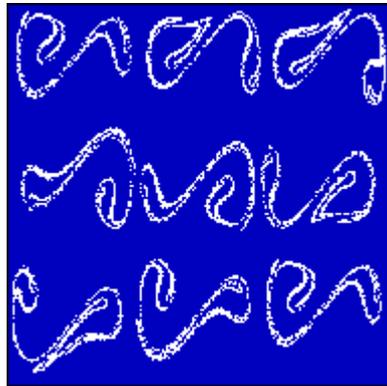


Dynamics Solver



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To the memory of my father

Acknowledgments

This version of *Dynamics Solver* has benefited greatly from critical comments made by friends from Euskal Herriko Unibertsitatea, Université Pierre et Marie Curie, and Universitat de Barcelona. For this help, the author is very grateful to Lluís Bel, Aníbal Hernández, Alfred Molina and Martín Rivas.

Introduction

Program Description

Dynamics Solver is intended to solve initial and boundary-value problems for continuous and discrete dynamical systems:

- single ordinary differential equations of arbitrary order,
- systems of any number of first-order ordinary differential equations,
- a rather large class of functional-differential equations and systems,
- iterated maps and recurrences in arbitrary dimensions,
- any problem that can be written in one of the aforementioned forms.

No programming is necessary: everything can be entered in user-friendly dialog boxes and complex graphics (and numerical) results can be easily and quickly obtained. The program has a powerful built-in compiler that automatically translates a large class of mathematical expressions written in a standard format into an internal code that can be executed very fast.

Apart from the dynamical system solution, one can compute any quantity involving the solution and its derivatives (for one or more values of the independent variables), the problem parameters and the initial conditions. For example, it is possible to draw phase-space portraits (including an optional direction field), Poincaré maps, Liapunov exponents, cobweb diagrams, histograms and bifurcation diagrams. The results can be projected (in perspective or not) along any direction and particular subspaces of the phase space (or the space of initial conditions) may be analyzed easily.

Very different kinds of results may be obtained in different graphics and text formats displayed in one or more windows. They can be sent to any Windows compatible printer or plotter and advanced output formats are available, including Encapsulated Postscript and user-defined formats. The results can be easily collected in a file in order to process them by means of other programs, such as Mathematica.

There are many programs to draw geometrical figures, but most of them are not appropriate to draw the figures that physicists often need when preparing lectures or research papers. One would not want to have to learn to use a very complex CAD program only to draw some figures in a collection of exercises. Simpler programs exist, but they are not able to draw mathematically defined curves (say a semi-cubic parabola) or to place elements at arbitrary points (often they can only be placed at some grid points), or to include data points. *Dynamics Solver* can be used to draw segments, arcs of circles and ellipses, arrows, arbitrary parametric curves in two and three dimensions, a large class of fractals, and points and lines from external data files (which can be generated by *Dynamics Solver* itself or any other program). The main goal is to have completely correct figures (not only artistic approximations) in a device-independent format that can be translated to standard formats (HPGL or Postscript, for instance) that, in turn, can be easily combined with output files from powerful text processors and formatters (we have TeX in mind, of course).

Each problem can be saved to and retrieved from a disk file. This “problem file“ can also be edited and used as a template for related problems. One can extend the program by providing more integration methods or additional mathematical functions.

A complete, context-sensitive, cross-referenced help system is available and each error message and dialog box has a help button that can be used to get the corresponding information. Furthermore, the program is highly configurable to better suit your needs and tastes.

Who May Use *Dynamics Solver*

Dynamics Solver is both a powerful professional tool to be used by physicists, mathematicians and engineers in their everyday work, and also a pedagogical tool to help learning and teaching about differential equations, continuous and discrete nonlinear dynamical systems, deterministic chaos, etc. Topics that were traditionally omitted in introductory courses because of their mathematical complexity

can now be analyzed easily. Even when exact solutions are known, it is often given by means of complex mathematical expressions that are not easily understood without explanatory graphics. Students can gain a more intuitive comprehension of many problems by analyzing them in a numerical laboratory, and *Dynamics Solver* is flexible enough to be an effective numerical laboratory in which many problems from physics and engineering can be easily tested. Note also that many examples discussed in books on computational physics and in papers published in journals like *American Journal of Physics*, *European Journal of Physics*, etc. can be analyzed, without programming, by means of *Dynamics Solver*. For instance, when teaching differential equations you can use *Dynamics Solver* in many different ways. Plenty of useful examples can be found in the excellent textbook of Borelli [1991], which can also be used to teach a full laboratory course on differential equations.

Program Versions

There are two versions of Dynamics Solver:

- a 16-bit edition which will run under Windows 3.1 (or compatible environments, including Windows for Workgroups 3.1+, Windows 95, Windows 98 and Windows NT 3.5 for Intel processors). Though it may run under Windows 3.1, it includes many interface enhancements of Windows 95/98/NT4, such as tabbed dialogs, toolbars, tooltips, status bar, spin controls and popup menus invoked with the right mouse button.
- a 32-bit edition which will run only under Windows 95, Windows 98 and Windows NT 4.0 and takes advantage of the enhanced possibilities of this environment, such as:
 - long file names,
 - use of the registry instead of .INI files,
 - bigger integers for repeated integration,
 - problem files automatically added to the *Documents* entry of the *Start* menu,
 - context help for dialog controls,
 - creation of new problem files from the *New* menu entry that opens when right-clicking in an *Explorer* or *My PC* folder (or even on the *Desktop*),
 - any problem that was open when the system is shut down will be reopened in the next Windows session,
 - more memory,
 - QuickView compatibility,

and more...

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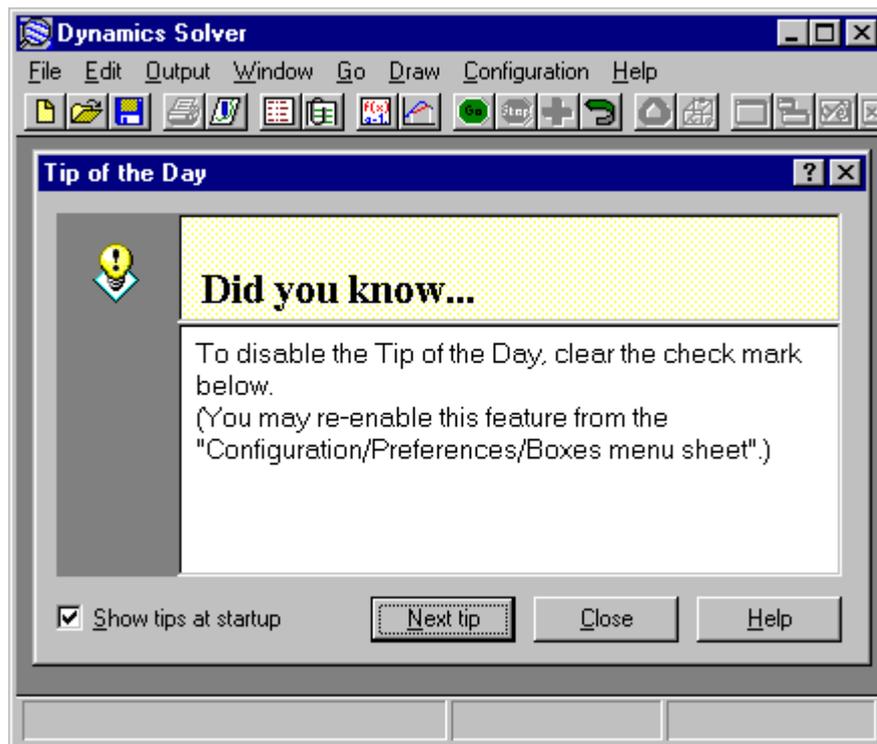
Introduction to *Dynamics Solver*

Tutorial

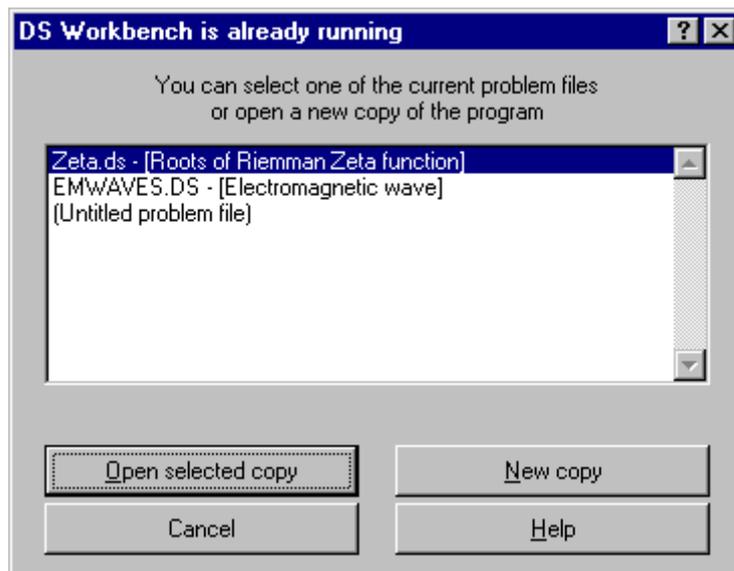
This section shows briefly some of the most basic possibilities of *Dynamics Solver*. The user is referred to the rest of the manual and help file to find both a complete description of each feature and a systematic discussion of the way in which a problem can be set up and input into *Dynamics Solver*. You may want to read this section from the program help system: you will be able to load the examples by clicking with the mouse in their name.

Starting *Dynamics Solver*

To start *Dynamics Solver* select it from the Start/Programs/Dynamics Solver menu (under Windows 3.1, double click with the left mouse button on the *Dynamics Solver* icon in the *Dynamics Solver* group of *Program Manager*). *Dynamics Solver* will start after a short initialization and you will see a window displaying a tip. Use the mouse to click the **C**lose button and the main window that lies behind will be visible.



If there is another instance of *Dynamics Solver* running when the program is started directly with no problem file appended after its file specification, or when the same problem file is being analyzed by another copy of the program, the following dialog will open:



You can select there one of the problem files which are already open or start a new copy of the program. It is also possible to cancel the operation. To start always a new copy of the program you may disable the **Check previous instances** entry of the **Preferences/Other** dialog box which opens when using the **Configuration/Preferences** menu.

Exiting *Dynamics Solver*

To exit *Dynamics Solver*, use the following procedure:

1. If the program is solving a problem, stop the solution by pressing **ESC** (or use the **Go/Stop!** menu or the corresponding  toolbar button¹).
2. Press **Alt+F4** (or use the **File/Quit** menu). If the current problem file has been changed you will get a warning and the opportunity to save the changes (see also **Saving and Loading a Problem**).

Getting Help

You can get help at any time by using the **Help/Index** menu entry or the **F1** key. The **Table of Contents** help topic will be displayed and from there you can go anywhere in the help system. (Consult your Windows manual or the **Help on Help** topic.)

It is also possible to have context sensitive help:

- While browsing through a menu press **F1** and the corresponding help topic will be displayed.
- From any dialog box press the **Help** button to get the corresponding help topic.
- Press **Shift+F1**. The mouse pointer will change and you can put it over a toolbar button or screen piece. Click the left button and the corresponding help topic will be shown. Use **ESC** to abandon the operation.
- In the 32-bit edition, dialog boxes have the familiar **?** button that allows getting the help topic corresponding to (most) dialog controls. It is also possible to right click the mouse to get the *What's*

¹ Every menu entry has a button that can be displayed in a toolbar and added to a popup menu that opens when pressing the right mouse button. The toolbars and buttons which will be displayed and the contents of the popup menu are controlled from the **Toolbar and popup menu** entry of the **Configuration** menu.

that menu from which the help topic is accessed. If there is no help for a control, try getting the help corresponding to its title or caption.

Using Problem Files

One of the most interesting features of *Dynamics Solver* is its ability to save a problem to the disk. You can retrieve it at any time to repeat or extend the analysis you have made so far. These files, called problem files in the manual and help file, can also be used as the starting point for a similar problem or to view and modify the problem as a whole. Over a hundred examples are included on the distribution disks in the form of problem files. Most probably they have been copied to your hard disk by the `Setup` (or `INSTALL`) program.

1. To save the current problem press `F2` or use the **File/Save** menu entry (or the corresponding  toolbar button).
2. To save the current problem under a different name use the **File/Save as** menu entry (or the corresponding  toolbar button).

In the second case, or if the problem has not been saved before, you will be prompted, in one of the standard dialog boxes which are often displayed by Windows programs, for a file name. Unless you give an explicit extension (which could be a single period), *Dynamics Solver* will add the default `.DS` extension. If there is already a file with the same name, a warning will be issued and you will have another opportunity before overwriting it.

A problem file can be loaded from the disk in different ways. If *Dynamics Solver* is already running, you can change the problem under study by using one of the following methods:

1. Press the `F3` key or use the **File/Open** menu entry or the corresponding  toolbar button. You will then be prompted, in one of the standard dialog boxes that are often displayed by Windows programs, for a file name. Unless you give an explicit extension (which could be a single period), *Dynamics Solver* will add the default `.DS` extension.
2. Pick a name from the list of the four most recently used problem files, which is displayed at the end of the **File** menu. This is a convenient way to resume the analysis of a problem.
3. Drag the problem file from *File Manager* and drop it over *Dynamics Solver*.

To start a new copy of *Dynamics Solver* with a problem file (you can run simultaneously as many copies as you want as long as Window has enough resources):

4. Start (double click on) the *Program Manager* icon representing to the problem file.
5. Start (double click on) the *File Manager* entry corresponding to the problem file.
6. From any program or utility that lets you run Windows applications and documents (such as the **File/Execute** menu entries in *File Manager* and *Program Manager*) you can simply enter as program to be run the complete file specification for the problem file. (You could append that file specification after the one corresponding to the executable program, `DSOLVER.EXE`; but this is not really necessary if the problem file has the default `.DS` extension, because Windows already knows it has to use *Dynamics Solver* to run files with that extension.)

A Simple Dynamical System

Our first example will be the well-known damped pendulum:

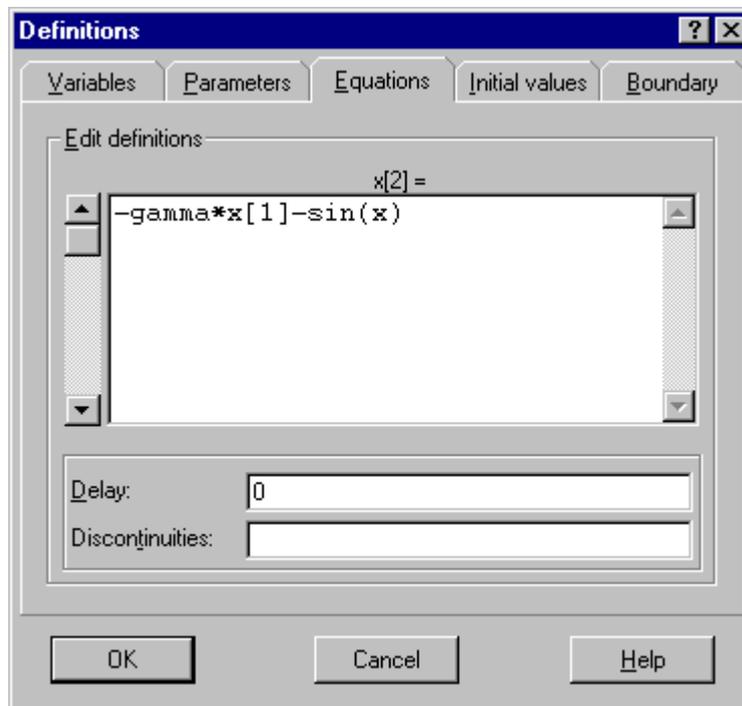
$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \sin x = 0.$$

To analyze this equation by means of *Dynamics Solver*, it must be solved for the highest order derivative:

$$\frac{d^2 x}{dt^2} = -\gamma \frac{dx}{dt} - \sin x.$$

The next step is to enter the problem into the program:

Press **Ctrl+E** (or use the **Edit/Equations** menu entry or the corresponding  toolbar button) to enter the following dialog box:



The header of the **Edit definitions** edit box, $x[2]$, indicates that the definition of $d^2 x / dt^2$ must be entered. This is the convention in *Dynamics Solver*. Derivative order appears between square brackets.

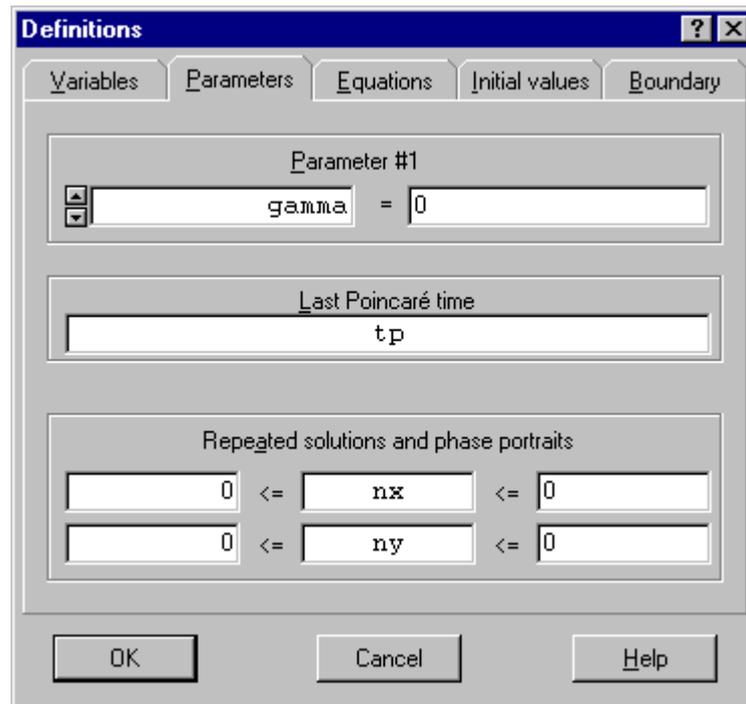
Type the right side of the previous equation in the form

$$-\text{gamma} * \text{x}[1] - \text{sin}(\text{x})$$

Note that, as usual in computing, the multiplication must be explicitly indicated as $*$, Greek letters are not allowed and the argument of the sine function appears between parentheses. According to the convention, $x[1]$ (or x') means dx / dt .

Press **Alt+P** (or use the mouse) to open the **Parameters** sheet² of the same dialog box:

² Most dialogs in *Dynamics Solver* are “tabbed dialogs”. Their controls (edit boxes, check boxes and so on) are arranged by category in different “sheets”. Each sheet has a “tab” (i.e., a control that looks like a tab on file folders) that identifies it. To activate (and make visible) a sheet you only have to select its tab by clicking on it with the mouse or by pressing the **Alt** key with the letter that appears underlined in the tab caption text.



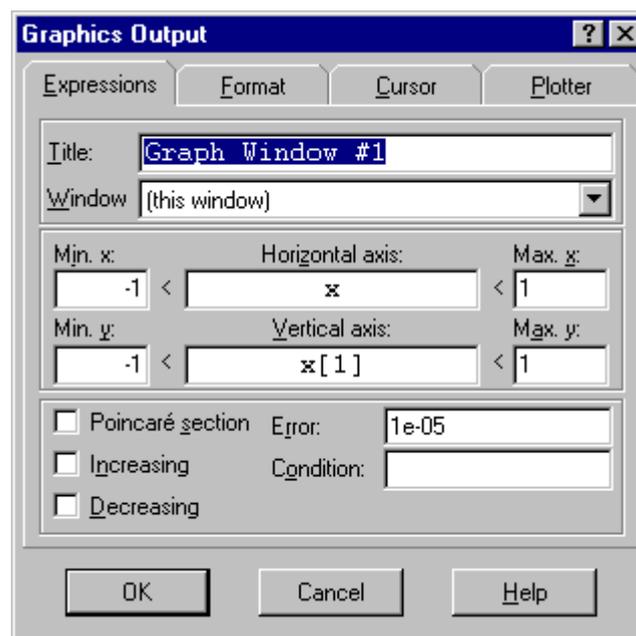
Type the name, gamma, and value, say 0, for the only parameter.

Press **Enter** or use the **OK** button to accept the definitions. The dialog box will close.

Graphics Output

The best *Dynamics Solver* feature is the ability to show the results of integration in graphics format. To take advantage of this possibility, one or more graphics windows must be created.

Press **Ctrl+G** (or use the **Output/New graph window** menu entry or the corresponding  toolbar button) to create a new graphics window. The **Expressions** sheet of the **Graphics Output** dialog box will open:



You can change many settings there, but press `Enter` or use the **OK** button to accept the default values. The dialog box will close.

Solving the Dynamical System

When a graphics window is active, you will see a flashing cross inside a circle in the center of the window. This is a cursor, the coordinates of which are displayed between parentheses in the middle panel of the status line³, and represent the initial values for the dependent variable, x , and its first derivative, $x[1]$. Because so far you have not chosen the initial conditions, they have the default null value.

Press the Left arrow. The cursor will move to a new point whose coordinates $(-0.01,0)$ are updated in the middle panel of the status line indicating that the initial value for x has changed from 0 to -0.01 .

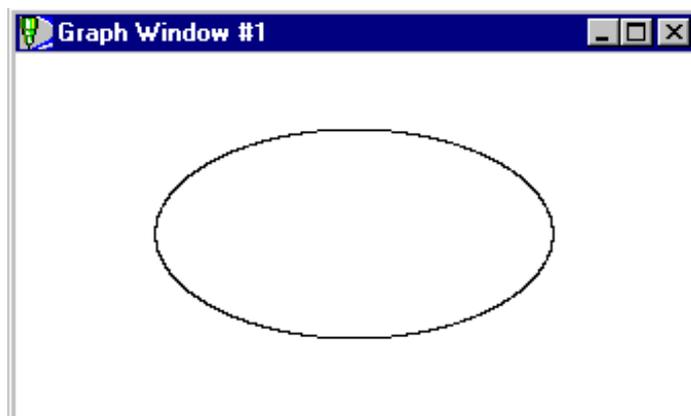
To change the initial condition for $x[1]$ from 0 to 0.01, press the Up arrow. Hold down the `Shift` key and press again the Up key: the value for $x[1]$ changes now faster!

Repeat the previous commands to select $(-0.5,0.3)$ as initial conditions.

(You can also use the mouse pointer to select initial conditions. Locate the mouse pointer at the desired point—the coordinates of the mouse pointer are permanently displayed in the right panel of the status line—and then double click on the left mouse button.)

To start the actual integration of the equation, press `Enter` (or use the **Go/Start** menu entry or the corresponding  toolbar button).

The solution appears as a curve in the $(x, dx/dt)$ space, the so-called phase space. Since we are integrating the undamped harmonic oscillator, the solution is periodic and appears on the screen as a clockwise curve that repeats over itself.



Press `ESC` (or use the **Go/Stop** menu entry or the corresponding  toolbar button) to stop the integration.

³ The status line is the last line in the main window. It is used to display:

1. In the left panel: numerical results, information and help lines, or the kind of graphics element being edited.
2. In the middle panel: the coordinates of the cursor indicating the initial condition (or the angle values of some graphics elements).
3. In the right panel: the coordinates of the mouse pointer.

Output Ranges

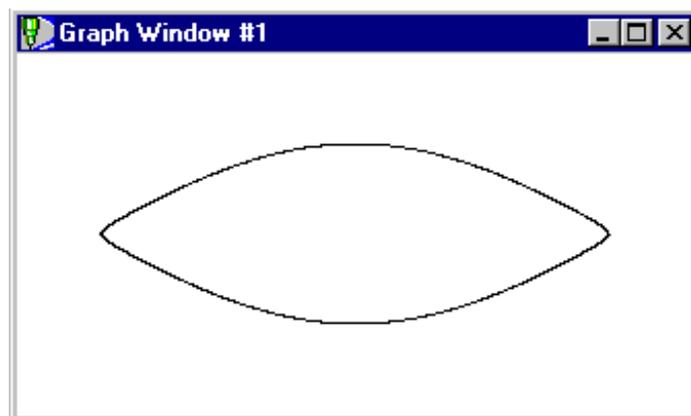
Press **Ctrl+F** (or use the **Output/Graphics format** menu entry or the corresponding  toolbar button) to open again the **Expressions** sheet of the **Graphics Output** dialog box:



Change the **Min. x**, **Max. x**, **Min. y** and **Max. y** entries and press **Enter** to select as new window the square $-4 < x, dx/dt < 4$.

Press **Del** (or use the **Window/Erase window** menu entry or the corresponding  toolbar button) to erase the screen, where the old curve is still being displayed.

Use the mouse (and the arrow keys, if necessary) to select the initial condition (3,0) and then press **Enter** to start the integration. The curve is no longer an ellipse, because now the pendulum starts from rest near its top position and the oscillation is harmonic only for small amplitudes.



