (and continue to do so today), the high pre-Columbian Mayan civilization had collapsed hundreds of years before the Spanish conquest. The Aztecs, on the other hand, were a thriving group as the Spanish arrived. Yet much knowledge about their culture has also been lost.

Much of the information in the rest of this section is summarized from two sources:

Calendrical Calculations;

an article on Mayan astronomy by A. D. Jenkins which is available on the Internet at

http://clunix.msu.edu/~vanhoose/astro/0101.html

The Maya had an elaborate writing system which is still being deciphered. They were also good mathematicians and astronomers. They had a number system which was base 20, as opposed to our base 10. They had accurate knowledge of the length of the year and the length of the synodic month. They were able, perhaps crudely by modern standards, to predict eclipses. Their knowledge of the apparent motions of the planet Venus may have surpassed that of any contemporary culture in the world. Although they counted days within the lunar cycle, they did not attempt to make their major calendars (the three supported by InterCal) track astronomical objects accurately. They seemed (in my opinion) to be fascinated with exact periodicity and relationships between cycles of differing periods. Their calendars were elaborate day counting and cycle-marking systems which paid only slight attention to such things as lunar months and solar years. Perhaps this lack of emphasis on longer time units is related to the view, held by many Mesoamerican cultures, that the **sun would not rise tomorrow** unless the people propitiated the gods through certain mandatory rituals. Some of these rituals were very bloody and a few involved human sacrifice.

Three calendars used by the Maya are supported by InterCal. These are the Long Count, the Tzolkin cycle, and the Haab cycle. The Tzolkin and Haab cycles were often used together to make a much longer cycle known as the Calendar Round. This latter cycle is also supported by InterCal. The Long Count is a pure day counting system, just like the Julian Day Number. The Haab cycle is roughly based (and the Maya knew how roughly) on the sun. The Tzolkin cycle is roughly based on the planet Venus. Both cycles had considerable religious significance attached to the various day numbers and names.

The Long Count—InterCal-style

1) The Long Count was nothing more than a five-vigesimal-digit number which counted days, starting from 0 and increasing by one every day. (Vigesimal means "base 20".) Note that only five digits were used. The Maya believed that when the value of the high-order vegisimal digit reached 13 the universe would be destroyed, a new universe created, and the Long Count would begin again at zero. Today, Long Count values are usually written as five decimal numbers separated by periods. Thus a typical Long Count date would be written as 12.9.0.14.19.

2) Each vegisimal digit in the Long Count can be in the range 0 through 19 inclusive, **except** for the second digit from the right, which was restricted to the range 0 through 17 inclusive. By making the second from the right digit base 18, a change of one in the third digit corresponded to a change in the value of the Long Count of 18*20 = 360, or roughly one year.

3) The time periods corresponding to a change of one in each digit had names as follows:
   a) 1 kin = 1 day (right-most digit)
   b) 1 uinal = 20 days (second digit from right)
   c) 1 tun = 360 days (third digit from right)
   d) 1 katun = 7200 days (fourth digit from right)—very roughly 20 years
   e) 1 baktun = 144,000 days (fifth digit from right)—very roughly 400 years

4) InterCal is designed **not** to be limited by arbitrary time boundaries, but only by computer-related limitations on the sizes of integers. Therefore InterCal uses an **8-vegisimal-digit** Long Count. Thus a typical Long Count is written 1.4.8.0.13.18.16.10, where the right-hand five vegisimal digits have their usual meanings. Admittedly, this rule ignores the Mayan belief in the destruction of the universe on 13.0.0.0.0. But the higher-order digits do not violate any rules of Mayan mathematics. In fact, the Maya even had words for longer time intervals which corresponded to higher-order digits as follows:
   a) 1 pictun = 2,880,000 days (sixth digit from right)—very roughly 8000 years
   b) 1 calabtun = 57,600,000 days (seventh digit from the right)
      very roughly 160,000 years
   c) 1 kinchiltun = 1,152,000,000 days (eighth digit from right)
      very roughly 3,200,000 years

   The Maya also had a word for the time period which would have corresponded to a ninth digit. That word is alautun and equals 23,040,000,000 days or very roughly 64,000,000 years. But InterCal cannot use this ninth digit because 23,040,000,000 exceeds the largest integer which can fit into a 32-bit word.

