fractint2

COLLABORATORS

	TITLE : fractint2		
ACTION	*NAME*	*DATE*	*SIGNATURE*
WRITTEN BY		July 20, 2024	

REVISION HISTORY

NUMBER	DATE	DESCRIPTION	NAME

Contents

Chapter 1

fractint2

1.1 Summary of Fractal Types

```
For detailed descriptions, select a hot-link below, see Fractal Types,
or use <F2> from the fractal type selection screen.
SUMMARY OF FRACTAL TYPES
```

```
 ant
    Generalized Ant Automaton as described in the July 1994 Scientific
    American. An ant wanders around the screen. A rule string (the first
    parameter) determines the ant's direction. When the ant leaves a cell of
    color k, it turns right if the kth symbol in the first parameter is a 1,
    or left otherwise. Then the color in the abandoned cell is incremented.
    The 2nd parameter is a maximum iteration to guarantee that the fractal
    will terminate. Also terminates if the edge is reached. You can slow down
    the ant to see her better using the <x> screen Orbit Delay - try 10.
```

```
 barnsleyj1
     z(0) = pixel;
     z(n+1) = (z-1)*c if real(z) >= 0, else
     z(n+1) = (z+1)*c
    Two parameters: real and imaginary parts of c
```

```
 barnsleyj2
     z(0) = pixel;
     if real(z(n)) * imag(c) + real(c) * imag(z((n)) >= 0
  z(n+1) = (z(n)-1)*c
     else
  z(n+1) = (z(n)+1)*c
   Two parameters: real and imaginary parts of c
 barnsleyj3
     z(0) = pixel;
     if real(z(n) > 0 then z(n+1) = (real(z(n))^2 - imag(z(n))^2 - 1)
  + i * (2*real(z((n)) * imag(z((n))) else
     z(n+1) = (real(z(n))^2 - imag(z(n))^2 - 1 + real(c) * real(z(n))
      + i * (2*real(z((n)) * imag(z((n)) + imag(c) * real(z(n))
   Two parameters: real and imaginary parts of c.
 barnsleym1
     z(0) = c = pixel;
     if real(z) >= 0 then
  z(n+1) = (z-1)*c
```

```
      else
  z(n+1) = (z+1)*c.
    Parameters are perturbations of z(0)


barnsleym2
      z(0) = c = pixel;
      if real(z)*imag(c) + real(c)*imag(z) >= 0
  z(n+1) = (z-1)*c
      else
  z(n+1) = (z+1)*c
    Parameters are perturbations of z(0)


barnsleym3
      z(0) = c = pixel;
      if real(z(n) > 0 then z(n+1) = (real(z(n))^2 - imag(z(n))^2 - 1)
   + i * (2*real(z((n)) * imag(z((n))) else
      z(n+1) = (real(z(n))^2 - imag(z(n))^2 - 1 + real(c) * real(z(n))
   + i * (2*real(z((n)) * imag(z((n)) + imag(c) * real(z(n))
    Parameters are perturbations of z(0)


bifurcation
    Pictorial representation of a population growth model.
      Let P = new population, p = oldpopulation, r = growth rate
      The model is: P = p + r*fn(p)*(1-fn(p)).
    Three parameters: Filter Cycles, Seed Population, and Function.



bif+sinpi
    Bifurcation variation: model is: P = p + r*fn(PI*p).
    Three parameters: Filter Cycles, Seed Population, and Function.


bif=sinpi
    Bifurcation variation: model is: P = r*fn(PI*p).
    Three parameters: Filter Cycles, Seed Population, and Function.


biflambda
    Bifurcation variation: model is: P = r*fn(p)*(1-fn(p)).
    Three parameters: Filter Cycles, Seed Population, and Function.


bifstewart
    Bifurcation variation: model is: P = (r*fn(p)*fn(p)) - 1.
    Three parameters: Filter Cycles, Seed Population, and Function.


bifmay
    Bifurcation variation: model is: P = r*p / ((1+p)^beta).
    Three parameters: Filter Cycles, Seed Population, and Beta.



cellular
    One-dimensional cellular automata or line automata.  The type of CA
    is given by kr, where k is the number of different states of the
    automata and r is the radius of the neighborhood.  The next generation
    is determined by the sum of the neighborhood and the specified rule.
    Four parameters: Initial String, Rule, Type, and Starting Row Number.
    For Type = 21, 31, 41, 51, 61, 22, 32, 42, 23, 33, 24, 25, 26, 27
        Rule =  4,  7, 10, 13, 16,  6, 11, 16,  8, 15, 10, 12, 14, 16 digits
chip
```

Chip attractor from Michael Peters - orbit in two dimensions.
```
    z(0) = y(0) = 0;
    x(n+1) = y(n) - sign(x(n)) * cos(sqr(ln(abs(b*x(n)-c))))
                              * arctan(sqr(ln(abs(c*x(n)-b))))
    y(n+1) = a - x(n)
```
Parameters are a, b, and c.

circle

Circle pattern by John Connett
```
    x + iy = pixel
    z = a*(x^2 + y^2)
    c = integer part of z
    color = c modulo(number of colors)
```

cmplxmarksjul

A generalization of the marksjulia fractal.
```
    z(0) = pixel;
    z(n+1) = (c^exp-1)*z(n)^2 + c.
```
Four parameters: real and imaginary parts of c,
and real and imaginary parts of exponent.

cmplxmarksmand

A generalization of the marksmandel fractal.
```
    z(0) = c = pixel;
    z(n+1) = (c^exp-1)*z(n)^2 + c.
```
Four parameters: real and imaginary parts of perturbation
of z(0), and real and imaginary parts of exponent.

complexnewton, complexbasin

Newton fractal types extended to complex degrees. Complexnewton
colors pixels according to the number of iterations required to
escape to a root. Complexbasin colors pixels according to which
root captures the orbit. The equation is based on the newton
formula for solving the equation z^p = r
```
    z(0) = pixel;
    z(n+1) = ((p - 1) * z(n)^p + r)/(p * z(n)^(p - 1)).
```
Four parameters: real & imaginary parts of degree p and root r.

diffusion

Diffusion Limited Aggregation. Randomly moving points
accumulate. Two parameters: border width (default 10), type.

dynamic

Time-discrete dynamic system.
```
    x(0) = y(0) = start position.
    y(n+1) = y(n) + f( x(n) )
    x(n+1) = x(n) - f( y(n) )
    f(k) = sin(k + a*fn1(b*k))
```
For implicit Euler approximation: x(n+1) = x(n) - f(y(n+1))
Five parameters: start position step, dt, a, b, and the function fn1.

fn(z)+fn(pix)
```
    c = z(0) = pixel;
    z(n+1) = fn1(z) + p*fn2(c)
```
Six parameters: real and imaginary parts of the perturbation
of z(0) and factor p, and the functions fn1, and fn2.

fn(z*z)
```
    z(0) = pixel;
    z(n+1) = fn(z(n)*z(n))
```

```
    One parameter: the function fn.

 fn*fn
     z(0) = pixel; z(n+1) = fn1(n)*fn2(n)
    Two parameters: the functions fn1 and fn2.

 fn*z+z
     z(0) = pixel; z(n+1) = p1*fn(z(n))*z(n) + p2*z(n)
    Five parameters: the real and imaginary components of
    p1 and p2, and the function fn.

 fn+fn
     z(0) = pixel;
     z(n+1) = p1*fn1(z(n))+p2*fn2(z(n))
    Six parameters: The real and imaginary components of
    p1 and p2, and the functions fn1 and fn2.

 formula
    Formula interpreter - write your own formulas as text files!


 frothybasin
    Pixel color is determined by which attractor captures the orbit.  The
    shade of color is determined by the number of iterations required to
    capture the orbit.
      Z(0) = pixel;  Z(n+1) = Z(n)^2 - C*conj(Z(n))
      where C = 1 + A*i, critical value of A = 1.028713768218725...

 gingerbread
    Orbit in two dimensions defined by:
      x(n+1) = 1 - y(n) + |x(n)|
      y(n+1) = x(n)
    Two parameters: initial values of x(0) and y(0).

 halley
     Halley map for the function: F = z(z^a - 1) = 0
     z(0) = pixel;
     z(n+1) = z(n) - R * F / [F' - (F" * F / 2 * F')]
     bailout when: abs(mod(z(n+1)) - mod(z(n)) < epsilon
    Four parameters: order a, real part of R, epsilon,
     and imaginary part of R.

 henon
    Orbit in two dimensions defined by:
      x(n+1) = 1 + y(n) - a*x(n)*x(n)
      y(n+1) = b*x(n)
    Two parameters: a and b

 hopalong
    Hopalong attractor by Barry Martin - orbit in two dimensions.
      z(0) = y(0) = 0;
      x(n+1) = y(n) - sign(x(n))*sqrt(abs(b*x(n)-c))
      y(n+1) = a - x(n)
    Parameters are a, b, and c.

 hypercomplex
    HyperComplex Mandelbrot set.
```

```
       h(0)   = (0,0,0,0)
       h(n+1) = fn(h(n)) + C.
       where "fn" is sin, cos, log, sqr etc.
    Two parameters: cj, ck
    C = (xpixel,ypixel,cj,ck)
```

hypercomplexj
    HyperComplex Julia set.
```
      h(0)   = (xpixel,ypixel,zj,zk)
      h(n+1) = fn(h(n)) + C.
      where "fn" is sin, cos, log, sqr etc.
    Six parameters: c1, ci, cj, ck
    C = (c1,ci,cj,ck)
```

icon, icon3d
    Orbit in three dimensions defined by:
```
      p = lambda + alpha * magnitude + beta * (x(n)*zreal - y(n)*zimag)
      x(n+1) = p * x(n) + gamma * zreal - omega * y(n)
      y(n+1) = p * y(n) - gamma * zimag + omega * x(n)
      (3D version uses magnitude for z)
      Parameters:  Lambda, Alpha, Beta, Gamma, Omega, and Degree
```

IFS
    Barnsley IFS (Iterated Function System) fractals. Apply
    contractive affine mappings.

julfn+exp
    A generalized Clifford Pickover fractal.
```
      z(0) = pixel;
      z(n+1) = fn(z(n)) + e^z(n) + c.
    Three parameters: real & imaginary parts of c, and fn
```

julfn+zsqrd
```
      z(0) = pixel;
      z(n+1) = fn(z(n)) + z(n)^2 + c
    Three parameters: real & imaginary parts of c, and fn
```

julia
    Classic Julia set fractal.
```
      z(0) = pixel; z(n+1) = z(n)^2 + c.
    Two parameters: real and imaginary parts of c.
```

julia_inverse
    Inverse Julia function - "orbit" traces Julia set in two dimensions.
```
      z(0) = a point on the Julia Set boundary; z(n+1) = +- sqrt(z(n) - c)
```
    Parameters: Real and Imaginary parts of c
          Maximum Hits per Pixel (similar to max iters)
          Breadth First, Depth First or Random Walk Tree Traversal
          Left or Right First Branching (in Depth First mode only)
       Try each traversal method, keeping everything else the same.
       Notice the differences in the way the image evolves. Start with
       a fairly low Maximum Hit limit, then increase it. The hit limit
       cannot be higher than the maximum colors in your video mode.

```
julia(fn||fn)
    z(0) = pixel;
    if modulus(z(n)) < shift value, then
        z(n+1) = fn1(z(n)) + c,
    else
        z(n+1) = fn2(z(n)) + c.
   Five parameters: real, imaginary portions of c, shift value,
                    fn1 and fn2.


julia4
   Fourth-power Julia set fractals, a special case
   of julzpower kept for speed.
    z(0) = pixel;
    z(n+1) = z(n)^4 + c.
   Two parameters: real and imaginary parts of c.

julibrot
   'Julibrot' 4-dimensional fractals.

julzpower
    z(0) = pixel;
    z(n+1) = z(n)^m + c.
   Three parameters: real & imaginary parts of c, exponent m

julzzpwr
    z(0) = pixel;
    z(n+1) = z(n)^z(n) + z(n)^m + c.
   Three parameters: real & imaginary parts of c, exponent m

kamtorus, kamtorus3d
   Series of orbits superimposed.
   3d version has 'orbit' the z dimension.
    x(0) = y(0) = orbit/3;
    x(n+1) = x(n)*cos(a) + (x(n)*x(n)-y(n))*sin(a)
    y(n+1) = x(n)*sin(a) - (x(n)*x(n)-y(n))*cos(a)
   After each orbit, 'orbit' is incremented by a step size.
   Parameters: a, step size, stop value for 'orbit', and
   points per orbit.

lambda
   Classic Lambda fractal. 'Julia' variant of Mandellambda.
    z(0) = pixel;
    z(n+1) = lambda*z(n)*(1 - z(n)).
   Two parameters: real and imaginary parts of lambda.

lambdafn
    z(0) = pixel;
    z(n+1) = lambda * fn(z(n)).
   Three parameters: real, imag portions of lambda, and fn

lambda(fn||fn)
    z(0) = pixel;
    if modulus(z(n)) < shift value, then
        z(n+1) = lambda * fn1(z(n)),
    else
        z(n+1) = lambda * fn2(z(n)).
```

    Five parameters: real, imaginary portions of lambda, shift value,
                       fn1 and fn2.

lorenz, lorenz3d
   Lorenz two lobe attractor - orbit in three dimensions.
   In 2d the x and y components are projected to form the image.
     z(0) = y(0) = z(0) = 1;
     x(n+1) = x(n) + (-a*x(n)*dt) + (a*y(n)*dt)
     y(n+1) = y(n) + (b*x(n)*dt) - (y(n)*dt) - (z(n)*x(n)*dt)
     z(n+1) = z(n) + (-c*z(n)*dt) + (x(n)*y(n)*dt)
   Parameters are dt, a, b, and c.