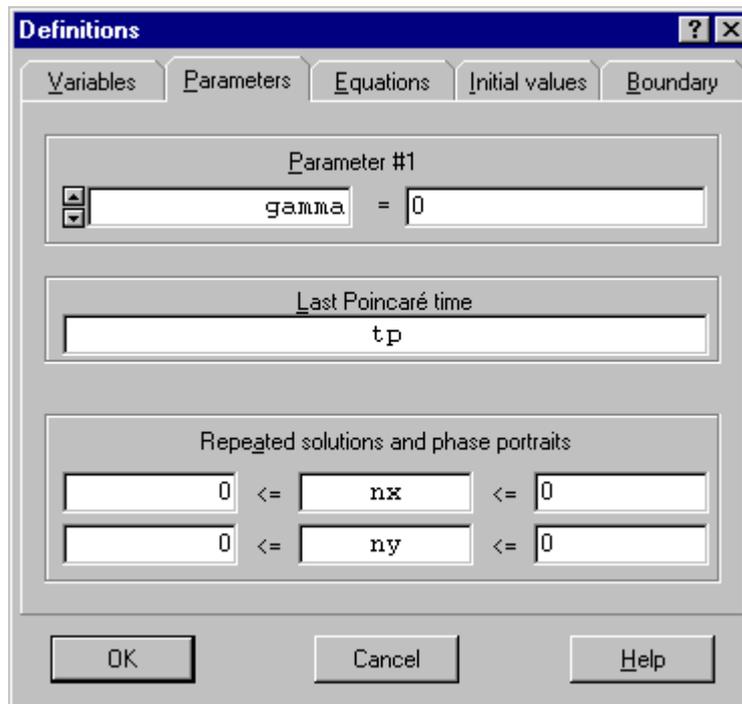
To see how the curve repeats itself, press **Del** to erase the screen while the integration is still being performed. The closed solution will be drawn again.

Press **Esc** to stop the integration.

Try different initial conditions by moving the cursor (other ways of entering initial conditions and choosing the window will be discussed later on) and using **Enter** and **Esc**.

You can also try different values of the parameter γ .

Press **Ctrl+A** (or use the **Edit/Parameters** menu entry or the corresponding  toolbar button) to open again the **Parameters** dialog sheet:



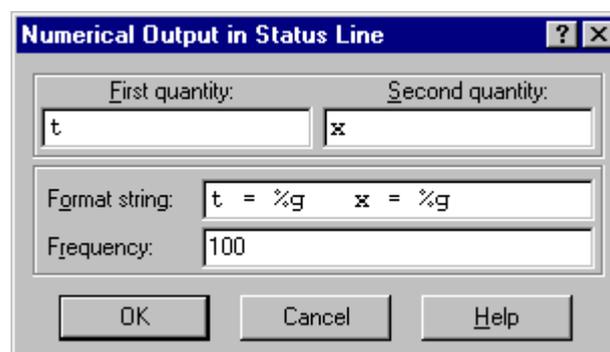
By changing the values for γ and repeating the integration (it will be useful to clear the screen from time to time as described before), you can see how the shape of the solution curve depends on γ . For $0 < \gamma < 1$, we obtain the damped harmonic oscillator that spirals toward the center. For $\gamma > 1$ one gets the overdamped cases.

If you want to save this first problem, use **F2** as described in **Saving and Loading a Problem**.

Numerical Output

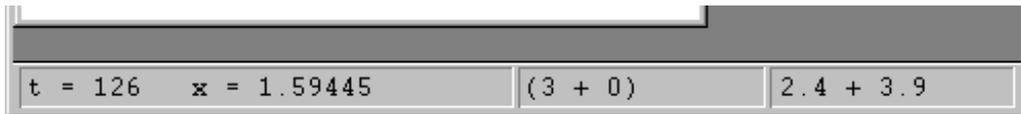
Now suppose you want numerical information about the solution while it is being drawn on the screen.

Press **Ctrl+S** (or use the **Output/Status line** menu entry or the corresponding  toolbar button) to open the **Numerical Output in Status Line** dialog box:



Change the default value of **Frequency** from 0 (which means no numerical output) to 100, to indicate that you want a numerical value every 100 points drawn on the screen. (Do not forget to press **Enter** to accept the new value.)

Press **Enter** to repeat the integration.



Now, while the solution is being computed and displayed, you can see in the left panel of the status line the changing numerical values of t and x . Numerical output is not restricted to the values of the independent variable and the dependent variable: it is possible to have numerical information on any desired expression, such as the energy of the system, for instance. Also, you can send numerical results to its own text window, to the printer or to an external file (to be processed by *Dynamics Solver* or other programs).

Output Quantities

You might prefer the values of t and x displayed in graphical format, rather than having them in numerical form; that is, a (t,x) plot of the solution instead of the default drawing of the $(x,dx/dt)$ phase space. You can, in fact, plot in each axis any expression made up of the solution (even at different values of time), its derivatives, and the parameters and initial conditions of the problem. (The full flexibility of the integrated expression compiler will be described later.)

Press **Ctrl+F** (or use the **Output/Graphics format** menu entry or the corresponding  toolbar button) to open again the **Expressions** sheet of the **Graphics Output** dialog box:



Change the **Horizontal axis** and **Vertical axis** entries to read t and x , respectively.

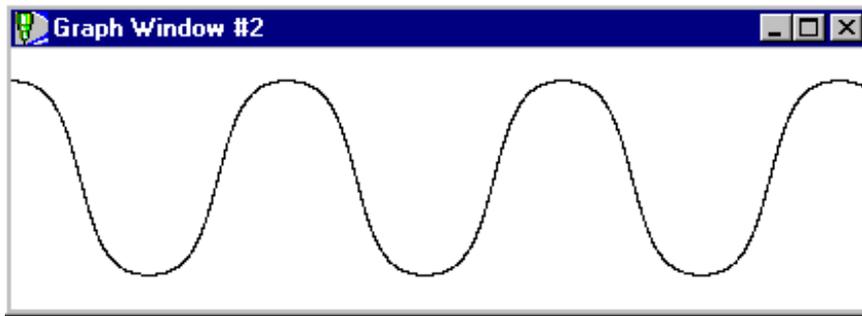
Change also the range displayed in the horizontal axis (which will now display the ever-increasing positive value of t) by setting **Min. x** to 0 and **Max. x** to 50 to display the range $0 < t < 50$. Set also **Min. y** to -4 and **Max. y** to 4 to display the range $-4 < x < 4$.

Press **Enter** to close the dialog box.

Press **Del** (or use the **Window/Erase window** menu entry or the corresponding  toolbar button) to erase the screen, where the old curve is still being displayed.

Press **Enter** to repeat the integration. When the solution runs off the screen, stop it with **Esc**.

You will see x displayed against t : it is clearly different from the familiar sinusoid.



Depending on the initial values you selected the last time, you may have noticed that the solution did not start at the position of the flashing cursor, which may be located outside the screen. This is not a bug in the program, but it is due to the fact that you have changed the meaning of the axes used to display results. When used to input initial conditions, the cursor position still refers to the default x and $x[1]$. It will be described later how to choose which variable, derivative, or parameter will have its initial value selected on each axis by the cursor.

Example Files

In the previous sections you have seen how directly a problem can be entered and solved in *Dynamics Solver*. It has also been shown that several settings (output window, expressions in axes, initial conditions, parameter values, numerical results, etc.) can be easily changed. But there are many other possibilities in the program. In order to get an overview of some of the more important ones, you will take advantage of the **File/Open** menu. Problem files will be used in the remaining of this tutorial to see what can be done in *Dynamics Solver*. The remaining sections in the manual (or the *Dynamics Solver help*) will show you how to do it.

To avoid typing definitions and setting options, you will load problem files from the `EXAMPLES` directory in your *Dynamics Solver* directory. When each example is needed, use the procedure explained in **Saving and Loading a Problem**.

Keep in mind that these examples could also be quickly entered by hand and the problem easily saved to the disk.

Systems of Equations

Dynamics Solver can solve not only a single equation, but also a system of equations. For instance, suppose you want to analyze the famous Lorenz equations:

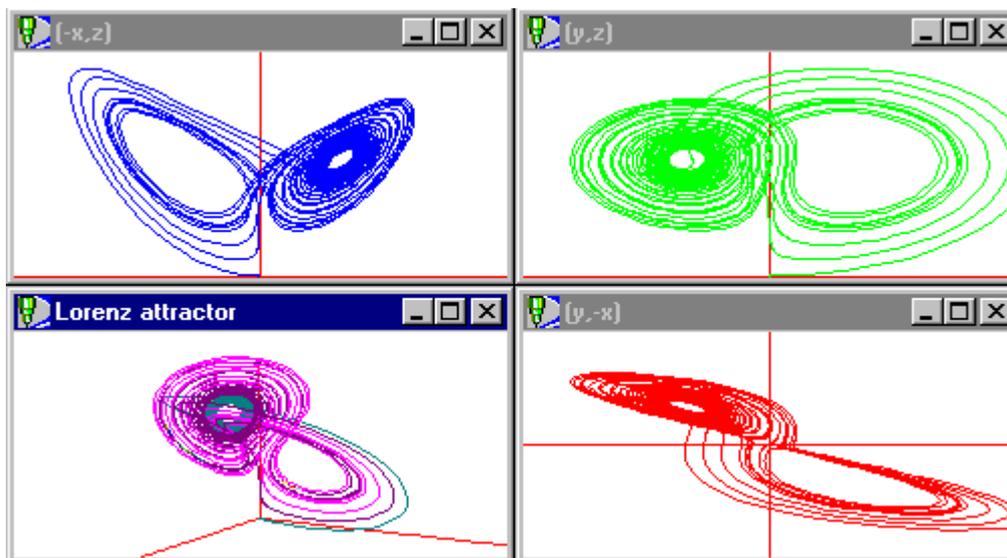
$$\begin{aligned}\frac{dx}{dt} &= \sigma (y - x), \\ \frac{dy}{dt} &= rx - y - xz, \\ \frac{dz}{dt} &= -bz + xy.\end{aligned}$$

Load `LORENZ.DS` in the `EXAMPLES\CHAOS` directory. If you are curious about how the problem has been entered into *Dynamics Solver*, you can examine the entries discussed in previous sections.

You already know how to start solving the problem, after having changed the initial conditions if necessary, by using `Enter`.

You will see different projections of a beautiful orbit that stays forever in a well-defined region, but never repeats itself. Recall that you can erase the current window at any moment without interrupting the

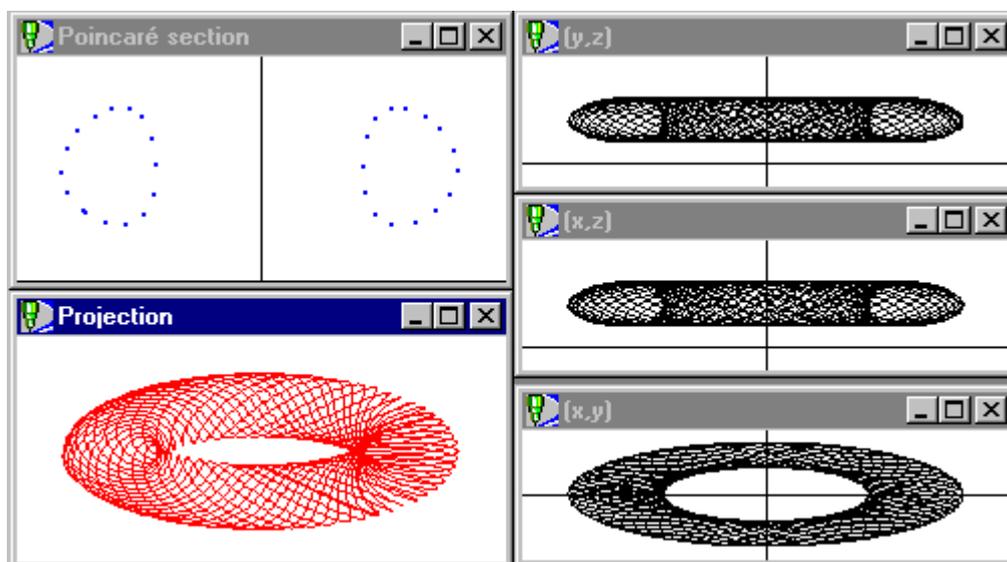
integration by pressing `Del`. To erase all the windows, press `Ctrl+Del` or use the **Window/Erase all** menu entry or the corresponding  toolbar button)



The orbit wanders around a complicated object (a so-called strange attractor) in the three-dimensional phase space, but you only see its projections on the different coordinate planes and one of the windows shows a projection along a view line.

Projections

To get a better idea of the shape of a three-dimensional orbit, you can use the projections on the three coordinate planes, (x,y) , (y,z) and (x,z) , as shown in three of the windows in `EXAMPLES\ODES\TORUS.DS`:



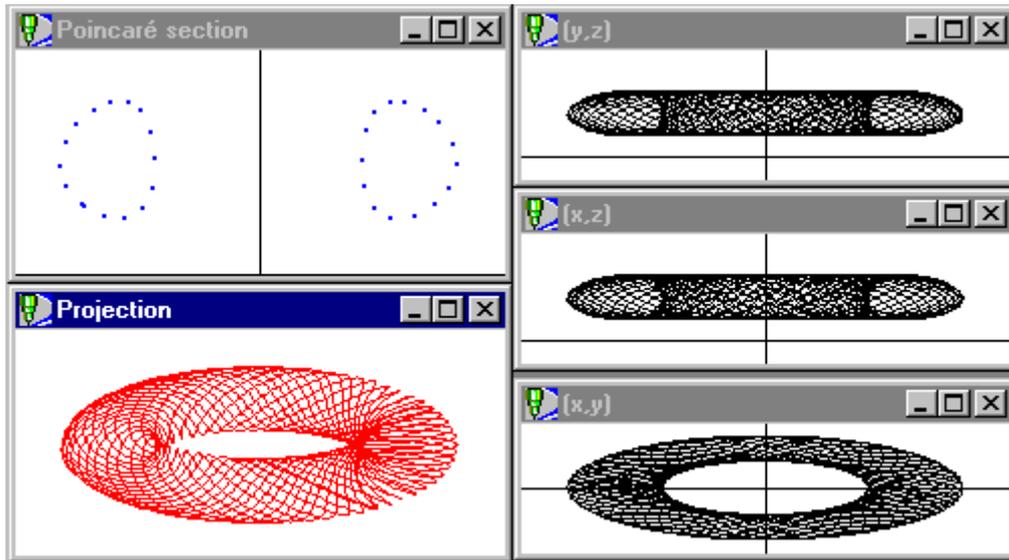
In *Dynamics Solver* you can also project the solution (in perspective or not) along an arbitrary direction, as shown in another window of the same problem.

From the previous figure it is rather clear that the orbit lies on a surface that has the form of a bagel (a mathematical torus).

Another example of the same projection techniques is shown in `EXAMPLES\CHAOS\LORENZ.DS`.

Poincaré Sections

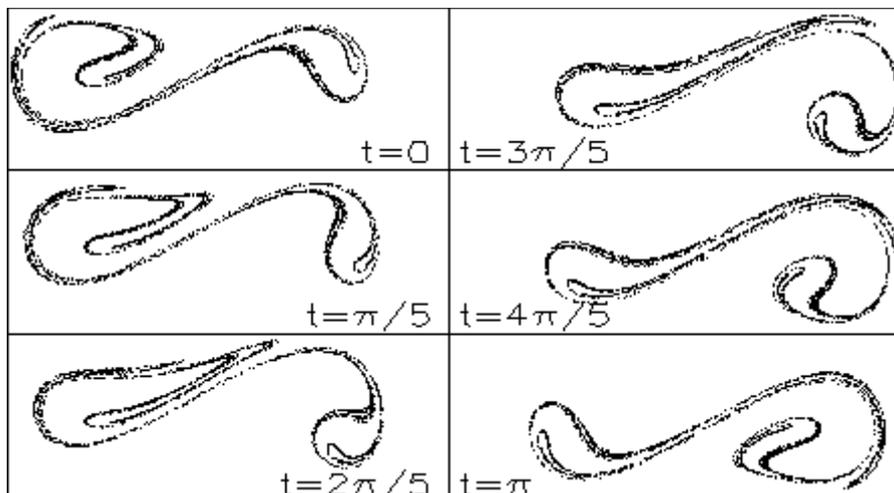
You can also see in the figure



that the section of the torus is not circular. Moreover, you can directly see this section in the window entitled *Poincaré section* where the (x,z) coordinates of the points of intersection of the orbit with the $y = 0$ plane are displayed.

Since successive solution points lying in this Poincaré section are located far away from each other, they will look odd if joined by a segment.

This intersection of the orbit with a plane is a very specific Poincaré section, but *Dynamics Solver* is able to compute rather general Poincaré sections. More interesting examples are computed in other problem files. For instance, one can see how works the “stretch-and-fold” mechanism in the strange attractor of Duffing equation (see `EXAMPLES\ODES\DUFFING2.DS`)



Furthermore, the same basic mechanism used by *Dynamics Solver* to draw Poincaré sections can be used to compute the periods of closed orbits and other quantities, as explained later.

Constrained Initial Conditions and Boundary Conditions

Projections and Poincaré sections are useful to obtain two-dimensional information from higher-dimensional systems. There is still another way to lower the dimension. In a Poincaré section one chooses from all the solutions only those points that satisfy a given condition. It is also possible to choose all the

points, but only from those solutions that satisfy a certain condition. In *Dynamics Solver*, this can be done by imposing a set of conditions on the initial values of the problem. This allows analyzing subspaces of the full phase space of the problem.

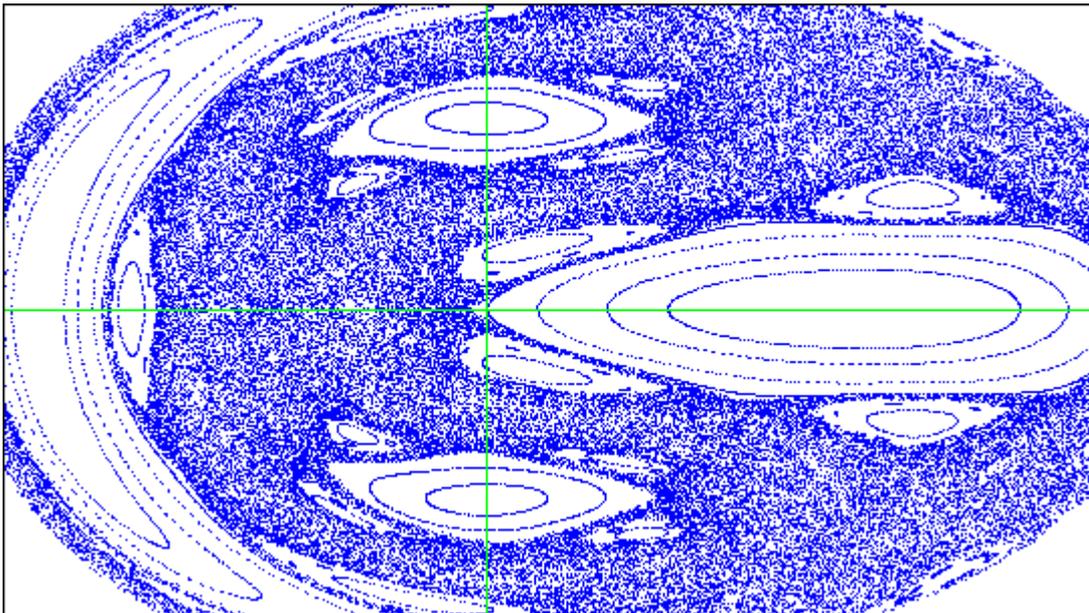
For example, consider the Hénon-Heiles system, i.e., the Hamiltonian system corresponding to the potential

$$V(x, y) = \frac{1}{2}(x^2 + y^2) + x^2y - \frac{1}{3}y^3.$$

The energy is a conserved quantity. If you are interested only in those solutions corresponding to a given value of the energy, you can choose at will the initial values for x , y and dy/dt , for instance, and then use the energy value and its definition to select dx/dt . You can easily instruct *Dynamics Solver* to do that automatically. The solutions corresponding to a certain energy value span the so-called “energy surface,” which has three dimensions. One can then take a Poincaré section of this energy surface by selecting, for instance, the points at which $x = 0$. This will give a two-dimensional plot that can be obtained by using the example in `EXAMPLES\CHAOS\HENONHEI.DS`.

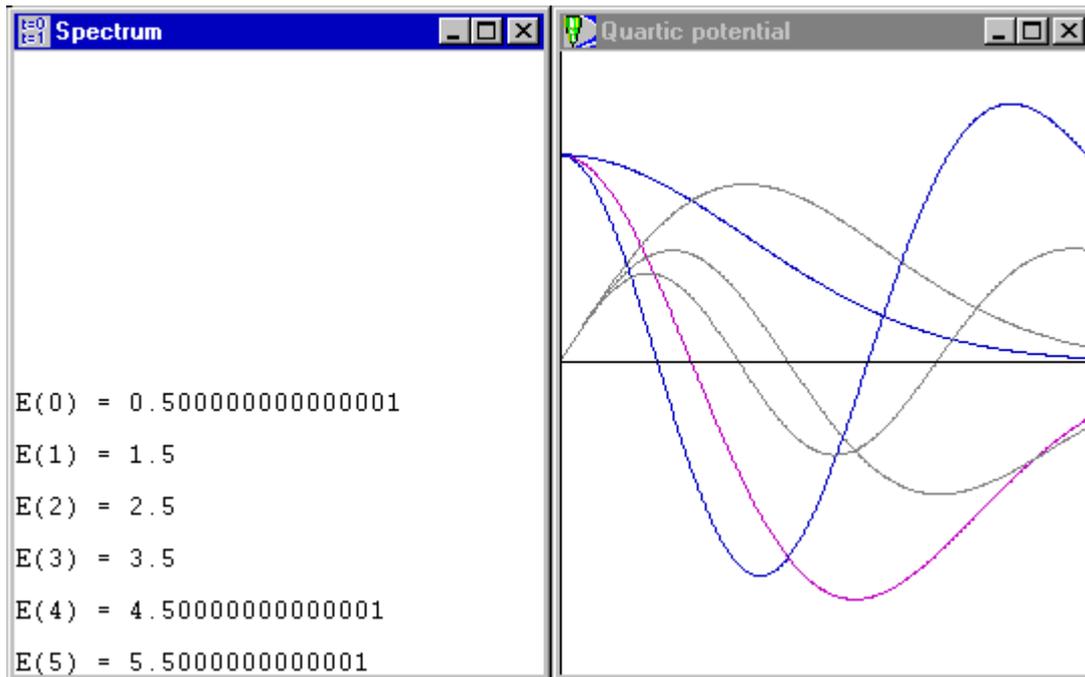
In the left panel of the status line you can see the values of time t and energy E . Since the last quantity should be conserved, you can check if it really remains unchanged; this gives you a measure of the integration quality.

If you now disable the numerical output (by using **Output/Status line** and setting the **Frequency** entry to zero), set the energy value to $E = 0.125$ (by using **Edit/Parameters**), and select different initial conditions, you can obtain the following classical plot:



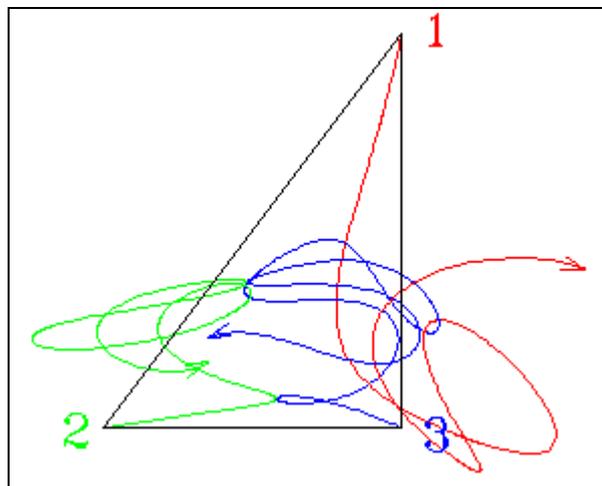
As will be explained in other sections, constrained initial conditions are useful not only to analyze energy surfaces and more general subspaces of the phase space of a problem, but also to solve in *Dynamics Solver* complex problems that do not appear in one of the simple forms discussed so far. Note also that, though only a single Poincaré condition can be imposed in *Dynamics Solver*, you can impose one additional condition for each initial value. These conditions need not be constants of motion of the problem. It is also possible to use constrained conditions that are not solved for the initial values.