To summarize, in InterCal the Long Count is written as 8 vegisimal digits, of which one (second from right) is actually base 18, not base 20.

5) InterCal allows the Long Count to be negative or zero. A negative Long Count is written exactly like any other, but the first digit is preceded by a minus sign. Thus a possible Long Count date in InterCal is -0.0.0.8.13.11.1.18.

6) The zero point of the Long Count is a critical piece of knowledge which has been lost. That is, no one knows for sure what Julian Day number corresponds to a Long Count of 0.0.0.0.0 (or, in InterCal, 0.0.0.0.0.0.0.0). Several specialists in Mayan archaeology have written papers advancing various possible correlations between the Long Count and the Julian Day Number. Dates for the zero point range from approximately 3400 B.C. to approximately 2600 B.C. InterCal permits the user to select from among several published correlations, or to specify his/her own. **As a default only,** InterCal uses the (unmodified) Goodman-Martinez-Thompson correlation, which states that noon on Long Count 0.0.0.0.0 occurred on Julian Day 584283.0 (August 11, 3114 B.C. Gregorian or September 6, 3114 B.C. Julian). In a paper published a few years before <u>Calendrical Calculations</u>, Dershowitz and Reingold stated that the modified Goodman-Martinez-Thompson correlation was the most commonly accepted one. Its date for Long Count 0.0.0.0.0 was two days later than the unmodified one. Additional astronomical evidence for this correlation was noted in the July 1997 issue of Sky and Telescope Magazine. InterCal Version 1.3 used the modified correlation as its default. However, in the last few years opinion seems to have moved in favor of the unmodified date, and <u>Calendrical Calculations</u> uses it.

7) Given a zero point correlation, (that is, given the Julian Day Number corresponding to 0.0.0.0.0.0.0.0), converting between Julian Day numbers and Long Count dates is a trivial problem of converting a decimal number (the Julian Day) to/from a number in another base. That is, to convert a Long Count to a Julian Day simply convert the vegisimal number into decimal, then add day0Mayan, where day0Mayan is the assumed Julian Day of 0.0.0.0.0.0.0.0. The reverse conversion is equally simple.

The Haab Cycle

The Haab cycle approximates the solar year, but is thought of as a cycle rather than a progression. That is, there is no year associated with a Haab date, just as there is no increasing count associated with our seven-day week. This calendar contained 18 months of 20 days each. At the end of the last month came an unlucky period of five days which belonged to no month. Thus the Haab is an ever-repeating cycle of 365 days.

The rules for the Haab calendar are as follows:

1) The months, in order, are: (The Guatemalan Maya orthography is used, as recommended in the Jenkins article.)

Pohp	(Mat)
Wo	(??)
Sip	(??)
Sotz’	(Bat)
Sek	(??)
Xul	(Dog)
Yaxk’in	(New Sun)
Mol	(Water)
Ch’en	(Black ??)
Yax	(Green ??)
Zak	(White ??)
Keh	(Red ??)
Mak	(??)
K’ank’in	(??)
Muwan	(Owl)
Pax	(??)
K’ayab	(Turtle)
Kumk’u	(??)

2) The unlucky five day period is called Wayeb.

3) Days in the months are numbered starting at 0, not 1. So the 20 days are numbered 0 through 19 inclusive.

4) InterCal treats Wayeb as a nineteenth month having five days numbered 0 through 4.

