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In[32]:= (*
FILE:   FalseEyeSeparation.nb
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HISTORY: Created 5/4/99

This Mathematica file is supplementary material for:

    Zachary Wartell,
    Larry F. Hodges and William Ribarsky. "Balancing Fusion, Image Depth, and
    Distortion in Stereoscopic Head-Tracked Displays."
    (To Appear) SIGGRAPH 99 Conference
    Proceedings,
    Annual Conference Series.   ACM SIGGRAPH, Addison Wesley, August 1999.

Various reviewers requested this Mathematica derivation,
to complement the manual derivation found in:

    Zachary Wartell,
    Larry Hodges, William Ribarsky.   "The Analytic Distortion Induced
    by False-Eye Separation in Head-Tracked Stereoscopic Displays."
    GVV Tech Report 99-01.   1999.

The Tech Report should be
    included with this file in the SIGGRAPH 99 Proceedings CD-ROM.

This
    file follows the outline of appendix in Tech Report 99-01.   The references are
    listed as "Ax.x" which refer
    to the corresponding section in the appendix of the tech report.
    See the diagram on page 11 of the tech report for the meaning of each variable.
    *)

Clear["*"]

Definitions = {
  (* Definitions from A1.1 *)
  Dx -> Ix + dx,
  Dz -> Iz + dz,
  Ax -> Ix - dx,
  Az -> Iz - dz,
  (* Definitions from A1.2 and A1.3 *)
  Hx -> (Ex - Ix - r*dx) * (Iz + r*dz) / (Iz - Ez + r*dz) + Ix + r*dx,
  Gx -> (Ex - Ix + r*dx) * (Iz - r*dz) / (Iz - Ez - r*dz) + Ix - r*dx
}

(*
**   Solves for Fx (A1.4)
*)

(* Solve for intersection of lines AG and DH in terms of A,D,G,H (Pages 12-13) *)
Solve[{(Gz - Az) ta + Az == (Hz - Dz) td + Dz, (Gx - Ax) ta + Ax == (Hx - Dx) td + Dx}, td, ta]
td = td /. %
td = First[td]
Fx = (Hx - Dx) td + Dx
Fx = Together[Fx]
Fx = Fx /. {Gz -> 0, Hz -> 0}

(* Solve for Denominator of Fx in terms of I,d,r,E (A1.4.1) *)
Den = Denominator[Fx]
Den = Den /. Definitions
Den = (Iz - Ez + r*dz) * (Iz - Ez - r*dz) * Den
Den = Cancel[Den]
Den = Factor[Den]

(* Solve for Numerator of Fx in terms of I,d,r,E (A1.4.2) *)
Num = Numerator[Fx]
Num = Num /. Definitions
Num = Cancel[(Iz - Ez + r*dz) * (Iz - Ez - r*dz) * Num]
Num = Factor[Num]

(* Solve for complete fraction of Fx (A1.4.3) *)
Fx = Num / Den

```

```
Fx = Collect[Numerator[Fx], {Ex, Ez}] / Collect[Denominator[Fx], {Ex, Ez}]
(* This is equivalent to A1.4.3,
```

in a slightly different

just

form. \*)

```
(*
** Solve for Fz (A1.6)
*)
```

```
(* Solve for Fz intersection of AG and DH in terms of A,D,G,H *)
Fz = (Hz - Dz) td + Dz
Fz = Together[Fz]
Fz = Fz /. {Hz -> 0, Gz -> 0}
```

```
(* Solve for Numerator of Fz in terms of I,d,r,E (A1.6.1) *)
Num = Numerator[Fz]
Num = Num /. Definitions
Num = Cancel[(Iz - Ez + r*dz) * (Iz - Ez - r*dz) * Num]
Num = Factor[Num]
```

```
(* Solve for Denominator of Fz in terms of I,
d,r,E (Note, here we repeat A1.4.1 instead of
performing A1.6.2) *)
Den = Denominator[Fz]
Den = Den /. Definitions
Den = Cancel[(Iz - Ez + r*dz) * (Iz - Ez - r*dz) * Den]
Den = Factor[Den]
```