Finally, *Dynamics Solver* may handle a very general class of boundary conditions. Run the example in `EXAMPLES\QUANTUM\HARMONIC.DS` to obtain numerically the well-known energy spectrum and wave functions of the quantum harmonic oscillator:

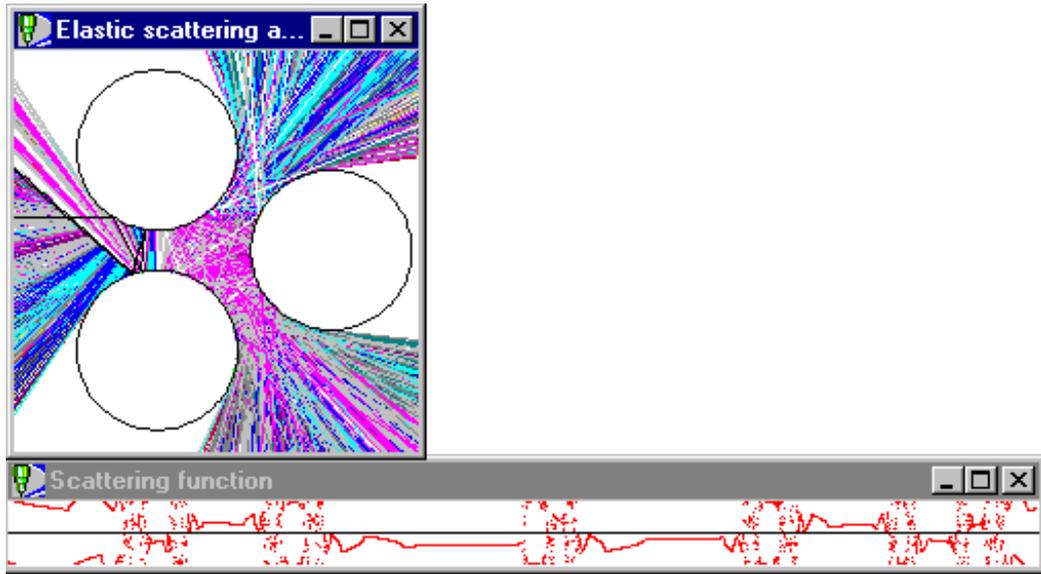


Numerical Simulation

Dynamics Solver is a good tool for numerical simulation of complex systems. For instance, by sending multiple output to a single window you may see how evolve three stars under their mutual gravitational attraction. A classical examples is solved in `EXAMPLES\MECH\BURREAU.DS`:



You may also see in real time how chaotic scattering happens in `EXAMPLES\CAHOS\DISKSCAT.DS`:



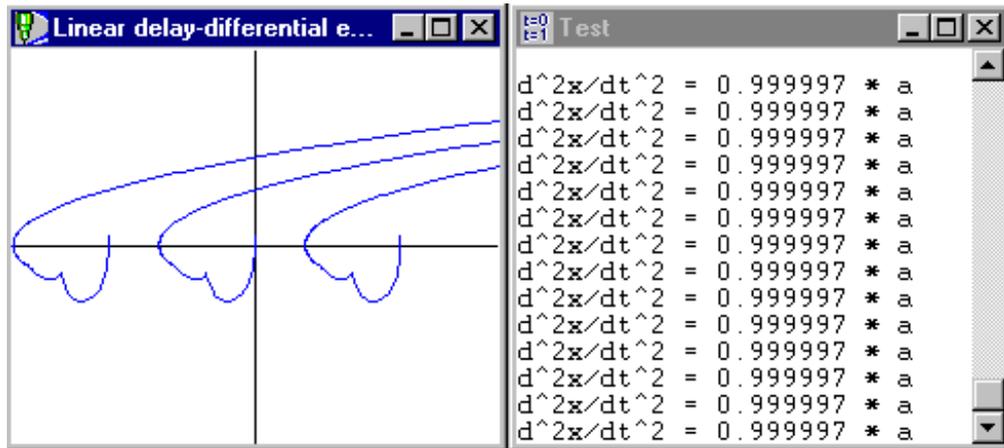
Functional-Differential Equations and Text Output

Dynamics Solver can also solve many functional-differential equations. For instance, some arguments can be taken at different values of the independent variable. Consider the following delay-differential equation as an example:

$$\dot{x}(t) = x(t) + x(t-1) + \frac{a}{2}.$$

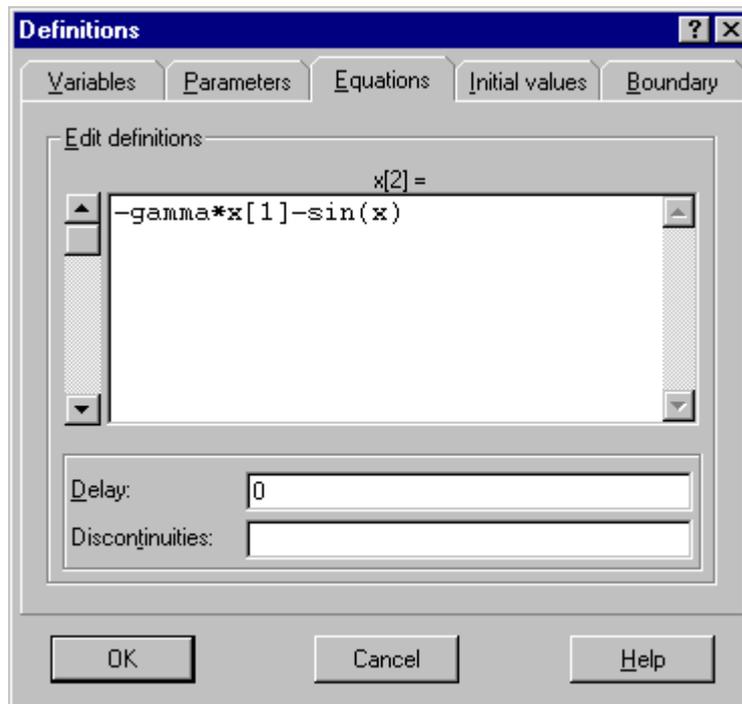
The only non-simultaneous element of this equation is the second term on the right side. It corresponds to a retarded value, $t-1$, of the independent variable. In consequence, in this problem the delay is constant and equal to 1. If you load `EXAMPLES\DELAY\DELAY1.DS`, you will see solutions corresponding to different nonlinear initial conditions in the form:

$$x(t) = x_0 + v_0(t+t^2), \text{ for } -1 \leq t \leq 0.$$



In the left window the solution is displayed in graphics format while the right window is used to display, in text format, a more complex quantity: the second derivative of the solution divided by the parameter a . On the other hand, you may be surprised that the solution plot does not start at the point where the cursor was located. This is because the solutions of delay-differential equations have a discontinuity in the first derivative of the solution at the point where the first step starts ($t=0$ in this example). As will be explained elsewhere, *Dynamics Solver* can deal with these discontinuities. In the example just loaded, this has been

accomplished simply by selecting 1 in the **Delay** entry, after using `Ctrl+E` (or the **Edit/Equations** menu entry or the corresponding  toolbar button) to enter the following dialog box:



Numerical Quadrature

Dynamics Solver is able to compute numerically an integral in the form:

$$I = \int_a^b f(t) dt.$$

In the example in `EXAMPLES\NUMERIC\ELLIPTIC.DS`, you can see the Legendre elliptic integral of first kind

$$F(u | m) = \int_0^u \frac{dv}{\sqrt{1 - m \sin^2 v}}$$

compared with the value returned by the built-in function `EllipticF`. The result appears in numerical form in a text window:

```

F(      0,0.35) = 0                (= 0                )
F(     0.1,0.35) = 0.100058308406611 (= 0.100058308406611)
F(     0.2,0.35) = 0.200465856018663 (= 0.200465856018663)
F(     0.3,0.35) = 0.301568679040681 (= 0.301568679040681)
F(     0.4,0.35) = 0.403705717550479 (= 0.403705717550479)
F(     0.5,0.35) = 0.507203524076695 (= 0.507203524076695)
F(     0.6,0.35) = 0.612368848706666 (= 0.612368848706666)
F(     0.7,0.35) = 0.719478402127162 (= 0.719478402127162)
F(     0.8,0.35) = 0.828765240551856 (= 0.828765240551856)
F(     0.9,0.35) = 0.94040157950294  (= 0.940401579502963)
F(      1,0.35) = 1.0544785484357    (= 1.0544785484357 )
F(     1.1,0.35) = 1.17098453149242  (= 1.17098453149242 )
F(     1.2,0.35) = 1.28978524551951  (= 1.28978524551951 )
F(     1.3,0.35) = 1.41061024239575  (= 1.41061024239575 )
F(     1.4,0.35) = 1.53305137237861  (= 1.53305137237861 )
F(     1.5,0.35) = 1.65657797218095  (= 1.65657797218095 )

```

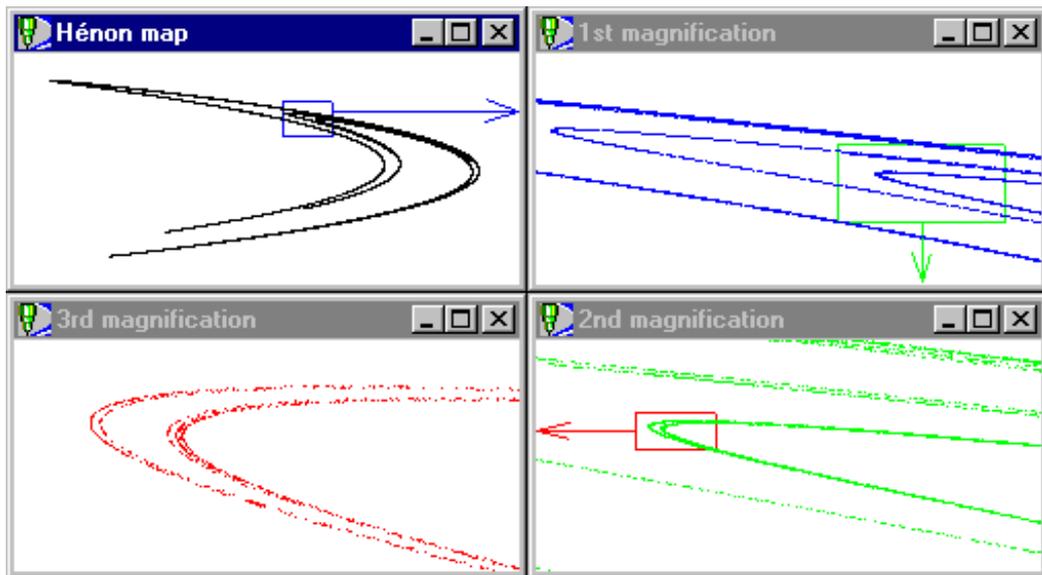
$$F(1.5708, 0.35) = 1.74435059748001 \quad (= 1.74435059697121)$$

Discrete Dynamical Systems

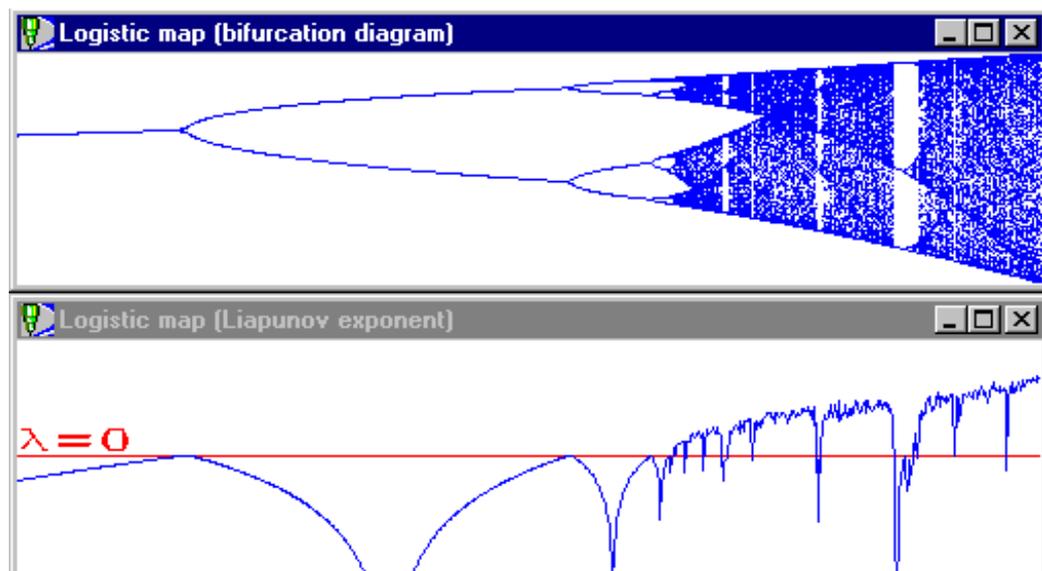
Dynamics Solver can also solve many discrete dynamical systems, which can be obtained by iterating maps in arbitrary dimensions or by using more general recurrence relations. Among other possibilities, one can compute and display their evolution, bifurcation diagrams, histograms, cobweb diagrams and Liapunov coefficients.

Many examples are include in the `EXAMPLES\ART` and `EXAMPLES\CHAOS` directories. Let us only mention here that one can display, among other things:

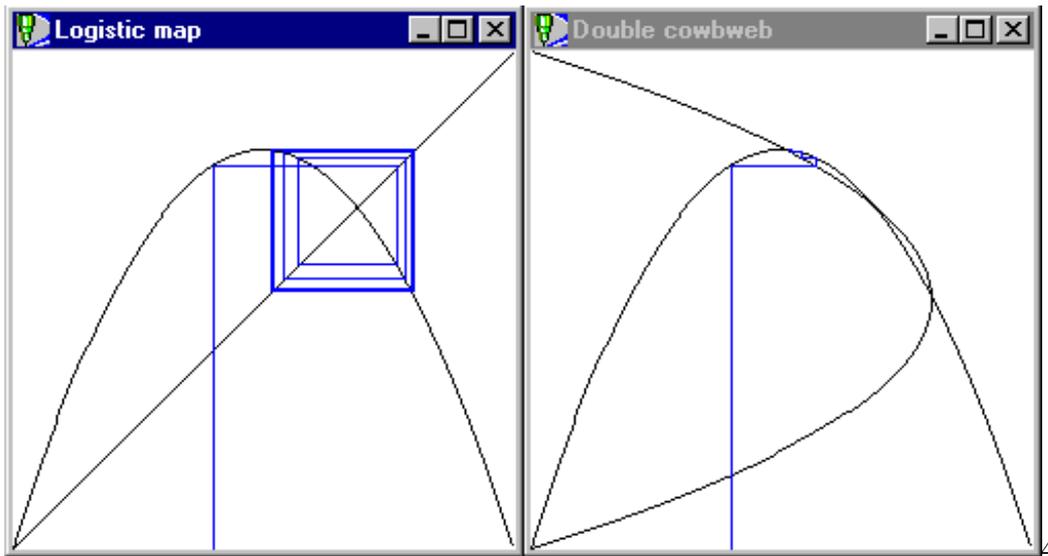
1. Different magnifications of strange attractors (`EXAMPLES\CHAOS\HENON.DS`):



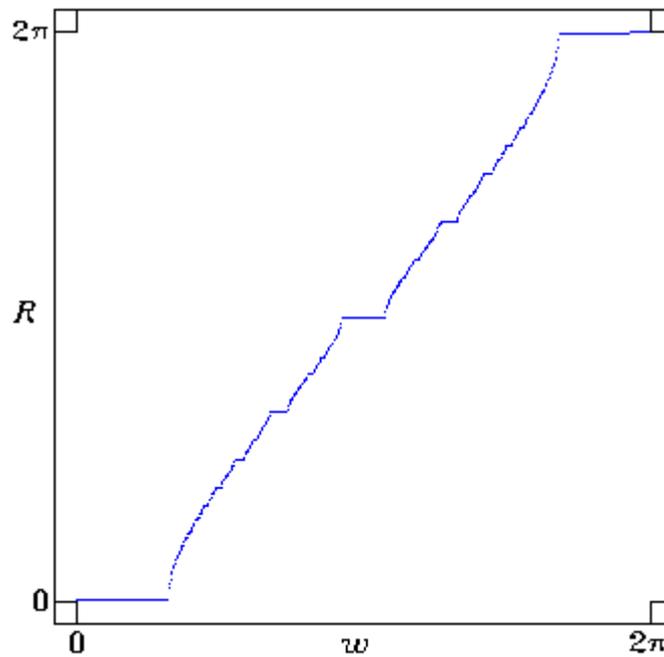
2. Bifurcation diagrams, including Liapunov exponents (`EXAMPLES\CHAOS\LIAPUNOV.DS`):



3. Single and double cobweb diagrams (`EXAMPLES\CHAOS\GRAPH.DS`):

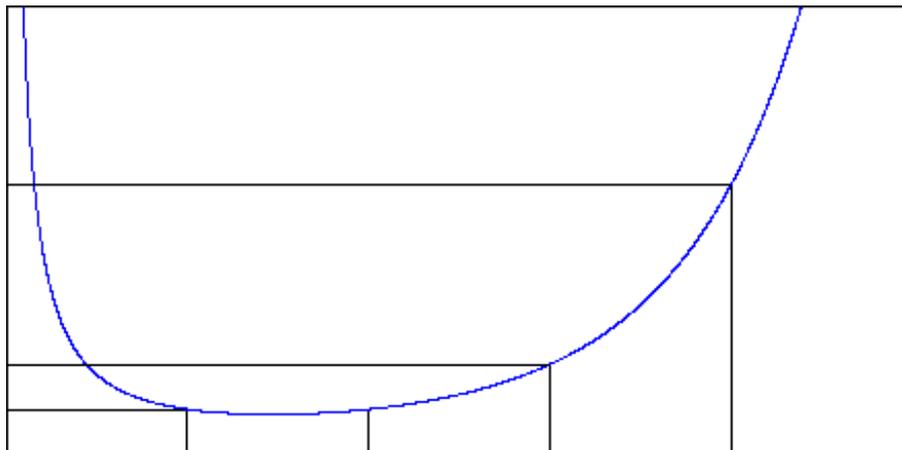


More sophisticated graphics, as the “devil's staircase” displayed below
(EXAMPLES\CHAOS\DEVIL.DS):



Function Graphs and Parametric Curves

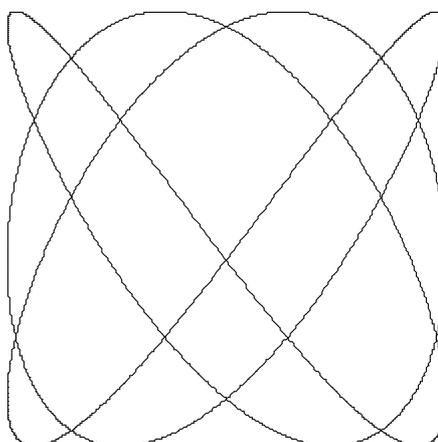
You can also use *Dynamics Solver* to plot the graph of a function by solving a trivial differential equation, such as $dy/dx = 0$, and inserting the function definition in the **Horizontal axis** and **Vertical axis** entries. For instance, in EXAMPLES\DRAWING\GAMMA.DS the graph of the Euler Γ function is drawn:



Parametric curves in the plane are also readily obtained in a similar way. For instance, `EXAMPLES\DRAWING\LISSAJOU.DS` lets you draw the Lissajous curves defined as:

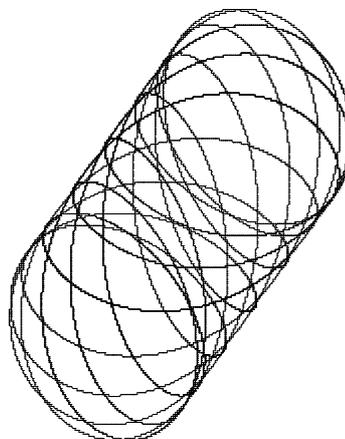
$$x = \cos(at + b),$$

$$y = \cos(ct + d).$$



In this example you can select c and d with the cursor and a and b by using the **Parameters** dialog sheet (which can be obtained by pressing `Ctrl+A`, the **Edit/Parameters** menu entry or the corresponding  toolbar button)).

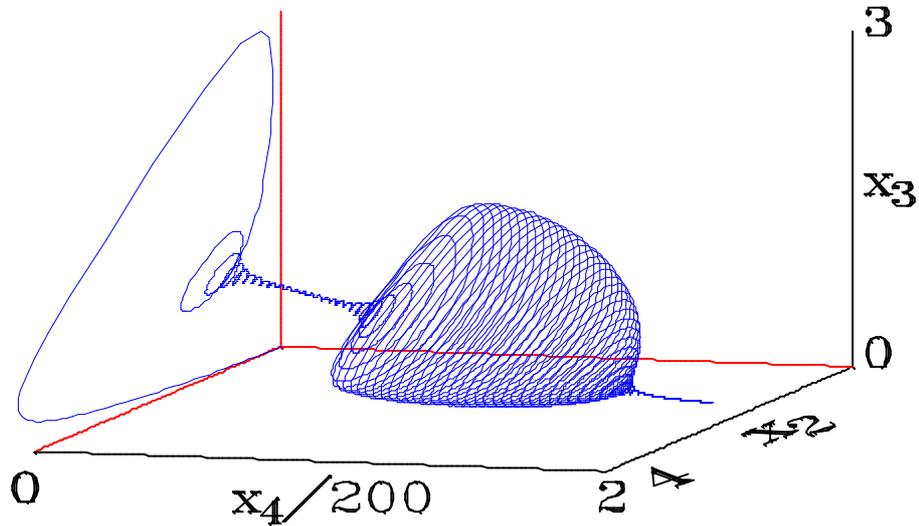
It is also possible to draw projections of three-dimensional curves. You can see Lissajous figures in three dimensions by loading `EXAMPLES\DRAWING\LISSAJO3.DS`.



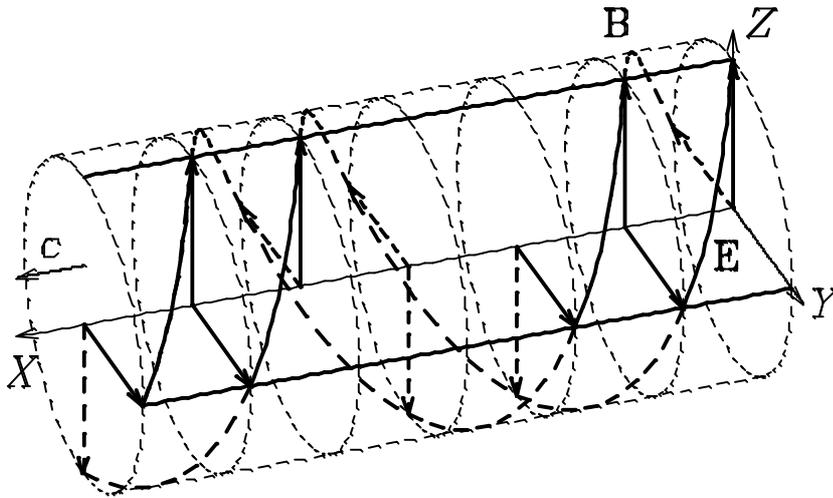
A better way to plot functions and to draw curves (and other graphics elements) is described in the next section of this tutorial: **Drawing with Dynamics Solver**.