5) Archaeologists are confident that they know the day in the Haab cycle corresponding to Long Count 0.0.0.0.0—it is 8 Kumk’u. Thus it is possible to convert Julian Day numbers into Haab cycle dates once you have agreed on a correlation between Julian Days and the Long Count. Similarly, it is possible to find the (infinite) set of all Julian Days corresponding to a given Haab date. The conversions amount to simple arithmetic modulo 365 and modulo 20. <u>Calendrical Calculations</u> gives the algorithms in detail, so I will not repeat them here.

The Tzolkin Cycle

In one way the Tzolkin cycle is similar to the Haab cycle. That is, it is not a progression, so there is no increasing count associated with it. However, the Tzolkin calendar (also called the sacred calendar) contained **two interacting cycles** within itself. Twenty day names continually cycle, and **at the same time** 13 numbers cycle. Thus the Tzolkin is an ever-repeating cycle of 20•13 = 260 days. To illustrate, 1 Imix is followed by 2 Ik' is followed by 3 Ak'bal, etc. After 13 Ben comes 1 Ix and then 2 Men, etc. After 260 days all combinations of numbers and names have occurred exactly once, and the cycle starts over. The fact that the Tzolkin cycle existed and had so much religious significance says a lot about the importance of the planet Venus to the Maya. They followed its motions closely, and knew that the average length of time Venus spends as the "Evening Star" before moving to the other side of the sun (as seen from Earth) to become the "Morning Star" is 260 days. Similarly, the average time spent as the "Morning Star" is 260 days.

The rules for the Tzolkin Calendar are as follows:

1) The day names (using the Guatemalan orthography again) are:

Imix	(Water lily)
Ik'	(Wind)
Ak'bal	(Night)
K'an	(Corn)
Chikchan	(Snake)
Kimi	(Death head)
Manik'	(Hand)
Lamat	(Venus)
Muluk	(Water)
Ok	(Dog)
Chuwen	(Frog)
Eb	(Skull)
Ben	(Cornstalk)
Ix	(Jaguar)
Men	(Eagle)
Kib	(Shell)
Kaban	(Earth)
Etz'nab	(Flint)
Kawak	(Storm cloud)
Ahaw	(Lord)

2) The numbers cycle from 1 to 13 inclusive. 0 is not used.

3) Archaeologists are confident that they know the day in the Tzolkin cycle corresponding to Long Count 0.0.0.0.0—it is 4 Ahaw. Thus it is possible to convert Julian Day numbers into Tzolkin cycle dates once you have agreed on a correlation between Julian Days and the Long Count. Similarly, it is possible to find the (infinite) set of all Julian Days corresponding to a given Tzolkin date. Because the day number and day names cycle **simultaneously** the formulas are a little more complicated than for the Haab cycle. They involve the simultaneous solution of two linear congruences ("equations" in modular

arithmetic). The conversion from Julian Day to Tzolkin date gives a unique solution. The conversion in the other direction has an infinite number of solutions separated by the cycle length of 260 days. Calendrical Calculations gives the algorithms in detail, so I will not repeat them here.

The Calendar Round

The Haab and Tzolkin dates were often used together to form a compound date. Because the Haab cycle contains 365 days and the Tzolkin cycle contains 260 days, you might think that there are 365•260 = 94900 combinations possible. But 365 and 260 have one common divisor, 5. Therefore after 94900/5 = 18980 days, both Haab and Tzolkin calendars have completed an integral number of cycles. So after 18980 days the pattern of Tzolkin and Haab pairs repeats. Four-fifths of the theoretically possible Haab-Tzolkin combinations never occur. 18980 days is about 12 1/2 days short of 52 tropical years and exactly 52•365. This long cycle is known as the Calendar Round. The completion of a cycle was the cause for ceremonies of major religious significance. The Aztecs, who shared many features of a common Mesoamerican culture with the Maya, are believed by some to have sacrificed tens of thousands of human victims at the last such ceremony held before the Conquest.

