INDUCTANCE

inductors in series :

$$L_T = L_1 + L_2 + \ldots + L_n$$

inductors in parallel :

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$$

A component exhibits the property of inductance when it opposes a change in the current through it. An inductor consists of a length of wire wound on a core to form a coil.

The following principles of electricity explain why a coil of wire exhibit the property of inductance :

1) **A magnetic field is generated by moving a charge** - current is a moving charge, thus a magnetic field exists in the space surrounding the current carrying conductor.

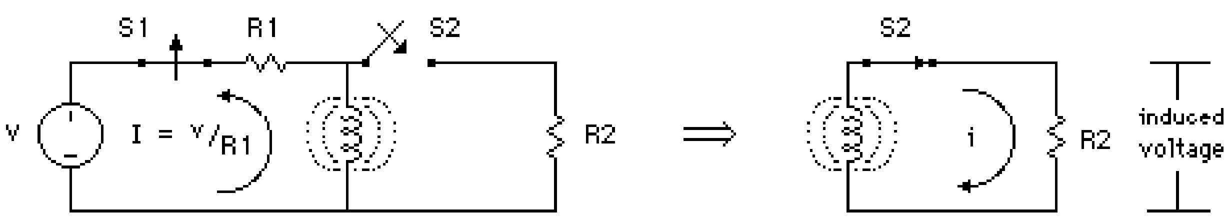
2) **When there is relative motion between a magnetic field and conductor, a voltage is induced in the conductor -** the magnetic lines of force from an expanding magnetic field move relative to the current carrying conductor. The magnetic lines are said to "cut" the conductor.

3) **The amount of induced voltage depends upon how rapidly the magnetic lines of force cut the conductor** - If the relative motion between a conductor and an expanding magnetic field is great, the induced voltage will be great.

When an inductor and series resistance are connected to a DC voltage source, the voltage source will begin to force current into the circuit. The instant current begins to flow, a magnetic field is generated surrounding the conductor (principle 1). As the voltage source forces more and more current into the circuit, the magnetic field expands outward from the center of the conductor, inducing (bringing about) a voltage in the conductor (principle 2).

After a sufficient amount of time, the current builds up to the steady-state Ohm's value of V/R. At this point the magnetic field has stopped expanding and is therefore constant. Since there is no longer any relative motion between the magnetic field and the inductor, the induced voltage is zero. Though there is no voltage across the inductor, there is energy stored in it in the form of a magnetic field.

The release of stored inductor energy is pictured in the below diagram. Assuming switch S1 has been closed for a long time, the inductor current has built up to the steady-state value of V/R1. Switch S2 is initially open. If switch S1 is opened and switch S2 is simultaneously closed, there will no longer be a voltage source and the inductor current will attempt to decrease. As a result, the magnetic field surrounding the inductor starts to collapse. This collapse causes motion relative to the inductor and, consequently, a voltage is induced in the inductor. Current will continue to flow during the time the field is collapsing. After a sufficient amount of time the magnetic field will be fully collapsed and the current wil be zero.



the voltage across an inductor is given by :

$$\boxed{v = L\frac{di}{dt}}$$

The above equation tells us that the voltage across the terminals of an inductor is proportional to the time rate of change of current in the inductor. Two observations can be made from this :

There cannot be an instantaneous rate of change in the current through the terminals of an inductor (it would require an infinite voltage).

If the current through the terminals is constant, the inductor voltage will be zero (the derivative of a constant = 0, di = 0).

The current through an inductor is derived from the voltage formula:
multiply both sides by dt, integrate both sides :

$$\boxed{i(t) = \int_{o}^{t} \frac{1}{L} v\,dt + i_o}$$

i(t) = current through the inductor at time t, in seconds
i(o) = initial current through the inductor
L = inductance of the inductor, in henries
v = voltage across the inductor