Drawing with *Dynamics Solver*

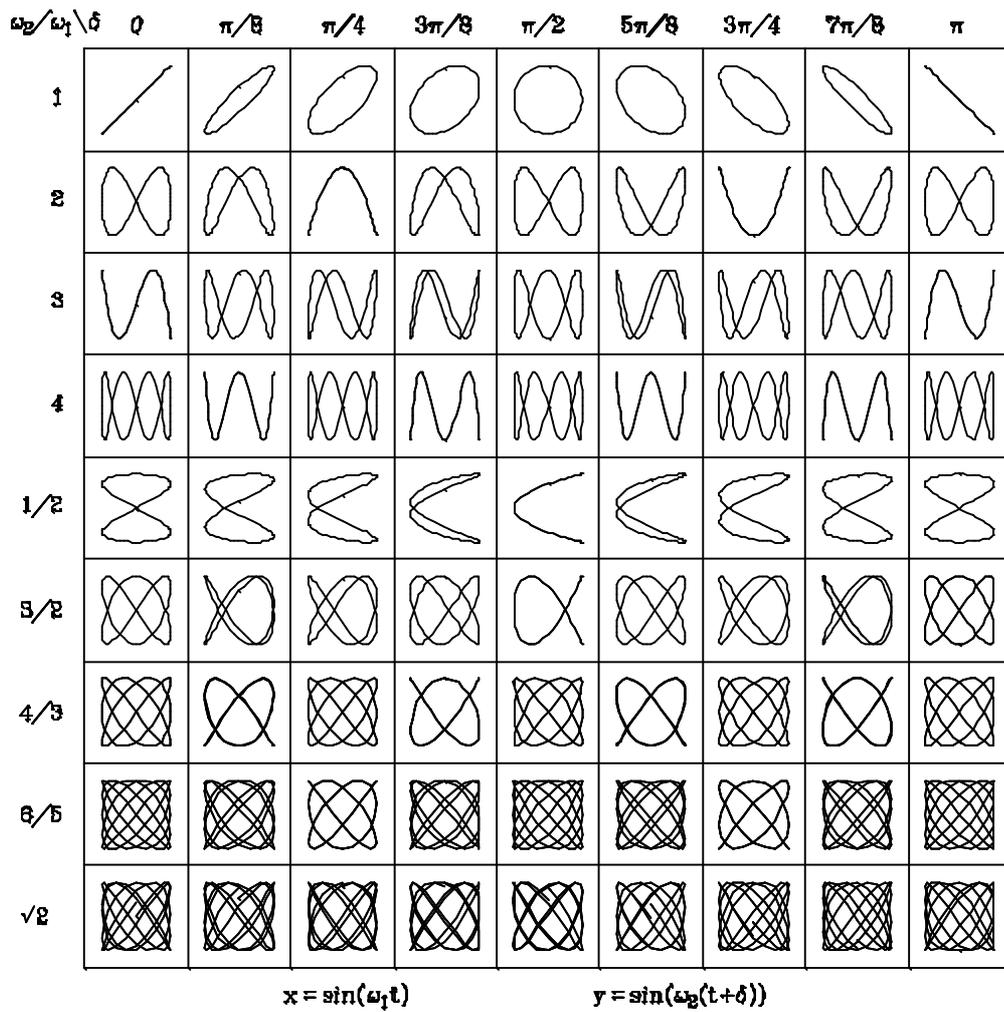
Dynamics Solver can be used to draw rather complex figures including segments, circles, ellipses, parametric curves in two and three dimensions, text strings (with Latin, Greek and Cyrillic letters), arrowheads, points and lines from external data files and a large class of fractal curves. This ability is very useful to add lettering and other informative elements to a problem solution (as illustrated in different example problem files)



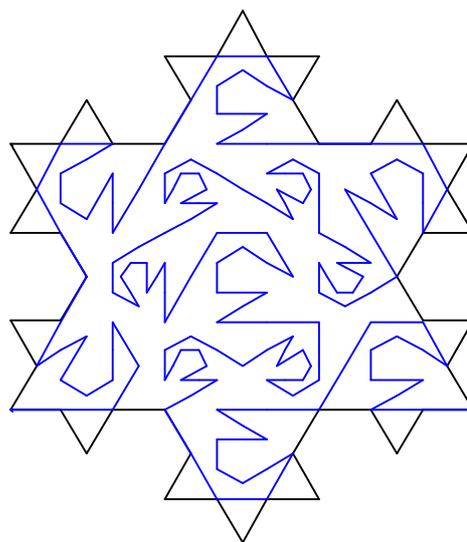
or when preparing drawings for courseware material and research papers:



Several interesting examples are included in the `EXAMPLES\DRAWING` and `EXAMPLES\FRACTAL` directories. Let us mention here another (better) way to draw Lissajous figures



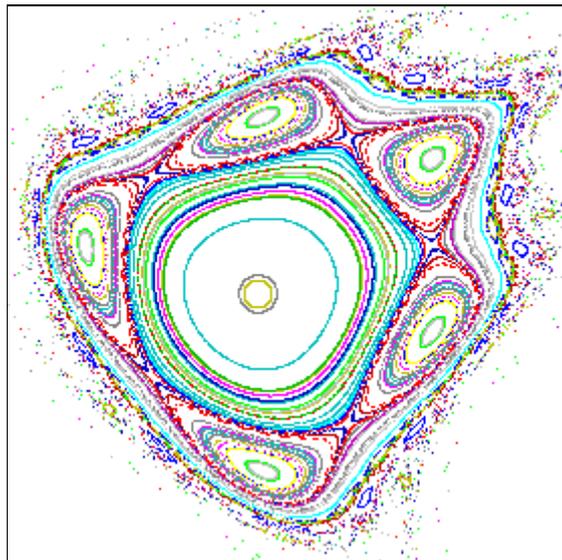
and a typical fractal curve:



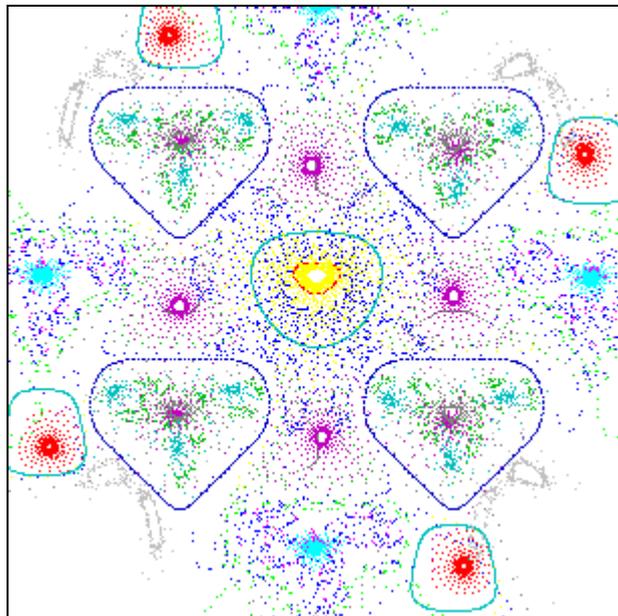
Decorative Examples

Dynamics Solver can also be used to get some artistic (!?) drawings as in many examples included in the EXAMPLES\ART and EXAMPLES\CHAOS directories. Let us only mention the following problem files:

1. EXAMPLES\CHAOS\HENONMAP.DS:



2. EXAMPLES\ART\COS1.DS:



High-Resolution Drawings

So that the user can see exactly what the screen looks like, the figures in this tutorial are direct screen dumps, and they have in consequence a rather low resolution. Needless to say, graphics screens can be printed, in different resolutions, by using standard Windows procedures.

On the other hand, *Dynamics Solver* can produce very high quality drawings (including lettering) by using the highest resolution available in your printer or plotter. You may also export graphics (and text) in standard formats (bitmaps, metafiles, Encapsulated Postscript, etc.) to be used by external word processors or text formatters (like T_EX and L^AT_EX).

This rather complex topic is described in detail in the **Printing** section.

Extending *Dynamics Solver*

Dynamics Solver has very powerful built-in features, but they can be extended in different ways to deal with very complex problems:

- New integration methods may be written in any programming language and used from *Dynamics Solver*. Examples in C, Pascal and FORTRAN are provided.
- Special functions (as well as very complex expressions that would be evaluated too slowly by the integrated compiler) may be written in any programming language and used from *Dynamics Solver*. Examples in C, Pascal and FORTRAN are provided.

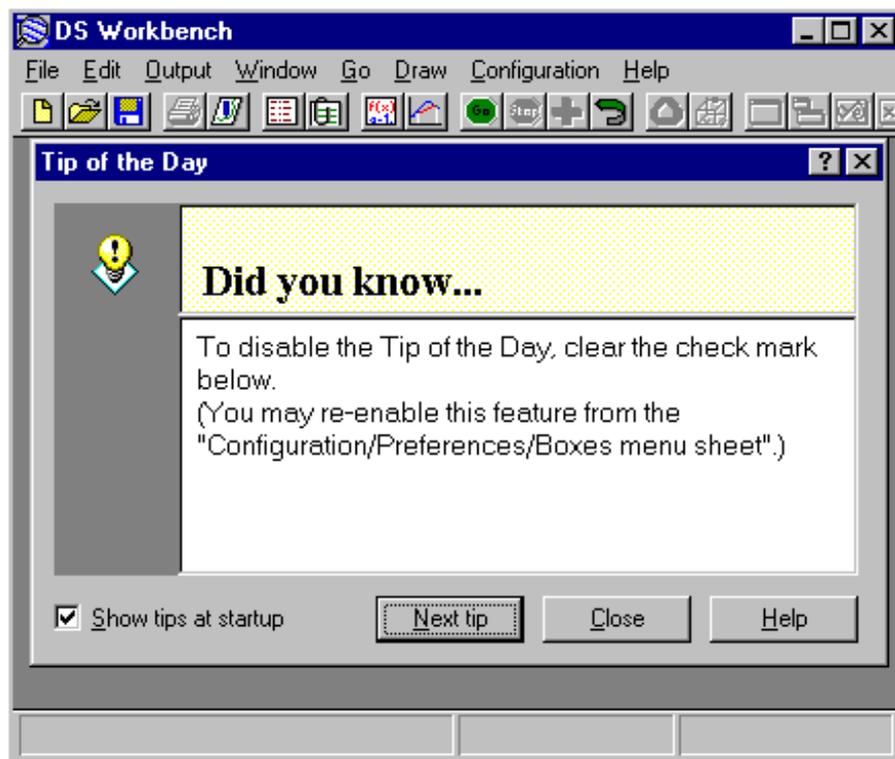
Manual

Running *Dynamics Solver*

This chapter describes the different ways to start the program and the problem files.

Starting *Dynamics Solver*

One usually starts *Dynamics Solver* by choosing its icon from the *Start/Programs/Dynamics Solver* menu (or, under Windows 3.1+, by double clicking with the left mouse button on the *Dynamics Solver* icon in the *Dynamics Solver* group of *Program Manager*). *Dynamics Solver* will start after a short initialization and you will see the tip window (if it is not disabled) over the main window:



You can also start *Dynamics Solver* (or, directly, a problem file⁴) by means of any of the usual procedures to start Windows applications and documents:

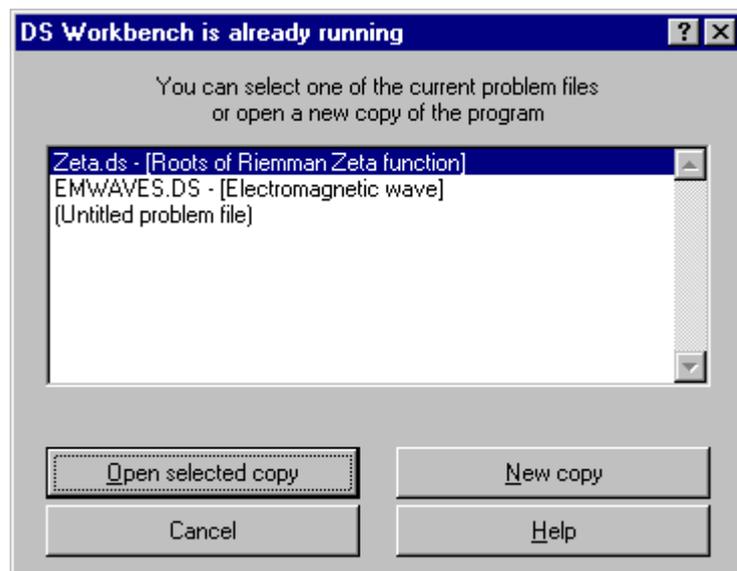
1. Icons in *Program Manager* groups (or *My PC* folders).
2. Entries in *File Manager*, *Start* menu or *Explorer*.
3. The *Start/Execute* menu or the *File/Execute* command in *Program Manager* or *File Manager*..
4. One of the many applications that let you run Windows programs and documents (even from DOS boxes).

⁴ A “problem file” is an ASCII file containing the data for a problem (differential system, discrete dynamical system or set of drawings) that can be analyzed by means of *Dynamics Solver*. Its default extension is *.DS*.

5. Under Windows 95/98/NT4, the program may be started from the command line in a DOS box. A problem file may be directly opened by using the `start` command.

Previous instances

If there is another instance of *Dynamics Solver* running when the program is started directly with no problem file appended after its file specification, or when the same problem file is being analyzed by another copy of the program, the following dialog will open:



You can select there one of the problem files which are already open or start a new copy of the program. It is also possible to cancel the operation. To start always a new copy of the program you may disable the **Check previous instances** entry of the **Preferences/Other** dialog box which opens when using the **Configuration/Preferences** menu.

Command line options

In most cases the command line used to run the program by any method includes only its file specification:

```
[path]dsolver[.exe]
```

(`c:\dsolver\dsolver.exe`, for instance).

The program path is optional if the program is in the current directory or in the PATH. Under Windows 95/98/NT4, if the program is correctly installed, it is also optional in all cases but running it from a DOS box, in which case

```
start dsolver[.exe]
```

will be enough.

To start a problem file directly the syntax is:

```
[path]dsolver[.exe] problem
```

where *problem* is the complete file specification of the problem file.

In many cases (short cuts in *My PC*, the *Start* menu or *Explorer*, *Program Manager* icons, the **Execute** entry of *Explorer*, *Program Manager*, or many other programs) the program specification is optional and it may be replaced by `start` in a DOS box under Windows 95/98/NT4.

When a problem file is opened you may change the corresponding definition in any way and Enter or use the Go/Start menu to start solving it. If you want the problem to be automatically solved after it is loaded, use one of the following command lines:

```
[path]dsolver[.exe] /r problem
```

```
[path]dsolver[.exe] /x problem
```

The difference lies in the fact that in the latter case (using */x*) the program will exit when the solution ends, unless an error happens. In the event of an error you will have an opportunity to see it and the program must be closed manually, as when */r* is used. To be useful, when using */x* you should probably make the program send some output to external files (see Output and output files).

To print a problem file, you may select the corresponding entry in the menu that opens when right clicking on the problem file icon in *My PC* or *Explorer*, or in the *File* menu of *Program Manager*. To print the same problem in the default printer you may also execute the program followed by */p* and the problem file name, in the form:

```
[path]dsolver[.exe] /p problem
```

Under Windows 95/98/NT4 you may drag the problem and drop it over any installed printer or, if you are an advanced user, execute the following equivalent command line:

```
[path]dsolver[.exe] /pt problem device driver output
```

where *device* is the printer name (say, HP Deskjet), *driver* the printer driver (for instance, HPDSKJET) and *output* the output port (LPT1:, FILE:, etc.)

Dynamical Systems

This chapter first describes the types of dynamical systems that can be analyzed with *Dynamics Solver* and then discuss how to write them into the appropriate mathematical form. Finally we describe the way to enter these dynamical systems into the program.

Single Equations and First-Order Systems

Apart from other classes of dynamical systems that will be described later, *Dynamics Solver* may handle directly, with no previous modification, with the following types of ordinary differential equations:

1. A single first-order equation written in the normal form

$$\frac{dy}{dx} = f(x, y), \quad (1)$$

as, for instance, the disintegration law:

$$\frac{dN}{dt} = -N. \quad (2)$$

Note that in Equation (1) the unknown is y while x is the independent variable, but in (2) the dependent variable is N and t is the independent variable.

2. A single equation of arbitrary order solved for the highest-order derivative:

$$\frac{d^n x}{dt^n} = f\left(t, x, \frac{dx}{dt}, \dots, \frac{d^{n-1}x}{dt^{n-1}}\right). \quad (3)$$

A well-known example is given by the equation of motion of the simple pendulum:

$$\frac{d^2\theta}{dt^2} + \gamma \frac{d\theta}{dt} + \frac{g}{l} \sin \theta = 0. \quad (4)$$

3. A system of n first-order equations written in normal form:

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(t, x_1, \dots, x_n), \\ \frac{dx_2}{dt} &= f_2(t, x_1, \dots, x_n), \\ &\dots \\ \frac{dx_n}{dt} &= f_n(t, x_1, \dots, x_n). \end{aligned} \quad (5)$$

In the previous chapter, for instance, we saw the Lorenz equations:

$$\begin{aligned} \frac{dx}{dt} &= \sigma (y - x), \\ \frac{dy}{dt} &= rx - y - xz, \\ \frac{dz}{dt} &= -bz + xy. \end{aligned} \quad (6)$$

Of course, it is not claimed that all the equations or systems of the previous types can be solved by *Dynamics Solver*, but only that they have the right form to be directly handled by the program. To have a reasonably good numerical solution, the functions appearing in their definition must be “well behaved,”

and there is no easy way to know if this is true or not for a given problem. (The interested user may refer to a textbook on numerical analysis; see the **Bibliography**.) Fortunately, many interesting problems arising in practice (in physics, mathematics and engineering) can be successfully analyzed by using *Dynamics Solver*.

Higher Order Systems

To handle systems of equations of order higher than first order, you can always rewrite them in the form of a first-order system by introducing additional variables. The idea is to treat each derivative but the last as a new variable. The procedure, which is fully described in any textbook on differential equations, can be easily understood in an example. Let us consider the equations of motion of a point particle of mass m moving in the gravitational field of another mass M held at rest at the origin of coordinates. Newton equations of this Kepler problem are

$$m \frac{d^2 \mathbf{r}}{dt^2} = -G \frac{mM}{r^2} \frac{\mathbf{r}}{r}. \quad (7)$$

As a consequence of the angular momentum conservation, you can always choose the coordinate system in such a way that the motion is confined to the (x,y) plane. The nontrivial components of (7) are, then, two second-order equations:

$$\begin{aligned} \frac{d^2 x}{dt^2} &= -\frac{GMx}{(x^2 + y^2)^{3/2}}, \\ \frac{d^2 y}{dt^2} &= -\frac{GMy}{(x^2 + y^2)^{3/2}}. \end{aligned} \quad (8)$$

To write this system in a form *Dynamics Solver* can understand, the first-order derivatives (i.e., the velocity components) must be considered as new variables. If one uses the names $u = dx/dt$ and $v = dy/dt$, system (8) can be written in the form of a system of four first-order equations:

$$\begin{aligned} \frac{dx}{dt} &= u, \\ \frac{dy}{dt} &= v, \\ \frac{du}{dt} &= -\frac{GMx}{(x^2 + y^2)^{3/2}}, \\ \frac{dv}{dt} &= -\frac{GMy}{(x^2 + y^2)^{3/2}}. \end{aligned} \quad (9)$$

In a similar way, you can always lower the order of equations by introducing new dependent variables and the corresponding new equations.

Equations Not Written in Normal Form

Dynamics Solver understand only equations written in “normal form,” that is, solved for the highest-order derivative. When this is not true the system must be rewritten. For example, let us assume that we do not know the elementary exact solution of the Friedmann equation for the radius R of a closed universe:

$$\left(\frac{dR}{dt}\right)^2 = \frac{1}{R} - 1 \quad (10)$$

Dynamics Solver understand only equations written in “normal form,” that is, solved for the highest order derivative. When this is not true the system must be rewritten. For example, let us assume that we do not know the elementary exact solution of the Friedmann equation for the radius R of a closed universe:

One can split Equation (10) in two equations of first order:

$$\frac{dR}{dt} = \sqrt{\frac{1}{R} - 1} \quad (11)$$

and

$$\frac{dR}{dt} = -\sqrt{\frac{1}{R} - 1} \quad (12)$$

The square root appearing in the right side of both equations will lead to numerical difficulties. If in the numerical integration and due to accumulated numerical errors the argument of the square root becomes a very tiny but negative number, which would raise an error. Moreover, the solution of (11) corresponds to an ever-increasing universe radius while in (12) the solution is always decreasing. However, the physical solution switches from one equation to the other: it starts with a null radius R (the “big bang”) and increases according to Equation (11) until $R = 1$, when both sides of Equation (11) vanish and the solution also satisfies (12) and will start decreasing until $R = 0$ (the “big crunch”).

In this particular case, all these problems can be solved with a bit of ingenuity by using the Backward function, as shown in `FRW1.DS`. It is enough to integrate forward according to Equation (12) and backward using Equation (11).

There is also another possibility: by taking the derivative of (10) you obtain:

$$\frac{d^2R}{dt^2} = -\frac{1}{2R^2} \quad (13)$$

which appears in normal form and can be used both for forward and backward integration, as in `FRW2.DS`.

Note, however, that although derivatives can be used in many cases to write the problem in normal form, the new equation or system is not equivalent to the original one. It has all the solutions of the latter and, in general, a lot more. In principle, the solutions of the original problem can always be selected by choosing those initial conditions that satisfy it (it will be shown in the next chapter that this can be easily done in *Dynamics Solver*). Nevertheless, in some cases the unavoidable numerical errors will give a numerical solution that is only a reasonable approximation to a solution of the derived system and does not satisfy the original equations. If this happens (and *Dynamics Solver* can be used to know if it actually happens), you cannot use this method of taking derivatives.

Functional-Differential Equations

Among the most original features of *Dynamics Solver* we may mention its ability to solve a large class of functional-differential equations. For instance, it can deal with differential-difference equations that can be written in normal form with right sides containing previous values of the variables. For instance, the Ikeda equation

$$\frac{dx}{dt}(t) = -x(t) + A^2(1 + 2B \cos x(t-r)), \quad (14)$$

has a single constant delay r , while the following retarded modification (with two constant delays) of the classical model of Kermack-McKendrick for epidemic propagation:

$$\begin{aligned}\frac{dx}{dt}(t) &= -x(t)y(t-1) + y(t-10), \\ \frac{dy}{dt}(t) &= x(t)y(t-1) - y(t), \\ \frac{dz}{dt}(t) &= y(t) - y(t-10).\end{aligned}\quad (15)$$

The delay may be variable, as in

$$\frac{d^2x}{dt^2}(t) = -a \frac{dx}{dt}(t - x(t)). \quad (16)$$

The only condition equations must satisfy is that *all the arguments appearing on the right sides must have known values when they are evaluated*. This is possible if they have been computed previously or if they are specified as part of the initial conditions of the problem, as discussed in the next chapter.

In some complex cases the already mentioned tricks of introducing new variables and taking derivatives of the equations can help to put the equations in the correct form. For instance, let us consider two electrons that are held at rest at positions $-x(0)$ and $x(0)$ and then released when $t = 0$. If the radiation reaction is neglected, the classical relativistic equation of motion of each electron is given, in appropriate dimensionless variables, by

$$\ddot{x}(t) = \frac{\left(1 - \dot{x}^2(t)\right)^{3/2}}{\left(x(t) + x(t-r)\right)^2} \frac{1 - \dot{x}(t-r)}{1 + \dot{x}(t-r)}, \quad (17)$$

where the delay r is given by the condition

$$x(t) + x(t-r) = r. \quad (18)$$

Since the delay is in fact an unknown, instead of the functional equation (18), it is better to use the differential equation one obtains by taking the derivative of (18). The final system now reads

$$\begin{aligned}\dot{x}(t) &= v(t), \\ \dot{v}(t) &= \frac{\left(1 - v^2(t)\right)^{3/2}}{\left(x(t) + x(t-r(t))\right)^2} \frac{1 - v(t-r(t))}{1 + v(t-r(t))}, \\ \dot{r}(t) &= \frac{v(t) + v(t-r(t))}{1 + v(t-r(t))}.\end{aligned}\quad (19)$$

with the initial constraint

$$r(0) = 2x(0). \quad (20)$$

Among the retarded arguments there can appear not only the solution but also some of its derivatives, as will be shown below.

Discontinuities in Functional-Differential Equations

The actual integration of delay-differential equations is performed in steps of length equal to the delay. This “method of steps” is able to solve a large class of functional-differential equations. Note that the step length will not be constant if the delay changes. An unavoidable characteristic of this method is that the solutions, in general, have discontinuities in their derivatives at the points separating contiguous steps. (This problem is discussed in the textbooks cited in the **Bibliography**.) In order to improve the quality of

the solution, any numerical procedure to integrate a functional-differential equation should be restarted when a discontinuity point is reached, at least in the case of the first few points.

Integro-Differential Equations and Definite Integrals

Some integro-differential equations may also be solved by using derivatives. For example, the Volterra equation

$$y'(x) = \left(c - ay(x) - c \int_0^x y(t) dt \right) y(x) \quad (21)$$

becomes

$$u''(x) = (c - au'(x) - cu(x))u'(x), \quad u(0) = 0, \quad (22)$$

by using

$$u(x) = \int_0^x y(t) dt.$$

An analogous method can be used for numerical quadrature: to compute

$$I = \int_a^b y(t) dt, \quad (23)$$

just call $y(x) = \int_0^x f(t) dt$ and integrate

$$y'(x) = f(x), \quad y(a) = 0 \quad (24)$$

from $x = a$ to $x = b$. Then, $I = y(b)$.

Discrete Dynamical Systems: Iterated Maps and Recurrences

Dynamics Solver is also able to deal with discrete dynamical systems that can be constructed (along with appropriate initial conditions) by repeated application of a map $f(n, x)$ in an arbitrary number of dimensions:

$$x_{n+1} = f(n, x_n), \quad (25)$$

where the independent variable is the integer index n . Notice that the last equation (as well as the dependent variable x) will have n different components in the case of n -dimensional maps. A very famous example of such a dynamical system is the “logistic equation”

$$x_{n+1} = r x_n (1 - x_n). \quad (26)$$

An example of two-dimensional map is the Hénon map

$$\begin{aligned} x_{n+1} &= 1 - a x_n^2 + y_n, \\ y_{n+1} &= b x_n. \end{aligned} \quad (27)$$

In the dynamical system there could appear values of the unknown(s) corresponding to previous values of the index. In this way we have recurrences in the form:

$$x_{n+1} = f(n, x_{n-r}, x_{n-r+1}, \dots, x_n). \quad (28)$$

For example, the Fibonacci numbers can be defined by the recurrence

$$F_{n+1} = F_n + F_{n-1}, \quad (F_0 = F_1 = 1). \quad (29)$$

Though this last kind of dynamical systems, recurrences, can be directly entered in *Dynamics Solver* (see the example DELAY\FIBONAC.DS), it is far more convenient to write them in the form of an equivalent iterated map by introducing new unknowns:

$$F_{n+1} = F_n + G_n, \quad G_{n+1} = F_n, \quad (F_0 = G_0 = 1). \quad (30)$$

Problem Parameters

As shown in previous sections, you can include parameters in the definition of equations and systems, and you can easily enter and change their values and names.