```
(* Solve for complete fraction (A1.6.3) *)
Fz = Cancel[Num / Den]
Fz = Collect[Numerator[Fz], {Ex, Ez}] / Collect[Denominator[Fz], {Ex, Ez}]
(* This is equivalent to A1.6.3,
```

in a slightly different

just

form. \*)

$$\text{Out}[1] = \left\{ \begin{aligned} &Dx \rightarrow dx + Ix, Dz \rightarrow dz + Iz, Ax \rightarrow -dx + Ix, Az \rightarrow -dz + Iz, \\ &Hx \rightarrow Ix + dx r + \frac{(Ex - Ix - dx r)(Iz + dz r)}{-Ez + Iz + dz r}, Gx \rightarrow Ix - dx r + \frac{(Ex - Ix + dx r)(Iz - dz r)}{-Ez + Iz - dz r} \end{aligned} \right\}$$

$$\text{Out}[2] = \left\{ \left\{ td \rightarrow -\frac{-Az Dx + Ax Dz + Az Gx - Dz Gx - Ax Gz + Dx Gz}{Az Dx - Ax Dz + Dz Gx - Dx Gz - Az Hx + Gz Hx + Ax Hz - Gx Hz} \right\} \right\}$$

$$\text{Out}[3] = \left\{ -\frac{-Az Dx + Ax Dz + Az Gx - Dz Gx - Ax Gz + Dx Gz}{Az Dx - Ax Dz + Dz Gx - Dx Gz - Az Hx + Gz Hx + Ax Hz - Gx Hz} \right\}$$

$$\text{Out}[4] = -\frac{-Az Dx + Ax Dz + Az Gx - Dz Gx - Ax Gz + Dx Gz}{Az Dx - Ax Dz + Dz Gx - Dx Gz - Az Hx + Gz Hx + Ax Hz - Gx Hz}$$

$$\text{Out}[5] = Dx - \frac{(-Az Dx + Ax Dz + Az Gx - Dz Gx - Ax Gz + Dx Gz)(-Dx + Hx)}{Az Dx - Ax Dz + Dz Gx - Dx Gz - Az Hx + Gz Hx + Ax Hz - Gx Hz}$$

$$\text{Out}[6] = \frac{Az Dx Gx - Ax Dx Gz - Ax Dz Hx - Az Gx Hx + Dz Gx Hx + Ax Gz Hx + Ax Dx Hz - Dx Gx Hz}{Az Dx - Ax Dz + Dz Gx - Dx Gz - Az Hx + Gz Hx + Ax Hz - Gx Hz}$$

$$\text{Out}[7] = \frac{Az Dx Gx - Ax Dz Hx - Az Gx Hx + Dz Gx Hx}{Az Dx - Ax Dz + Dz Gx - Az Hx}$$

$$\text{Out}[8] = Az Dx - Ax Dz + Dz Gx - Az Hx$$

$$\text{Out}[9] = (\text{dx} + \text{Ix}) (-\text{dz} + \text{Iz}) - (-\text{dx} + \text{Ix}) (\text{dz} + \text{Iz}) + (\text{dz} + \text{Iz}) \left( \text{Ix} - \text{dx} r + \frac{(\text{Ex} - \text{Ix} + \text{dx} r) (\text{Iz} - \text{dz} r)}{-\text{Ez} + \text{Iz} - \text{dz} r} \right) -$$

$$(-\text{dz} + \text{Iz}) \left( \text{Ix} + \text{dx} r + \frac{(\text{Ex} - \text{Ix} - \text{dx} r) (\text{Iz} + \text{dz} r)}{-\text{Ez} + \text{Iz} + \text{dz} r} \right)$$

$$\text{Out}[10] = (-\text{Ez} + \text{Iz} - \text{dz} r) (-\text{Ez} + \text{Iz} + \text{dz} r)$$