The inductor stores energy in the form of a magnetic field. The amount of charge stored by an inductor is derived from is given by the following :

$$p = vi = \left(L\frac{di}{dt}\right)i$$

power equals dw/dt :

$$\frac{dw}{dt} = \left(L\frac{di}{dt}\right)i$$

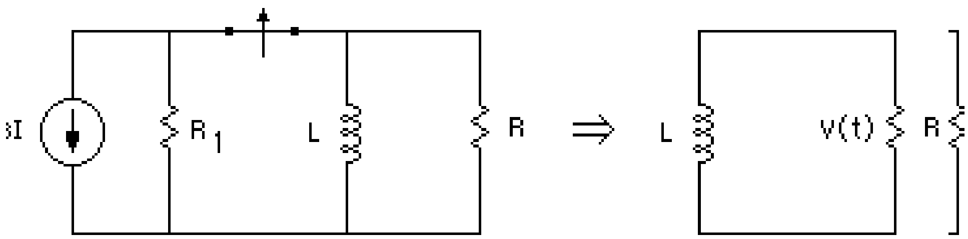
rearranging terms to isolate dw on the lefthand side, and integrating :

$$\boxed{W = \frac{1}{2}L\,I^2}$$

W = energy stored in joules (J)
L = inductance in Henries (H)
I = current in amps (A)
a henry is defined as the amount of inductance that induces one volt in a conductor when the current through the conductor is changing at the rate of one ampere per second.

NATURAL RESPONSE OF AN RL CIRCUIT

The natural response of an RL circuit deals with the behavior of a circuit that can be reduced to one equivalent resistance and one equivalent inductance. It is assumed that the current source generates a constant current of I amps and that the switch has been closed a long time. Because the current is constant, the voltage in the inductive branch is zero (v = L di/dt, derivative of a constant (i) is zero, therefore v = 0). Because the voltage is zero, the inductor is acting as a short circuit and therefore all the current appears in the inductive branch. The inductor is then isolated from sources, as in the diagram below :



switch is thrown... circuit reduces to ...

The natural response is the study of the collapsing of an inductor's magnetic field. When the switch is thrown, magnetic field begins to collapse.

The key to analyzing the natural response of an RL circuit is to determine the initial current, i(t). All subsequent calculations follow from knowing i(t).

To find i(t), apply Kirchoff's Voltage Law around the closed loop (see right hand side diagram, above):

$$V_L + V_R = 0$$

$$L\frac{di}{dt} + Ri = 0$$

rearrange terms :

$$\frac{di}{i} = -\frac{R}{L}dt$$

integrate both sides :

$$\ln i_{(t)} - \ln i_{(o)} = -\frac{R}{L}t$$

simplify :

$$e^{\left[\ln\frac{i_{(t)}}{i_{(o)}}\right]} = e^{\left[-\frac{R}{L}t\right]}$$

rearrange terms :

$$\boxed{i_{(t)} = I_{(o)}\, e^{\left[-\frac{R}{L}t\right]}}$$

i(t) = current through the load resistor
I(o) = initial current through the inductor
R/L = resistance, in ohms, divided by the inductance, in henries
t = time in seconds

From V = IR, the voltage v(t) is found :

$$\boxed{v(t) = I(o) R e^{\left[-\frac{R}{L}t\right]}}$$

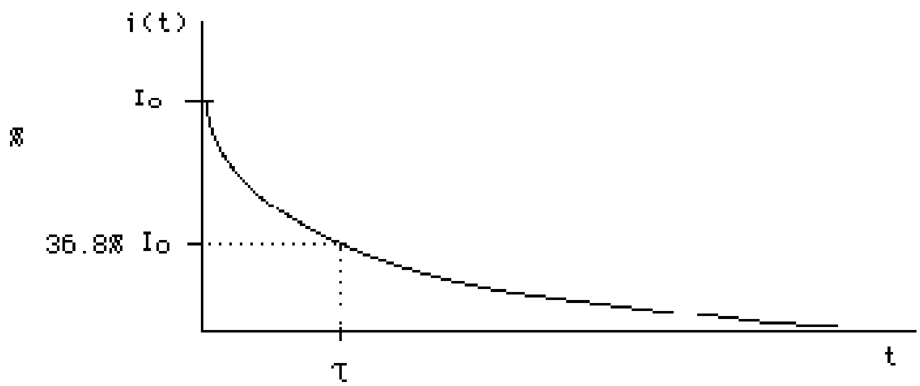
The voltage across the inductor will equal the voltage across the resistor, v(t).

The result of dividing the resistance by the inductance (R/L) is refered to as the time constant (t).

The natural response of an RL circuit is an exponential decay of the initial current.
The rate of decay is governed by the time constant, R/L.

After one time constant, the inductor current will have decayed to 36.8% of its initial value.