If you are planning to do an exhaustive analysis of a system you should expend some time identifying the essential parameters in the problem. For instance, in the pendulum equation two parameters appear: the acceleration of gravity g and the pendulum length l , but none of them is essential. To start with, only its quotient g/l appears and, by choosing its square root as time unit, even this disappears to let

$$\frac{d^2\theta}{dt^2} + \sin\theta = 0. \quad (31)$$

This shows that, except for a trivial change of scale, there is essentially only one mathematical pendulum. In a similar way, some of the examples already mentioned can be simplified by carefully choosing the units of measure. Other examples, such as the Friedmann equation (10), have already been presented in terms of appropriate dimensionless quantities.

Of course, in many problems there are essential parameters that cannot be eliminated in this way. For instance, if you add a friction term to the pendulum equation,

$$\frac{d^2\theta}{dt^2} + \frac{1}{Q} \frac{d\theta}{dt} + \sin\theta = 0. \quad (32)$$

you cannot simultaneously get rid of the quality factor Q and the quotient g/l . In fact, the solution of (26) changes qualitatively with Q , as you probably know.

Complex Numbers

Dynamics Solver does not directly understand complex numbers. So, you must separate the real and imaginary parts of every complex quantity. For example, let us consider the following complex system, which arises in the modeling of a very long Fabry-Pérot cavity. The unknowns are the real quantity x and the complex f . The dynamical equation for x is

$$\frac{d^2x}{dt^2} + \frac{1}{Q} \frac{dx}{dt} + x + x_0 = A \sin^2 \theta |f|^2, \quad (33)$$

while the evolution equation for f is a functional one

$$f(t) = 1 + \cos\theta e^{ix(t-r)} f(t-r), \quad (34)$$

where the delay r is given in

$$r = r_0 + \frac{x(t) + x(t-r) - 2x_s}{c}. \quad (35)$$

A , Q , θ , x_s , r_0 and c are free parameters and x_0 is defined as

$$x_0 + x_s = \frac{A \sin^2 \theta}{1 - \cos^2 \theta \cos x_s}. \quad (36)$$

The initial conditions are $x = \text{const.}$ and

$$f = \frac{1}{1 - \cos \theta e^{ix}} \quad \text{for } t \leq 0. \quad (37)$$

To deal with this problem:

1. The complex quantity f is written as $f = g + i h$.
2. The velocity $v = dx/dt$ is introduced to lower the order of Equation (33):

$$\begin{aligned} \frac{dx}{dt} &= v, \\ \frac{dv}{dt} &= A \sin^2 \theta (g^2 + h^2) - \left(\frac{v}{Q} + x + x_0 \right). \end{aligned} \quad (38)$$

3. A differential equation for the delay r is obtained by taking the derivative of the functional equation (35):

$$\frac{dr}{dt} = \frac{1 - v(t)/c}{1 - v(t-r)/c} \quad (39)$$

with the initial condition:

$$r(0) = r_0 + 2 \frac{x(0) - x_s}{c}. \quad (40)$$

4. In the same way, the real and imaginary parts of the functional equation for f are differentiated to obtain two differential equations and the corresponding initial conditions. The actual expressions are cumbersome (but *Dynamics Solver* has no problem to deal with them) and may be found in the example file `FP.DS`.

Entering the Dynamical System into *Dynamics Solver*

After having set up the dynamical system in the appropriate form, you can enter them into *Dynamics Solver* as follows:

1. Use the Edit/**Type** menu (or press `Ctrl+Y`) to enter the **Dynamical System Type** dialog box.

Select the type of **Dynamical system** to be solved: **Single ODE** for a single equation of arbitrary order, **System of ODEs** for an arbitrary number of first-order equations or **Map (iteration)** for a discrete dynamical system.

Even if you have a single differential equation of order n , you can always rewrite it in the form of a first-order system, by introducing $n-1$ new variables (the first $n-1$ derivatives of the unknown) and equations, as discussed previously. So **System of ODEs** will work even in this case, but then you will have to write down n equations. By using **Single ODE**, you only have to write one equation. If you have more than one equation of higher order than the first, you have no choice however: the problem must be rewritten in the form of a first-order system. **Single ODE** only works for a single equation.

In this dialog box you should also choose (in **Dimension/Order**) the order of the single equation, the number of first-order equations or the dimension of the iterated map to be analyzed.

2. Use the Edit/**Variables** menu (or press `Ctrl+V`) to enter the **Variables** dialog box.

Change, if needed, the collective and individual names for the independent and dependent variables and their initial values. As any identifier in *Dynamics Solver*, the name of any variable in the problem must be

a capital or lowercase letter optionally followed by one to seven digits or letters. In this context the underscore character, `_`, is considered to be a letter. So, `t`, `s1`, `Time` or `time0` are OK, but `alongname` is too long, `1t` starts with a digit and `á` is not an ASCII character. See **Mathematical Expressions**.

If you have a single equation, the derivatives of the only unknown will be denoted with the same collective name followed by the derivative order between square brackets. So if you chose `theta` for the angle in the pendulum equation (4), `theta[1]` (or `theta'`) will its first derivative and so on. In the case of a system, the number appearing in square brackets indicates the corresponding subindex. In consequence if you choose the default `x`, `x[1]` will denote the first dependent variable, `x[2]` the second one, and so on..

You may also select a fake value (a space, for instance) for the collective name if you prefer to use individual names for variables or derivatives. In this form, you can use, for example, `x` (instead of `x[1]`) for the first unknown, `y` (and not `x[2]`) for the second one, and so on.

3. In the **Parameters** sheet of the same **Variables** dialog box (which can also be accessed form the **Edit/Parameters** menu, or by pressing `Ctrl+A`) you can select the names and values of the parameters.

In the same dialog, you can select the name for the last value of the independent variable for which the condition defining a Poincaré section was true.

Additionally you can select the names and ranges for two integer variables (indices) that will allow performing repeated integrations in an automatic way (to get phase portraits and to deal with complex problems).

As will be discussed later, you can also use as parameters of your problem the initial value of the independent and dependent variables and the two additional “initial values” described in the next chapter.

4. In the **Equations** sheet of the same **Variables** dialog box (which can also be accessed form the **Edit/Equations** menu, or by pressing `Ctrl+E`) you can enter the expressions defining the right sides of Equations (1), (3), (5), (25) or (28).

Dealing with Discontinuities

If there is a single constant delay or if all delays are constant and multiple of a single value, you should enter this value in the **Delay** entry of the **Equations** sheet of the **Variables** dialog box (which can also be accessed form the **Edit/Equations** menu, or pressing `Ctrl+E`). *Dynamics Solver* will use this value to perform the integration in intervals having exactly this length. This will improve the solution quality through the discontinuities of its derivatives.

If the delay is not constant the previous procedure is useless. In many cases (but not always: refer to the textbooks in the **Bibliography**) the first derivative is discontinuous only at the start of the first step, the second derivative is discontinuous only at the start of the first and second steps, and so on. If this is the case, only the first few discontinuity points matter when a numerical procedure is being used. If you are able to find (by hand, or using *Dynamics Solver*) the points at which the first discontinuities arise, enter these values separated by spaces in the **Discontinuities** entry of the same dialog sheet. When *Dynamics Solver* reaches one of these points, it will restart the integration to improve the solution quality.

If even this is not feasible, you may still obtain meaningful results with *Dynamics Solver* in some particular cases by simply ignoring the discontinuities. This may be the case if the system has an attractor (an asymptotically stable equilibrium point or limit cycle, or a chaotic attractor, for instance) and all the interesting solutions approach it. You may be able to obtain the attractor, despite the additional error induced in the transient by discontinuities. In many interesting practical cases, this error is small enough to be completely neglected, but do not assume this in your problem.

Initial Values

We saw in **Dynamical Systems** how to define the equation(s) corresponding to a dynamical system and the way to enter them into *Dynamics Solver*; but in order to have a well-posed mathematical problem initial conditions or boundary conditions are also needed. This chapter describes the form in which initial conditions must be specified and entered into *Dynamics Solver*. **Boundary Conditions** will be considered in the next chapter.

See also **Repeated Integration/Iteration** and **Phase Portraits**,

Initial Conditions for Ordinary Differential Equations

In the case of a first-order differential equation in normal form,

$$\frac{dx}{dt} = f(t, x), \quad (41)$$

a well-posed initial value problem requires the initial value of the unknown at the initial value of the independent variable:

$$x(t_0) = x_0. \quad (42)$$

For a single equation of order n ,

$$\frac{d^n x}{dt^n} = f\left(t, x, \frac{dx}{dt}, \dots, \frac{d^{n-1}x}{dt^{n-1}}\right), \quad (43)$$

the n initial values of the unknown and its first $n-1$ derivatives must be given:

$$\begin{aligned} x(t_0) &= x_0, \\ \frac{dx}{dt}(t_0) &= \left(\frac{dx}{dt}\right)_0, \\ &\dots \\ \frac{d^{n-1}x}{dt^{n-1}}(t_0) &= \left(\frac{d^{n-1}x}{dt^{n-1}}\right)_0. \end{aligned} \quad (44)$$

A system of n first-order equations,

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(t, x_1, \dots, x_n), \\ \frac{dx_2}{dt} &= f_2(t, x_1, \dots, x_n), \\ &\dots \\ \frac{dx_n}{dt} &= f_n(t, x_1, \dots, x_n), \end{aligned} \quad (45)$$

needs the n initial values of the n unknowns:

$$\begin{aligned} x_1(t_0) &= x_{10}, \\ x_2(t_0) &= x_{20}, \\ &\dots \\ x_n(t_0) &= x_{n0}. \end{aligned} \quad (46)$$

Constrained Initial Conditions

In some cases you may want to have the value of one or some initial values defined in terms of the values of other initial conditions or parameters. For instance, if you want to explore a subspace of the phase space of the problem, you must make sure that initial conditions lay in that subspace.

In the Kepler problem discussed in **Dynamical Systems**, you may want to parameterize it in terms of the two main constants of motion: the energy

$$E = \frac{1}{2}(u^2 + v^2) - \frac{1}{\sqrt{x^2 + y^2}} \quad (47)$$

and the angular momentum

$$L = xv - yu. \quad (48)$$

(A system of units has been selected in which $m = 1$ and $GM = 1$. Recall the discussion in **Choosing Parameters**.) The definitions of these constants of motion can be solved for the velocity to give:

$$u = \frac{x\sqrt{2\sqrt{x^2 + y^2} + 2E(x^2 + y^2)} - L^2 - Ly}{x^2 + y^2}, \quad (49)$$

$$v = \frac{y\sqrt{2\sqrt{x^2 + y^2} + 2E(x^2 + y^2)} - L^2 + Lx}{x^2 + y^2}.$$

You can now then select at will the initial values of the coordinates x and y and then compute the initial components of the velocity by using Equation (49). Actually, you are not completely free when selecting x_0 and y_0 , because they must satisfy the condition that the argument in the first square root of Equation (49) cannot be negative! If you try values that do not satisfy this condition, *Dynamics Solver* will issue an error message.

Constrained initial conditions must also be used when additional variables have been introduced or the original equations have been differentiated. This has been already discussed in **Dynamical Systems**, in which these procedures have been suggested to rewrite equations in a more appropriate form. For example, if you take the derivative of the Friedmann equation,

$$\left(\frac{dR}{dt}\right)^2 = \frac{1}{R} - 1, \quad (50)$$

to obtain

$$\frac{d^2R}{dt^2} = -\frac{1}{2R^2}, \quad (51)$$

you can choose the initial value of R , but its derivative must satisfy the original equation:

$$\left(\frac{dR}{dt}\right)_0 = \pm \sqrt{\frac{1}{R_0} - 1}. \quad (52)$$

Sometimes, the constraints are conserved along the exact solution of the system. This is the case in the two examples described above, because the constraints were constants of motion (i.e., first integrals) of the resulting system. It should be obvious that the inherent errors of every numerical calculation will degrade the accuracy of the imposed constraints as the independent variable increases. In some cases this is not a real problem, but if the constraints are unstable, the error can grow very quickly, leading to an unacceptable solution. This problem may be completely unavoidable due to mathematical instabilities and the approximate nature of numerical computations.

You should always (or, at least, in some preliminary runs) monitor from time to time the conserved constraints to see if they are stable or not. This task can be easily carried out in *Dynamics Solver*, as described in **Output Parameters**.

In fact, this kind of problem can arise even in very simple cases. Often it is practically impossible to compute numerically an unstable solution. Suppose you want to compute:

$$y'' = 100y, \quad y(0) = 1, \quad y'(0) = -10. \quad (53)$$

The solution is $y = \exp(-10x)$, but the general solution of (53) is $y = A \exp(-10x) + B \exp(10x)$ for arbitrary constants A and B . Because of numerical errors, the solution, which starts as an approximation to $\exp(-10x)$, becomes an approximation to something like $A \exp(-10x) + B \exp(10x)$ with very low values for $A-1$ and B . Since the second term grows exponentially while the first decreases, the numerical solution will quickly go to infinity, while the exact one goes to 0! (See the example in `ODES\UNSTABLE.DS`.)

Nevertheless, in many cases good approximations to unstable solutions can be computed by integrating backward in time. For example, the so-called “stable manifold” of a saddle point in a two-dimensional dynamical system (refer to the textbooks in the **Bibliography**, if you do not understand this paragraph) is an unstable solution, but a reasonable approximation can be easily computed integrating backward from a point near the equilibrium point. See, for instance, the example in `MECH\PENDULUM.DS` where the “stable manifold” and the “unstable manifold” of a saddle point can be computed by starting very close to the fixed point and integrating in both directions.

See also **Boundary Conditions**.

Initial Conditions for Functional-Differential Equations

With functional-differential equations, it is not enough to specify a finite number of initial conditions: full functions must be provided instead. The full discussion of this topic lies out of the scope of this manual, but there is a rule of thumb that will work in most practical cases: you must provide all the information needed to have known values for the arguments in the functions defining the equations for every value of the independent variable in the range you are interested in. In other words, you must specify the values that will be needed but not actually computed in the integration process.

For instance, suppose you want to solve Ikeda's delay-differential equation

$$\frac{dx}{dt}(t) = -x(t) + A^2(1 + 2B \cos x(t-r)), \quad (54)$$

for $t \geq t_0$. Due to the delay r , to evaluate the right side of Equation (54) when $t_0 \leq t \leq t_0 + r$, the values of the solution x for $t_0 - r \leq t \leq t_0$ must be known. Since they will not be computed in the integration process, they must be provided in the form of initial conditions that in this case will be precisely the value of x in the interval $t_0 - r \leq t \leq t_0$. Notice that to evaluate Equation (54) for $t_0 + r \leq t \leq t_0 + 2r$ the corresponding values of $x(t-r)$ have already been calculated in the previous interval, so no initial condition is needed here.

Remember also the problem of discontinuities.

See also **Entering Initial Functions and Constrained Initial Conditions**.

Initial Conditions for Discrete Dynamical Systems

In the case of iterated n -dimensional maps, the initial conditions of a discrete dynamical system are the set of values the n unknowns (or coordinates of the unknown) for the initial value of the independent variable (the index). In this respect, these systems are not different from system of first-order equations and you can refer to the discussion in **Initial Conditions for Ordinary Differential Equations**.

For more general recurrences in which the new values of the unknowns depend at each step of values from more than one previous step (as in the example of Fibonacci numbers discussed in **Discrete Dynamical Systems: Iterated Maps and Recurrences**), one has to provide all the values necessary to start the iteration. In this respect, these systems are not different from functional-differential equations and the initial conditions can be entered by using initial functions as discussed in **Initial Conditions for Functional-Differential Equations**. But in this case, unlike in the case of differential equations, the system has still a finite dimension and it can be easily rewritten as an iterated map with needs only ordinary initial conditions which are easier to enter and change (even on the screen): see **Discrete Dynamical Systems: Iterated Maps and Recurrences**.

Entering Initial Values

In the **Initial values** sheet of the **Variables** dialog box (which can also be accessed from the Edit/**Initial conditions** menu, or pressing `Ctrl+I`) you can enter the initial values for the independent and dependent variables. (The names of these initial values are selected in the **Variables** sheet of the same dialog box.)

You should note here that for a single equation of order n , for a system of n first-order equations or for a n -dimensional map you can in fact enter $n+2$ initial values. The two additional “initial values” are not true initial conditions but additional parameters, the values of which can be graphically selected on the screen, as described in the next section.

Entering Initial Values on the Screen

In many cases you will change very often the initial values to see different solutions of the same problem. You may also want to change often the value of some parameters to see their influence. In either case, *Dynamics Solver* lets you choose two of these values in a very intuitive and fast way when you are already integrating the system. This is accomplished by moving (with the mouse or the cursor keys) a cursor on the graphics screen, as described in detail in **Selecting Initial Conditions on the Screen**. But, before doing it, you must choose which values will be selected on the screen. Since the screen is two-dimensional, only two values can be simultaneously selected in this way. The initial values or parameters that will be selected on the screen are chosen in the **Variable in axis X** and **Variable in axis Y** entries of the **Cursor** sheet of every graphics window.

In the same place you can use the **Horizontal step** and **Vertical step** entries to change, if necessary, the value of the elementary step that the cursor will advance on the screen when selecting initial conditions by using the cursor keys (instead of the mouse pointer).

Solutions Arriving to a Predefined Point

If you want to “final conditions”, i.e., compute the solution finishing at a predefined point (for instance, to draw the stable manifolds of saddle points), the easiest method is to start at that point (or very close to it, if it is an equilibrium point) and integrate backward in time. Of course, this is not possible for delay-differential equations. You might also use **Boundary Conditions**.

Multiple Sets of Initial Conditions

You may select consecutively as many sets of initial conditions as you please, but in some cases (while studying the sensitive dependence on initial conditions that defines deterministic chaos, for example) it is better to see in real time the evolution of more than one solution. This can be achieved in the following way:

1. If you want to integrate m sets of initial conditions of a system of n equations (or a single equation of order n) define a new system of $n \times m$ equations by repeating m times the n equations in the system with different names for the dependent variables. A single equation must be first converted to the equivalent system of first-order equations.

2. Define a window to collect the output corresponding to each set of initial conditions and redirect the output from all these windows to one of them.

Let us consider the interesting example in CHAOS\DUFFING4.DS. One has to solve the Duffing equation

$$\ddot{x} + \gamma \dot{x} - ax(1 - x^2) = f \cos t, \quad (55)$$

for two sets of very near initial conditions to see how evolve the distance between them. One starts by defining the equivalent system

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= -\gamma x + ax(1 - x^2) + f \cos t, \end{aligned} \quad (56)$$

And then duplicates it as follows:

$$\begin{aligned} \dot{x}_1 &= v_1, \\ \dot{v}_1 &= -\gamma x_1 + ax_1(1 - x_1^2) + f \cos t, \\ \dot{x}_2 &= v_{21}, \\ \dot{v}_2 &= -\gamma x_2 + ax_2(1 - x_2^2) + f \cos t. \end{aligned} \quad (57)$$

The two sets of initial conditions are applied to (x_1, v_1) and (x_2, v_2) , and a window is defined to display each of these pairs of coordinates, but the output of one of them is redirected to the other, which will then display both solutions simultaneously.

See **Output Ranges**.

Entering Initial Functions and Constrained Initial Conditions

Use the **Initial values** sheet of the **Variables** dialog box (which can also be accessed from the Edit/**Initial conditions** menu, or pressing **Ctrl+I**) to enter both constrained initial conditions for ordinary differential equations and initial functions for functional-differential systems and general recurrences. In a single equation of order n , in a system of n first-order equations or in a n -dimensional discrete dynamical system, exactly n functions must be specified. They will be used to provide all the unknown initial values in the case of a delay-differential equation or general recurrence. In the case of ordinary differential equations and iterated maps, these initial functions are not generally necessary. If defined, they are applied to the currently selected initial values to find the actual initial values *Dynamics Solver* will use. In this way, the effective initial values are computed (in terms of other initial values, parameters, etc.) just before integration starts. If the constraint equations are not solved for the initial values, or you have constraints involving more than one value of the independent variable, you have to use **Boundary Conditions**.

Reading Initial Conditions from Data Files

In the **Initial values** sheet of the **Variables** dialog box (which can also be accessed from the Edit/**Initial Conditions** menu, or pressing **Ctrl+I**) you can enter both constrained initial conditions for ordinary differential equations and initial functions for functional-differential systems. But this functions can be

also used to read the actual initial values from an external input file by using `read(i)` (or `read(resetread(i,i))`, if you want to start reading again from the beginning of the file). This possibility is especially useful when **Repeated Integration/Iteration** is being performed.

Boundary Conditions

A problem that can be solved by means of *Dynamics Solver* is not completely defined by specifying the corresponding equations as discussed in **Equations**. A second element is necessary in order to have a well-posed mathematical problem: initial conditions or boundary conditions. This chapter is devoted to the form in which boundary conditions must be first set up mathematically and then entered into *Dynamics Solver*.

See also **Boundary Conditions, Repeated Integration/Iteration and Phase Portraits**,

Boundary Conditions for Ordinary Differential Equations

A very general boundary-value problem for a single equation of order n ,

$$\frac{d^n x}{dt^n} = f\left(t, x, \frac{dx}{dt}, \dots, \frac{d^{n-1}x}{dt^{n-1}}\right),$$

is a set of n conditions that the values of the unknown and its first $n-1$ derivatives at given, but not necessarily equal, times must satisfy:

$$\begin{aligned} \Phi_1\left(t_1, x(t_{11}), \frac{dx}{dt}(t_{12}), \dots, \frac{d^{n-1}x}{dt^{n-1}}(t_{1n})\right) &= 0, \\ \Phi_2\left(t_2, x(t_{21}), \frac{dx}{dt}(t_{22}), \dots, \frac{d^{n-1}x}{dt^{n-1}}(t_{2n})\right) &= 0, \\ &\dots \\ \Phi_n\left(t_n, x(t_{n1}), \frac{dx}{dt}(t_{n2}), \dots, \frac{d^{n-1}x}{dt^{n-1}}(t_{nm})\right) &= 0. \end{aligned}$$

It might even appear derivatives of order higher than $n-1$. As the notation suggests the independent variable may be evaluated at any number of points, not just the initial and final ones (intermediary values may also appear). On the other hand, it may happen that all these values are the same, in which case we can think of these conditions as constrained initial conditions that are not necessarily solved for the initial values of the unknown and its derivatives. (See also **Constrained Initial Conditions**.)

In an analogous way, a system of n first-order equations,

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(t, x_1, \dots, x_n), \\ \frac{dx_2}{dt} &= f_2(t, x_1, \dots, x_n), \\ &\dots \\ \frac{dx_n}{dt} &= f_n(t, x_1, \dots, x_n). \end{aligned}$$

needs the n boundary conditions on the values of the n unknowns at given times:

$$\begin{aligned} \Phi_1(t_1, x_1(t_{11}), x_2(t_{12}), \dots, x_n(t_{1n})) &= 0, \\ \Phi_2(t_2, x_1(t_{21}), x_2(t_{22}), \dots, x_n(t_{2n})) &= 0, \\ &\dots \\ \Phi_n(t_n, x_1(t_{n1}), x_2(t_{n2}), \dots, x_n(t_{nn})) &= 0. \end{aligned}$$

Derivatives could appear instead of the unknowns and the independent variable may be evaluated at any number of points, not just the initial and final ones (intermediary values may also appear). If all these values happen to be the same, we can think of these conditions as constrained initial conditions that are not necessarily solved for the initial values of the unknown and its derivatives. (See also **Constrained Initial Conditions**.)

Boundary conditions may be also imposed to discrete dynamical systems in exactly the same way.

See also **Solving Boundary-value Problems**.

Defining Boundary Conditions

You may use *Dynamics Solver* to solve very general boundary-value problems, because you may specify any n general expressions as **Conditions** in the **Boundary** sheet of the **Variables** dialog box (which can also be accessed from the Edit/**Boundary Conditions** menu, or pressing Ctrl+Z). In the same sheet you specify the **Point** until which the integrations necessary to compute the conditions must be performed. It must be equal or greater (smaller) than the largest (smallest) value of the independent variable appearing in **Conditions** when integrating forward (backward). The value **Iterations** indicates the maximum number of times the program must try to integrate the system before it announces the procedure has failed. Finally, **Error** indicates the maximum absolute value a **Condition** may have to be considered satisfied.

If **Iterations** is 1 or greater, the program will:

1. Use the current initial values (see **Entering Initial Values**).
2. Apply initial functions (see **Entering Initial Functions and Constrained Initial Conditions**), if any. In most cases these functions are not used, but they may help guess good initial values.
3. Solve the problem according to the current settings until the value **Point** of the independent variable. No graphics or text output happens, but you are remembered in the status line that this process is in progress and the ESC key may be used to stop it. This step is not performed if **Point** coincides with the starting integration point.
4. Evaluate the **Conditions**. They are completely general expressions and they may involve the initial values, the values at **Point** or any intermediate value if the corresponding interpolated solution is available.
5. If the **Conditions** have an absolute value over **Error** their values are used to estimate new (and hopefully better) initial values and steps 2 to 5 are repeated; but if the number of repetitions exceeds **Iterations** an error is issued and the procedure ends.
6. If the **Conditions** are met, the current initial values are used to perform the actual integration is performed, with normal output, until the maximum value indicated in the **Range** dialog sheet.