lorenz3d1
   Lorenz one lobe attractor - orbit in three dimensions.
   The original formulas were developed by Rick Miranda and Emily Stone.
     z(0) = y(0) = z(0) = 1; norm = sqrt(x(n)^2 + y(n)^2)
     x(n+1) = x(n) + (-a*dt-dt)*x(n) + (a*dt-b*dt)*y(n)
        + (dt-a*dt)*norm + y(n)*dt*z(n)
     y(n+1) = y(n) + (b*dt-a*dt)*x(n) - (a*dt+dt)*y(n)
        + (b*dt+a*dt)*norm - x(n)*dt*z(n) - norm*z(n)*dt
     z(n+1) = z(n) +(y(n)*dt/2) - c*dt*z(n)
   Parameters are dt, a, b, and c.
lorenz3d3
   Lorenz three lobe attractor - orbit in three dimensions.
   The original formulas were developed by Rick Miranda and Emily Stone.
     z(0) = y(0) = z(0) = 1; norm = sqrt(x(n)^2 + y(n)^2)
     x(n+1) = x(n) +(-(a*dt+dt)*x(n) + (a*dt-b*dt+z(n)*dt)*y(n))/3
        + ((dt-a*dt)*(x(n)^2-y(n)^2)
        + 2*(b*dt+a*dt-z(n)*dt)*x(n)*y(n))/(3*norm)
     y(n+1) = y(n) +((b*dt-a*dt-z(n)*dt)*x(n) - (a*dt+dt)*y(n))/3
        + (2*(a*dt-dt)*x(n)*y(n)
        + (b*dt+a*dt-z(n)*dt)*(x(n)^2-y(n)^2))/(3*norm)
     z(n+1) = z(n) +(3*x(n)*dt*x(n)*y(n)-y(n)*dt*y(n)^2)/2 - c*dt*z(n)
   Parameters are dt, a, b, and c.

lorenz3d4
   Lorenz four lobe attractor - orbit in three dimensions.
   The original formulas were developed by Rick Miranda and Emily Stone.
     z(0) = y(0) = z(0) = 1;
     x(n+1) = x(n) +(-a*dt*x(n)^3
        + (2*a*dt+b*dt-z(n)*dt)*x(n)^2*y(n) + (a*dt-2*dt)*x(n)*y(n)^2
        + (z(n)*dt-b*dt)*y(n)^3) / (2 * (x(n)^2+y(n)^2))
     y(n+1) = y(n) +((b*dt-z(n)*dt)*x(n)^3 + (a*dt-2*dt)*x(n)^2*y(n)
        + (-2*a*dt-b*dt+z(n)*dt)*x(n)*y(n)^2
        - a*dt*y(n)^3) / (2 * (x(n)^2+y(n)^2))
     z(n+1) = z(n) +(2*x(n)*dt*x(n)^2*y(n) - 2*x(n)*dt*y(n)^3 - c*dt*z(n))
   Parameters are dt, a, b, and c.

lsystem
   Using a turtle-graphics control language and starting with
   an initial axiom string, carries out string substitutions the
   specified number of times (the order), and plots the resulting.

lyapunov
   Derived from the Bifurcation fractal, the Lyapunov plots the Lyapunov
   Exponent for a population model where the Growth parameter varies between
   two values in a periodic manner.

```
magnet1j
    z(0) = pixel;
   [  z(n)^2 + (c-1)   ] 2
     z(n+1) =  | --------------- |
   [  2*z(n) + (c-2)   ]
   Parameters: the real and imaginary parts of c

magnet1m
    z(0) = 0; c = pixel;
   [  z(n)^2 + (c-1)   ] 2
     z(n+1) =  | --------------- |
   [  2*z(n) + (c-2)   ]
   Parameters: the real & imaginary parts of perturbation of z(0)

magnet2j
    z(0) = pixel;
   [  z(n)^3 + 3*(C-1)*z(n) + (C-1)*(C-2)           ] 2
     z(n+1) =  | ------------------------------------------- |
   [  3*(z(n)^2) + 3*(C-2)*z(n) + (C-1)*(C-2) + 1 ]
   Parameters: the real and imaginary parts of c

magnet2m
    z(0) = 0; c = pixel;
   [  z(n)^3 + 3*(C-1)*z(n) + (C-1)*(C-2)           ] 2
     z(n+1) =  | ------------------------------------------- |
   [  3*(z(n)^2) + 3*(C-2)*z(n) + (C-1)*(C-2) + 1 ]
   Parameters: the real and imaginary parts of perturbation of z(0)

mandel
   Classic Mandelbrot set fractal.
    z(0) = c = pixel;
    z(n+1) = z(n)^2 + c.
   Two parameters: real & imaginary perturbations of z(0)

mandel(fn||fn)
    c = pixel;
    z(0) = p1
    if modulus(z(n)) < shift value, then
       z(n+1) = fn1(z(n)) + c,
    else
       z(n+1) = fn2(z(n)) + c.
   Five parameters: real, imaginary portions of p1, shift value,
                    fn1 and fn2.

mandelcloud
   Displays orbits of Mandelbrot set:
    z(0) = c = pixel;
    z(n+1) = z(n)^2 + c.
   One parameter: number of intervals

mandel4
   Special case of mandelzpower kept for speed.
    z(0) = c = pixel;
    z(n+1) = z(n)^4 + c.
   Parameters: real & imaginary perturbations of z(0)
```

```
mandelfn
    z(0) = c = pixel;
    z(n+1) = c*fn(z(n)).
  Parameters: real & imaginary perturbations of z(0), and fn

manlam(fn||fn)
    c = pixel;
    z(0) = p1
    if modulus(z(n)) < shift value, then
        z(n+1) = fn1(z(n)) * c, else
        z(n+1) = fn2(z(n)) * c.
  Five parameters: real, imaginary parts of p1, shift value, fn1, fn2.

Martin
  Attractor fractal by Barry Martin - orbit in two dimensions.
    z(0) = y(0) = 0;
    x(n+1) = y(n) - sin(x(n))
    y(n+1) = a - x(n)
  Parameter is a (try a value near pi)

mandellambda
    z(0) = .5; lambda = pixel;
    z(n+1) = lambda*z(n)*(1 - z(n)).
  Parameters: real & imaginary perturbations of z(0)

mandphoenix
  z(0) = c = pixel, y(0) = 0;
  For degree = 0:
    z(n+1) = z(n)^2 + c.x + c.y*y(n), y(n+1) = z(n)
  For degree >= 2:
    z(n+1) = z(n)^degree + c.x*z(n)^(degree-1) + c.y*y(n)
    y(n+1) = z(n)
  For degree <= -3:
    z(n+1) = z(n)^|degree| + c.x*z(n)^(|degree|-2) + c.y*y(n)
    y(n+1) = z(n)
  Three parameters: real & imaginary perturbations of z(0), and degree.

mandphoenixclx
  z(0) = c = pixel, y(0) = 0;
  For degree = 0:
    z(n+1) = z(n)^2 + c + p2*y(n), y(n+1) = z(n)
  For degree >= 2:
    z(n+1) = z(n)^degree + c*z(n)^(degree-1) + p2*y(n), y(n+1) = z(n)
  For degree <= -3:
    z(n+1) = z(n)^|degree| + c*z(n)^(|degree|-2) + p2*y(n), y(n+1) = z(n)
  Five parameters: real & imaginary perturbations of z(0), real &
    imaginary parts of p2, and degree.

manfn+exp
  'Mandelbrot-Equivalent' for the julfn+exp fractal.
    z(0) = c = pixel;
    z(n+1) = fn(z(n)) + e^z(n) + C.
  Parameters: real & imaginary perturbations of z(0), and fn

manfn+zsqrd
  'Mandelbrot-Equivalent' for the Julfn+zsqrd fractal.
    z(0) = c = pixel;
```

```
    z(n+1) = fn(z(n)) + z(n)^2 + c.
  Parameters: real & imaginary perturbations of z(0), and fn

manowar
    c = z1(0) = z(0) = pixel;
    z(n+1) = z(n)^2 + z1(n) + c;
    z1(n+1) = z(n);
  Parameters: real & imaginary perturbations of z(0)
manowarj
    z1(0) = z(0) = pixel;
    z(n+1) = z(n)^2 + z1(n) + c;
    z1(n+1) = z(n);
  Parameters: real & imaginary parts of c

manzpower
  'Mandelbrot-Equivalent' for julzpower.
    z(0) = c = pixel;
    z(n+1) = z(n)^exp + c; try exp = e = 2.71828...
  Parameters: real & imaginary perturbations of z(0), real &
  imaginary parts of exponent exp.

manzzpwr
  'Mandelbrot-Equivalent' for the julzzpwr fractal.
    z(0) = c = pixel
    z(n+1) = z(n)^z(n) + z(n)^exp + C.
  Parameters: real & imaginary perturbations of z(0), and exponent

marksjulia
  A variant of the julia-lambda fractal.
    z(0) = pixel;
    z(n+1) = (c^exp-1)*z(n)^2 + c.
  Parameters: real & imaginary parts of c, and exponent

marksmandel
  A variant of the mandel-lambda fractal.
    z(0) = c = pixel;
    z(n+1) = (c^exp-1)*z(n)^2 + c.
  Parameters: real & imaginary parts of perturbations of z(0),
  and exponent

marksmandelpwr
  The marksmandelpwr formula type generalized (it previously
  had fn=sqr hard coded).
    z(0) = pixel, c = z(0) ^ (z(0) - 1):
    z(n+1) = c * fn(z(n)) + pixel,
  Parameters: real and imaginary perturbations of z(0), and fn

newtbasin
  Based on the Newton formula for finding the roots of z^p - 1.
  Pixels are colored according to which root captures the orbit.
    z(0) = pixel;
    z(n+1) = ((p-1)*z(n)^p + 1)/(p*z(n)^(p - 1)).
  Two parameters: the polynomial degree p, and a flag to turn
  on color stripes to show alternate iterations.

newton
  Based on the Newton formula for finding the roots of z^p - 1.
```

```
   Pixels are colored according to the iteration when the orbit
   is captured by a root.
     z(0) = pixel;
     z(n+1) = ((p-1)*z(n)^p + 1)/(p*z(n)^(p - 1)).
   One parameter: the polynomial degree p.

phoenix
   z(0) = pixel, y(0) = 0;
   For degree = 0: z(n+1) = z(n)^2 + p1.x + p2.x*y(n), y(n+1) = z(n)
   For degree >= 2:
    z(n+1) = z(n)^degree + p1.x*z(n)^(degree-1) + p2.x*y(n), y(n+1) = z(n)
   For degree <= -3:
    z(n+1) = z(n)^|degree| + p1.x*z(n)^(|degree|-2) + p2.x*y(n), y(n+1) = z(n)
   Three parameters: real parts of p1 & p2, and degree.

phoenixcplx
   z(0) = pixel, y(0) = 0;
   For degree = 0: z(n+1) = z(n)^2 + p1 + p2*y(n), y(n+1) = z(n)
   For degree >= 2:
     z(n+1) = z(n)^degree + p1*z(n)^(degree-1) + p2*y(n), y(n+1) = z(n)
   For degree <= -3:
     z(n+1) = z(n)^|degree| + p1*z(n)^(|degree|-2) + p2*y(n), y(n+1) = z(n)
   Five parameters: real & imaginary parts of p1 & p2, and degree.
pickover
   Orbit in three dimensions defined by:
     x(n+1) = sin(a*y(n)) - z(n)*cos(b*x(n))
     y(n+1) = z(n)*sin(c*x(n)) - cos(d*y(n))
     z(n+1) = sin(x(n))
   Parameters: a, b, c, and d.
plasma
   Random, cloud-like formations.  Requires 4 or more colors.
   A recursive algorithm repeatedly subdivides the screen and
   colors pixels according to an average of surrounding pixels
   and a random color, less random as the grid size decreases.
   Four parameters: 'graininess' (.5 to 50, default = 2), old/new
   algorithm, seed value used, 16-bit out output selection.

popcorn
   The orbits in two dimensions defined by:
     x(0) = xpixel, y(0) = ypixel;
     x(n+1) = x(n) - h*sin(y(n) + tan(3*y(n))
     y(n+1) = y(n) - h*sin(x(n) + tan(3*x(n))
   are plotted for each screen pixel and superimposed.
   One parameter: step size h.

popcornjul
   Conventional Julia using the popcorn formula:
     x(0) = xpixel, y(0) = ypixel;
     x(n+1) = x(n) - h*sin(y(n) + tan(3*y(n))
     y(n+1) = y(n) - h*sin(x(n) + tan(3*x(n))
   One parameter: step size h.

quadruptwo
   Quadruptwo attractor from Michael Peters - orbit in two dimensions.
     z(0) = y(0) = 0;
     x(n+1) = y(n) - sign(x(n)) * sin(ln(abs(b*x(n)-c)))
                                * arctan(sqr(ln(abs(c*x(n)-b))))
```

```
      y(n+1) = a - x(n)
   Parameters are a, b, and c.

quatjul
   Quaternion Julia set.
     q(0)   = (xpixel,ypixel,zj,zk)
     q(n+1) = q(n)*q(n) + c.
   Four parameters: c, ci, cj, ck
   c = (c1,ci,cj,ck)

quat
   Quaternion Mandelbrot set.
     q(0)   = (0,0,0,0)
     q(n+1) = q(n)*q(n) + c.
   Two parameters: cj,ck
   c = (xpixel,ypixel,cj,ck)

rossler3D
   Orbit in three dimensions defined by:
     x(0) = y(0) = z(0) = 1;
     x(n+1) = x(n) - y(n)*dt -   z(n)*dt
     y(n+1) = y(n) + x(n)*dt + a*y(n)*dt
     z(n+1) = z(n) + b*dt + x(n)*z(n)*dt - c*z(n)*dt
   Parameters are dt, a, b, and c.

sierpinski
   Sierpinski gasket - Julia set producing a 'Swiss cheese triangle'
     z(n+1) = (2*x,2*y-1) if y > .5;
   else (2*x-1,2*y) if x > .5;
   else (2*x,2*y)
   No parameters.

spider
     c(0) = z(0) = pixel;
     z(n+1) = z(n)^2 + c(n);
     c(n+1) = c(n)/2 + z(n+1)
   Parameters: real & imaginary perturbation of z(0)

sqr(1/fn)
     z(0) = pixel;
     z(n+1) = (1/fn(z(n))^2
   One parameter: the function fn.

sqr(fn)
     z(0) = pixel;
     z(n+1) = fn(z(n))^2
   One parameter: the function fn.

test
   'test' point letting us (and you!) easily add fractal types via
   the c module testpt.c.  Default set up is a mandelbrot fractal.
   Four parameters: user hooks (not used by default testpt.c).

tetrate
     z(0) = c = pixel;
     z(n+1) = c^z(n)
   Parameters: real & imaginary perturbation of z(0)
```

```
threeply
    Threeply attractor by Michael Peters - orbit in two dimensions.
      z(0) = y(0) = 0;
      x(n+1) = y(n) - sign(x(n)) * (abs(sin(x(n))*cos(b)
                                    +c-x(n)*sin(a+b+c)))
      y(n+1) = a - x(n)
    Parameters are a, b, and c.

tim's_error
    A serendipitous coding error in marksmandelpwr brings to life
    an ancient pterodactyl!  (Try setting fn to sqr.)
      z(0) = pixel, c = z(0) ^ (z(0) - 1):
      tmp = fn(z(n))
      real(tmp) = real(tmp) * real(c) - imag(tmp) * imag(c);
      imag(tmp) = real(tmp) * imag(c) - imag(tmp) * real(c);
      z(n+1) = tmp + pixel;
    Parameters: real & imaginary perturbations of z(0) and function fn

unity
      z(0) = pixel;
      x = real(z(n)), y = imag(z(n))
      One = x^2 + y^2;
      y = (2 - One) * x;
      x = (2 - One) * y;
      z(n+1) = x + i*y
    No parameters.
```

1.2 Fractal Types

A list of the fractal types and their mathematics can be found in the
{Summary of Fractal Types}. Some notes about how Fractint calculates
them are in "A Little Code" in Fractals and the PC.