Rules for the Calendar Round:

1) The set of all valid Haab-Tzolkin pairs can be determined from the fact that 4 Ahaw 8 Kumk'u is known to be the Calendar Round date corresponding to a Long Count of 0.0.0.0.0.

2) Given an agreed-upon correlation between Julian Days and the Long Count, a Julian Day number can be uniquely converted into a Calendar Round date. Given a **valid** Calendar Round date, the infinite set of Julian Days corresponding to that date can be determined. The solutions are separated by the length of the cycle, 18980 days. The derivation of the equations is complicated, but once they have been derived the calculation of solutions is very simple. Both the derivations and the final formulas are found in Calendrical Calculations and are not repeated here.

   An attempt to convert an invalid Calendar Round date (an impossible Haab-Tzolkin combination) results in congruences which have no solution. InterCal senses when that occurs and puts up an Alert screen. Details are in the Users' Guide.

Chinese Calendar

First, The Excuses

I start with a long, involved explanation of why the accuracy of InterCal for the Chinese calendar is not guaranteed.

The Chinese calendar is a **physical** luni-solar calendar. That is, it tracks both the sun and the moon, and it uses the real astronomical bodies in the calendar rules, not simplified mathematical formulas which approximate the motions. The only other physical calendar presently implemented by InterCal is the Islamic. But there, a mathematical approximation to the true calendar was used. That had to be done, because the Islamic months start only after a new crescent moon has been **observed** by a designated cleric. Given the complexities which determine when a new crescent moon can be first observed, combined with additional potential interference from the weather, it is impossible for a computer to predict the **true** Islamic calendar in advance, and a horribly difficult task to tabulate the actual past months dating back to the Hegira. So, as already noted, InterCal uses a simple (but good) mathematical approximation for the Islamic calendar. **The Chinese take a different approach.** They are willing to trust the calculations of astronomers concerning the positions of the real sun and moon. So it is theoretically possible to calculate the Chinese calendar well into the past and future.

Although it might be *theoretically possible* to calculate the Chinese calendar for any date, there are numerous practical problems.

1) No matter how accurate the equations are today, they will lose accuracy as you calculate further and further into the past or future. The equations do not give the positions of the sun and moon in closed form. Such closed-form solutions do not exist because there are more than two bodies in the solar system. Instead, the equations consist of time series expansions centered on the present, which lose accuracy as the time difference from the present grows.

   To explain further, the orbits of the earth and moon are not fixed. Tidal interactions between the earth and moon and perturbations caused by other bodies in the solar system slowly change the orbits. The equations attempt to account for these changes. But because there are many bodies in the solar system, the equations cannot include all effects. So the motions of the earth and moon over **very long** time periods are essentially unpredictable.

2) The derivation of the equations requires knowledge of the position and velocity of every body in the solar system at some moment in time. Reasonably accurate, but not perfectly precise, position and velocity information exists only for the planets and a few asteroids. Astronomers do the best they can by observing the solar system over many years and constantly improving the accuracy of their observations. So the calculations of lunar and solar positions

depend on a huge number of observations extending over many decades and performed by thousands of people. All of the data include some observational errors. Thus, even in the present, the equations do not have **exactly** correct inputs. To make matters more complicated, different astronomers have used different sets of equations. The differences are due partly to different choices of observations and different estimates of the observational errors, partly due to disagreements on the very best way to handle the numerical solutions, and partly due to differences in simplifying approximations which arise because of the different intended uses of the equations.

3) Even in the simplified form of the equations InterCal is forced to use (because of processing power limitations of Macs and PC's), the equations are quite complex. The equations in InterCal **might** have actual coding errors. A keypunch error or copying error in one of the dozens of floating-point numbers in the equations could lead to subtle errors which produce erroneous calendar dates only once in a while, or only far from the present where I had no tables to compare to. Such errors are possible in the basic codes used by the astronomers as well as in InterCal.