Let us assume that we have to solve the following (admittedly artificial) problem:

$$\begin{aligned} \frac{d^4 x}{dt^4} + 5 \frac{d^2 x}{dt^2} + 4x &= 0, \\ x(0) &= 1, \\ \frac{d^2 x}{dt^2} \left(\frac{\pi}{2} \right) + \frac{d^2 x}{dt^2}(\pi) &= 2, \quad (58) \\ x \left(\frac{\pi}{2} \right) &= -\frac{d^6 x}{dt^6} \left(\frac{\pi}{2} \right), \\ \frac{dx}{dt}(\pi) &= -1. \end{aligned}$$

We would choose **Point** equal to 3.14159265358979, **Iterations** to, say, 15, **Error** as 1E-7 (for instance) and the following **Conditions**:

```
x0-1
x(2,1.570796326794897)+x[2]-2
x(0,1.570796326794897)+x(6,1.570796326794897)
x[1]+1
```

After a short computation one would obtain a good approximation to the solution:

$$\cos t - \sin t - \sin 2t.$$

Solving Boundary-Value Problems

Though the Newton method used by *Dynamics Solver* to find the initial values that satisfy a given set of boundary conditions converges quadratically when it starts from a *close enough* set of values, it should be clear that:

- Not every boundary-value problem has a solution. In fact, the existence of a solution is often the exception. Recall, for instance, that the fact that some boundary-value problems for the Schrödinger equation have solutions only for certain values of the energy is one of the main results of Quantum Mechanics.
- Even if the problem has a solution, the method is not guaranteed to find the solution except if we start *close enough*.

The interactive nature of *Dynamics Solver* helps deal with the last problem. The user may easily change the initial values from which the Newton search starts. Often it is convenient to start with high values for **Iterations** and **Error** and once the solution has been determined with low accuracy one can select lower values for those parameters in order to refine the solution.

As for the first problem, if one is interested in finding the values of the energy (or any other parameter) for which the problem has a solution, this can be easily performed by including the energy (or the parameter) among the dynamical variables. One simply converts the problem into a system of $n+1$ equations of first order for the original variables and the parameter and includes a trivial equation (null derivative) for the latter.

In `QUANTUM\QOSCILL.DS` you may see how to guess good starting values for the initial conditions, which are then used in `QUANTUM\HARMONIC.DS` and `QUANTUM\QUARTIC.DS` to find accurate values for the energy spectrum. In `QUANTUM\SPECTRUM.DS` repeated integration is used as an alternative, but far less convenient, way to compute the spectrum. Finally `QUANTUM\PENK.DS` shows how to deal with boundary conditions that should be imposed at infinity and how to get high accuracy in the energy values.

Solution Options

Although in many cases the default values will work fine, at least to start analyzing the problem, you will have to change at some time the values of some settings related to the solution method. This chapter is devoted to describe them.

Solution Range

Dynamics Solver will start computing the solution at the initial value of the independent variable that you have selected in the **Initial values** dialog sheet. You may want the solution to end when a certain value for the independent variable is reached, or when the solution becomes too large, or any other condition is satisfied. As explained below, all these possibilities can be readily implemented in *Dynamics Solver*. You may also want to compute but exclude from output an initial transient, say for $t_0 < t < t_2$, in order to see only the asymptotic behavior of the solution. This is especially useful to analyze attractors and can be accomplished easily in *Dynamics Solver*. However, it must be stressed that the interactive nature of *Dynamics Solver* gives you the best way to select the effective integration range: simply press a key to end the integration or erase the screen to forget a transient.

In special cases you might want to end the computation (maybe to continue with another set of initial conditions) when some conditions that have to be computed at run time are met.

Selecting the Integration Range

After selecting the initial value of the independent variable at which solution starts as described in **Initial Values**, you may use the **Range** dialog sheet (which can be activated from the Edit/**Range** menu entry or by pressing `Ctrl+R`) to choose among other things:

1. The **Last value** defining the end point of the integration interval.
2. The **Infinity** value to stop the integration when it “runs wild.” If the absolute value of any dependent variable or derivative reaches this value, the integration ends. In fact, the integration halts when
 - The user presses the `Esc` key or uses the **Go/Stop** menu entry.
 - An error occurs.
 - The **Last value** is reached by the independent variable.
 - The absolute value of any dependent variable reaches the **Infinity** value.
 - The first argument of a `break` function in an expression is not equal to zero.
 - The first argument of a `continue` function in an expression is equal to zero.

In all but the first two cases the next iteration (if any has been scheduled by using the **Repeated solutions and phase portraits** entries in the **Parameters** dialog sheet) starts.

3. If you want to exclude from output a transient interval, enter its end value in the **First Value** entry. *Dynamics Solver* will compute the values from the initial value for the independent variable (in the **Initial values** dialog sheet) on, but will start output only when the independent variable reaches that **First Value**.

In most cases you want to see the complete integration range; that is, usually you will have a **First Value** equal to the initial value for the independent variable. Since it would be rather awkward to change **First Value** every time the initial value for the independent variable is changed, a special meaning is attached in *Dynamics Solver* to the value 0 of **First Value**. When **First Value** is 0, *Dynamics Solver* will interpret that you want the output started precisely at the initial value for the independent variable. This lets you change the later value freely without worrying about **First Value**. In the rather improbable case that you

want precisely a null **First Value** and a negative initial value for the independent variable, you must select in **First Value** a tiny non-null value, as 1E-307 for instance.

In the case of discrete dynamical systems, you can select in the same dialog box, the parameters to draw bifurcation diagrams.

Selecting When to Have Output

You may want to get (graphics and text) output in different windows precisely every time *Dynamics Solver* computes a new solution point. If this is the case, set off the **Interpolate** entry in the **Range** dialog sheet.

Since the integration step length is automatically selected by the program (unless a fixed step size method is used), successive solution points will be unevenly separated and, worse, the distance between them can be large. If this happens and successive points are being joined by a segment to have a continuous solution, the latter would appear in the form of a rather awkward polygonal. You could lower **Tolerance** or **Max. h** in the **Method** dialog sheet to force *Dynamics Solver* to compute more points, but this would increase the computational effort unnecessarily. A better alternative is available.

As discussed previously, the numerical code used by *Dynamics Solver* returns not only isolated solution points but also a global piecewise polynomial approximation to the solution. You can take advantage of this important feature to have dense output. Set on the **Interpolate** entry in the **Range** dialog sheet and select in the **Step** entry of the same sheet the independent variable interval between two successive output points. Then the expressions in **Horizontal axis** and **Vertical axis** (in the **Graphics Output** dialog box) of each graphics window and those corresponding to each text window (as defined in the **Text Output** dialog box) and the will be evaluated and output by intervals of length **Step**. To do this, *Dynamics Solver* needs to store at least the last computed point in memory, so you cannot have **Saved points** set to zero.

Notice, however, that in some cases the rate of change of plotted quantities is much higher in some points than in other places of the same solution. If **Interpolate** is selected in and the interpolation step in **Step** is moderate, successive solution points can be located far away and look odd when the velocity is high. If you try correcting this by lowering the value in **Step**, you can find that the program spends too much time when plotting the part of the solution corresponding to low velocity. If this happens, you can try disabling the interpolation, because often the integration routine automatically computes more points when the velocity is higher. You can then use **Max. h** and **Tolerance** (in the **Method** dialog sheet) to control how far away consecutive points are plotted. (See the example in `DIPOLE.DS`).

It is possible to skip the output of a graphics or text result, maybe because it lies outside the range of interest and will only waste memory. See **Skip function**.

Forcing the Program to Wait

Normally *Dynamics Solver* will output the integration results as fast as it can, and this is what you will prefer in most cases. But sometimes you may want to have the time to see each new point is plotted or each new numerical value. Then, use the **Pause** entry in the **Range** dialog sheet to select the time (in tenths of second) the program will wait after each solution point is computed and before integration continues. By default this value is null (i.e., there is no pause). By default this value is null (i.e., there is no pause). The status line will display `Pausing` and you may press any key to stop the current pause (if the value is negative, you *must* press a key to break the pause). You are reminded to do this by the bottom line of the screen. See the examples in `CHAOS\DISKS.DS` and `GRAPH.DS`.

Furthermore, in numerical simulations you may want the real time between two successive outputs to be proportional to the corresponding range of the independent variable (or index). For instance, when simulating the motion of a body around a gravitational center, you may want the output to be faster when the body is nearer the center, because as a consequence of Kepler's second laws the velocity will be higher there. You may simulate the real time to some extent by setting a non-zero value in the **Real time** entry of the **Range** or **Iteration Range** dialog boxes. Since the simulation of real time has some overhead and the time resolution of Windows is rather limited, you may need to experiment somewhat with the

value in **Real time** in order to get good results. Better results are obtained with a fast computer (and by checking in the **Interpolate** entry in the Range dialog sheet for a continuous system). See the examples in MECH\KEPLER.DS and R3BODY1.DS.

Storing the Solution in Memory

Dynamics Solver computes (if **Saved points** is different from zero, as it is by default) not only some solution points, but also a piecewise polynomial approximation to the global solution. This allows you integrating many functional-differential equations, to compute periods, Poincaré sections and derivatives of the solution and to have a dense graphical output with a moderate computation effort. The same possibility is useful for advanced purposes and to deal with general recurrences, if you do not desire to write them in the form of an iterated map (see **Discrete Dynamical Systems: Iterated Maps and Recurrences**).

The number of points saved in memory is the value in the **Saved points** entry of the **Range** dialog sheet (which can be activated from the Edit/**Range** menu entry or by pressing `Ctrl+R`).

The maximum value that can be used here is bounded by the amount of free memory but usually a rather low value will be enough.

Integration Step and Integration Error

Numerical codes to solve differential equations proceed by steps. After computing the approximations to the solution for a certain number of points, t_0, t_1, \dots, t_n , a new step is performed to find the approximation to the solution at next point $t_{n+1} = t_n + h$. In general (but not always because of the finite precision arithmetic used by any computer), choosing a smaller step length h gives a better approximation to the real solution, but more time will be spent because more points must be computed. In practice, a compromise between numerical accuracy and integration time must always be reached. There is no general rule to do that, only practice and knowledge of the problem will help, but *Dynamics Solver* lets you try different choices with great ease.

The exact relation between integration step length and solution error is unknown except in academic examples. Furthermore, in different pieces of a single solution, different integration errors occur with the same step length. So in general you would prefer to select the accuracy goal the integration routine must meet instead of the step length. This is precisely the approach taken in *Dynamics Solver*. The program will automatically decrease the integration step if the error is too big, and when the error becomes smaller, the step will increase to save computational effort.

As a final remark, notice that “local errors” are being addressed here; that is, the errors that the approximate nature of the numerical code produces when a single step is computed, even if the starting point is completely exact. The error that is accumulated along the full process of computing the solution in a given interval is called, for obvious reasons, the “global error,” and its relation with the integration step is even more involved. Nevertheless, smaller integration steps would give smaller errors, as long as the step does not become too small.

The previous discussion does not apply to discrete dynamical systems because their solution is straightforward and the only error is due to the finite precision of computer arithmetic.

Selecting the Integration Method

To solve discrete dynamical systems *Dynamics Solver* simply uses their definition, because the solution process is trivial. To integrate differential equations, *Dynamics Solver* uses the routine selected in the **Integration code entry** of the **Method** dialog sheet (which can be activated from the Edit/**Method** menu entry or by pressing `Ctrl+H`). In the same sheet you choose the **Tolerance**, **Step h** and **Max. h** values, as well as any other parameter of the integration code.

Currently 17 integration codes are distributed with *Dynamics Solver*, but you can easily add your own method and install it by using the Configuration/**External codes** menu. See **Writing External Integration Codes**.

First of all, note that the default method, **Dormand-Prince 8(5,3)**, and the alternative **Dormand-Prince 5(4)** are the most general, robust and flexible methods in *Dynamics Solver*. They will work in most cases and you will not use another method except in special cases, because the remaining routines have limitations that do not apply to these two codes. For instance, if retarded values of the variables (in delay-differential equations or computing periods) or accurate interpolated values (when computing Poincaré sections or derivatives of the solution) are needed, these are the method of choice. Nevertheless, for stiff problems you should use **Implicit Gear** and for quick solution of very regular problems you could try **Extrapolation**. Of course, you are free to experiment and to compare methods. *Dynamics Solver* makes it easy.

The available codes can be classified as follows:

Code	Type	Order	Step size	Interpolation
Dormand-Prince 8(5,3)	embedded Runge-Kutta	8(5,3)	variable	7th order polynomial
Dormand-Prince 5(4)	embedded Runge-Kutta	5(4)	variable	4th order polynomial
Dormand-Prince 8	embedded Runge-Kutta	8(5,3)	variable	Hermite (3rd order)
Extrapolation	extrapolation	variable	variable	Hermite (3rd order)
Cash-Karp 5	embedded Runge-Kutta	5(4)	variable	Hermite (3rd order)
Runge-Kutta-Fehlberg	embedded Runge-Kutta	4(5)	variable	Hermite (3rd order)
Adams-Bashforth-Moulton	predictor-corrector	4	variable	Hermite (3rd order)
Implicit Gear	implicit	4	variable	Hermite (3rd order)
Gear	predictor-corrector	5	fixed	Hermite (3rd order)
Hamming	predictor-corrector	4	fixed	Hermite (3rd order)
Runge-Kutta 4	Runge-Kutta	4	fixed	Hermite (3rd order)
Adams-Bashforth-Moulton 4	predictor-corrector	4	fixed	Hermite (3rd order)
Milne	predictor-corrector	4	fixed	Hermite (3rd order)
Runge-Kutta 2	elementary	2	fixed	Linear (1st order)
Adams-Bashforth-Moulton 2	elementary	2	fixed	Linear (1st order)
Heun	elementary	2	fixed	Linear (1st order)
Euler	elementary	1	fixed	Linear (1st order)

Type of the Integration Code

Although predictor-corrector methods are faster than Runge-Kutta of the same order and extrapolation methods are, in general, even faster, Runge-Kutta methods are more reliable and robust and continue working when the others fail. Elementary methods can often be classified in more than one type and should not be used except for pedagogical purposes (i.e., to see the reason not to use them). Implicit methods are codes of choice for stiff systems.

Order of the Integration Code

In a method of order n , the local error in a step of length h is $O(h^{n+1})$. In embedded Runge-Kutta methods two simultaneous methods of different order are used. The first one actually integrates the problem. The second one, whose order appears between parentheses, is used to estimate the error in order to change appropriately the integration step. Extrapolation methods typically have a very high variable order. Roughly speaking, higher order means faster and more accurate methods. Methods of order 3 and below are described in the table as elementary and, as discussed before, should not be used except in special cases.

Step Size of the Integration Code

Methods with a variable step size choose automatically the step length to keep the estimated relative error of each step below **Tolerance** (in the **Method** dialog sheet). They are much faster and easier to use than

methods that use the fixed step length in **Step h** in the same dialog. (For a complete discussion including the way in which the estimated error can be used to change the step size, see Hairer [1993].)

Interpolation in the Integration Code

Numerical methods compute only the (approximated) values of the solution at some discrete points. To estimate the remaining values, interpolation must be used at least in the following cases:

1. Dense output. Graphics output looks often awkward if successive points are simply joined with a segment. Of course, the problem can be solved if more points are computed by choosing a lower **Tolerance** (or **Step h**, in fixed step methods), but interpolation is much faster.
2. Computing Poincaré sections.
3. Computing derivatives.
4. Solving equations with retarded arguments.

The linear interpolation of elementary methods is very poor and a waste of time for dense graphics output.

The Hermite interpolation uses a third order spline that passes through two consecutive points with prescribed tangents. This gives acceptable dense output except with the very long steps that may arise when using **Extrapolation** and a moderate **Tolerance**.

The interpolation of **Dormand-Prince 8(5,3)** and **Dormand-Prince 5(4)** methods is very good. It should be used when computing derivatives and Poincaré sections and to solve equations with retarded arguments.

In the following sections you can find a short description of each routine. To obtain more information, you should refer to the help file describing each method: it can be activated from the **Examine** button of the **Install External Methods** dialog box (which is accessed from the Configuration/**External codes** menu). The help file corresponding to the currently selected method is also available from the **Help** menu.

Dormand-Prince 8(5,3)

This integration code is based on the embedded Runge-Kutta method of order 8 with automatic step size control developed by Prince and Dormand in 1981, as described in Hairer [1993]. It is a continuous method that provides a global piecewise polynomial approximation of seventh order to the solution. The latter is thus available for dense graphical output, to solve a large class of functional-differential equations, to calculate derivatives of the solution and to compute Poincaré sections. This routine and **Dormand-Prince 5(4)** are the most robust and flexible methods in *Dynamics Solver*. They will work in most cases and you should use one of them except in special cases. Note that the remaining routines have limitations that do not apply to these two codes. This method can be used when high precision results are needed, because for stringent **Tolerances** (say $1E-7$ and below), the results are usually better and faster than those provided by **Dormand-Prince 5(4)**. Since this method is too complex to be described here, you should refer to the mentioned reference.

Dormand-Prince 5(4)

This is an **embedded Runge-Kutta method** with automatic step size control of order 5(4) developed by Dormand and Prince in 1980. It is a continuous method that provides a global piecewise polynomial approximation of fourth order to the solution. The latter is thus available for dense graphical output, to solve a large class of functional-differential equations, to calculate derivatives of the solution and to compute Poincaré sections. This default value and **Dormand-Prince 8(5,3)** are the most robust and flexible methods in *Dynamics Solver*. They will work in most cases and you should use one of them except in special cases. Note that the remaining routines have limitations that do not apply to these two codes. This method is described in detail in Hairer [1993].

Dormand-Prince 8

This **embedded Runge-Kutta method** is included for pedagogical purposes, because it is **Dormand-Prince 8(5,3)** with a poorer interpolation routine. It uses a third order Hermite interpolator. This fact and

might give poor results when integrating functional-differential equations or computing Poincaré sections and derivatives (only up to the third one) with moderate values of **Tolerance**. For the same reason, graphics output often looks awkward or inconsistent. Fourth order derivatives of the interpolated solution are unavailable. Since this method is too complex to be describes here, you should refer to Hairer [1993].

Extrapolation

A variable order extrapolation method with automatic step size control based on the code ODEX from the previous reference. It uses the explicit midpoint rule and polynomial extrapolation. It can be used when very high precision results are needed. But, the current version uses a third order Hermite interpolator. This fact and the typical long steps of this method can give poor results when integrating functional-differential equations or computing Poincaré sections and derivatives (only up to the third one) with moderate values of **Tolerance**. For the same reason, graphics output often looks awkward or inconsistent. This is not usually a problem for low values (say 1E-8 and below) of **Tolerance**. Fourth order derivatives of the interpolated solution are unavailable.

Still, for low values of **Tolerance**, the results are often far better and much faster than those provided by **Dormand-Prince 8(5,3)**.

For a complete discussion including the way in which the order and step size are controlled, see Hairer [1993].

Cash-Karp 5

A fifth order **embedded Runge-Kutta method** with step size control. Hermite interpolation is used. It is not very different from **Dormand-Prince 5(4)**, but the interpolation is poorer. It is discussed in detail in Press [1992] (but the routine in Dynamics Solver uses the step size control of Hairer [1993]).

Runge-Kutta-Fehlberg

The classical **embedded Runge-Kutta-Fehlberg method** of order 4(5) with automatic step size control. Fourth order derivatives of the solution are unavailable due to the use of a third order Hermite interpolation.

Adams-Bashforth-Moulton

The classical fourth order Adams-Bashforth predictor and Adams-Moulton corrector with automatic step size control. Hermite interpolation is used for retarded values. Fourth order derivatives of the **solution** are unavailable. The final and the first three steps are computed by means of **Runge-Kutta 4**.

Implicit Gear

The fourth-order implicit method of Gear with automatic step size control. It is specially suited to deal with stiff systems (especially with moderate tolerances) and is described in Engeln [1996]. (Try it with `NUMERIC\OREGO.DS`, or `NUMERIC\EULER.DS` for instance.) Hermite interpolation is used for retarded values and interpolation.

Fourth order derivatives of the solution are unavailable and it cannot be used to integrate backward.

Gear

The classical fifth order predictor-corrector method of Gear with fixed step size. The first and final steps are computed by using a **Runge-Kutta 4** method with a step equal to **Step h** divided by 10. Hermite interpolation is used for retarded values and interpolation.

It can be used for pedagogical purposes. Fourth order derivatives of the solution are unavailable.

Hamming

The classical fourth order predictor-corrector method of Hamming with fixed step size. The first and final steps are computed by using a **Runge-Kutta 4** method with a step equal to **Step h** divided by 2. Hermite interpolation is used for retarded values and interpolation.

It can be used for pedagogical purposes. Fourth order derivatives of the solution are unavailable.

Runge-Kutta 4

The classical fourth order Runge-Kutta method with fixed step size. Hermite interpolation is used for retarded values. It can be used for pedagogical purposes and to test difficult cases, because it is very reliable (and slow). Fourth order derivatives of the solution are unavailable.

This is the most popular and most widely used routine. Many people do not feel comfortable until they have tested at least some special cases with it. (But nowadays one should probably use instead **Dormand-Prince 5(4)**.)

Adams-Bashforth-Moulton 4

The classical fourth order Adams-Bashforth predictor and Adams-Moulton corrector with fixed step size. Hermite interpolation is used for retarded values. It can be used only for pedagogical purposes. Fourth order derivatives of the **solution** are unavailable. The final and the first three steps are computed by means of **Runge-Kutta 4**.

Milne 4

The classical fourth order predictor-corrector method of Milne with fixed step size. The first and final steps are computed by using a **Runge-Kutta 4** method. Hermite interpolation is used for retarded values and interpolation.

It should be used only for pedagogical purposes, because it is unstable. (See, for instance, R3BODY.DS with equal to **Step h** = 0.003, 0.001.) Fourth order derivatives of the **solution** are unavailable.

Runge-Kutta 2

The complete family of second order Runge-Kutta method with linear interpolation and fixed step length. It should be used only for pedagogical purposes. Only first derivatives of the solution are available.

This method has an additional parameter, Ω . For $\Omega = 1$ this method is the modified Euler method, for $\Omega = 1/2$ the Heun method and for $\Omega = 2/3$ the method in the family having the optimal error bound.

Adams-Bashforth-Moulton 2

The classical second order Adams-Bashforth predictor and Adams-Moulton corrector with fixed step size. Linear interpolation is used for retarded values. It should be used only for pedagogical purposes. Only first derivatives of the **solution** are available. The first and final steps are computed by means of **Heun**.

Heun

The classical second order Heun method with fixed step size. Linear interpolation to compute retarded values. It is both a second order Runge-Kutta and a predictor-corrector method. It should be used only for pedagogical purposes. Only first derivatives of the solution can be computed.

Euler

The elementary first-order Euler method with fixed step size. Linear interpolation to compute retarded values. It should be used only for pedagogical purposes, because it is very unstable and very slow. Only first derivatives of the solution can be computed.

Selecting the Integration Step

When solving differential equations in *Dynamics Solver* by means of a variable step method, you do not select directly the value of the integration step, but rather the accuracy the solution must have.

Set in the **Tolerance** entry of the **Method** dialog sheet (which can be activated from the Edit/**Method** menu entry or by pressing `Ctrl+H`) the maximum error *Dynamics Solver* must accept.

The exact meaning of this parameter is rather technical and lies beyond the scope of this manual. The interested user must refer to the complete explanation given in Hairer [1993]. Suffice it to say that the Dormand-Prince method used by default (see **Selecting the Integration Method**), computes simultaneously two approximations of each solution point. By comparing them, the routine is able to make an estimate of the integration error. If it finds that this probable error is greater than the value in **Tolerance**, the integration step length will be decreased. If, on the contrary, the estimated error is lower than **Tolerance**, the step length will increase, though it will never be larger than the value **Max. h**.

In fact, the best practical rule to use to know if the solution just computed is accurate enough, is to repeat the calculation with a lower **Tolerance** to see if the results change significantly. Just as in other branches of numerical calculus, integrating differential equations is something of an art and requires some experimentation. Previous experience with numerical calculus and knowledge of the concrete problem are the best aids in this task. *Dynamics Solver* is designed precisely to help try different values and to increase knowledge of the problem quickly.

The program automatically increases or decreases the effective integration step to fulfill the specified **Tolerance**. You cannot directly set the step length; but two related values can be controlled from the same dialog sheet. Though in most cases the default values will work fine, in some instances you may need to change them. The first value, **Step h**, is the starting step length. When *Dynamics Solver* tries for the first time to find the step length appropriate for the current setting of **Tolerance**, it must start by using a given step to estimate the magnitude of the error and, then, the effective step length. This is precisely the meaning of **Step h**. Most often any value will be adequate here, but sometimes an error occurs when starting or continuing the integration. The most frequent cause is some singularity in the equations, but it is also possible to have an initial value that is too high for the integration step, **Step h**. In this case, try lowering it.