Fractint starts by default with the Mandelbrot set. You can change that by
using the command-line argument "TYPE=" followed by one of the
fractal type names, or by using the <T> command and
selecting the type - if parameters are needed, you will be prompted for
them.

In the text that follows, due to the limitations of the ASCII character
set, "a*b" means "a times b", and "a^b" means "a to the power b".

Press <PageDown> for type selection list.
Select a fractal type:

The Mandelbrot Set
Julia Sets
Inverse Julias
Newton domains of attraction
Newton
Complex Newton
Lambda Sets
Mandellambda Sets
Plasma Clouds

```
Lambdafn
Mandelfn
Barnsley Mandelbrot/Julia Sets
Barnsley IFS Fractals
Sierpinski Gasket
Quartic Mandelbrot/Julia
Distance Estimator
Pickover Mandelbrot/Julia Types
Pickover Popcorn
Dynamic System
Quaternion
Peterson Variations
Unity
Circle
Scott Taylor / Lee Skinner Variations
Kam Torus
Bifurcation
Orbit Fractals
Lorenz Attractors
Rossler Attractors
Henon Attractors
Pickover Attractors
Martin Attractors
Gingerbreadman
Test
Formula
Julibrots
Diffusion Limited Aggregation
Magnetic Fractals
L-Systems
Lyapunov Fractals
fn||fn Fractals
Halley
Cellular Automata
Phoenix
Frothy Basins
Icon
Hypercomplex
```

1.3 The Mandelbrot Set

```
(type=mandel)
```

This set is the classic: the only one implemented in many plotting
programs, and the source of most of the printed fractal images published
in recent years. Like most of the other types in Fractint, it is simply a
graph: the x (horizontal) and y (vertical) coordinate axes represent
ranges of two independent quantities, with various colors used to
symbolize levels of a third quantity which depends on the first two. So
far, so good: basic analytic geometry.

Now things get a bit hairier. The x axis is ordinary, vanilla real
numbers. The y axis is an imaginary number, i.e. a real number times i,
where i is the square root of -1. Every point on the plane -- in this
case, your PC's display screen -- represents a complex number of the form:

```
    x-coordinate + i * y-coordinate
```

If your math training stopped before you got to imaginary and complex
numbers, this is not the place to catch up. Suffice it to say that they
are just as "real" as the numbers you count fingers with (they're used
every day by electrical engineers) and they can undergo the same kinds of
algebraic operations.

OK, now pick any complex number -- any point on the complex plane -- and
call it C, a constant. Pick another, this time one which can vary, and
call it Z. Starting with Z=0 (i.e., at the origin, where the real and
imaginary axes cross), calculate the value of the expression

```
    Z^2 + C
```

Take the result, make it the new value of the variable Z, and calculate
again. Take that result, make it Z, and do it again, and so on: in
mathematical terms, iterate the function Z(n+1) = Z(n)^2 + C. For certain
values of C, the result "levels off" after a while. For all others, it
grows without limit. The Mandelbrot set you see at the start -- the solid-
colored lake (blue by default), the blue circles sprouting from it, and
indeed every point of that color -- is the set of all points C for which
the magnitude of Z is less than 2 after 150 iterations (150 is the default
setting, changeable via the <X> options screen or "maxiter=" parameter).
All the surrounding "contours" of other colors represent points for which
the magnitude of Z exceeds 2 after 149 iterations (the contour closest to
the M-set itself), 148 iterations, (the next one out), and so on.

We actually don't test for the magnitude of Z exceeding 2 - we test the
magnitude of Z squared against 4 instead because it is easier. This value
(FOUR usually) is known as the "bailout" value for the calculation, because
we stop iterating for the point when it is reached. The bailout value can
be changed on the <Z> options screen but the default is usually best. See
also Bailout Test.

Some features of interest:

1. Use the <X> options screen to increase the maximum number of iterations.
Notice that the boundary of the M-set becomes more and more convoluted (the
technical terms are "wiggly," "squiggly," and "utterly bizarre") as the Z-
magnitudes for points that were still within the set after 150 iterations turn
out to exceed 2 after 200, 500, or 1200. In fact, it can be proven that
the true boundary is infinitely long: detail without limit.

2. Although there appear to be isolated "islands" of blue, zoom in -- that
is, plot for a smaller range of coordinates to show more detail -- and
you'll see that there are fine "causeways" of blue connecting them to the
main set. As you zoomed, smaller islands became visible; the same is true
for them. In fact, there are no isolated points in the M-set: it is
"connected" in a strict mathematical sense.

3. The upper and lower halves of the first image are symmetric (a fact
that Fractint makes use of here and in some other fractal types to speed
plotting). But notice that the same general features -- lobed discs,
spirals, starbursts -- tend to repeat themselves (although never exactly)
at smaller and smaller scales, so that it can be impossible to judge by

eye the scale of a given image.

4. In a sense, the contour colors are window-dressing: mathematically, it
is the properties of the M-set itself that are interesting, and no
information about it would be lost if all points outside the set were
assigned the same color. If you're a serious, no-nonsense type, you may
want to cycle the colors just once to see the kind of silliness that other
people enjoy, and then never do it again. Go ahead. Just once, now. We
trust you.

1.4 Julia Sets

(type=julia)

These sets were named for mathematician Gaston Julia, and can be generated
by a simple change in the iteration process described for the
Mandelbrot Set. Start with a
specified value of C, "C-real + i * C-imaginary"; use as the initial value
of Z "x-coordinate + i * y-coordinate"; and repeat the same iteration,
Z(n+1) = Z(n)^2 + C.

There is a Julia set corresponding to every point on the complex plane --
an infinite number of Julia sets. But the most visually interesting tend
to be found for the same C values where the M-set image is busiest, i.e.
points just outside the boundary. Go too far inside, and the corresponding
Julia set is a circle; go too far outside, and it breaks up into scattered
points. In fact, all Julia sets for C within the M-set share the
"connected" property of the M-set, and all those for C outside lack it.

Fractint's spacebar toggle lets you "flip" between any view of the M-set
and the Julia set for the point C at the center of that screen. You can
then toggle back, or zoom your way into the Julia set for a while and then
return to the M-set. So if the infinite complexity of the M-set palls,
remember: each of its infinite points opens up a whole new Julia set.

Historically, the Julia sets came first: it was while looking at the M-set
as an "index" of all the Julia sets' origins that Mandelbrot noticed its
properties.

The relationship between the Mandelbrot set and Julia set can
hold between
other sets as well. Many of Fractint's types are "Mandelbrot/Julia" pairs
(sometimes called "M-sets" or "J-sets". All these are generated by
equations that are of the form z(k+1) = f(z(k),c), where the function
orbit is the sequence z(0), z(1), ..., and the variable c is a complex
parameter of the equation. The value c is fixed for "Julia" sets and is
equal to the first two parameters entered with the "params=Creal/Cimag"
command. The initial orbit value z(0) is the complex number corresponding
to the screen pixel. For Mandelbrot sets, the parameter c is the complex
number corresponding to the screen pixel. The value z(0) is c plus a
perturbation equal to the values of the first two parameters. See
the discussion of Mandellambda Sets.
This approach may or may not be the
"standard" way to create "Mandelbrot" sets out of "Julia" sets.

Some equations have additional parameters. These values are entered as the
third or fourth params= value for both Julia and Mandelbrot sets. The
variables x and y refer to the real and imaginary parts of z; similarly,
cx and cy are the real and imaginary parts of the parameter c and fx(z)
and fy(z) are the real and imaginary parts of f(z). The variable c is
sometimes called lambda for historical reasons.

NOTE: if you use the "PARAMS=" argument to warp the M-set by starting with
an initial value of Z other than 0, the M-set/J-sets correspondence breaks
down and the spacebar toggle no longer works.

1.5   Julia Toggle Spacebar Commands

The spacebar toggle has been enhanced for the classic Mandelbrot and Julia
types. When viewing the Mandelbrot, the spacebar turns on a window mode that
displays the Inverse Julia corresponding to the cursor position in a window.
Pressing the spacebar then causes the regular Julia escape time fractal
corresponding to the cursor position to be generated. The following keys
take effect in Inverse Julia mode.

```
<Space>      Generate the escape-time Julia Set corresponding to the cursor
             position. Only works if fractal is a "Mandelbrot" type.
<n>          Numbers toggle - shows coordinates of the cursor on the
             screen. Press <n> again to turn off numbers.
<p>          Enter new pixel coordinates directly
<h>          Hide fractal toggle. Works only if View Windows is turned on
             and set for a small window (such as the default size.) Hides
             the fractal, allowing the orbit to take up the whole screen.
             Press <h> again to uncover the fractal.
<s>          Saves the fractal, cursor, orbits, and numbers.
<<> or <,>   Zoom inverse julia image smaller.
<>> or <.>   Zoom inverse julia image larger.
<z>          Restore default zoom.
```

The Julia Inverse window is only implemented for the classic Mandelbrot
(type=mandel). For other "Mandelbrot" types <space> turns on the cursor
without the Julia window, and allows you to select coordinates of the
matching Julia set in a way similar to the use of the zoom box with the
Mandelbrot/Julia toggle in previous Fractint versions.

1.6   Inverse Julias

(type=julia_inverse)

Pick a function, such as the familiar Z(n) = Z(n-1) squared plus C
(the defining function of the Mandelbrot Set). If you pick a point Z(0)
at random from the complex plane, and repeatedly apply the function to it,
you get a sequence of new points called an orbit, which usually either
zips out toward infinity or zooms in toward one or more "attractor" points
near the middle of the plane. The set of all points that are "attracted"
to infinity is called the "Basin of Attraction" of infinity. Each of the
other attractors also has its own Basin of Attraction. Why is it called

a Basin? Imagine a lake, and all the water in it "draining" into the
attractor. The boundary between these basins is called the Julia Set of
the function.

The boundary between the basins of attraction is sort of like a
repeller; all orbits move away from it, toward one of the attractors.
But if we define a new function as the inverse of the old one, as for
instance $Z(n) = sqrt(Z(n-1)$ minus $C)$, then the old attractors become
repellers, and the former boundary itself becomes the attractor! Now,
starting from any point, all orbits are drawn irresistibly to the Julia
Set! In fact, once an orbit reaches the boundary, it will continue to
hop about until it traces the entire Julia Set! This method for drawing
Julia Sets is called the Inverse Iteration Method, or IIM for short.

Unfortunately, some parts of each Julia Set boundary are far more
attractive to inverse orbits than others are, so that as an orbit
traces out the set, it keeps coming back to these attractive parts
again and again, only occasionally visiting the less attractive parts.
Thus it may take an infinite length of time to draw the entire set.
To hasten the process, we can keep track of how many times each pixel
on our computer screen is visited by an orbit, and whenever an orbit
reaches a pixel that has already been visited more than a certain number
of times, we can consider that orbit finished and move on to another one.
This "hit limit" thus becomes similar to the iteration limit used in the
traditional escape-time fractal algorithm. This is called the Modified
Inverse Iteration Method, or MIIM, and is much faster than the IIM.

Now, the inverse of Mandelbrot's classic function is a square root, and
the square root actually has two solutions; one positive, one negative.
Therefore at each step of each orbit of the inverse function there is
a decision; whether to use the positive or the negative square root.
Each one gives rise to a new point on the Julia Set, so each is a good
choice. This series of choices defines a binary decision tree, each
point on the Julia Set giving rise to two potential child points.
There are many interesting ways to traverse a binary tree, among them
Breadth first, Depth first (left or negative first), Depth first (right
or positive first), and completely at random. It turns out that most
traversal methods lead to the same or similar pictures, but that how the
image evolves as the orbits trace it out differs wildly depending on the
traversal method chosen. As far as we know, this fact is an original
discovery by Michael Snyder, and version 18.2 of FRACTINT was its first
publication.

Pick a Julia constant such as $Z(0) = (-.74543, .11301)$, the popular
Seahorse Julia, and try drawing it first Breadth first, then Depth first
(right first), Depth first (left first), and finally with Random Walk.

Caveats: the video memory is used in the algorithm, to keep track of
how many times each pixel has been visited (by changing it's color).
Therefore the algorithm will not work well if you zoom in far enough that
part of the Julia Set is off the screen.

Bugs: Not working with Disk Video.
        Not resumeable.

The <J> key toggles between the Inverse Julia orbit and the

corresponding Julia escape time fractal.

1.7 Newton domains of attraction

(type=newtbasin)

The Newton formula is an algorithm used to find the roots of polynomial
equations by successive "guesses" that converge on the correct value as
you feed the results of each approximation back into the formula. It works
very well -- unless you are unlucky enough to pick a value that is on a
line BETWEEN two actual roots. In that case, the sequence explodes into
chaos, with results that diverge more and more wildly as you continue the
iteration.