The net result of inaccuracies and downright errors is that no two independently-written programs are likely to produce identical results for all times. Most of the time this doesn't matter, because in carefully written and tested programs the differences from one program to the next should be on the order of fractions of seconds to (at worst) minutes. But if a calendar-critical astronomical event such as new moon happens to fall very close to midnight, differences of seconds or minutes can mean a whole day's difference in when a month is determined to begin!

Until recently (and probably to this day, but I am not sure) the Chinese solved the practical problems by assigning one observatory the responsibility for determining the calendar. Tables showing the starts of months for the next hundred years or so have been produced and distributed by the designated authority. Such tables are occasionally recalculated in light of improvements in the equations and/or the calculating techniques. But InterCal does not operate by storing large tables. If it did, it would be limited to only a few centuries around the present. So InterCal uses its own set of equations and calculates the Chinese calendar on the fly. This has the following important consequences:

1) The accuracy of the InterCal version of the Chinese calendar is not guaranteed. It has been checked against all the dates given in a checklist in <u>Calendrical Calculations</u>, and against a few other published date comparisons. But agreement in a few cases is no guarantee of complete agreement with published tables from China (which I do not have access to) or to accuracy beyond the range of the tables.

2) The equations used by InterCal (and in fact by the astronomers) eventually blow up, producing absolutely nonsensical results when you get too far away

from the present. For that reason, when the Chinese calendar is the primary system, InterCal attempts to prevent the user from entering dates outside a range of about 6000 years on either side of the year 2000 A.D. Gregorian. Thus, the Chinese is the **only** calendar implemented by InterCal which has a narrower range of use than the limit imposed by the use of 32-bit integers. More details on this limitation are provided in the Users' Guide.

3) A very high number of floating-point calculations are required. I have taken great care to minimize the calculation of Chinese dates, using caching techniques. Nevertheless, InterCal 1.4 and any later versions which are not PowerPC-native have poor performance. On a Mac IIci without a floating-point co-processor it takes well over 90 seconds to put the initial display on the screen! The same delay occurs every time the program needs to calculate parameters for a year not in the cache. Performance on Power PC's and 680x0 Macs which **do** have a floating point co-processor is tolerable, but calculating parameters for a new Chinese year still takes 3 or 4 seconds. Someday I hope to have a Power-PC native version, which should not show any noticeable performance degradation from InterCal 1.3 and earlier.

Finally, the Rules

These rules (and other information) have been obtained from Calendrical Calculations. Keep in mind that the basic rules, as stated here, were finalized in 1645 A.D. Also, note that the older the Chinese table, the older and less accurate was the theory of the earth's and moon's motions used to create the table. So even if InterCal were perfectly accurate, it would disagree with old tables. Once again, InterCal only tries to be mathematically accurate, because being historically accurate is just too big a job.

The basic rules are #3 (which ties the calendar to the moon), #5 (which ties it to the sun), and #6 which simultaneously determines New Year's and the leap month in 13-month years. Rule #6 is quite tricky and mistakes in its interpretation will lead to errors. That is why there is so much explanatory text accompanying it. The rest of the rules either provide basic definitions (Rules 1&2) or fill in necessary details.

1) All astronomical events (such as new moon or solar longitudes having specific values) refer to **apparent** positions of the sun and moon, **not mean** positions. The last major rule change (which occurred in 1645) switched from the use of mean solar positions to apparent ones. Apparent lunar motions had been in use since 619 A.D. A thorough understanding of the Chinese calendar requires some knowledge of positional astronomy, where the terms "mean" and "apparent" have well-defined meanings. Speaking simplistically, apparent positions are what are actually observed. Mean motions are smoothed, with many periodic effects deliberately ignored.

Note that the Chinese calendar, like the Elliott Super, uses astronomical new moon. The Islamic (and many centuries ago the Jewish) use(d) observations of the new crescent, which occurs later than astronomical new moon.