$$\left( (\text{dx} + \text{Ix}) (-\text{dz} + \text{Iz}) - (-\text{dx} + \text{Ix}) (\text{dz} + \text{Iz}) + (\text{dz} + \text{Iz}) \left( \text{Ix} - \text{dx} r + \frac{(\text{Ex} - \text{Ix} + \text{dx} r) (\text{Iz} - \text{dz} r)}{-\text{Ez} + \text{Iz} - \text{dz} r} \right) - \right.$$

$$\left. (-\text{dz} + \text{Iz}) \left( \text{Ix} + \text{dx} r + \frac{(\text{Ex} - \text{Ix} - \text{dx} r) (\text{Iz} + \text{dz} r)}{-\text{Ez} + \text{Iz} + \text{dz} r} \right) \right)$$

$$\text{Out}[11] = -2$$

$$(\text{dz Ex Ez Iz} - \text{dx Ez}^2 \text{ Iz} - \text{dz Ez Ix Iz} - \text{dz Ex Iz}^2 + 2 \text{dx Ez Iz}^2 + \text{dz Ix Iz}^2 - \text{dx Iz}^3 - \text{dz Ex Ez Iz} r +$$

$$\text{dx Ez}^2 \text{ Iz} r + \text{dz Ez Ix Iz} r - \text{dx Ez Iz}^2 r + \text{dz}^3 \text{ Ex} r^2 - \text{dx dz}^2 \text{ Ez} r^2 - \text{dz}^3 \text{ Ix} r^2 + \text{dx dz}^2 \text{ Iz} r^2)$$

$$\text{Out}[12] = -2 (\text{dz Ex} - \text{dx Ez} - \text{dz Ix} + \text{dx Iz}) (\text{Ez Iz} - \text{Iz}^2 - \text{Ez Iz} r + \text{dz}^2 r^2)$$

$$\text{Out}[13] = \text{Az Dx Gx} - \text{Ax Dz Hx} - \text{Az Gx Hx} + \text{Dz Gx Hx}$$

$$\text{Out}[14] = (\text{dx} + \text{Ix}) (-\text{dz} + \text{Iz}) \left( \text{Ix} - \text{dx} r + \frac{(\text{Ex} - \text{Ix} + \text{dx} r) (\text{Iz} - \text{dz} r)}{-\text{Ez} + \text{Iz} - \text{dz} r} \right) -$$

$$(-\text{dx} + \text{Ix}) (\text{dz} + \text{Iz}) \left( \text{Ix} + \text{dx} r + \frac{(\text{Ex} - \text{Ix} - \text{dx} r) (\text{Iz} + \text{dz} r)}{-\text{Ez} + \text{Iz} + \text{dz} r} \right) -$$

$$(-\text{dz} + \text{Iz}) \left( \text{Ix} - \text{dx} r + \frac{(\text{Ex} - \text{Ix} + \text{dx} r) (\text{Iz} - \text{dz} r)}{-\text{Ez} + \text{Iz} - \text{dz} r} \right) \left( \text{Ix} + \text{dx} r + \frac{(\text{Ex} - \text{Ix} - \text{dx} r) (\text{Iz} + \text{dz} r)}{-\text{Ez} + \text{Iz} + \text{dz} r} \right) +$$

$$(\text{dz} + \text{Iz}) \left( \text{Ix} - \text{dx} r + \frac{(\text{Ex} - \text{Ix} + \text{dx} r) (\text{Iz} - \text{dz} r)}{-\text{Ez} + \text{Iz} - \text{dz} r} \right) \left( \text{Ix} + \text{dx} r + \frac{(\text{Ex} - \text{Ix} - \text{dx} r) (\text{Iz} + \text{dz} r)}{-\text{Ez} + \text{Iz} + \text{dz} r} \right)$$