This fractal type shows the results for the polynomial Z^n - 1, which has
n roots in the complex plane. Use the <T>ype command and enter "newtbasin"
in response to the prompt. You will be asked for a parameter, the "order"
of the equation (an integer from 3 through 10 -- 3 for x^3-1, 7 for x^7-1,
etc.). A second parameter is a flag to turn on alternating shades showing
changes in the number of iterations needed to attract an orbit. Some
people like stripes and some don't, as always, Fractint gives you a
choice!

The coloring of the plot shows the "basins of attraction" for each root of
the polynomial -- i.e., an initial guess within any area of a given color
would lead you to one of the roots. As you can see, things get a bit weird
along certain radial lines or "spokes," those being the lines between
actual roots. By "weird," we mean infinitely complex in the good old
fractal sense. Zoom in and see for yourself.

This fractal type is symmetric about the origin, with the number of
"spokes" depending on the order you select. It uses floating-point math if
you have an FPU, or a somewhat slower integer algorithm if you don't have
one.

See also: Newton

1.8 Newton

(type=newton)

The generating formula here is identical to that for newtbasin,
but the
coloring scheme is different. Pixels are colored not according to the root
that would be "converged on" if you started using Newton's formula from
that point, but according to the iteration when the value is close to a
root. For example, if the calculations for a particular pixel converge to
the 7th root on the 23rd iteration, NEWTBASIN will color that pixel using
color #7, but NEWTON will color it using color #23.

If you have a 256-color mode, use it: the effects can be much livelier
than those you get with type=newtbasin, and color cycling becomes, like,

downright cosmic. If your "corners" choice is symmetrical, Fractint
exploits the symmetry for faster display.

The applicable "params=" values are the same as newtbasin. Try "params=4."
Other values are 3 through 10. 8 has twice the symmetry and is faster. As
with newtbasin, an FPU helps.

1.9   Complex Newton

(type=complexnewton/complexbasin)

Well, hey, "Z^n - 1" is so boring when you can use "Z^a - b" where "a" and
"b" are complex numbers! The new "complexnewton" and "complexbasin"
fractal types are just the old newton and newtbasin fractal types with
this little added twist. When you select these fractal types, you are
prompted for four values (the real and imaginary portions of "a" and "b").
If "a" has a complex portion, the fractal has a discontinuity along the
negative axis - relax, we finally figured out that it's *supposed* to be
there!

1.10   Lambda Sets

(type=lambda)

This type calculates the Julia set of the formula lambda*Z*(1-Z). That is,
the value Z[0] is initialized with the value corresponding to each pixel
position, and the formula iterated. The pixel is colored according to the
iteration when the sum of the squares of the real and imaginary parts
exceeds 4.

Two parameters, the real and imaginary parts of lambda, are required. Try
0 and 1 to see the classical fractal "dragon". Then try 0.2 and 1 for a
lot more detail to zoom in on.

It turns out that all quadratic Julia-type sets can be calculated using
just the formula z^2+c (the "classic" Julia"), so that this type is
redundant, but we include it for reason of it's prominence in the history
of fractals.

1.11   Mandellambda Sets

(type=mandellambda)

This type is the "Mandelbrot equivalent" of the lambda set.
A comment is
in order here. Almost all the Fractint "Mandelbrot" sets are created from
orbits generated using formulas like z(n+1) = f(z(n),C), with z(0) and C
initialized to the complex value corresponding to the current pixel. Our
reasoning was that "Mandelbrots" are maps of the corresponding "Julias".
Using this scheme each pixel of a "Mandelbrot" is colored the same as the

Julia set corresponding to that pixel. However, Kevin Allen informs us
that the MANDELLAMBDA set appears in the literature with z(0) initialized
to a critical point (a point where the derivative of the formula is zero),
which in this case happens to be the point (.5,0). Since Kevin knows more
about Dr. Mandelbrot than we do, and Dr. Mandelbrot knows more about
fractals than we do, we defer! Starting with version 14 Fractint
calculates MANDELAMBDA Dr. Mandelbrot's way instead of our way. But ALL
THE OTHER "Mandelbrot" sets in Fractint are still calculated OUR way!
(Fortunately for us, for the classic Mandelbrot Set these two methods are
the same!)

Well now, folks, apart from questions of faithfulness to fractals named in
the literature (which we DO take seriously!), if a formula makes a
beautiful fractal, it is not wrong. In fact some of the best fractals in
Fractint are the results of mistakes! Nevertheless, thanks to Kevin for
keeping us accurate!

(See description of "initorbit=" command in Image Calculation Parameters
for a way to experiment with different orbit intializations).

1.12   Circle

(type=circle)

This fractal types is from A. K. Dewdney's "Computer Recreations" column
in "Scientific American". It is attributed to John Connett of the
University of Minnesota.

(Don't tell anyone, but this fractal type is not really a fractal!)

Fascinating Moire patterns can be formed by calculating x^2 + y^2 for
each pixel in a piece of the complex plane. After multiplication by a
magnification factor (the parameter), the number is truncated to an integer
and mapped to a color via color = value modulo (number of colors). That is,
the integer is divided by the number of colors, and the remainder is the
color index value used. The resulting image is not a fractal because all
detail is lost after zooming in too far. Try it with different resolution
video modes – the results may surprise you!

1.13   Plasma Clouds

(type=plasma)

Plasma clouds ARE real live fractals, even though we didn't know it at
first. They are generated by a recursive algorithm that randomly picks
colors of the corner of a rectangle, and then continues recursively
quartering previous rectangles. Random colors are averaged with those of
the outer rectangles so that small neighborhoods do not show much change,
for a smoothed-out, cloud-like effect. The more colors your video mode
supports, the better. The result, believe it or not, is a fractal
landscape viewed as a contour map, with colors indicating constant
elevation. To see this, save and view with the <3> command

(see '3D' Images)
and your "cloud" will be converted to a mountain!

You've GOT to try color cycling on these (hit "+" or "-").
If you
haven't been hypnotized by the drawing process, the writhing colors will
do it for sure. We have now implemented subliminal messages to exploit the
user's vulnerable state; their content varies with your bank balance,
politics, gender, accessibility to a Fractint programmer, and so on. A
free copy of Microsoft C to the first person who spots them.

This type accepts four parameters.

The first determines how abruptly the colors change. A value of .5 yields
bland clouds, while 50 yields very grainy ones. The default value is 2.

The second determines whether to use the original algorithm (0) or a
modified one (1). The new one gives the same type of images but draws
the dots in a different order. It will let you see
what the final image will look like much sooner than the old one.

The third determines whether to use a new seed for generating the
next plasma cloud (0) or to use the previous seed (1).

The fourth parameter turns on 16-bit .POT output which provides much
smoother height gradations. This is especially useful for creating
mountain landscapes when using the plasma output with a ray tracer
such as POV-Ray.

With parameter three set to 1, the next plasma cloud generated will be
identical to the previous but at whatever new resolution is desired.

Zooming is ignored, as each plasma-cloud screen is generated randomly.

The random number seed used for each plasma image is displayed on the
<tab> information screen, and can be entered with the command line
parameter "rseed=" to recreate a particular image.

The algorithm is based on the Pascal program distributed by Bret Mulvey as
PLASMA.ARC. We have ported it to C and integrated it with Fractint's
graphics and animation facilities. This implementation does not use
floating-point math. The algorithm was modified starting with version 18
so that the plasma effect is independent of screen resolution.

Saved plasma-cloud screens are EXCELLENT starting images for fractal
"landscapes" created with the 3D commands.

1.14 Lambdafn

(type=lambdafn)

Function=[sin|cos|sinh|cosh|exp|log|sqr|...]) is specified with this type.
Prior to version 14, these types were lambdasine, lambdacos, lambdasinh,
lambdacos, and lambdaexp. Where we say "lambdasine" or some such below,
the good reader knows we mean "lambdafn with function=sin".)

These types calculate the Julia set of the formula lambda*fn(Z), for
various values of the function "fn", where lambda and Z are both complex.
Two values, the real and imaginary parts of lambda, should be given in the
"params=" option. For the feathery, nested spirals of LambdaSines and the
frost-on-glass patterns of LambdaCosines, make the real part = 1, and try
values for the imaginary part ranging from 0.1 to 0.4 (hint: values near
0.4 have the best patterns). In these ranges the Julia set "explodes". For
the tongues and blobs of LambdaExponents, try a real part of 0.379 and an
imaginary part of 0.479.

A coprocessor used to be almost mandatory: each LambdaSine/Cosine
iteration calculates a hyperbolic sine, hyperbolic cosine, a sine, and a
cosine (the LambdaExponent iteration "only" requires an exponent, sine,
and cosine operation)! However, Fractint now computes these
transcendental functions with fast integer math. In a few cases the fast
math is less accurate, so we have kept the old slow floating point code.
To use the old code, invoke with the float=yes option, and, if you DON'T
have a coprocessor, go on a LONG vacation!

1.15   Halley

(type=halley)

The Halley map is an algorithm used to find the roots of polynomial
equations by successive "guesses" that converge on the correct value as
you feed the results of each approximation back into the formula. It works
very well -- unless you are unlucky enough to pick a value that is on a
line BETWEEN two actual roots. In that case, the sequence explodes into
chaos, with results that diverge more and more wildly as you continue the
iteration.

This fractal type shows the results for the polynomial Z(Z^a - 1), which
has a+1 roots in the complex plane. Use the <T>ype command and enter
"halley" in response to the prompt. You will be asked for a parameter, the
"order" of the equation (an integer from 2 through 10 -- 2 for Z(Z^2 - 1),
7 for Z(Z^7 - 1), etc.). A second parameter is the relaxation coefficient,
and is used to control the convergence stability. A number greater than
one increases the chaotic behavior and a number less than one decreases the
chaotic behavior. The third parameter is the value used to determine when
the formula has converged. The test for convergence is
||Z(n+1)|^2 - |Z(n)|^2| < epsilon. This convergence test produces the
whisker-like projections which generally point to a root.

1.16   Phoenix

(type=phoenix, mandphoenix, phoenixcplx, mandphoenixclx)

The phoenix type defaults to the original phoenix curve discovered by
Shigehiro Ushiki, "Phoenix", IEEE Transactions on Circuits and Systems,
Vol. 35, No. 7, July 1988, pp. 788-789. These images do not have the
X and Y axis swapped as is normal for this type.

The mandphoenix type is the corresponding Mandelbrot set image of the
phoenix type. The spacebar toggles between the two as long as the
mandphoenix type has an initial z(0) of (0,0). The mandphoenix is not
an effective index to the phoenix type, so explore the wild blue yonder.

To reproduce the Mandelbrot set image of the phoenix type as shown in
Stevens' book, "Fractal Programming in C", set initorbit=0/0 on the
command line or with the <g> key. The colors need to be rotated one
position because Stevens uses the values from the previous calculation
instead of the current calculation to determine when to bailout.

The phoenixcplx type is implemented using complex constants instead of the
real constants that Stevens used. This recreates the mapping as
originally presented by Ushiki.

The mandphoenixclx type is the corresponding Mandelbrot set image of the
phoenixcplx type. The spacebar toggles between the two as long as the
mandphoenixclx type has a perturbation of z(0) = (0,0). The mandphoenixclx
is an effective index to the phoenixcplx type.

1.17   fn||fn Fractals

(type=lambda(fn||fn), manlam(fn||fn), julia(fn||fn), mandel(fn||fn))

Two functions=[sin|cos|sinh|cosh|exp|log|sqr|...]) are specified with
these types. The two functions are alternately used in the calculation
based on a comparison between the modulus of the current Z and the
shift value. The first function is used if the modulus of Z is less
than the shift value and the second function is used otherwise.

The lambda(fn||fn) type calculates the Julia set of the formula
lambda*fn(Z), for various values of the function "fn", where lambda
and Z are both complex. Two values, the real and imaginary parts of
lambda, should be given in the "params=" option. The third value is
the shift value. The space bar will generate the corresponding
"psuedo Mandelbrot" set, manlam(fn||fn).

The manlam(fn||fn) type calculates the "psuedo Mandelbrot" set of the
formula fn(Z)*C, for various values of the function "fn", where C
and Z are both complex. Two values, the real and imaginary parts of
Z(0), should be given in the "params=" option. The third value is
the shift value. The space bar will generate the corresponding
julia set, lamda(fn||fn).

The julia(fn||fn) type calculates the Julia set of the formula
fn(Z)+C, for various values of the function "fn", where C
and Z are both complex. Two values, the real and imaginary parts of
C, should be given in the "params=" option. The third value is
the shift value. The space bar will generate the corresponding
mandelbrot set, mandel(fn||fn).

The mandel(fn||fn) type calculates the Mandelbrot set of the formula
fn(Z)+C, for various values of the function "fn", where C
and Z are both complex. Two values, the real and imaginary parts of

Z(0), should be given in the "params=" option. The third value is
the shift value. The space bar will generate the corresponding
julia set, julia(fn||fn).

1.18   Mandelfn

(type=mandelfn)

Function=[sin|cos|sinh|cosh|exp|log|sqr|...]) is specified with this type.
Prior to version 14, these types were mandelsine, mandelcos, mandelsinh,
mandelcos, and mandelexp. Same comment about our lapses into the old
terminology as above!

These are "pseudo-Mandelbrot" mappings for the LambdaFn
Julia functions.
They map to their corresponding Julia sets via the spacebar command in
exactly the same fashion as the original M/J sets. In general, they are
interesting mainly because of that property (the function=exp set in
particular is rather boring). Generate the appropriate "Mandelfn" set,
zoom on a likely spot where the colors are changing rapidly, and hit the
spacebar key to plot the Julia set for that particular point.

Try "FRACTINT TYPE=MANDELFN CORNERS=4.68/4.76/-.03/.03 FUNCTION=COS" for a
graphic demonstration that we're not taking Mandelbrot's name in vain
here. We didn't even know these little buggers were here until Mark
Peterson found this a few hours before the version incorporating Mandelfns
was released.

Note: If you created images using the lambda or mandel "fn" types prior to
version 14, and you wish to update the fractal information in the "*.fra"
file, simply read the files and save again. You can do this in batch mode
via a command line such as:

    "fractint oldfile.fra savename=newfile.gif batch=yes"

For example, this procedure can convert a version 13 "type=lambdasine"
image to a version 14 "type=lambdafn function=sin" GIF89a image. We do
not promise to keep this "backward compatibility" past version 14 - if you
want to keep the fractal information in your *.fra files accurate, we
recommend conversion. See GIF Save File Format.