2) By "day" is meant the period from midnight to midnight **Beijing time**. **This meaning is used throughout all the rules.** By "Beijing time" is meant "mean solar time at the longitude of Beijing" before 1929 Gregorian, and "Beijing standard time" from 1929 on. These two times differ by several minutes.

   Note that the use of Beijing time means that some Chinese months will start a day later than they would if Greenwich time were being used. Calendars which show phases of the moon get their dates from almanacs, which in turn use Greenwich time. So you will see (about one-third of the time) a discrepancy between new moon shown on a typical commercial calendar and the start of a Chinese month.

3) Months begin on any day during which a new moon occurs. Days within the month are numbered starting with 1 (like all the calendars implemented by InterCal except the Mayan Haab). Chinese months are almost always either 29 or 30 days long, but there is no repetitive pattern to the month lengths.

4) Months are **named** using numbers from 1 to 12 inclusive. In leap years, a thirteenth month is added. It can come after (theoretically) any of the 12 months (the details are in Rule #6). The leap month is **named** "leap n" if it is inserted after month "n". Thus one possible sequence of month **names** would be: One, Two, Three, Four, leap Four, Five, Six, Seven, Eight, Nine, Ten, Eleven, Twelve.

5) The winter solstice (solar celestial longitude exactly 270 degrees) always occurs in the month **named** Eleven. (For an explanation of celestial longitude, see a basic astronomy text). Note that the month named Eleven may be either the eleventh or twelfth month of the year.

6) **Leap Month And New Year's Determination**—Any day during which the celestial longitude of the sun passes through an exact multiple of 30 degrees is called a "major solar term". From one winter solstice to the next there are exactly 12 major solar terms, counting one (but not both) of the winter solstices. This follows naturally from the fact that 360/30 equals 12. But from one winter solstice to the next there can be either 12 or 13 new moons. Relationships between the orbits of the Earth and the Moon preclude fewer than 12 or more than 13. The new moons **on or before** two consecutive winter solstices begin month Eleven of two different (consecutive) years (Rule #5). When these new moons are spaced twelve lunar cycles apart, the **eleven** new moons **between** them are, in order, the first days of month Twelve of the first year and months One through Ten of the second year. In this case there are **no leap months**

inserted, regardless of whether or not any of the months have no major solar terms. When the new moons on or before the winter solstices are spaced thirteen lunar cycles apart, the **twelve** new moons **between** them include the first days of month Twelve of the first year and months One through Ten of the second year. But there are only eleven names for the twelve months. Somewhere in the sequence Eleven, Twelve, One, Two,…Ten, Eleven **exactly one leap month** must be inserted. The first month Eleven is guaranteed **not** to be a leap month because by definition it contains the winter solstice, which is a major solar term. <u>Starting with the first new moon after the first month Eleven</u> , **sequentially** check for a major solar term in each month. If one is found, the month's name is one more than the previous month's name (except that One follows Twelve). The **first** time a month is encountered which has no major solar term, **that month is the leap month.** At least one such month is guaranteed to exist when there are 13 lunar cycles between the new moons starting the two months Eleven. If the leap month follows a month named "n" it is named "Leap n". The search then stops. All the subsequent months are then immediately named in sequence regardless of whether or not they contain major solar terms.

<u>Calendrical Calculations</u> contains a diagram illustrating this process. That diagram should help confused readers understand rule #6. It is admittedly complex and confusing, but it is not vague or ambiguous. It uniquely determines the name of every month and uniquely determines Chinese New Year's (day 1 of month One) for every year. One reason it is confusing is because of phrases like "the **first** time…no major solar term" and "at least one is guaranteed to exist". At first glance one might think that Chinese months have exactly one major solar term **unless** they have none, and that between any two winter solstices there can be at most one month without a major solar term. But that is not the case. The reason is that **times of day** of major solar terms and new moons are **irrelevant** to the rules of the calendar. Two astronomical events important to the calendar which happen hours apart can still fall on the same day and the ordering might then seem to be disturbed. For example, suppose there is a sequence "major solar term / new moon / new moon / major solar term". Usually the first new moon in such a sequence starts a month with no major solar term. But if it falls on the same day as the first major solar term, then the first new moon is in the same month as that major solar term and there is no month in that sequence without a major term. Because of this type of situation, it is not unusual for a month to contain zero major solar terms and a nearby month to contain two (one near the beginning and one near the end).