You can also control the maximum value of the integration step with the **Max. h** entry of the same dialog sheet. The main reason to change this entry will be discussed in **Interactive Solution**: if, when trying to stop an integration the `Esc` key (or the **Go/Stop** menu entry), does not get immediate attention, it may be due to having values too high for **Step** or **Max. h**. You could also find convenient to select lower values for those entries in order not to miss points in Poincaré sections (or satisfying some special conditions).

In methods with fixed step size, its value is entered in **Step h**, and **Max. h** and **Tolerance** are ignored.

Output Parameters

If we have specified a well-posed mathematical problem (a dynamical system and a set of initial or boundary conditions) and selected the right solution parameters, as described in previous chapters, *Dynamics Solver* is able to start solving (approximately, of course) the problem; but it would be completely worthless right now because we have also to indicate what kind of results we want, and in what format we want them. In other words, we have to select the output options.

The good news is that *Dynamics Solver* is able to output any general mathematical expression involving the solution and the parameters of the problem, and it can do it in different formats. This lets you analyze any quantity related to the problem, for instance, the energy or the angular momentum of a mechanical system. Furthermore, you can analyze at any time even previously computed solution points (as long as they are still in memory).

Probably the most useful output format in *Dynamics Solver* is the graphical output. You will see the solution (or the desired quantities) plotted on one or some windows as it is being computed by the program. You can easily stop and continue the integration, and select and change the part of the solution space to be seen on the screen. The results can be viewed (in perspective or not) from any direction, and they can be plotted in any available color, in the form of discrete points or as a continuous curve. Usually you will use the continuous line, but the discrete format will be preferable for discrete dynamical systems, or if the solution points are unrelated or if you want to have the solution points at regular intervals of time, for example. This stroboscopic feature lets you have simultaneous information on three quantities in a two-dimensional screen. You can open several windows to view different quantities or different views of the same quantities. It is also possible to draw the direction field of the equation or system under study. Finally, you may have multiple output in a single window (to see how evolve multiple bodies, or to solve simultaneously for more than one set of initial conditions, for instance,).

The graphics screen can be printed (in different ways), and for higher quality drawings you can use an HPGL compatible plotter, a PostScript printer or any other device for which an appropriate driver is available. You can also add letters and other graphical elements.

It is also possible to have any number of quantities in text format and one can send them to an external file. The output file can be the printer (if you want a usually long list of points) or any ASCII file. The latter is especially useful for additional numerical or graphical processing by means of *Dynamics Solver* or other programs. Since the output format is very flexible, you will be able to use the results obtained with *Dynamics Solver* in most programs.

It is also possible to have simultaneous output of one or two additional quantities in numerical format at the bottom of the screen, in the status line. This can be useful to see the current value of the time (or of the actual independent variable) or of any important quantity (such as the energy) that you want to monitor. For instance, this lets you test if a conserved quantity is really kept constant by the numerical algorithm. This is often a good test of the quality of the integration method and lets you see if the system obtained from the original problem, by taking derivatives or by introducing additional variables, is being integrated appropriately (see **Dynamical Systems** and **Initial Values**). You may use any two general quantities and select the preferred frequency output and the format to be used.

See also **Graphics Elements, Printing, Advanced Procedures, Phase Portraits, Projections, Direction Field, Poincaré Sections, Bifurcation Diagrams, Repeated Integration/Iteration, Constrained Initial Conditions, Periods, Liapunov Exponents, Histograms, Cobwebs and Some Examples.**

Graphics Output

Use the Output/**New graph window** menu entry (or press `Ctrl+G`) to open a new window for graphics output. After creating the new window, *Dynamics Solver* will open the **Graphics Output** dialog box (which you can invoke at any time from the Output/**Graphics format** menu or by using `Ctrl+F`). In its four sheets you can define and control the graphics output.

You have to enter in the **Horizontal axis** and **Vertical axis** entries the expressions to be displayed in each axis. By default they are x and $x[1]$ for equations and $x[1]$ and $x[2]$ for systems, but you may want to

display other variable, derivative or full expression. The default settings are intended to plot the phase space (position vs. velocity) of a two-dimensional dynamical system, but by selecting appropriate expressions, you can see on the screen the evolution of a single variable or more general quantity in a (t,x) diagram. In a three-dimensional problem you may prefer the projection of the solution in a different coordinate plane, etc. Any two expressions can be used, and they can involve the independent variable, the solution and its derivatives at the current or previous values of the independent variable, the parameters in the problem, and the initial conditions.

It is easy to undervalue the power and flexibility of having the full solution through the interpolated functions $x(i, t)$. Try using it. As an advanced use, you can easily draw pseudo phase spaces by plotting $x(t)$ vs. $x(t-T)$ (see Moon [1987]). For example, very interesting results are obtained with Lorenz equations by setting **Horizontal axis** and **Vertical axis** to $x[1]$ and $x(1, t-1)$, respectively. Note that, since one of the values is retarded, you must set **First value** in the **Range** dialog sheet to 1.

You need a graphics window for each pair of quantities you want displayed on the screen. By default, they will be plotted in their own window (and the **Window** entry of the **Graphics Output** dialog box will have its default value: `this window`). But in some cases you will prefer to output to a single window more than one pair of expressions, in order to see how several members of a system, for instance. This can be easily accomplished: you still need a graphics window for each pair of quantities, but you may select in the aforementioned **Window** entry the caption of another graphics window. In this case, all output parameters (color, pen width and so on) will continue to be selected in their window, but the corresponding graphics output will be redirected to the window whose caption is selected. Notice that plotter output will not be redirected (you may get this, at your own risk, by specifying the same plotter for different windows).

Output Ranges

You can select in the same **Graphics Output** dialog box (which you can invoke at any time from the **Output/Graphics format** menu or by using `Ctrl+F`) the ranges displayed in the two axes:

1. In **Min. x**, set the minimum value along the x -axis.
2. In **Max. x**, set the maximum value along the x -axis.
3. In **Min. y**, set the minimum value along the y -axis.
4. In **Max. y**, set the maximum value along the y -axis.

Note also that, as explained in the next chapter, these values can also be changed in a more intuitive way by zooming in or out on the current window or selecting with the mouse a portion of the screen.

You can also select (by using Windows standard methods for the mouse and the cursor keys) the part of the physical screen to be used for graphical output. In some cases (when displaying special solutions that are expected to have a certain form, circular for instance) you may want to have always a 1:1 aspect ratio by selecting in the **Aspect ratio** entry of the **Format** sheet in the same dialog box one of the following values:

Value	Meaning
Arbitrary	Ignore aspect ratio
Adjust X range	Change Min. x and Max. x
Adjust Y range	Change Min. y and Max. y
Square	The window is always square
Adjust width	Change the window width
Adjust height	Change the window height

In very special cases, especially when using repeated integration or drawing bifurcation diagrams, you want to prescribe exact values, in pixels, for the width and/or height of the client part of the output window, instead of choosing the window size with the mouse or keyboard. This can be achieved by entering the corresponding value(s) in the **Width** and **Height** entries of the same dialog sheet. Keep in mind that null values for these entries are ignored and that Windows sets minimum values for window dimensions.

Remember that you need a graphics window for each pair of quantities you want displayed on the screen. By default, they will be plotted in their own window (and the **Window** entry of the **Graphics Output** dialog box will have its default value: `this window`). But in some cases you will prefer to output to a single window more than one pair of expressions, in order to see how several members of a system, for instance. This can be easily accomplished: you still need a graphics window for each pair of quantities, but you may select in the aforementioned **Window** entry the caption of another graphics window. In this case, all output parameters (color, pen width and so on) will continue to be selected in their window, but the corresponding graphics output will be redirected to the window whose caption is selected. Notice that plotter output will not be redirected (you may get this, at your own risk, by specifying the same plotter for different windows). The **Min. x**, **Max. x**, **Min. Y** and **Max. y** values of the window that will receive the output will be used. If you need different values for each pair of quantities, you have to scale the corresponding expressions.

In `MECH\BURRAU.DS` the evolution of three bodies is displayed in a single window while in `CHAOS\DUFFING4.DS` you may see how to use this feature to solve simultaneously for more than one single set of initial conditions.

Screen and Solution Attributes

You can change the appearance of the screen and the solution being drawn on it by changing the following settings in the **Format** sheet:

1. Use the **Color** button to open the **Colors** dialog box (which you can also invoke at any time from the **Output/Color** menu, from or by using `Ctrl+O`). There you can select the colors and pen widths used to display solutions, axes, direction fields and background. The new colors will be used for the next solution curve or if the **Window/Erase** or **Erase all** entries (or the equivalent `Del` and `Ctrl+Del` keys) are used. You can also use the `PenColor`, `PenWidth`, `HSV` and `RGB` functions in the **Horizontal axis** and **Vertical axis** entries of the **Graphics Output** dialog box.
2. Enter in the **Axes** entry a list of straight lines to be drawn on screen. They can be used as rudimentary axes or to mark interesting points. Each line is indicated by means of a set of four numbers, a , b , c and d , separated by spaces. They give the axis start (a,b) and end (c,d) . Sets of coordinates corresponding to different lines are also separated by spaces. This entry is provided mainly for backward compatibility. An alternative and more powerful alternative is provided by the use of graphics elements.
3. Use the **Continuous line** entry to indicate if you want successive points drawn on the screen joined by a segment. The solution looks better with this setting selected, unless you are solving a discrete dynamical system, drawing a Poincaré section or you want a stroboscopic analysis of the solution.

Plotting Functions and Curves

Dynamics Solver to be used as a general tool for evaluating and plotting functions and parametric curves in two and three dimensions: you may use a **graphics element** of type **2-curve** or **3-curve**, but there is also another, more convoluted way, that however allows using the arrow keys and the mouse cursor to select some function parameters.

Since you have no actual equation to integrate, you can select the simplest one: $y' = 0$. *Dynamics Solver* is obviously able to integrate such a simple equation with very high accuracy, so you should choose a rather low value for **Max. h** in the **Method** dialog sheet, say 1, to avoid a long wait each time you want to stop the integration.

To plot or evaluate the function $f(t)$ it is enough to choose t in the **Horizontal axis** entry and $f(t)$ in **Vertical axis**. For a parametric curve in the plane in the form $x = f(t)$, $y = g(t)$, you must choose $f(t)$ and $g(t)$. In a similar way, for a projection of a three-dimensional curve given in the form $x = f(t)$, $y = g(t)$, $z = h(t)$, you must choose $f(t)$, $g(t)$ and $h(t)$.

$= h(t)$, the right values are $\text{proj}_x(f(t), g(t), h(t))$ and $\text{proj}_y(f(t), g(t), h(t))$. You should also define the range of t by means of **First value** and **Last value** in the **Range** dialog sheet.

As usual, you can have parameters and two of them, $x_0[1]$ and $x_0[2]$, can be selected with the cursor.

An alternative and better way is using a graphics element of type 2-curve or 3-curve.

Saving and Loading Graphics and Text Screens

The current screen can be saved to the disk or to the clipboard (and retrieved from there) by using the **Window** menu:

1. Use **Get screen** (or press **Ctrl+C** or **Ctrl+Ins**) to save to the clipboard the current graphics or text screen. The graphics formats saved to the clipboard are controlled in the **Copy to clipboard** entries of the **Graphics** sheet of the **Preferences** dialog box.
2. Use **Put screen** (or press **Ctrl+V** or **Shift+Ins**) to load from the clipboard a graphics screen. The **Mode** dialog box will open to let you select the format to be pasted and the way to handle the previous size and contents of the screen.
3. Use **Save screen** to save to the disk the current graphics or text screen, after selecting the file name and type in the corresponding **Save Screen as** dialog box.
4. Use **Load screen** to load from the disk a graphics screen, after selecting the file name in the corresponding **Load Screen From** dialog box. The **Mode** dialog box will open to let you select the way to handle the previous size and contents of the screen.

The Windows standard formats for graphics files that are copied to the clipboard are controlled from the, **Bitmap output** and **Save output in memory** entries in the **Format** sheet and from the **Copy to clipboard entry** in the **Preferences/Graphics** dialog sheet.

In the metafile format, the integration/iteration results will be saved only if the **Save output in memory** entry in the **Format** sheet was not null when they were computed.

In the bitmap format, you could get only a partial window snapshot if the **Bitmap output** entry in the **Format** sheet was **No bitmap** when they were computed and the current windows is not completely visible.

Notice that, unlike graphics windows, text windows may be directly saved but not restored. You can still retrieve them from the disk or from the clipboard to any text editor or to an *Dynamics Solver* **Edit Window**.

Text Output

Use the **Output/New text window** menu entry (or press **Ctrl+T**) to open a new window for text output. After creating the new window, *Dynamics Solver* will open the **Text Output** dialog box (which you can invoke at any time from the **Output/Text format** menu or by using **Ctrl+X**). There you can define and control the expressions to be output, along with the corresponding format strings and other control parameters.

If the **Lines in memory** is not null, the lines currently held in the text window can be saved to the clipboard or to an external text file (see **Saving and Loading Graphics and Text Screens**).

You may also want to save to the disk the complete (and often long) text output as described in **Sending Output to External Files**.

It is possible to skip the output of a graphics result, maybe because it lies outside the range of interest and will only waste memory (and disk space if it is being sent to an external file). See **Skip function**.

Numerical Output

Dynamics Solver can use the last line of the screen (the status line) to display (among other useful information) the numerical values of two expressions which are entered, along with the corresponding format strings and other control parameters in the **Numerical Output in Status Line** dialog box, which is accessed from the Output/**Status line** menu (or by pressing `Ctrl+S`).

This possibility is handy while solving a problem, at least, to monitor conserved quantities and constraints (refer to **Constrained Initial Conditions**), to know where the solution is if it is not visible on the screen or it changes slowly and to know the current value of any variable or expression. Nevertheless, you should take into account that the information in the status line disappears easily, because that line has many functions.

You can also issue a single numerical x value in its own message box by using `message(x)`.

To have a more permanent numeric output use **Text Output**.

Sending the Results to an External File

If you want to save in an external text file one or several numerical quantities (they can be completely generic expressions) in order to process them by means of another program or from *Dynamics Solver* as a data graphics element, proceed as follows:

1. Use the Output/**New text window** menu entry (or press `Ctrl+T`) to open a new window for graphics output. After creating the new window, *Dynamics Solver* will open the **Text Output** dialog box (which you can invoke at any time from the Output/**Text format** menu or by using `Ctrl+X`). There you can define and control the expressions to be output, along with the corresponding format strings and other control parameters.
2. Though this is not important for the text seen on the screen, you should probably make sure that the `\r\n` control characters end the last format string.
3. Set on the **Capture** entry in the same dialog box and use **Output file** and **Browse** to select the name and location of the text file.
4. Use the **Separator** entry to enter the string to be inserted after each run (which would correspond to a polygonal line if drawn on the screen with the **Continuous line** option selected in). If the text file is to be used for a data graphics element, the default `\r\n` control characters are fine, but for other programs you might have to change this.
5. Select in **Output frequency** the number of points that have to be draw on the screen to have a line output to the file. Since *Dynamics Solver* is able to generate many solution points very quickly, you might want to select here a high value (say form 10 to 1000) in order to avoid generating very long output files.
6. You might want to minimize the text window to avoid seeing a long list of numbers, which slow down the global operation.

A more primitive (but sometimes useful) way of sending output to external files is using the mathematical functions `openwrite`, `write` and `deletewrite` to write to output files.

See also **Reading *Dynamics Solver's* Output in Mathematica**

Reading Values from the Keyboard or External Files

It is possible to use `input(i)` to ask the user for a single numerical value in an **Input Value** dialog box.

On the other hand, a primitive but sometimes useful way of reading input from external files is using the mathematical functions `openread`, `read` and `resetread` to read from input files.

Reading *Dynamics Solver's* Output in Mathematica

The output from *Dynamics Solver* can be easily read in Mathematica. Let us assume that you use *Dynamics Solver* to generate a data file, `C:\W\PENDULUM.DAT`, by using the `PENDULUM.DS` problem file. If you use the procedure described in **Selecting Output to the Status Line** (make sure that the `\r\n` control characters end the last format string and that **Separator** has its default value: the `\r\n` characters) and you send for each solution point its two coordinates, x and y , you can easily read the points in Mathematica by using the following command:

```
data = ReadList["C:\\W\\PENDULUM.DAT", {Real,Real},
               RecordSeparators->{"\n\n"}, RecordLists->True];
```

The result data will contain a list of lists in the form:

```
{{{x1,y},{x2,y2},...,{xn,yn}},{x1,y},{x2,y2},...,{xn,yn}},{x1,y},{x2,y2},...,{xn,yn}}.
```

Each sublist, $\{x_1, y_1, \dots, x_n, y_n\}$, contains the coordinates of the points generated in a run (which starts when you use `Enter` or `Go/Start` and end when using `Esc` or `Go/Stop`) and they would be plot as a polygonal line in a graphics window if a continuous line is used for output).

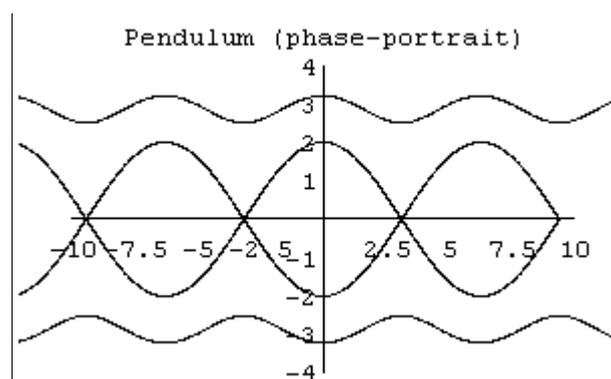
In the same way you can easily write a function that automatically reads and displays (with your preferred graphics options) the data file:

```
PlotDS[filename_,opts___] := Module[{r},
  r = ReadList[filename,{Real,Real},
              RecordSeparators->{"\n\n"},
              RecordLists->True];
  r = Map[ListPlot[#,DisplayFunction->Identity]&,r];
  Apply[Show,Union[{r,DisplayFunction->${DisplayFunction}},{opts}]]
]
```

So using the command

```
PlotDS["C:\\W\\PENDULUM.DAT",
       PlotRange->{{-10,10},{-4,4}},
       PlotLabel->"Pendulum (phase portrait)"]
```

you will get the following drawing:



Notice the use of an explicit `PlotRange` option to clip the data. You might need to select in a Mathematica menu the **Make Lines Thin** entry to have thin lines as shown in the previous figure.

Also, remember to use in *Dynamics Solver* the **Output frequency** option (in the **Text Output** dialog box) to control the number of points, because Mathematica is rather slow plotting long lists.

Format String

Format strings are used to send numerical output to text windows, external files and the status line, and to read data from external files. They are ASCII character strings which may contain special escape characters that start with a backslash, \, and are translated according to the following table:

Sequence	C form	Value	Meaning
\n		<i>n</i>	Explicit decimal value
\a	\a	7	Bell character
\b	\b	8	Backspace
\e		27	Esc character
\f	\f	12	Form feed
\n	\n	10	New line (line feed)
\r	\r	13	Carriage return
\on	\n	<i>n</i>	Explicit octal value
\t	\t	9	Horizontal Tab
\v	\v	11	Vertical Tab
\xn	\xn	<i>n</i>	Explicit hexadecimal value
\z		0	Null character

When reading from input files, the string remaining after this translation is interpreted as a **C** format string that is directly passed to the standard **scanf** routine which returns a double precision number. This number is represented in the string as indicated in a format field of the following form:

```
%[Width]lg
```

where the *Width* element is optional and, if present and equal to *n*, indicates that at most *n* characters are to be read.

When writing numbers, the string remaining after that translation is interpreted as a **C** format string that is directly passed to the standard **sprintf** routine along with one or more double precision number. Each of these numbers is represented in the string as indicated in a format field of the following form:

```
%[Flags][Width][Precision]Type
```

where the elements that appear between square brackets are optional. The meaning of these fields is as follows (*n* is a decimal number):

Flags	
-	Left justify the result
+	Always prefix with + or -
<i>space</i>	Prefix with a space if nonnegative.
#	Print always a decimal point and do not remove trailing zeroes

Width	
<i>n</i>	Print at least <i>n</i> characters padded with spaces
0 <i>n</i>	Print at least <i>n</i> characters padded with zeroes

Precision	
.0	Print no decimal point in e, E, and f formats
. <i>n</i>	Print <i>n</i> decimal places

Type	
e	The number is printed in the form [-]d[.ddd...]e{+/-}ddd
E	The number is printed in the form [-]d[.ddd...]E{+/-}ddd
f	The number is printed in the form [-]ddd[.ddd...]

g	Best of e o f without trailing zeroes
G	Best of E o f without trailing zeroes

Example:

```
x = %10.2f\n
```

Print a number after `x =` , right-justified in a field of 10 characters, padded with blanks and followed by a new line character. The latter is useful when you are sending the results to a file. The value is presented in the form `[-]dd.dd` with exactly two digits after the decimal point.

Interactive Solution

After setting a problem (as described in **Dynamical Systems, Initial Values, Boundary Conditions, Solution Options, and Output Parameters**), you are ready to start solving it. This chapter describes how to do that in an interactive and user-friendly way.

Selecting Initial Conditions on the Screen

Before starting solving the problem, remember that if the current *Dynamics Solver* window is a graphics window, a cursor in the form of a flashing cross inside a circle will appear on the screen. Its coordinates are permanently indicated in the middle panel of the bottom line. They are two initial conditions or parameters whose value can be changed by merely moving the cursor with the mouse pointer or the direction keys. The two initial conditions selected in this way are by default x and $x[1]$ for an equation and $x[1]$ and $x[2]$ for a system, but this can be changed in the **Variable in axis X** and **Variable in axis Y** entries of the **Cursor** dialog sheet of the current graphics windows.

This possibility is very useful to interactively change initial conditions (or parameters). You can select easily the solution starting at a certain point, or arriving to it. Simply start from the selected point and integrate forward or backward. It can also be used to find the coordinates of interesting points in the solution window.

To move the cursor (and change the initial conditions or parameters) you can use the direction keys, which will move the cursor by an amount equal to the corresponding elementary step value selected in the entries **Horizontal step** and **Vertical step** of the **Cursor** dialog sheet. Holding down the `Ctrl` key while using a direction key will move the cursor by a length equal to ten times the corresponding elementary step value. The cursor elementary step length can be doubled (halved) by pressing `*` (`/`) in the numeric keypad.

You may also use the mouse pointer (whose coordinates are displayed in the right panel of the bottom line. Locate it at the desired location and double click on the left button. (A single click will be enough if you have set off the **Double click to select** option in the **Other** sheet of the Preferences/**Other** dialog.)

Starting, Stopping and Resuming the Solution

To start solving a problem, press `Enter` or use the **Go/Start** menu. First, *Dynamics Solver* will try to compile all the expressions defined in the problem. In the event of an error, you will be prompted to correct it. A **Compiler Error** dialog displaying the type and probable location of the error will be opened. You can then correct it and continue solving the problem, or stop the compilation at this point.

After a successful compilation, the different kinds of output will start (refer to **Output Parameters**). Most of the menus are disabled while the computation is being performed, but you can, for instance, erase one (or all) windows as described in **Erasing and Refreshing Windows**.

To stop the solution, press `Esc` or use the **Go/Stop** menu. Please note that, to have the possibility of a smooth continuation, the `Esc` key is only read after each solution point is computed. This can suppose several, or even many, points plotted on the screen and, thus, the action is not always immediate if **Interpolation** is on (in the **Range** dialog) and **Step** low, or if **Max. h** (in the **Method** dialog) is high, or if the **Tolerance** is very low.

In fact, the integration halts when any of the following things happens:

- The user presses the `Esc` key or uses the **Go/Stop** menu entry.
- An error happens.
- The **Last value** is reached by the independent variable.
- The absolute value of any dependent variable reaches the **Infinity** value.
- The first argument of a `break` function in an expression is not equal to zero.

- The first argument of a `continue` function in an expression is equal to zero.

In all but the first two cases the next iteration (if any has been scheduled by using the **Repeated solutions and phase portraits** entries in the **Parameters** dialog sheet) starts.

The solution might also be paused if the **Pause** (see **Forcing the Program to Wait**) is not null. If that value is negative, you will have to press a key to get the next solution point. You are reminded to do this by the bottom line of the main window. See the examples in `CHAOS\DISKS.DS` and `GRAPH.DS`.

When the integration is over but you are still in the output window, you can select other initial conditions and start solving again. It is possible to have automatically repeated solutions for different initial values or settings (see **Repeated Integration/Iteration**).

Selecting the Solution Direction

Most ordinary differential equations can be easily solved for decreasing values of the independent variable: you simply use the **Go/Backward** menu entry (or press `Ctrl+U`) and then **Go/Continue** or **Go/Start**. Use again the menu (or press again the key combination) to resume solving forward. Note that most functional-differential equations can not be solved backward because previous solution values must be known at each step. If you try solving backward one of these equations you will get an error, most probably an **'argument out of range' Interpolation Error**.

In the case of discrete dynamical systems, *Dynamics Solver* will always iterate forward, because most maps are not invertible. If the particular map you are analyzing is invertible, you have to invert it by hand and then solved the inverse map forward.