1.19   Barnsley Mandelbrot/Julia Sets

(type=barnsleym1/.../j3)

Michael Barnsley has written a fascinating college-level text, "Fractals
Everywhere," on fractal geometry and its graphic applications. (See
Bibliography.) In it, he applies the principle of the M and J
sets to more general functions of two complex variables.

We have incorporated three of Barnsley's examples in Fractint. Their
appearance suggests polarized-light microphotographs of minerals, with

patterns that are less organic and more crystalline than those of the M/J
sets. Each example has both a "Mandelbrot" and a "Julia" type. Toggle
between them using the spacebar.

The parameters have the same meaning as they do for the "regular"
Mandelbrot and Julia. For types M1, M2, and M3, they are used to "warp"
the image by setting the initial value of Z. For the types J1 through J3,
they are the values of C in the generating formulas.

Be sure to try the <O>rbit function while plotting these types.

1.20 Barnsley IFS Fractals

(type=ifs)

One of the most remarkable spin-offs of fractal geometry is the ability to
"encode" realistic images in very small sets of numbers -- parameters for
a set of functions that map a region of two-dimensional space onto itself.
In principle (and increasingly in practice), a scene of any level of
complexity and detail can be stored as a handful of numbers, achieving
amazing "compression" ratios... how about a super-VGA image of a forest,
more than 300,000 pixels at eight bits apiece, from a 1-KB "seed" file?

Again, Michael Barnsley and his co-workers at the Georgia Institute of
Technology are to be thanked for pushing the development of these iterated
function systems (IFS).

When you select this fractal type, Fractint scans the current IFS file
(default is FRACTINT.IFS, a set of definitions supplied with Fractint) for
IFS definitions, then prompts you for the IFS name you wish to run. Fern
and 3dfern are good ones to start with. You can press <F6> at the
selection screen if you want to select a different .IFS file you've
written.

Note that some Barnsley IFS values generate images quite a bit smaller
than the initial (default) screen. Just bring up the zoom box, center it
on the small image, and hit <Enter> to get a full-screen image.

To change the number of dots Fractint generates for an IFS image before
stopping, you can change the "maximum iterations" parameter on the <X>
options screen.

Fractint supports two types of IFS images: 2D and 3D. In order to fully
appreciate 3D IFS images, since your monitor is presumably 2D, we have
added rotation, translation, and perspective capabilities. These share
values with the same variables used in Fractint's other 3D facilities; for
their meaning see Rectangular Coordinate Transformation.
You can enter these values from the command line using:

rotation=xrot/yrot/zrot   (try 30/30/30)
shift=xshift/yshift   (shifts BEFORE applying perspective!)
perspective=viewerposition   (try 200)

Alternatively, entering <I> from main screen will allow you to modify
these values. The defaults are the same as for regular 3D, and are not

always optimum for 3D IFS. With the 3dfern IFS type, try
rotation=30/30/30. Note that applying shift when using perspective changes
the picture -- your "point of view" is moved.

A truly wild variation of 3D may be seen by entering "2" for the stereo
mode (see Stereo 3D Viewing),
putting on red/blue "funny glasses", and watching the fern develop
with full depth perception right there before your eyes!

This feature USED to be dedicated to Bruce Goren, as a bribe to get him to
send us MORE knockout stereo slides of 3D ferns, now that we have made it
so easy! Bruce, what have you done for us *LATELY* ?? (Just kidding,
really!)

Each line in an IFS definition (look at FRACTINT.IFS with your editor for
examples) contains the parameters for one of the generating functions,
e.g. in FERN:

```
   a    b     c    d   e     f     p

 _____
   0    0     0   .16  0     0    .01
 .85   .04  -.04  .85  0    1.6   .85
 .2   -.26  .23   .22  0    1.6   .07
-.15   .28  .26   .24  0    .44   .07
```

The values on each line define a matrix, vector, and probability:

```
    matrix    vector  prob
    |a b|      |e|      p
    |c d|      |f|
```

The "p" values are the probabilities assigned to each function (how often
it is used), which add up to one. Fractint supports up to 32 functions,
although usually three or four are enough.

3D IFS definitions are a bit different. The name is followed by (3D) in
the definition file, and each line of the definition contains 13 numbers:
a b c d e f g h i j k l p, defining:

```
    matrix    vector  prob
    |a b c|    |j|      p
    |d e f|    |k|
    |g h i|    |l|
```

;You can experiment with changes to IFS definitions interactively by using
;Fractint's <Z> command. After selecting an IFS definition, hit <Z> to
;bring up the IFS editor. This editor displays the current IFS values, lets
;you modify them, and lets you save your modified values as a text file
;which you can then merge into an XXX.IFS file for future use with
;Fractint.
;
The program FDESIGN can be used to design IFS fractals - see FDESIGN.

You can save the points in your IFS fractal in the file ORBITS.RAW which is
overwritten each time a fractal is generated. The program Acrospin can
read this file and will let you view the fractal from any angle using
the cursor keys. See Acrospin.

1.21   Sierpinski Gasket

(type=sierpinski)

Another pre-Mandelbrot classic, this one found by W. Sierpinski around
World War I. It is generated by dividing a triangle into four congruent
smaller triangles, doing the same to each of them, and so on, yea, even
unto infinity. (Notice how hard we try to avoid reiterating "iterating"?)

If you think of the interior triangles as "holes", they occupy more and
more of the total area, while the "solid" portion becomes as hopelessly
fragile as that gasket you HAD to remove without damaging it -- you
remember, that Sunday afternoon when all the parts stores were closed?
There's a three-dimensional equivalent using nested tetrahedrons instead
of triangles, but it generates too much pyramid power to be safely
unleashed yet.

There are no parameters for this type. We were able to implement it with
integer math routines, so it runs fairly quickly even without an FPU.

1.22   Quartic Mandelbrot/Julia

(type=mandel4/julia4)

These fractal types are the moral equivalent of the original M and J sets,
except that they use the formula $Z(n+1) = Z(n)^4 + C$, which adds
additional pseudo-symmetries to the plots. The "Mandel4" set maps to the
"Julia4" set via -- surprise! -- the spacebar toggle. The M4 set is kind
of boring at first (the area between the "inside" and the "outside" of the
set is pretty thin, and it tends to take a few zooms to get to any
interesting sections), but it looks nice once you get there. The Julia
sets look nice right from the start.

Other powers, like $Z(n)^3$ or $Z(n)^7$, work in exactly the same fashion. We
used this one only because we're lazy, and $Z(n)^4 = (Z(n)^2)^2$.

1.23   distance estimator

(distest=nnn/nnn)

This used to be type=demm and type=demj. These types have not died, but
are only hiding! They are equivalent to the mandel and julia types with
the "distest=" option selected with a predetermined value.

The Distance Estimator Method
can be used to produce higher quality images of M and J sets,
especially suitable for printing in black and white.

If you have some *.fra files made with the old types demm/demj, you may
want to convert them to the new form. See the Mandelfn
section for directions to carry out the conversion.

1.24   Pickover Mandelbrot/Julia Types

(type=manfn+zsqrd/julfn+zsqrd, manzpowr/julzpowr, manzzpwr/julzzpwr,
manfn+exp/julfn+exp – formerly included man/julsinzsqrd and
man/julsinexp which have now been generalized)

These types have been explored by Clifford A. Pickover, of the IBM Thomas
J. Watson Research center. As implemented in Fractint, they are regular
Mandelbrot/Julia set pairs that may be plotted with or without the
biomorph option Pickover used to create organic-looking
beasties (see
below). These types are produced with formulas built from the functions
z^z, z^n, sin(z), and e^z for complex z. Types with "power" or "pwr" in
their name have an exponent value as a third parameter. For example,
type=manzpower params=0/0/2 is our old friend the classical Mandelbrot,
and type=manzpower params=0/0/4 is the Quartic Mandelbrot. Other values of
the exponent give still other fractals. Since these WERE the original
"biomorph" types, we should give an example. Try:

    FRACTINT type=manfn+zsqrd biomorph=0 corners=-8/8/-6/6 function=sin

to see a big biomorph digesting little biomorphs!

1.25   Pickover Popcorn

(type=popcorn/popcornjul)

Here is another Pickover idea. This one computes and plots the orbits of
the dynamic system defined by:

    x(n+1) = x(n) – h*sin(y(n)+tan(3*y(n))
    y(n+1) = y(n) – h*sin(x(n)+tan(3*x(n))

with the initializers x(0) and y(0) equal to ALL the complex values within
the "corners" values, and h=.01. ALL these orbits are superimposed,
resulting in "popcorn" effect. You may want to use a maxiter value less
than normal – Pickover recommends a value of 50. As a bonus,
type=popcornjul shows the Julia set generated by these same equations with
the usual escape-time coloring. Turn on orbit viewing with the "O"
command, and as you watch the orbit pattern you may get some insight as to
where the popcorn comes from. Although you can zoom and rotate popcorn,
the results may not be what you'd expect, due to the superimposing of
orbits and arbitrary use of color. Just for fun we added type popcornjul,
which is the plain old Julia set calculated from the same formula.

1.26   Dynamic System

(type=dynamic, dynamic2)

These fractals are based on a cyclic system of differential equations:
    x'(t) = –f(y(t))
    y'(t) = f(x(t))

These equations are approximated by using a small time step dt, forming
a time-discrete dynamic system:
    x(n+1) = x(n) - dt*f(y(n))
    y(n+1) = y(n) + dt*f(x(n))
The initial values x(0) and y(0) are set to various points in the plane,
the dynamic system is iterated, and the resulting orbit points are plotted.

In fractint, the function f is restricted to:
      f(k) = sin(k + a*fn1(b*k))
The parameters are the spacing of the initial points, the time step dt,
and the parameters (a,b,fn1) that affect the function f.
Normally the orbit points are plotted individually, but for a negative
spacing the points are connected.

This fractal is similar to the Pickover Popcorn.

A variant is the implicit Euler approximation:
    y(n+1) = y(n) + dt*f(x(n))
    x(n+1) = x(n) - dt*f(y(n+1))
This variant results in complex orbits. The implicit Euler approximation
is selected by entering dt<0.

There are two options that have unusual effects on these fractals. The
Orbit Delay value controls how many initial points are computed before
the orbits are displayed on the screen. This allows the orbit to settle
down. The outside=summ option causes each pixel to increment color every
time an orbit touches it; the resulting display is a 2-d histogram.

These fractals are discussed in Chapter 14 of Pickover's "Computers,
Pattern, Chaos, and Beauty".

1.27 Mandelcloud

(type=mandelcloud)

This fractal computes the Mandelbrot function, but displays it differently.
It starts with regularly spaced initial pixels and displays the resulting
orbits. This idea is somewhat similar to the Dynamic System.

There are two options that have unusual effects on this fractal. The
Orbit Delay value controls how many initial points are computed before
the orbits are displayed on the screen. This allows the orbit to settle
down. The outside=summ option causes each pixel to increment color every
time an orbit touches it; the resulting display is a 2-d histogram.

This fractal was invented by Noel Giffin.

1.28 Peterson Variations

(type=marksmandel, marksjulia, cmplxmarksmand, cmplxmarksjul, marksmandelpwr,
tim's_error)

These fractal types are contributions of Mark Peterson. MarksMandel and
MarksJulia are two families of fractal types that are linked in the same
manner as the classic Mandelbrot/Julia sets: each MarksMandel set can be
considered as a mapping into the MarksJulia sets, and is linked with the
spacebar toggle. The basic equation for these sets is:

$$Z(n+1) = ((lambda\text{\textasciicircum}exp-1) * Z(n)\text{\textasciicircum}2) + lambda$$

where Z(0) = 0.0 and lambda is (x + iy) for MarksMandel. For MarksJulia,
Z(0) = (x + iy) and lambda is a constant (taken from the MarksMandel
spacebar toggle, if that method is used). The exponent is a positive
integer or a complex number. We call these "families" because each value
of the exponent yields a different MarksMandel set, which turns out to be
a kinda-polygon with (exponent) sides. The exponent value is the third
parameter, after the "initialization warping" values. Typically one would
use null warping values, and specify the exponent with something like
"PARAMS=0/0/5", which creates an unwarped, pentagonal MarksMandel set.

In the process of coding MarksMandelPwr formula type, Tim Wegner
created the type "tim's_error" after making an interesting coding mistake.

1.29   Unity

(type=unity)

This Peterson variation began with curiosity about other "Newton-style"
approximation processes. A simple one,

   One = (x * x) + (y * y); y = (2 - One) * x;   x = (2 - One) * y;

produces the fractal called Unity.

One of its interesting features is the "ghost lines." The iteration loop
bails out when it reaches the number 1 to within the resolution of a
screen pixel. When you zoom a section of the image, the bailout criterion
is adjusted, causing some lines to become thinner and others thicker.

Only one line in Unity that forms a perfect circle: the one at a radius of
1 from the origin. This line is actually infinitely thin. Zooming on it
reveals only a thinner line, up (down?) to the limit of accuracy for the
algorithm. The same thing happens with other lines in the fractal, such as
those around |x| = |y| = (1/2)^(1/2) = .7071

Try some other tortuous approximations using the TEST stub and
let us know what you come up with!

1.30   Scott Taylor / Lee Skinner Variations

(type=fn(z*z), fn*fn, fn*z+z, fn+fn, fn+fn(pix), sqr(1/fn), sqr(fn), spider,
tetrate, manowar)

Two of Fractint's faithful users went bonkers when we introduced the
"formula" type, and came up with all kinds of variations on escape-time
fractals using trig functions. We decided to put them in as regular

types, but there were just too many! So we defined the types with variable
functions and let you, the overwhelmed user, specify what the functions
should be! Thus Scott Taylor's "z = sin(z) + z^2" formula type is now the
"fn+fn" regular type, and EITHER function can be one of sin, cos, tan, cotan,
sinh, cosh, tanh, cotanh, exp, log, sqr, recip, ident, conj, flip, cosxx,
asin, asinh, acos, acosh, atan, atanh, sqrt, abs, or cabs.