Rule #6 implies that the leap month may occur late in the first year (and be called Leap Eleven or Leap Twelve) or sometime before month Eleven in the second year (and be called Leap One, Leap Two, etc. through Leap Ten). The latter case is by far the most common. This has led to an erroneous statement of the leap year rule: "The leap month is the first month of the year not containing a major solar term." Though wrong, this rule gives a correct result such a high percentage of the time that considerable confusion has resulted. There are other erroneous "rules" that have been stated regarding the determination of Chinese New Year's. Because of the rarity of leap months falling very late in a year, these erroneous rules **usually** give correct results. Once again, this situation has allowed much misinformation to go unnoticed and unchallenged. The fact is, Rules 4, 5, and 6 determine year boundaries. To state the New Year's rule in plain English, New Year's Day is the first day of month One. You have to carefully apply rules 4, 5, and 6 to determine when that is. There are no shortcuts.

Due to the relationship of the earth's and moon's orbits there can never be two leap months in any contiguous 25-month period. Thus, there are never two consecutive leap years.

The leap year and leap month pattern does not repeat.

If mean solar and lunar positions were used, leap months could (and did under the old rules) occur after any month. But with apparent positions, because the earth is moving faster near perihelion in early January, Leap 1 and Leap 12 do not occur near the present time. Also, Leap 11 is quite rare. But Leap 11 does occasionally occur, and when it does all the common shortcut rules fail. In some cases thousands of years in the past or future, InterCal does occasionally produce Leap One and Leap Twelve. I believe this circumstance is probably due either to subtle errors in InterCal or to the inherent inaccuracy of the equations (especially the transformation to Universal Time from Dynamic Time). However, there is a slight possibility that the effect is real.

7) In the traditional Chinese calendar, years are not counted linearly. Instead there is a 60-year cycle consisting of two subcycles which increment together much like the Mayan Tzolkin cycle. The subcycles have periods of 10 and 12 years. Since the least common multiple of 10 and 12 is 60, the double cycle repeats every 60 years. InterCal treats the Chinese calendar as essentially linear, by numbering the 60-year cycles and including the cycle number in the year designation. In keeping with the rules for the other calendars, I have arbitrarily chosen **not** to use a cycle 0. Thus cycle -1 is followed by cycle 1.

8) The beginning of cycle 1 (day 1 of month 1 of year 1 of cycle 1) was at midnight Beijing time on the day on which midnight Greenwich time had Julian Day Number 758325.5. That day was Wednesday, February 15, 2637 B.C. Gregorian or March 8, 2637 B.C. Julian.

9) The major solar terms have been given names as follows (they are listed in their usual order within the Chinese year):

330°	Yushui	Rain Water
0°	Chunfen	Spring Equinox

30°	Guyu	Grain Rain
60°	Xiaoman	Grain Full
90°	Xiazhi	Summer Solstice
120°	Dashu	Great Heat
150°	Chushu	Limit of Heat
180°	Qiufen	Autumnal Equinox
210°	Shuangjiang	Descent of Frost
240°	Xiaoxue	Slight Snow
270°	Dongzhi	Winter Solstice
300°	Dahan	Great Cold

10) Although they do not affect any of the calendrical calculations, **minor** solar terms have been defined and are displayed on traditional Chinese calendars. InterCal follows this tradition. Minor solar terms are days during which the longitude of the sun in degrees passes through a value which equals exactly 15 modulo 30. As with the major terms, Chinese months may contain 0, 1, or 2 minor solar terms, with 1 being the usual case.