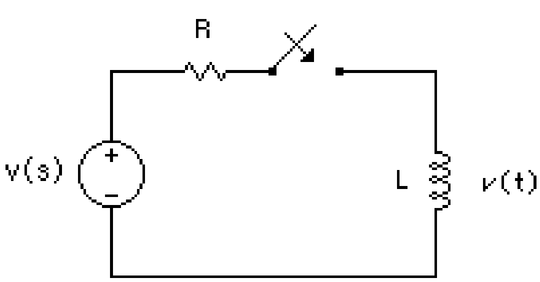
After five time constants, it can be assumed that the inductor's magnetic field is completely collapsed and that the current is zero (0.07% of initial value).



(one time constant, R/L)

STEP RESPONSE OF AN RL CIRCUIT

The step response of an RL circuit deals with the behavior of a circuit that can be reduced to one equivalent resistance and one equivalent inductor. The inductor is placed into a resistive circuit, as in the diagram below :



The step response is the study of the build up of an magnetic field about an inductor. The response of the circuit due to the sudden application of a constant voltage or current source is studied. When the switch is thrown, current will begin to flow through the inductor.

From the above circuit it can be seen that when the switch is closed, Kirchoff's voltage law requires that the component voltages equal the source voltage :

$$V_S = (Ri) + \left(L\frac{di}{dt}\right)$$

solve for di/dt :

$$\frac{di}{dt} = \frac{-Ri + V_S}{L} = \frac{-R}{L}\left(i - \frac{V_S}{R}\right)$$

separate variables :

$$\frac{di}{i - V_S/R} = \frac{-R}{L}dt$$

set up the integration :

$$\int_{I_o}^{i(t)} \frac{di}{i - V_S/R} = \frac{-R}{L}\int_0^t dt$$

The limits of the first integral are the initial current, Io, and the current at any time, t. Integrating results in :

$$\ln(i_{(t)} - V_S/R) - \ln(I_o - V_S/R) = -\frac{R}{L}t$$

rearrange terms, both sides e raised to the ... :

$$e^{\left[\ln\left(\frac{i_{(t)} - V_S/R}{I_o - V_S/R}\right)\right]} = e^{\left[-\frac{R}{L}t\right]}$$

simplify, rearrange :

i(t) = current through the inductor at time t, in seconds
Io = initial current (at t=0)
Vs = source voltage

To determine the voltage across the inductor, recall that v = L(di/dt) :

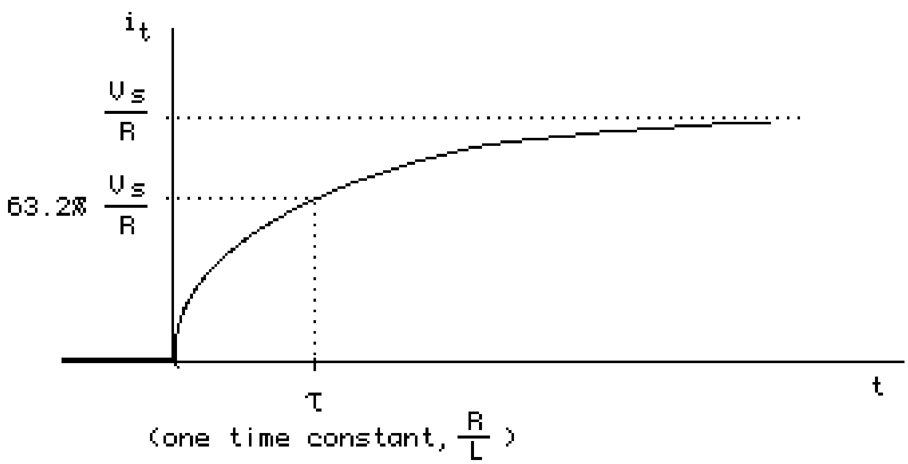
$$\boxed{v(t) = \left(v_S - I_0 R\right) e^{\left[-\frac{R}{L} t\right]}}$$

The step response of an RL circuit tells us that when a constant voltage source is suddenly applied through a resistance to an inductor, the current increases exponentially from zero to a final value of Vs/R amps.

The step response also tells us that the voltage jumps to Vs volts the instant the switch is closed (Vs - IoR if Io≠0) and then decays exponentially to zero.

The rates of both the current buildup and the voltage decay are governed by the time constant, R/L.

After one time constant, the inductor current will have increased to 63.2% of its final value :



After one time constant, the capacitor voltage will have decayed to 36.8% of its inital value :