Plus we give you 4 parameters to set, the complex
coefficients of the two functions! Thus the innocent-looking "fn+fn" type
is really 256 different types in disguise, not counting the damage
done by the parameters!

 Some functions that require further explanation:

 conj()   - returns the complex conjugate of the argument. That is, changes
            sign of the imaginary component of argument: (x,y) becomes (x,-y)
 ident() - identity function. Leaves the value of the argument unchanged,
            acting like a "z" term in a formula.

 flip()   - Swap the real and imaginary components of the complex number.
            e.g. (4,5) would become (5,4)

Lee informs us that you should not judge fractals by their "outer"
appearance. For example, the images produced by z = sin(z) + z^2 and z =
sin(z) - z^2 look very similar, but are different when you zoom in.

1.31 Kam Torus

(type=kamtorus, kamtorus3d)

This type is created by superimposing orbits generated by a set of
equations, with a variable incremented each time.

    x(0) = y(0) = orbit/3;
    x(n+1) = x(n)*cos(a) + (x(n)*x(n)-y(n))*sin(a)
    y(n+1) = x(n)*sin(a) - (x(n)*x(n)-y(n))*cos(a)

After each orbit, 'orbit' is incremented by a step size. The parameters
are angle "a", step size for incrementing 'orbit', stop value for 'orbit',
and points per orbit. Try this with a stop value of 5 with sound=x for
some weird fractal music (ok, ok, fractal noise)! You will also see the
KAM Torus head into some chaotic territory that Scott Taylor wanted to
hide from you by setting the defaults the way he did, but now we have
revealed all!

The 3D variant is created by treating 'orbit' as the z coordinate.

With both variants, you can adjust the "maxiter" value (<X> options
screen or parameter maxiter=) to change the number of orbits plotted.

1.32 Bifurcation

(type=bifxxx)

The wonder of fractal geometry is that such complex forms can arise from
such simple generating processes. A parallel surprise has emerged in the
study of dynamical systems: that simple, deterministic equations can yield
chaotic behavior, in which the system never settles down to a steady state
or even a periodic loop. Often such systems behave normally up to a
certain level of some controlling parameter, then go through a transition
in which there are two possible solutions, then four, and finally a
chaotic array of possibilities.

This emerged many years ago in biological models of population growth.
Consider a (highly over-simplified) model in which the rate of growth is
partly a function of the size of the current population:

New Population = Growth Rate * Old Population * (1 - Old Population)

where population is normalized to be between 0 and 1. At growth rates less
than 200 percent, this model is stable: for any starting value, after
several generations the population settles down to a stable level. But for
rates over 200 percent, the equation's curve splits or "bifurcates" into
two discrete solutions, then four, and soon becomes chaotic.

Type=bifurcation illustrates this model. (Although it's now considered a
poor one for real populations, it helped get people thinking about chaotic
systems.) The horizontal axis represents growth rates, from 190 percent
(far left) to 400 percent; the vertical axis normalized population values,
from 0 to 4/3. Notice that within the chaotic region, there are narrow
bands where there is a small, odd number of stable values. It turns out
that the geometry of this branching is fractal; zoom in where changing
pixel colors look suspicious, and see for yourself.

Three parameters apply to bifurcations: Filter Cycles, Seed Population,
and Function or Beta.

Filter Cycles (default 1000) is the number of iterations to be done before
plotting maxiter population values. This gives the iteration time to settle
into the characteristic patterns that constitute the bifurcation diagram,
and results in a clean-looking plot. However, using lower values produces
interesting results too. Set Filter Cycles to 1 for an unfiltered map.

Seed Population (default 0.66) is the initial population value from which
all others are calculated. For filtered maps the final image is independent
of Seed Population value in the valid range (0.0 < Seed Population < 1.0).

Seed Population becomes effective in unfiltered maps - try setting Filter
Cycles to 1 (unfiltered) and Seed Population to 0.001 ("PARAMS=1/.001" on
the command line). This results in a map overlaid with nice curves. Each
Seed Population value results in a different set of curves.

Function (default "ident") is the function applied to the old population
before the new population is determined. The "ident" function calculates
the same bifurcation fractal that was generated before these formulae
were generalized.

Beta is used in the bifmay bifurcations and is the power to which the

denominator is raised.

Note that fractint normally uses periodicity checking to speed up
bifurcation computation. However, in some cases a better quality image
will be obtained if you turn off periodicity checking with "periodicity=no";
for instance, if you use a high number of iterations and a smooth
colormap.

Many formulae can be used to produce bifurcations. Mitchel Feigenbaum
studied lots of bifurcations in the mid-70's, using a HP-65 calculator
(IBM PCs, Fractals, and Fractint, were all Sci-Fi then !). He studied
where bifurcations occurred, for the formula r*p*(1-p), the one described
above. He found that the ratios of lengths of adjacent areas of
bifurcation were four and a bit. These ratios vary, but, as the growth
rate increases, they tend to a limit of 4.669+. This helped him guess
where bifurcation points would be, and saved lots of time.

When he studied bifurcations of r*sin(PI*p) he found a similar pattern,
which is not surprising in itself. However, 4.669+ popped out, again.
Different formulae, same number ? Now, THAT's surprising ! He tried many
other formulae and ALWAYS got 4.669+ - Hot Damn !!! So hot, in fact, that
he phoned home and told his Mom it would make him Famous ! He also went on
to tell other scientists. The rest is History...

(It has been conjectured that if Feigenbaum had a copy of Fractint, and
used it to study bifurcations, he may never have found his Number, as it
only became obvious from long perusal of hand-written lists of values,
without the distraction of wild color-cycling effects !).

We now know that this number is as universal as PI or E. It appears in
situations ranging from fluid-flow turbulence, electronic oscillators,
chemical reactions, and even the Mandelbrot Set - yup, fraid so:
"budding" of the Mandelbrot Set along the negative real axis occurs at
intervals determined by Feigenbaum's Number, 4.669201660910.....

Fractint does not make direct use of the Feigenbaum Number (YET !).
However, it does now reflect the fact that there is a whole sub-species of
Bifurcation-type fractals. Those implemented to date, and the related
formulae, (writing P for pop[n+1] and p for pop[n]) are :

```
  bifurcation  P =  p + r*fn(p)*(1-fn(p))  Verhulst Bifurcations.
  biflambda    P =      r*fn(p)*(1-fn(p))  Real equivalent of Lambda Sets.
  bif+sinpi    P =  p + r*fn(PI*p)         Population scenario based on...
  bif=sinpi    P =      r*fn(PI*p)         ...Feigenbaum's second formula.
  bifstewart   P =      r*fn(p)*fn(p) - 1  Stewart Map.
  bifmay       P =      r*p / ((1+p)^b)    May Map.
```

It took a while for bifurcations to appear here, despite them being over a
century old, and intimately related to chaotic systems. However, they are
now truly alive and well in Fractint!

1.33   Orbit Fractals

Orbit Fractals are generated by plotting an orbit path in two or three
dimensional space.

See Lorenz Attractors, Rossler Attractors,Henon Attractors, Pickover Attractors,
Gingerbreadman, and Martin Attractors.

The orbit trajectory for these types can be saved in the file ORBITS.RAW
by invoking
Fractint with the "orbitsave=yes" command-line option. This file will
be overwritten each time you generate a new fractal, so rename it if you
want to save it. A nifty program called Acrospin can read these files and
rapidly rotate them in 3-D - see Acrospin.

1.34 Lorenz Attractors

(type=lorenz/lorenz3d)

The "Lorenz Attractor" is a "simple" set of three deterministic equations
developed by Edward Lorenz while studying the non- repeatability of
weather patterns. The weather forecaster's basic problem is that even
very tiny changes in initial patterns ("the beating of a butterfly's
wings" - the official term is "sensitive dependence on initial
conditions") eventually reduces the best weather forecast to rubble.

The lorenz attractor is the plot of the orbit of a dynamic system
consisting of three first order non-linear differential equations. The
solution to the differential equation is vector-valued function of one
variable. If you think of the variable as time, the solution traces an
orbit. The orbit is made up of two spirals at an angle to each other in
three dimensions. We change the orbit color as time goes on to add a
little dazzle to the image. The equations are:

    dx/dt = -a*x + a*y
    dy/dt = b*x - y   -z*x
    dz/dt = -c*z + x*y

We solve these differential equations approximately using a method known
as the first order taylor series. Calculus teachers everywhere will kill
us for saying this, but you treat the notation for the derivative dx/dt as
though it really is a fraction, with "dx" the small change in x that
happens when the time changes "dt". So multiply through the above
equations by dt, and you will have the change in the orbit for a small
time step. We add these changes to the old vector to get the new vector
after one step. This gives us:

        xnew = x + (-a*x*dt) + (a*y*dt)
        ynew = y + (b*x*dt) - (y*dt) - (z*x*dt)
        znew = z + (-c*z*dt) + (x*y*dt)

        (default values: dt = .02, a = 5, b = 15, c = 1)

We connect the successive points with a line, project the resulting 3D
orbit onto the screen, and voila! The Lorenz Attractor!

We have added two versions of the Lorenz Attractor. "Type=lorenz" is the
Lorenz attractor as seen in everyday 2D. "Type=lorenz3d" is the same set
of equations with the added twist that the results are run through our

perspective 3D routines, so that you get to view it from different angles
(you can modify your perspective "on the fly" by using the <I> command.)
If you set the "stereo" option to "2", and have red/blue funny glasses on,
you will see the attractor orbit with depth perception.

Hint: the default perspective values (x = 60, y = 30, z = 0) aren't the
best ones to use for fun Lorenz Attractor viewing. Experiment a bit –
start with rotation values of 0/0/0 and then change to 20/0/0 and 40/0/0
to see the attractor from different angles.– and while you're at it, use a
non-zero perspective point Try 100 and see what happens when you get
inside the Lorenz orbits. Here comes one – Duck! While you are at it,
turn on the sound with the "X". This way you'll at least hear it coming!

Different Lorenz attractors can be created using different parameters.
Four parameters are used. The first is the time-step (dt). The default
value is .02. A smaller value makes the plotting go slower; a larger value
is faster but rougher. A line is drawn to connect successive orbit values.
The 2nd, third, and fourth parameters are coefficients used in the
differential equation (a, b, and c). The default values are 5, 15, and 1.
Try changing these a little at a time to see the result.

1.35   Rossler Attractors

(type=rossler3D)

This fractal is named after the German Otto Rossler, a non-practicing
medical doctor who approached chaos with a bemusedly philosophical
attitude. He would see strange attractors as philosophical objects. His
fractal namesake looks like a band of ribbon with a fold in it. All we can
say is we used the same calculus-teacher-defeating trick of multiplying
the equations by "dt" to solve the differential equation and generate the
orbit. This time we will skip straight to the orbit generator – if you
followed what we did above with type Lorenz you can easily
reverse engineer the differential equations.

```
        xnew = x – y*dt –   z*dt
        ynew = y + x*dt + a*y*dt
        znew = z + b*dt + x*z*dt – c*z*dt
```

Default parameters are dt = .04, a = .2, b = .2, c = 5.7

1.36   Henon Attractors

(type=henon)

Michel Henon was an astronomer at Nice observatory in southern France. He
came to the subject of fractals via investigations of the orbits of
astronomical objects. The strange attractor most often linked with
Henon's name comes not from a differential equation, but from the world of
discrete mathematics – difference equations. The Henon map is an example
of a very simple dynamic system that exhibits strange behavior. The orbit
traces out a characteristic banana shape, but on close inspection, the

shape is made up of thicker and thinner parts. Upon magnification, the
thicker bands resolve to still other thick and thin components. And so it
goes forever! The equations that generate this strange pattern perform the
mathematical equivalent of repeated stretching and folding, over and over
again.

```
        xnew =  1 + y - a*x*x
        ynew =  b*x
```

The default parameters are a=1.4 and b=.3.

1.37   Pickover Attractors

(type=pickover)

Clifford A. Pickover of the IBM Thomas J. Watson Research center is such a
creative source for fractals that we attach his name to this one only with
great trepidation. Probably tomorrow he'll come up with another one and
we'll be back to square one trying to figure out a name!

This one is the three dimensional orbit defined by:

```
        xnew = sin(a*y) - z*cos(b*x)
        ynew = z*sin(c*x) - cos(d*y)
        znew = sin(x)
```

Default parameters are: a = 2.24, b = .43, c = -.65, d = -2.43

1.38   Gingerbreadman

(type=gingerbreadman)

This simple fractal is a charming example stolen from "Science of Fractal
Images", p. 149.

```
        xnew = 1 - y + |x|
        ynew = x
```

The initial x and y values are set by parameters, defaults x=-.1, y = 0.

1.39   Martin Attractors

(type=hopalong/martin)

These fractal types are from A. K. Dewdney's "Computer Recreations" column
in "Scientific American". They are attributed to Barry Martin of Aston
University in Birmingham, England.

Hopalong is an "orbit" type fractal like lorenz. The image is obtained by
iterating this formula after setting z(0) = y(0) = 0:

```
      x(n+1) = y(n) - sign(x(n))*sqrt(abs(b*x(n)-c))
      y(n+1) = a - x(n)
```
Parameters are a, b, and c. The function "sign()" returns 1 if the argument
is positive, -1 if argument is negative.

This fractal continues to develop in surprising ways after many iterations.

Another Martin fractal is simpler. The iterated formula is:
```
      x(n+1) = y(n) - sin(x(n))
      y(n+1) = a - x(n)
```
The parameter is "a". Try values near the number pi.

Michael Peters has based the HOP program on variations of these Martin types.
You will find three of these here: chip, quadruptwo, and threeply.

1.40   Icon

(type=icon/icon3d)

  This fractal type was inspired by the book "Symmetry in Chaos"
  by Michael Field and Martin Golubitsky (ISBN 0-19-853689-5, Oxford Press)

  To quote from the book's jacket,

    "Field and Golubitsky describe how a chaotic process eventually can
    lead to symmetric patterns (in a river, for instance, photographs of
    the turbulent movement of eddies, taken over time, often reveal
    patterns on the average."

  The Icon type implemented here maps the classic population logistic
  map of bifurcation fractals onto the complex plane in Dn symmetry.