$$\text{Out}[15] = -2 (\text{dz Ex Ez Ix Iz} - \text{dx Ez}^2 \text{ Ix Iz} - \text{dz Ez Ix}^2 \text{ Iz} - \text{dz Ex}^2 \text{ Iz}^2 + \text{dx Ex Ez Iz}^2 +$$

$$\text{dz Ex Ix Iz}^2 + \text{dx Ez Ix Iz}^2 - \text{dx Ex Iz}^3 + \text{dx dz}^2 \text{ Ex Ez} r - \text{dx}^2 \text{ dz Ez}^2 r - \text{dx dz}^2 \text{ Ez Ix} r +$$

$$\text{dx}^2 \text{ dz Ez Iz} r - \text{dz Ex Ez Ix Iz} r + \text{dx Ez}^2 \text{ Ix Iz} r + \text{dz Ez Ix}^2 \text{ Iz} r - \text{dx Ez Ix Iz}^2 r +$$

$$\text{dz}^3 \text{ Ex}^2 r^2 - 2 \text{dx dz}^2 \text{ Ex Ez} r^2 + \text{dx}^2 \text{ dz Ez}^2 r^2 - \text{dz}^3 \text{ Ex Ix} r^2 + \text{dx dz}^2 \text{ Ez Ix} r^2 +$$

$$\text{dx dz}^2 \text{ Ex Iz} r^2 - \text{dx}^2 \text{ dz Ez Iz} r^2)$$

$$\text{Out}[16] = -2 (\text{dz Ex} - \text{dx Ez} - \text{dz Ix} + \text{dx Iz})$$

$$(\text{Ez Ix Iz} - \text{Ex Iz}^2 + \text{dx dz Ez} r - \text{Ez Ix Iz} r + \text{dz}^2 \text{ Ex} r^2 - \text{dx dz Ez} r^2)$$

$$\text{Out}[17] = \frac{\text{Ez Ix Iz} - \text{Ex Iz}^2 + \text{dx dz Ez} r - \text{Ez Ix Iz} r + \text{dz}^2 \text{ Ex} r^2 - \text{dx dz Ez} r^2}{\text{Ez Iz} - \text{Iz}^2 - \text{Ez Iz} r + \text{dz}^2 r^2}$$

$$\text{Out}[18] = \frac{\text{Ez} (\text{Ix Iz} + \text{dx dz} r - \text{Ix Iz} r - \text{dx dz} r^2) + \text{Ex} (-\text{Iz}^2 + \text{dz}^2 r^2)}{-\text{Iz}^2 + \text{dz}^2 r^2 + \text{Ez} (\text{Iz} - \text{Iz} r)}$$

$$\text{Out}[19] = \text{Dz} - \frac{(-\text{Az Dx} + \text{Ax Dz} + \text{Az Gx} - \text{Dz Gx} - \text{Ax Gz} + \text{Dx Gz}) (-\text{Dz} + \text{Hz})}{\text{Az Dx} - \text{Ax Dz} + \text{Dz Gx} - \text{Dx Gz} - \text{Az Hx} + \text{Gz Hx} + \text{Ax Hz} - \text{Gx Hz}}$$

$$\text{Out}[20] = \frac{-\text{Az Dz Gx} + \text{Ax Dz Gz} + \text{Az Dz Hx} - \text{Dz Gz Hx} - \text{Az Dx Hz} + \text{Az Gx Hz} - \text{Ax Gz Hz} + \text{Dx Gz Hz}}{-\text{Az Dx} + \text{Ax Dz} - \text{Dz Gx} + \text{Dx Gz} + \text{Az Hx} - \text{Gz Hx} - \text{Ax Hz} + \text{Gx Hz}}$$

$$\text{Out}[21] = \frac{-\text{Az Dz Gx} + \text{Az Dz Hx}}{-\text{Az Dx} + \text{Ax Dz} - \text{Dz Gx} + \text{Az Hx}}$$

$$\text{Out}[22] = -\text{Az Dz Gx} + \text{Az Dz Hx}$$