The names are:

315°	Lichun	Beginning of Spring
345°	Jingzhe	Waking of Insects
15°	Qingming	Pure Brightness
45°	Lixia	Beginning of Summer
75°	Mangzhong	Grain in Ear
105°	Xiaoshu	Slight Heat
135°	Liqiu	Beginning of Autumn
165°	Bailu	White Dew
195°	Hanlu	Cold Dew
225°	Lidong	Beginning of Winter
255°	Daxue	Great Snow
285°	Xiaohan	Slight Cold

11) Years within each 60-year cycle are numbered from 1 to 60, and strictly increase even in cycles whose **cycle** number is negative. Thus, year -1 1 is followed by -1 2. Eventually, year -1 60 is followed by year 1 1. Each of the 60 years in a cycle is given a double name, corresponding to its position in the two subcycles. The part of the name corresponding to the 10-year subcycle is called the "celestial stem". The possible values (which Dershowitz and Reingold say have no translation although at least one user has claimed otherwise) are:

(1) Jia
(2) Yi
(3) Bing
(4)Ding
(5) Wu
(6) Ji
(7) Geng
(8) Xin
(9) Ren
(10) Gui

The part of the name corresponding to the 12-year subcycle is called the "terrestrial branch". These are translated as the names of various real or imaginary animals as follows:

(1) Rat	(7) Horse
(2) Ox	(8) Sheep
(3) Tiger	(9) Monkey
(4) Hare	(10) Fowl
(5) Dragon	(11) Dog
(6) Snake	(12) Pig

To determine the two names calculate their indices in the above lists as follows:

a) Set y = the year within the 60-year cycle (from 1 through 60 inclusive).
b) Celestial Stem index = y modulo 10.
c) If the result comes out zero, set the Celestial Stem index to 10. The result will then always be in the range 1 to 10 inclusive.
d) Terrestrial Branch index = y modulo 12.
e) If the result comes out zero, set the Terrestrial Branch index to 12. The result will then always be in the range 1 to 12 inclusive.

12) Formerly, months were named using the same double cycle of 60 names specified in Rule #11 for years. Traditional calendars often include these names. InterCal displays them in the upper right portion of the month display, immediately below the year. The rules are as follows:

a) Calculate n = 12•y + m +50, where 'y' is the year number within the 60-year cycle (1 through 60 inclusive) and 'm' is the month **name** (in the range 1 to 12 inclusive). Leap months are given the same name as the month they follow. As an example, in a year having a Leap Four both the month Four and the month Leap Four have the same double name, obtained by putting m = 4 in the above formula.
b) Celestial Stem index = n modulo 10.
c) If the result comes out zero, set the Celestial Stem index to 10. The result will then always be in the range 1 to 10 inclusive.
d) Terrestrial Branch index = n modulo 12.
e) If the result comes out zero, set the Terrestrial Branch index to 12. The result will then always be in the range 1 to 12 inclusive.

13) Formerly, days were also named using the same system. InterCal also displays a day name, directly underneath the year and month names. See the User's Guide for an explanation of which day within the month has its name chosen for display. The rules for calculating the name are:

a) Calculate D = floor(Julian Day Number of the date in question) - 9 [for non-programmers and non-mathematicians, the "floor" function returns the algebraically largest integer less than or equal to its argument—that is, floor(13.6) = 13; floor(-4.8) = -5; floor (102.0) = 102]

b) Celestial Stem index = D modulo 10.
c) If the result comes out zero, set the Celestial Stem index to 10. The result will then always be in the range 1 to 10 inclusive.
d) Terrestrial Branch index = D modulo 12.
e) If the result comes out zero, set the Terrestrial Branch index to 12. The result will then always be in the range 1 to 12 inclusive.

14) Formerly, even **hours** were named in the same manner as years, months, and days. InterCal does not use hours and does not attempt to include hour names.