  The initial points plotted are the more chaotic initial orbits, but
  as you wait, delicate webs will begin to form as the orbits settle
  into a more periodic pattern. Since pixels are colored by the number
  of times they are hit, the more periodic paths will become clarified
  with time. These fractals run continuously.

There are 6 parameters: Lambda, Alpha, Beta, Gamma, Omega, and Degree
    Omega 0 = Dn, or dihedral (rotation + reflectional) symmetry
          !0 = Zn, or cyclic (rotational) symmetry
    Degree = n, or Degree of symmetry

1.41   Quaternion

(type=quat,quatjul)

These fractals are based on quaternions. Quaternions are an extension of
complex numbers, with 4 parts instead of 2. That is, a quaternion Q
equals a+ib+jc+kd, where a,b,c,d are reals. Quaternions have rules for
addition and multiplication. The normal Mandelbrot and Julia formulas
can be generalized to use quaternions instead of complex numbers.

There is one complication. Complex numbers have 2 parts, so they can
be displayed on a plane. Quaternions have 4 parts, so they require 4
dimensions to view. That is, the quaternion Mandelbrot set is actually a
4-dimensional object. Each quaternion C generates a 4-dimensional Julia set.

One method of displaying the 4-dimensional object is to take a 3-dimensional
slice and render the resulting object in 3-dimensional perspective.
Fractint isn't that sophisticated, so it merely displays a 2-dimensional
slice of the resulting object. (Note: Now Fractint is that sophisticated!
See the Julibrot type!)

In fractint, for the Julia set, you can specify the four parameters
of the quaternion constant: c=(c1,ci,cj,ck), but the 2-dimensional slice
of the z-plane Julia set is fixed to (xpixel,ypixel,0,0).

For the Mandelbrot set, you can specify the position of the c-plane slice:
(xpixel,ypixel,cj,ck).

These fractals are discussed in Chapter 10 of Pickover's "Computers,
Pattern, Chaos, and Beauty".

See also HyperComplex and Quaternion and Hypercomplex Algebra

;
;

1.42 HyperComplex

(type=hypercomplex,hypercomplexj)

These fractals are based on hypercomplex numbers, which like quaternions
are a four dimensional generalization of complex numbers. It is not
possible to fully generalize the complex numbers to four dimensions without
sacrificing some of the algebraic properties shared by real and complex
numbers. Quaternions violate the commutative law of multiplication, which
says z1*z2 = z2*z1. Hypercomplex numbers fail the rule that says all non-zero
elements have multiplicative inverses - that is, if z is not 0, there
should be a number 1/z such that (1/z)*(z) = 1. This law holds most of the
time but not all the time for hypercomplex numbers.

However hypercomplex numbers have a wonderful property for fractal purposes.
Every function defined for complex numbers has a simple generalization
to hypercomplex numbers. Fractint's implementation takes advantage of this
by using "fn" variables - the iteration formula is
    h(n+1) = fn(h(n)) + C.
where "fn" is the hypercomplex generalization of sin, cos, log, sqr etc.
You can see 3D versions of these fractals using fractal type Julibrot.
Hypercomplex numbers were brought to our attention by Clyde Davenport,
author of "A Hypercomplex Calculus with Applications to Relativity",
ISBN 0-9623837-0-8.

See also Quaternion and Quaternion and Hypercomplex Algebra

1.43   Cellular Automata

(type=cellular)

These fractals are generated by 1-dimensional cellular automata. Consider
a 1-dimensional line of cells, where each cell can have the value 0 or 1.
In each time step, the new value of a cell is computed from the old value
of the cell and the values of its neighbors. On the screen, each horizontal
row shows the value of the cells at any one time. The time axis proceeds
down the screen, with each row computed from the row above.

Different classes of cellular automata can be described by how many different
states a cell can have (k), and how many neighbors on each side are examined
(r). Fractint implements the binary nearest neighbor cellular automata
(k=2,r=1), the binary next-nearest neighbor cellular automata (k=2,r=2),
and the ternary nearest neighbor cellular automata (k=3,r=1) and several
others.

The rules used here determine the next state of a given cell by using the
sum of the states in the cell's neighborhood. The sum of the cells in the
neighborhood are mapped by rule to the new value of the cell. For the
binary nearest neighbor cellular automata, only the closest neighbor on
each side is used. This results in a 4 digit rule controlling the
generation of each new line: if each of the cells in the neighborhood is
1, the maximum sum is 1+1+1 = 3 and the sum can range from 0 to 3, or 4
values. This results in a 4 digit rule. For instance, in the rule 1010,
starting from the right we have 0->0, 1->1, 2->0, 3->1. If the cell's
neighborhood sums to 2, the new cell value would be 0.

For the next-nearest cellular automata (kr = 22), each pixel is determined
from the pixel value and the two neighbors on each side. This results in
a 6 digit rule.

For the ternary nearest neighbor cellular automata (kr = 31), each cell
can have the value 0, 1, or 2. A single neighbor on each side is examined,
resulting in a 7 digit rule.

kr	#'s in rule	example rule	kr	#'s in rule	example rule
21	4	1010	42	16	2300331230331001
31	7	1211001	23	8	10011001
41	10	3311100320	33	15	021110101210010
51	13	2114220444030	24	10	0101001110
61	16	3452355321541340	25	12	110101011001
22	6	011010	26	14	00001100000110
32	11	21212002010	27	16	0010000000000110

The starting row of cells can be set to a pattern of up to 16 digits or to a
random pattern. The borders are set to zeros if a pattern is entered or are
set randomly if the starting row is set randomly.

A zero rule will randomly generate the rule to use.

Hitting the space bar toggles between continuously generating the cellular
automata and stopping at the end of the current screen.

Recommended reading:

"Computer Software in Science and Mathematics", Stephen Wolfram, Scientific
American, September, 1984.
"Abstract Mathematical Art", Kenneth E. Perry, BYTE, December, 1986.
"The Armchair Universe", A. K. Dewdney, W. H. Freeman and Company, 1988.
"Complex Patterns Generated by Next Nearest Neighbors Cellular Automata",
Wentian Li, Computers & Graphics, Volume 13, Number 4.

1.44   Ant Automaton

(type=ant)

This fractal type is the generalized Ant Automaton described in the "Computer
Recreations" column of the July 1994 Scientific American. The article
attributes this automaton to Greg Turk of Stanford University, Leonid A.
Bunivomitch of the Georgia Institute of Technology, and S. E. Troubetzkoy of
the University of Bielefeld.

The ant wanders around the screen, starting at the middle. A rule string,
which the user can input as Fractint's first parameter, determines the ant's
direction. This rule string is stored as a double precision number in our
implementation. Only the digit 1 is significant -- all other digits are
treated as 0. When the ant leaves a cell (a pixel on the screen) of color k,
it turns right if the kth symbol in the rule string is a 1, or left otherwise.
Then the color in the abandoned cell is incremented. Fractint's 2nd parameter
is a maximum iteration to guarantee that the fractal will terminate. Our
implementation also terminates when the edge is reached. You can slow down
the ant to see her better using the <x> screen Orbit Delay – try 10.

Try rule string 10. In this case, the ant moves in a seemingly random pattern,
then suddenly marches off in a straight line. This happens for many other
rule strings. The default 1100 produces symmetrical images.

If the screen initially contains an image, the path of the ant changes. To
try this, generate a fractal, and press <Ctrl-a>. Note that images
seeded with an image are not (yet) reproducible in PAR files. When started
using the <Ctrl-a> keys, after the ant is finished the default fractal
type reverts to that of the underlying fractal.

~Label=ANTCOMMANDS
Special keystrokes are in effect during the ant's march. The <space> key
toggles a step-by-step mode. When in this mode, press <enter> to see each step
of the ant's progress. When orbit delay (on <x> screen) is set to 1, the step
mode is the default.

If you press the right or left arrow diuring the ant's journey, you can
adjust the orbit delay factor with the arrow keys (increment by 10) or
ctrl-arrow keys (increment by 100). Press any other key to get out of the
orbit delay adjustment mode. Higher values cause slower motion. Changed values
are not saved after the ant is finished, but you can set the orbit delay
value in advance from the <x> screen.

1.45   Test

```
(type=test)
```

This is a stub that we (and you!) use for trying out new fractal types.
"Type=test" fractals make use of Fractint's structure and features for
whatever code is in the routine 'testpt()' (located in the small source
file TESTPT.C) to determine the color of a particular pixel.

If you have a favorite fractal type that you believe would fit nicely into
Fractint, just rewrite the C function in TESTPT.C (or use the prototype
function there, which is a simple M-set implementation) with an algorithm
that computes a color based on a point in the complex plane.

After you get it working, send your code to one of the authors and we
might just add it to the next release of Fractint, with full credit to
you. Our criteria are: 1) an interesting image and 2) a formula
significantly different from types already supported. (Bribery may also
work. THIS author is completely honest, but I don't trust those other
guys.) Be sure to include an explanation of your algorithm and the
parameters supported, preferably formatted as you see here to simplify
folding it into the documentation.

1.46 Formula

```
(type=formula)
```

This is a "roll-your-own" fractal interpreter - you don't even need a
compiler!

To run a "type=formula" fractal, you first need a text file containing
formulas (there's a sample file - FRACTINT.FRM - included with this
distribution). When you select the "formula" fractal type, Fractint scans
the current formula file (default is FRACTINT.FRM) for formulas, then
prompts you for the formula name you wish to run. After prompting for any
parameters, the formula is parsed for syntax errors and then the fractal
is generated. If you want to use a different formula file, press <F6> when
you are prompted to select a formula name.

There are two command-line options that work with type=formula
("formulafile=" and "formulaname="), useful when you are using this
fractal type in batch mode.

The following documentation is supplied by Mark Peterson, who wrote the
formula interpreter:

Formula fractals allow you to create your own fractal formulas. The
general format is:

```
   Mandelbrot(XAXIS) { z = Pixel:  z = sqr(z) + pixel, |z| <= 4 }
      |    |    |          |        |
    Name    Symmetry    Initial   Iteration   Bailout
        Condition        Criteria
```

Initial conditions are set, then the iterations performed while the
bailout criteria remains true or until 'z' turns into a periodic loop.

All variables are created automatically by their usage and treated as
complex. If you declare 'v = 2' then the variable 'v' is treated as a
complex with an imaginary value of zero.

```
    Predefined Variables (x, y)
    ----------------------------------------------
    z           used for periodicity checking
    p1          parameters 1 and 2
    p2          parameters 3 and 4
    p3          parameters 5 and 6
    pixel       screen coordinates
         LastSqr    Modulus from the last sqr() function
         rand       Complex random number

         Precedence
         --------------------------------------------
         1          sin(), cos(), sinh(), cosh(), cosxx(), tan(), cotan(),
                    tanh(), cotanh(), sqr(), log(), exp(), abs(), conj(),
                    real(), imag(), flip(), fn1(), fn2(), fn3(), fn4(),
                    srand(), asin(), asinh(), acos(), acosh(), atan(),
                    atanh(), sqrt(), cabs()
         2          - (negation), ^ (power)
         3          * (multiplication), / (division)
         4          + (addition), - (subtraction)
         5          = (assignment)
         6          < (less than), <= (less than or equal to)
                    > (greater than), >= (greater than or equal to)
                    == (equal to), != (not equal to)
         7          && (logical AND), || (logical OR)
```

Precedence may be overridden by use of parenthesis. Note the modulus squared
operator |z| is also parenthetic and always sets the imaginary component to
zero. This means 'c * |z - 4|' first subtracts 4 from z, calculates the
modulus squared then multiplies times 'c'. Nested modulus squared operators
require overriding parenthesis: c * |z + (|pixel|)|

The functions fn1(...) to fn4(...) are variable functions - when used,
the user is prompted at run time (on the <Z> screen) to specify one of
sin, cos, sinh, cosh, exp, log, sqr, etc. for each required variable function.

Most of the functions have their conventional meaning, here are a few notes on
others that are not conventional. The function cosxx() duplicates a bug in the
version 16 cos() function. Then abs(x+iy) = abs(x)+i*abs(y), flip(x+iy) =
y+i*x, and |x+iy| = x*x+y*y.

The formulas are performed using either integer or floating point
mathematics depending on the <F> floating point toggle. If you do not
have an FPU then type MPC math is performed in lieu of traditional
floating point.

The 'rand' predefined variable is changed with each iteration to a new
random number with the real and imaginary components containing a value
between zero and 1. Use the srand() function to initialize the random
numbers to a consistent random number sequence. If a formula does not
contain the srand() function, then the formula compiler will use the system
time to initialize the sequence. This could cause a different fractal to be
generated each time the formula is used depending on how the formula is

written.

Remember that when using integer math there is a limited dynamic range, so
what you think may be a fractal could really be just a limitation of the
integer math range. God may work with integers, but His dynamic range is
many orders of magnitude greater than our puny 32 bit mathematics! Always
verify with the floating point <F> toggle.

The possible values for symmetry are:

XAXIS, XAXIS_NOPARM
YAXIS, YAXIS_NOPARM
XYAXIS, XYAXIS_NOPARM
ORIGIN, ORIGIN_NOPARM
PI_SYM, PI_SYM_NOPARM
XAXIS_NOREAL
XAXIS_NOIMAG

These will force the symmetry even if no symmetry is actually present, so try
your formulas without symmetry before you use these.

For mathematical formulas of functions used in the parser language, see
Trig Identities

1.47   Frothy Basins

(type=frothybasin)

Frothy Basins, or Riddled Basins, were discovered by James C. Alexander of
the University of Maryland. The discussion below is derived from a two page
article entitled "Basins of Froth" in Science News, November 14, 1992 and
from correspondence with others, including Dr. Alexander.

The equations that generate this fractal are not very different from those
that generate many other orbit fractals.

```
    Z(0) = pixel;
    Z(n+1) = Z(n)^2 - C*conj(Z(n))
    where C = 1 + A*i
```

One of the things that makes this fractal so interesting is the shape of
the dynamical system's attractors. It is not at all uncommon for a
dynamical system to have non-point attractors. Shapes such as circles are
very common. Strange attractors are attractors which are themselves
fractal. What is unusual about this system, however, is that the
attractors intersect. This is the first case in which such a phenomenon
has been observed. The attractors for this system are made up of line
segments which overlap to form an equilateral triangle. This attractor
triangle can be seen by using the "show orbits" option (the 'o' key) or
the "orbits window" option (the ctrl-'o' key).