$$\text{Out}[23] = -(-\text{dz} + \text{Iz}) (\text{dz} + \text{Iz}) \left( \text{Ix} - \text{dx} r + \frac{(\text{Ex} - \text{Ix} + \text{dx} r) (\text{Iz} - \text{dz} r)}{-\text{Ez} + \text{Iz} - \text{dz} r} \right) +$$

$$(-\text{dz} + \text{Iz}) (\text{dz} + \text{Iz}) \left( \text{Ix} + \text{dx} r + \frac{(\text{Ex} - \text{Ix} - \text{dx} r) (\text{Iz} + \text{dz} r)}{-\text{Ez} + \text{Iz} + \text{dz} r} \right)$$

$$\text{Out}[24]= 2 \left( dz^3 \text{Ex Ez r} - dx dz^2 \text{Ez}^2 \text{r} - dz^3 \text{Ez Ix r} + dx dz^2 \text{Ez Iz r} - dz \text{Ex Ez Iz}^2 \text{r} + dx \text{Ez}^2 \text{Iz}^2 \text{r} + \right. \\ \left. dz \text{Ez Ix Iz}^2 \text{r} - dx \text{Ez Iz}^3 \text{r} \right)$$

$$\text{Out}[25]= 2 \text{Ez} (dz - \text{Iz}) (dz + \text{Iz}) (dz \text{Ex} - dx \text{Ez} - dz \text{Ix} + dx \text{Iz}) \text{r}$$

$$\text{Out}[26]= -\text{Az Dx} + \text{Ax Dz} - \text{Dz Gx} + \text{Az Hx}$$

$$\text{Out}[27]= - (dx + \text{Ix}) (-dz + \text{Iz}) + (-dx + \text{Ix}) (dz + \text{Iz}) - (dz + \text{Iz}) \left( \text{Ix} - dx \text{r} + \frac{(\text{Ex} - \text{Ix} + dx \text{r}) (\text{Iz} - dz \text{r})}{-\text{Ez} + \text{Iz} - dz \text{r}} \right) + \\ (-dz + \text{Iz}) \left( \text{Ix} + dx \text{r} + \frac{(\text{Ex} - \text{Ix} - dx \text{r}) (\text{Iz} + dz \text{r})}{-\text{Ez} + \text{Iz} + dz \text{r}} \right)$$

$$\text{Out}[28]= 2 \left( dz \text{Ex Ez Iz} - dx \text{Ez}^2 \text{Iz} - dz \text{Ez Ix Iz} - dz \text{Ex Iz}^2 + 2 dx \text{Ez Iz}^2 + dz \text{Ix Iz}^2 - dx \text{Iz}^3 - dz \text{Ex Ez Iz r} + \right. \\ \left. dx \text{Ez}^2 \text{Iz r} + dz \text{Ez Ix Iz r} - dx \text{Ez Iz}^2 \text{r} + dz^3 \text{Ex r}^2 - dx dz^2 \text{Ez r}^2 - dz^3 \text{Ix r}^2 + dx dz^2 \text{Iz r}^2 \right)$$

$$\text{Out}[29]= 2 (dz \text{Ex} - dx \text{Ez} - dz \text{Ix} + dx \text{Iz}) (\text{Ez Iz} - \text{Iz}^2 - \text{Ez Iz r} + dz^2 \text{r}^2)$$

$$\text{Out}[30]= - \frac{\text{Ez} (dz - \text{Iz}) (dz + \text{Iz}) \text{r}}{-\text{Ez Iz} + \text{Iz}^2 + \text{Ez Iz r} - dz^2 \text{r}^2}$$

$$\text{Out}[31]= \frac{\text{Ez} (-dz + \text{Iz}) (dz + \text{Iz}) \text{r}}{\text{Iz}^2 - dz^2 \text{r}^2 + \text{Ez} (-\text{Iz} + \text{Iz r})}$$