The number of attractors present is dependant on the value of A, the
imaginary part of C. For values where A <= 1.028713768218725..., there
are three attractors. When A is larger than this critical value, two of
attractors merge into one, leaving only two attractors. An interesting

variation on this fractal can be generated by applying the above mapping
twice per each iteration. The result is that some of the attractors are
split into two parts, giving the system either six or three attractors,
depending on whether A is less than or greater than the critical value.

These are also called "Riddled Basins" because each basin is riddled with
holes. Which attractor a point is eventually pulled into is extremely
sensitive to its initial position. A very slight change in any direction
may cause it to end up on a different attractor. As a result, the basins
are thoroughly intermingled. The effect appears to be a frothy mixture that
has been subjected to lots of stirring and folding.

Pixel color is determined by which attractor captures the orbit. The shade
of color is determined by the number of iterations required to capture the
orbit. In Fractint, the actual shade of color used depends on how many
colors are available in the video mode being used. If 256 colors are
available, the default coloring scheme is determined by the number of
iterations that were required to capture the orbit. An alternative
coloring scheme can be used where the shade is determined by the
iterations required divided by the maximum iterations. This method is
especially useful on deeply zoomed images. If only 16 colors are
available, then only the alternative coloring scheme is used. If fewer
than 16 colors are available, then Fractint just colors the basins without
any shading.

1.48   Julibrots

(type=julibrot)

The Julibrot fractal type uses a general-purpose renderer for visualizing
three dimensional solid fractals. Originally Mark Peterson developed
this rendering mechanism to view a 3-D sections of a 4-D structure he
called a "Julibrot". This structure, also called "layered Julia set" in
the fractal literature, hinges on the relationship between the Mandelbrot
and Julia sets. Each Julia set is created using a fixed value c in the
iterated formula $z^2 + c$. The Julibrot is created by layering Julia sets
in the x-y plane and continuously varying c, creating new Julia sets as z is
incremented. The solid shape thus created is rendered by shading the surface
using a brightness inversely proportional to the virtual viewer's eye.

Starting with Fractint version 18, the Julibrot engine can be used
with other Julia formulas besides the classic $z^2 + c$. The first field on
the Julibrot parameter screen lets you select which orbit formula to use.

You can also use the Julibrot renderer to visualize 3D cross sections of
true four dimensional Quaternion and Hypercomplex fractals.

The Julibrot Parameter Screens

Orbit Algorithm - select the orbit algorithm to use. The available
   possibilities include 2-D Julia and both mandelbrot and Julia variants
   of the 4-D Quaternion and Hypercomplex fractals.

Orbit parameters - the next screen lets you fill in any parameters
   belonging to the orbit algorithm. This list of parameters is not

necessarily the same as the list normally presented for the orbit
algorithm, because some of these parameters are used in the Julibrot
layering process.

From/To Parameters
These parameters allow you to specify the "Mandelbrot" values used to
generate the layered Julias. The parameter c in the Julia formulas will
be incremented in steps ranging from the "from" x and y values to the
"to" x and y values. If the orbit formula is one of the "true" four
dimensional fractal types quat, quatj, hypercomplex, or hypercomplexj,
then these numbers are used with the 3rd and 4th dimensional values.

The "from/to" variables are different for the different kinds of orbit
algorithm.

    2D Julia sets – complex number formula z' = f(z) + c
        The "from/to" parameters change the values of c.
    4D Julia sets – Quaternion or Hypercomplex formula z' = f(z) + c
        The four dimensions of c are set by the orbit parameters.
        The first two dimensions of z are determined by the corners values.
        The third and fourth dimensions of z are the "to/from" variables.
    4D Mandelbrot sets – Quaternion or Hypercomplex formula z' = f(z) + c
        The first two dimensions of c are determined by the corners values.
        The third and fourth dimensions of c are the "to/from" variables.

Distance between the eyes – set this to 2.5 if you want a red/blue
   anaglyph image, 0 for a normal greyscale image.

Number of z pixels – this sets how many layers are rendered in the screen
   z-axis. Use a higher value with higher resolution video modes.

The remainder of the parameters are needed to construct the red/blue
picture so that the fractal appears with the desired depth and proper 'z'
location. With the origin set to 8 inches beyond the screen plane and the
depth of the fractal at 8 inches the default fractal will appear to start
at 4 inches beyond the screen and extend to 12 inches if your eyeballs are
2.5 inches apart and located at a distance of 24 inches from the screen.
The screen dimensions provide the reference frame.

1.49 Diffusion Limited Aggregation

(type=diffusion)

This type begins with a single point in the center of the screen.
Subsequent points move around randomly until coming into contact with the
first point, at which time their locations are fixed and they are colored
randomly. This process repeats until the fractals reaches the edge of the
screen. Use the show orbits function to see the points' random motion.

One unfortunate problem is that on a large screen, this process will tend
to take eons. To speed things up, the points are restricted to a box
around the initial point. The first and only parameter to diffusion
contains the size of the border between the fractal and the edge of the
box. If you make this number small, the fractal will look more solid and
will be generated more quickly.

Diffusion was inspired by a Scientific American article a couple of years
back which includes actual pictures of real physical phenomena that behave
like this.

Thanks to Adrian Mariano for providing the diffusion code and
documentation. Juan J. Buhler added the additional options.

1.50   Lyapunov Fractals

(type=lyapunov)

The Bifurcation fractal illustrates what happens in a simple population
model as the growth rate increases. The Lyapunov fractal expands that model
into two dimensions by letting the growth rate vary in a periodic fashion
between two values. Each pair of growth rates is run through a logistic
population model and a value called the Lyapunov Exponent is calculated for
each pair and is plotted. The Lyapunov Exponent is calculated by adding up
log | r – 2*r*x| over many cycles of the population model and dividing by the
number of cycles. Negative Lyapunov exponents indicate a stable, periodic
behavior and are plotted in color. Positive Lyapunov exponents indicate
chaos (or a diverging model) and are colored black.

Order parameter.
Each possible periodic sequence yields a two dimensional space to explore.
The Order parameter selects a sequence. The default value 0 represents the
sequence ab which alternates between the two values of the growth parameter.
On the screen, the a values run vertically and the b values run
horizontally. Here is how to calculate the space parameter for any desired
sequence. Take your sequence of a's and b's and arrange it so that it starts
with at least 2 a's and ends with a b. It may be necessary to rotate the
sequence or swap a's and b's. Strike the first a and the last b off the list
and replace each remaining a with a 1 and each remaining b with a zero.
Interpret this as a binary number and convert it into decimal.

An Example.
I like sonnets. A sonnet is a poem with fourteen lines that has the
following rhyming sequence: abba abba abab cc. Ignoring the rhyming couplet
at the end, let's calculate the Order parameter for this pattern.

```
  abbaabbaabab          doesn't start with at least 2 a's
  aabbaababab           rotate it
  1001101010            drop the first and last, replace with 0's and 1's
  512+64+32+8+2 = 618
```

An Order parameter of 618 gives the Lyapunov equivalent of a sonnet. "How do
I make thee? Let me count the ways..."

Population Seed.
When two parts of a Lyapunov overlap, which spike overlaps which is strongly
dependent on the initial value of the population model. Any changes from
using a different starting value between 0 and 1 may be subtle. The values 0
and 1 are interpreted in a special manner. A Seed of 1 will choose a random
number between 0 and 1 at the start of each pixel. A Seed of 0 will suppress
resetting the seed value between pixels unless the population model diverges

in which case a random seed will be used on the next pixel.

Filter Cycles.
Like the Bifurcation model, the Lyapunov allow you to set the number of
cycles that will be run to allow the model to approach equilibrium before
the lyapunov exponent calculation is begun. The default value of 0 uses one
half of the iterations before beginning the calculation of the exponent.

Reference.
A.K. Dewdney, Mathematical Recreations, Scientific American, Sept. 1991

1.51 Magnetic Fractals

(type=magnet1m/.../magnet2j)

These fractals use formulae derived from the study of hierarchical
lattices, in the context of magnetic renormalisation transformations.
This kinda stuff is useful in an area of theoretical physics that deals
with magnetic phase-transitions (predicting at which temperatures a given
substance will be magnetic, or non-magnetic). In an attempt to clarify
the results obtained for Real temperatures (the kind that you and I can
feel), the study moved into the realm of Complex Numbers, aiming to spot
Real phase-transitions by finding the intersections of lines representing
Complex phase-transitions with the Real Axis. The first people to try
this were two physicists called Yang and Lee, who found the situation a
bit more complex than first expected, as the phase boundaries for Complex
temperatures are (surprise!) fractals.

And that's all the technical (?) background you're getting here! For more
details (are you SERIOUS ?!) read "The Beauty of Fractals". When you
understand it all, you might like to rewrite this section, before you
start your new job as a professor of theoretical physics...

In Fractint terms, the important bits of the above are "Fractals",
"Complex Numbers", "Formulae", and "The Beauty of Fractals". Lifting the
Formulae straight out of the Book and iterating them over the Complex
plane (just like the Mandelbrot set) produces Fractals.

The formulae are a bit more complicated than the Z^2+C used for the
Mandelbrot Set, that's all. They are :

```
          [        ] 2
          |  Z^2 + (C-1)  |
  MAGNET1 : | ------------- |
          |  2*Z + (C-2)  |
          [        ]


          [                ] 2
          |  Z^3 + 3*(C-1)*Z + (C-1)*(C-2)         |
  MAGNET2 : | ----------------------------------- |
          |  3*(Z^2) + 3*(C-2)*Z + (C-1)*(C-2) + 1  |
          [                ]
```

These aren't quite as horrific as they look (oh yeah ?!) as they only
involve two variables (Z and C), but cubing things, doing division, and

eventually squaring the result (all in Complex Numbers) don't exactly
spell S-p-e-e-d ! These are NOT the fastest fractals in Fractint !

As you might expect, for both formulae there is a single related
Mandelbrot-type set (magnet1m, magnet2m) and an infinite number of related
Julia-type sets (magnet1j, magnet2j), with the usual toggle between the
corresponding Ms and Js via the spacebar.

If you fancy delving into the Julia-types by hand, you will be prompted
for the Real and Imaginary parts of the parameter denoted by C. The
result is symmetrical about the Real axis (and therefore the initial image
gets drawn in half the usual time) if you specify a value of Zero for the
Imaginary part of C.

Fractint Historical Note: Another complication (besides the formulae) in
implementing these fractal types was that they all have a finite attractor
(1.0 + 0.0i), as well as the usual one (Infinity). This fact spurred the
development of Finite Attractor logic in Fractint. Without this code you
can still generate these fractals, but you usually end up with a pretty
boring image that is mostly deep blue "lake", courtesy of Fractint's
standard Periodicity Logic. See Finite Attractors for more
information on this aspect of Fractint internals.

(Thanks to Kevin Allen for Magnetic type documentation above).

1.52   L-Systems

(type=lsystem)

These fractals are constructed from line segments using rules specified in
drawing commands. Starting with an initial string, the axiom,
transformation rules are applied a specified number of times, to produce
the final command string which is used to draw the image.

Like the type=formula fractals, this type requires a separate data file.
A sample file, FRACTINT.L, is included with this distribution. When you
select type lsystem, the current lsystem file is read and you are asked
for the lsystem name you wish to run. Press <F6> at this point if you wish
to use a different lsystem file. After selecting an lsystem, you are asked
for one parameter - the "order", or number of times to execute all the
transformation rules. It is wise to start with small orders, because the
size of the substituted command string grows exponentially and it is very
easy to exceed your resolution. (Higher orders take longer to generate
too.) The command line options "lname=" and "lfile=" can be used to over-
ride the default file name and lsystem name.

Each L-System entry in the file contains a specification of the angle, the
axiom, and the transformation rules. Each item must appear on its own
line and each line must be less than 160 characters long.

The statement "angle n" sets the angle to 360/n degrees; n must be an
integer greater than two and less than fifty.

"Axiom string" defines the axiom.

Transformation rules are specified as "a=string" and convert the single
character 'a' into "string." If more than one rule is specified for a
single character all of the strings will be added together. This allows
specifying transformations longer than the 160 character limit.
Transformation rules may operate on any characters except space, tab or
'}'.

Any information after a ; (semi-colon) on a line is treated as a comment.

Here is a sample lsystem:

```
Dragon {     ; Name of lsystem, { indicates start
  Angle 8  ; Specify the angle increment to 45 degrees
  Axiom FX   ; Starting character string
  F=       ; First rule:  Delete 'F'
  y=+FX--FY+   ; Change 'y' into  "+fx--fy+"
  x=-FX++FY-   ; Similar transformation on 'x'
}    ; final } indicates end
```

The standard drawing commands are:
    F Draw forward
    G Move forward (without drawing)
    + Increase angle
    - Decrease angle
    | Try to turn 180 degrees. (If angle is odd, the turn
      will be the largest possible turn less than 180 degrees.)

These commands increment angle by the user specified angle value. They
should be used when possible because they are fast. If greater flexibility
is needed, use the following commands which keep a completely separate
angle pointer which is specified in degrees.

    D Draw forward
    M Move forward
    nn Increase angle nn degrees
    /nn Decrease angle nn degrees

Color control:
    Cnn Select color nn
    <nn Increment color by nn
    >nn decrement color by nn

Advanced commands:
    !   Reverse directions (Switch meanings of +, - and , /)
    @nnn Multiply line segment size by nnn
    nnn may be a plain number, or may be preceded by
        I for inverse, or Q for square root.
        (e.g. @IQ2 divides size by the square root of 2)
    [Push. Stores current angle and position on a stack
    ]   Pop. Return to location of last push

Other characters are perfectly legal in command strings. They are ignored
for drawing purposes, but can be used to achieve complex translations.

The characters '+', '-', '<', '>', '[', ']', '|', '!', '@', '/', '',
and 'c' are reserved symbols and cannot be redefined. For example, c=f+f
and <= , are syntax errors.

The integer code produces incorrect results in five known instances, Peano2
with order >= 7, SnowFlake1 with order >=6, and SnowFlake2, SnowFlake3, and
SnowflakeColor with order >= 5. If you see strange results, switch to the
floating point code.