Erasing and Refreshing Windows

1. To erase the current window, use `Del` or the **Window/Erase window** menu entry.
2. To erase all windows, use `Ctrl+Del` or the **Window/Erase all** menu entry.
3. To redraw the current window, use `D` or the **Draw/Refresh window** menu entry.
4. To redraw the current window, use `Ctrl+D` or the **Draw/Refresh all** menu entry.

Zooming a Graphics Window

The ranges displayed in a graphics window can be selected in the **Graphics Output** dialog box, but you can also zoom in a window piece in two different ways.

If you want to select a rectangular piece of a window and amplify it to fill the window:

1. Locate the mouse at one of the new rectangle corners,
2. While holding down the `Shift` key and the left mouse button, move the pointer to the other corner on the same rectangle diagonal. The pointer location will be displayed, as usual, on the right panel of the status (bottom) line. Furthermore, the two corners of the new window will be displayed in the format $(x_1, y_1) - (x_2, y_2)$ inside the rectangle that is currently selected.
3. If you release the left mouse button before the `Shift` key, the rectangle will be the new window. If, on the contrary, the `Shift` key is released before the left button, the previous window ranges will be retained.

If you want only to zoom in provisionally to see some detail and then recover easily the previous window, proceed as follows:

1. Select the **Action/Move** around mode for the cursor. (This can also be accomplished by pressing `M`.)

2. Locate the cursor (with the mouse pointer or the direction keys, as described in **Selecting Initial Conditions on the Screen**) in the center of the area you want to amplify.
3. Use Zoom, step and order/**Zoom in** (or press I). The amplification level is controlled in the **Zooming factor** of the **Cursor** dialog sheet for that window. The default value is 10. If you prefer to see a larger area, instead of peering into smaller details, use Zoom, step and order/**Zoom out**.
4. Repeat steps 2 and 3 as many times as necessary.
5. To recover the original window, use Zoom, step and order/**Actual size** (or press Z).
6. You should probably use Action/**Initial conditions** (or press X) to be able to select initial conditions on the screen to start solving the problem again.

Advanced Procedures

This chapter describes how to get some special kinds of results that often result very useful.

See also **Constrained Initial Conditions, Seeing and Editing the Whole Problem, Printing, Using Graphics Elements, Writing External Integration Codes, Adding Mathematical Functions and Constants, Reading Dynamics Solver's Output in Mathematica and Saving and Loading Graphics and Text Screens.**

Repeated Integration/Iteration

Dynamics Solver can be instructed to automatically repeat the solving process any number of times. Since the dynamical systems solved by *Dynamics Solver* are deterministic, you might think that a repeated solution will necessarily reproduce the first one. This is not always the case, however, because *Dynamics Solver* can be instructed to select different values of initial conditions or parameters before starting each new solution.

Each solution (each run) is identified by the value of one or two index variables whose names (by default x_1 and y_1) and ranges are selected in the entries of the **Parameters** dialog box. The integration/iteration process is repeated for each integer value of these index variables in the selected ranges. Since these ranges have, by default, null starting and ending values, the process is usually executed exactly once, as you will want most often.

If one or both ranges are not empty, the solution process will be repeated. If the names and ranges for the index variables are $x_1 \leq x_2$ and $y_1 \leq y_2$, the whole solution process can be understood in the following pseudo-code:

```
for  $x_1$  from  $x_1$  to  $x_2$  by steps of 1 do
  for  $y_1$  from  $y_1$  to  $y_2$  by steps of 1 do
    integrate/iterate once
```

If you take advantage of the possibility to use **Initial functions** (in the **Initial values** dialog sheet) you can obtain different solutions each time. You can even read them from external data files: see **Reading Initial Functions from Data Files**. (See also **Constrained Initial Conditions**.) You may use as initial function any expression, so it is possible, for instance:

1. Change parameters from run to run. Though **Initial functions** are only applied to the dependent variables, nothing prevents you from deciding that one or some of the parameters in the problem are dependent variables. You simply define an higher **Order/Dimension** (in **Type**) and enter fake equations (null derivatives in differential equations and identity transformations in iterated maps) for the new dependent variables/parameters.
2. Use the `random` function to analyze a problem by changing at random initial values or parameters.
3. Use expressions in the form $x_1 + n_x * (x_2 - x_1) / 10$ and $y_1 + n_y * (y_2 - y_1) / 10$ (or more complex expressions) to draw phase portraits (see **Drawing Phase Portraits**).
4. Use something like `read(i)` to read the initial values from external input files. You will have the opportunity to enter the file name the first time the program needs it.
5. Use the last results of a previous integration to determine, in a simple or complex expression, the new initial conditions (or parameters), because their value is still in the corresponding variables.

Notice that you must make sure that each integration is automatically ended in a reasonable amount of time. This can be achieved by using the **Last value** entry (or **Infinity** in some cases) in the **Range** menu sheet). The functions `break` and `continue` are also very useful to check for termination conditions.

If you need only one non-empty range, select the corresponding to the first index variable, because an interrupted calculation can be resumed more smoothly (as described in **Starting, Stopping and Resuming the Solution**). A resumed computation starts with the next value (if more than one) of the first index variable and uses all the possible values of the second index. If you only use the second index, resuming the integration will be equivalent to starting it, all index values will be used in turn.

The most usual application for this feature will be the drawing of phase portraits (see the example in ODES\PHASE.DS and HENONMAP.DS), but, with a bit of ingenuity, the possibilities are infinite. Let us only mention the drawing of basins of attraction (CHAOS\BAS.DS and BASIN.DS) and the analysis of chaotic scattering (CHAOS\DISKSCAT.DS and SCAT.DS). This is also the only way to get bifurcation diagrams of continuous dynamical systems (CHAOS\LORENZL.DS). Often one can take advantage of the PenColor, PenWidth, HSV and RGB functions, as shown in the CHAOS\HENONMAP.DS, and ART\COS1.DS examples.

Drawing Phase Portraits

The best way to draw phase spaces is to use your knowledge of the problem, which you can get by trying different initial conditions to know how the phase space will look. Then you wisely choose a number of initial conditions and integrate them (often both forward and backward, as discussed below). The portrait will most probably look far better than any automatic drawing.

Anyway, you can get automatically phase portraits as a particular case of repeated integration.

Let us assume that the displayed ranges (selected in **Graphics Output**) are (x_1, x_2) and (y_1, y_2) , and you want $n \times m$ evenly spaced initial conditions in the phase portrait.

1. Choose in the entries of the **Parameters** dialog box the names of the two index variables that will be used to draw the phase portrait. In most cases the default n_x and n_y names will be fine.
2. Make sure that the corresponding ranges go from 0 to $n-1$ and from 0 to $m-1$, respectively.
3. Use expressions in the form $x_1 + n_x * (x_2 - x_1) / n$ and $y_1 + n_y * (y_2 - y_1) / m$ in the **Initial functions** entry of the **Initial values** dialog sheet. Sometimes you will prefer using $x_1 + (n_x + 0.5) * (x_2 - x_1) / (n + 1)$ and $y_1 + (n_y + 0.5) * (y_2 - y_1) / (m + 1)$. In other cases more complex functions will be necessary (to have logarithmic spacing, instead of linear spacing, for instance). A even more complex example is given in ODES\PHASE.DS.
4. In many cases, to have complete solution curves, you will run the complete process twice, once for each solution direction selected in the **Go/Backward** menu. Notice that backward integration is only possible for ordinary differential equations. The process will most probably fail with functional-differential equations and *Dynamics Solver* does not iterate backward discrete dynamical systems (see **Selecting the Solution Direction**).

You must make sure that each integration is automatically ended in a reasonable amount of time. This can be achieved by using the **Last value** entry (or **Infinity** in some cases) in the **Range** menu sheet). The functions `break` and `continue` are also very useful to check for termination conditions.

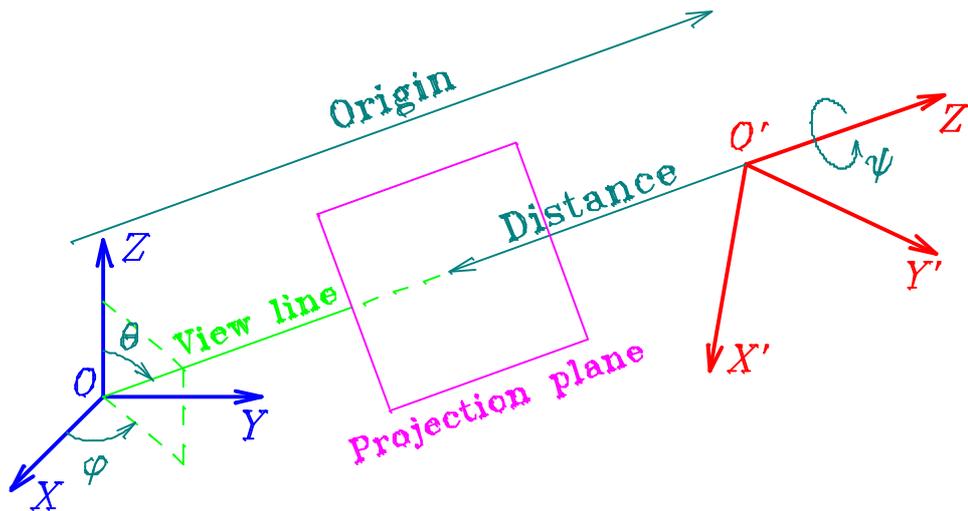
Often one can take advantage of the PenColor, PenWidth, HSV and RGB functions, as shown in the CHAOS\HENONMAP.DS and ART\COS1.DS examples.

See also the ODES\PHASE.DS, CHAOS\STANDARD.DS example.

Using Projections

You can use the **Three-Variable functions** `projx` and `projy` (in the form `projx(x, y, z)`, `projy(x[1], x[2], x[3])` etc.) to have projections in any expression output in a graphics or text window. The view point and other projection options are selected in the **View Point** dialog box.

The projection functions give the x and y coordinates in a new reference system defined as follows:



1. One goes from the original system S to a new one, S_1 , by means of a counterclockwise rotation of angle $0^\circ \leq \mathbf{Phi} < 360^\circ$ around the Z axis, as described by the rotation matrix

$$\begin{pmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. A second counterclockwise rotation of angle $0^\circ \leq \mathbf{Theta} \leq 180^\circ$ around the Y_1 axis is used to define system S_2 :

$$\begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix}.$$

As a consequence, the view line lies along the direction of the new Z_2 axis, defined in the original system by the polar angles (**Theta**,**Phi**).

3. A counterclockwise rotation of angle $0^\circ \leq \mathbf{Psi} < 360^\circ$ around the Z_2 axis changes the orientation of the projection axes and defines a new system, S_3 ,

$$\begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

4. A fourth system, S_4 , is obtained from S_3 by a translation of value **Origin** along the $Z_3 (= Z_2=Z_4)$ axis, so that $x_4 = x_3$, $y_4 = y_3$ and $z_4 = z_3 - \mathbf{Origin}$. The view point is the origin of the new system and its position with respect to S is given by the polar coordinates **Origin**, **Theta**, and **Phi**. The value of **Origin** is ignored unless the following value is different from zero.
5. If **Distance** is null, the projection functions return parallel projections, i.e., the components along the new X_4 and Y_4 axes. If, on the contrary, **Distance** is different from 0, true perspective projections are returned and the opposite of **Distance** is the z value where the projection plane intersects perpendicularly the axis Z_4 . The values returned by the projection functions are thus $x' = -x_4 \mathbf{Distance}/z_4$ and $y' = -y_4 \mathbf{Distance}/z_4$.

The easiest way to understand the result is to look at the sample cube displayed in the **View Point** dialog box, whose orientation can be changed by dragging it with the mouse.

The default values are **Phi** = 30° , **Theta** = 60° and **Psi** = 90° , and other common view lines are as follows:

View line	Proj. plane	Phi	Theta	Psi
-----------	-------------	-----	-------	-----

Z	(X,Y)	0°	0°	0°
X	(Y,Z)	0°	90°	90°
Y	(Z,X)	90°	90°	180°

To see the axes with the same projection settings, use three 3-curves with trivial definitions: $(\tau, 0, 0)$, $(0, \tau, 0)$ and $(0, 0, \tau)$.

See the examples ODES\AUTOCAT.DS, TORUS.DS and CHAOS\LORENZ.DS.

Drawing the Direction Field

With one- and two-dimensional systems, drawing the associated direction field (i.e., a set of short segments which are tangent to the solution curves) may help understand the qualitative properties of the problem. To display it, select the **Direction field** entry of the **Format** dialog sheet. In the same dialog you can select the distance between contiguous segments (in the **Cell** entry) and the segment length (in **Length**). These two quantities are measured in pixels, and their default values will be right in most cases. Once the equations have been compiled (see **Starting, Stopping and Resuming the Solution**) or when the window is erased), the direction field is displayed in the color selected in the **Colors** dialog.

Unless you do not care about strange results, in order to guarantee that the solution curves are tangent to the direction field you should use this option only in one of the following three cases:

1. With a single first-order equation, $y' = f(x,y)$, to display the independent variable x in the X axis and the dependent variable y in the Y axis. The slope of the direction field at each point will be $y' = dy/dx$.
2. With a single second-order equation, $y'' = f(x,y,y')$, to display the dependent variable y in the X axis and its derivative y' in the Y axis. The slope of the direction field at each point will be y''/y' .
3. With a system of two first-order equations, $y' = f(x,y,z)$, $z' = g(x,y,z)$, to display the first dependent variable y in the X axis and the second dependent variable z in the Y axis. The slope of the direction field at each point will be z'/y' .

Since many derivatives must be computed, the drawing of the direction field may be a bit long in slow machines. Pressing **ESC** will stop the process.

See the example in MECH\PENDULUM.DS.

Using Poincaré Sections

To analyze orbits in three or more dimensions you can use, apart from projections, the so-called “Poincaré sections.” In this case, you are interested not in the full solution but only in the solution points that satisfy an additional condition. For instance, often you will want to plot the intersection points of the solution with a given plane (or a more general surface) in the phase space. In *Dynamics Solver*, the Poincaré section can be defined by any condition that can be put in the form $f(\dots) = 0$, f being any function of the independent variable, the solution components, the parameters, and the initial conditions.

Dynamics Solver is able to find the value of the independent variable for which a certain condition written in the very general form $f(\dots) = 0$ is satisfied. You may impose one Poincaré condition for each graphics or text window you are using for output. You only have to set the corresponding definitions and options in the **Graphics Output** or **Text Output** dialog box.

1. Enter in **Condition** the expression $f(\dots)$ (without the $= 0$ part) that must be zero.
2. Select **Increasing** if you want the value of the independent variable for which $f(\dots)$ is null while increasing from negative to positive values.
3. Select **Decreasing** if you want the value of the independent variable for which $f(\dots)$ is null while decreasing from positive to negative values.
4. Specify in **Error** the tolerance the root finding routine uses to consider that $f(\dots)$ vanishes.

5. Specify in **Last Poincaré time** (in the **Parameters** dialog sheet) the name of the value of the independent variable for which the condition is satisfied. The default name is t_p .

Notice that **Increasing** and **Decreasing** are completely independent settings; you can set both or only one of them as you wish.

In the previous sentences $f(\dots)$ stands for any expression containing the independent variable, the solution, the parameters and the initial conditions. However, since the point to be found does not necessarily correspond to any of the actually computed points, the solution and its derivatives (or the different dependent variables) are not directly available as x , $x[1]$, etc., but through the interpolated value $x(i, t)$ that, as usual, returns the n -th derivative or component of the computed solution. (Remember that in the solution points actually computed by *Dynamics Solver*, $x = x(0, t)$, $x[1] = x(1, t)$, etc., but the solution $x(i, t)$ is defined not only for those points, but also for any value of t in the range still in memory. In fact, $x(i, t)$ is the global piecewise polynomial approximation to the solution that has been already discussed in **Interpolated Solution**.) Notice also that this global solution is stored only if **Saved points** (in the **Range** dialog box) has not been set to 0, because the value in that entry is precisely the number of computed points stored in memory.

For instance, to compute the intersections of the solution of a three-dimensional system with the plane $2x - y + az = 13$ you must enter $2*x(1, t) - x(2, t) + a*x(3, t) - 13$ if you have the default names $x[1]$, $x[2]$ and $x[3]$ for the unknowns, or $2*x(t) - y(t) + a*z(t) - 13$ if you have changed the names to x , y and z . *Dynamics Solver* will complain if you try $2*x[1] - x[2] + a*x[3] - 13$ or $2*x - y + a*z - 13$.

The program will monitor continuously the value of $f(\dots)$ and when its sign changes according to the settings **Increasing** and/or **Decreasing**, it will try to find the value of the independent variable for which $f(\dots) = 0$ within the tolerance selected in **Error**. If the value is found, it is stored under the name selected in **Last Poincaré time** and it can be then used in any expression.

In some cases a sign change can be missed if **Interpolate** (in the **Range** dialog box) is not set or **Step** (in the same dialog box) is too high. On the other hand, if the latter value is too small the program can spend a long time testing the condition sign.

The tolerance in the root determination of the condition defining the Poincaré section is set in **Error**. With lower tolerance, the error is smaller, of course, but it would be meaningless to compute these zeroes with better approximation than the solution. A rule of thumb is to take it equal or slightly higher than the **Tolerance** (in the **Method** dialog box) used by the integration method.

A common mistake is to forget writing the expression in such a way that a sign change actually happens when the condition is null. In particular, special care must be taken with discontinuous functions and operators ($\%$ and mod , for instance). So, to find the points at which $x = 0 \text{ mod } 2\pi$, you must type something like $(x + \pi) \% (2 * \pi) - \pi$, instead of the more natural condition $x \% (2 * \pi)$, which will be always nonnegative.

The value for which **Condition** is true is computed, if **Increasing** or **Decreasing** are set, even if you do not want the Poincaré section drawn on the screen or sent to a text window: you may want this value computed only to find the period of a limit cycle or periodic orbit, for instance. (See the examples in `MECH\R3BODY.P.DS` and `ODES\VDPOLP.DS` and the discussion in **Computing Periods**.)

Even if this value is being computed, the output routines work as usual, drawing or writing the expressions selected in the **Graphics Output** or **Text Output** dialog box every time a new solution point is computed or, if **Interpolate** is set (in the **Range** dialog box), every interval of length **Step**. If you want to draw the expressions only at the points in which the **Condition** is true (i.e., if you want the Poincaré section on the screen or in the text window):

1. Set on the **Poincaré section** entry to have output every time the **Condition** defining the section is met (and not according to the settings in the **Range** dialog box).
2. In the case of a graphics window, turn off the **Continuous line** option in the **Format** sheet. This is not truly necessary, but successive points in the Poincaré section correspond normally to rather different values of the independent variable and usually are not very close. Seeing they joined by a segment would be odd in most cases.

See the examples in `ODES\TORUS.DS`, and `CHAOS\HENONHEI.DS`.

In the special case of Poincaré sections of the type $t = b \bmod a$ (i.e., a “stroboscopic” Poincaré section for the values $t = n a + b$) can be found by using a condition in the form $t \% a = b$ or, better, by merely using **First value** = b and **Step** = a (in the **Range** dialog box), without searching for any root. See the example in CHAOS\DUFFING.DS and DUFFING1.DS.

See also **Computing Periods**.

Computing Periods

This topic can be understood as a special application of the possibility to compute **Poincaré Sections**.

1. Start from a point located in the periodic solution. If you do not know any point in the periodic solution you can use *Dynamics Solver* to locate it by trying different initial conditions. If it is an attracting limit cycle (thus, an isolated periodic solution), you can easily locate one of its points by means of *Dynamics Solver*: simply run the problem until the solution is redrawing once and again the periodic solution and stop the integration. You can use a **Text Window** (and in bidimensional systems the status line) to get the coordinates of a point, but it is easier to simply enter in **First value** (in the **Range** dialog box) a value high enough to make sure that the output starts only when the periodic orbit has been reached.
2. Use a **Text Window** to get the numerical value of the period.
3. Enter in **Condition** an expression $f(\dots)$ (without the $= 0$ part) that has a null value when an orbit repeats over itself. (See below.)
4. Select **Increasing** if you want the value of the independent variable for which $f(\dots)$ is null while increasing from negative to positive values.
5. Select **Decreasing** if you want the value of the independent variable for which $f(\dots)$ is null while decreasing from positive to negative values. Note that **Increasing** and **Decreasing** are completely independent settings; you can set both or only one of them, as you wish.
6. If necessary change in **Error** the tolerance the root finding routine uses to consider that $f(\dots)$ vanishes. Make it equal or slightly higher than the **Tolerance** (in the **Method** dialog box) used by the integration routine.
7. If necessary, change in **Last Poincaré time** (in the **Parameters** dialog sheet) the name of the value of the independent variable for which the condition is satisfied. The default name is t_p .
8. Set on the **Poincaré section** entry to have output every time the **Condition** defining the section is met (and not according to the settings in the **Range** dialog box).

$f(\dots)$ can be any expression containing the independent variable, the solution, the parameters and the initial conditions. However, since the point to be found does not necessarily correspond to any of the actually computed points, the solution and its derivatives (or the different dependent variables) are not directly available as x , $x[1]$, etc., but as $x(i, t)$ that always (and not only in this context) means the n -th derivative or component of the computed solution. So, in the solution points actually computed by *Dynamics Solver*, $x = x(0, t)$, $x[1] = x(1, t)$, etc., but the solution $x(i, t)$ is defined not only for those points, but also for any value of t in the range still in memory. In fact, this $x(i, t)$ is just the global piecewise polynomial approximation to the solution that has been already discussed (refer to **Interpolated Solution**). Note that this global solution is stored only if **Saved points** (in the **Range** dialog box) has not been set to 0, because the value in that menu entry is precisely the number of computed points stored in memory.

The program will monitor the value of $f(\dots)$ and when its sign changes in a selected sense, it will try to find the value of the independent variable for which $f(\dots) = 0$ within the tolerance selected in **Error**. If the value is found, it is stored under the name selected in **Last Poincaré time** and it can be then used in any expression.

A sign change can be missed if **Interpolate** (in the **Range** dialog box) is not set or **Step** (in the same dialog box) is too high. On the other hand, if the latter value is too small the program can spend a long time testing the condition sign.

Note also that you must write the expression in such a way that a sign change happens when the condition is null. For instance, you could be tempted to use for a bidimensional system a condition of the form $\text{abs}(x(0, t) - x(0, 0)) + \text{abs}(x(1, t) - x(1, 0))$, if the integration starts from a null value of the independent variable, or $\text{abs}(x(0, t) - x(0, 100)) + \text{abs}(x(1, t) - x(1, 100))$, if **First value** has been set to 100, because these functions will be null if and only if the orbit repeats over itself. Unfortunately, this will not work, because such an expression is never negative. Use, instead, $x(0, t) - x(0, 0)$ or $x(0, t) - x(0, 100)$. Of course, x can repeat before a full period has elapsed, but you can control if the true necessary and sufficient condition (such as the one discussed before) is (nearly) null to know if the independent variable value correspond to a period.

See the examples in ODES\VDPOLP.DS and MECH\R3BODYP.DS (in the latter case a very simple **Condition**, $y(t)$, is used).

Drawing Bifurcation Diagrams

The only way to get bifurcation diagrams (see Strogatz [1994] for the definition of bifurcation diagrams) of continuous dynamical systems is the use of **Repeated Integration** as shown in CHAOS\LORENZL.DS.

In the case of discrete dynamical systems an easier method is provided by *Dynamics Solver*:

1. Enter the name of the bifurcation parameter in the **Bifurcation parameter** entry of the **Parameters** dialog sheet.
2. Select in the **First value** and **Last value** entries of the **Iteration Range** dialog sheet the index range for which output will occur in each run (i.e., for each value of the bifurcation parameter). You might want to try first low values for these entries to avoid having to wait very long before you know if everything else is right.
3. Select the **Begin** and **End** values for the bifurcation parameter in the **Iteration Range** dialog sheet.
4. Select the **Step** value (in the **Iteration Range** dialog sheet) that will increment the bifurcation parameter after each run. You might want to try first a rather high value (say **(End-Begin)/100**) and once everything else is right, a value small enough to avoid having void vertical lines in the window. It may be convenient to adjust accordingly the dimension of the output window. To help in this task, its dimensions (in pixels) are drawn in the status line when it is resized. You may also prescribe exact values, in pixels, for the width and/or height of the client part of the output window, instead of choosing the window size with the mouse or keyboard. This can be achieved by entering the corresponding value(s) in the **Width** and **Height** entries of the same dialog sheet. Keep in mind that null values for these entries are ignored and that Windows sets minimum values for window dimensions.
5. Set in the **Horizontal axis** entry of the **Graphics Output** dialog sheet the name of the bifurcation parameter. You will probably want to make the values in the **Min. X** and **Max. X** entries equal to those on **Begin** and **End**.
6. Set in the **Vertical axis** entry of the **Graphics output** dialog sheet the name of expression to be analyzed for each value of the bifurcation parameter. It will be usually, but not necessarily, a variable. It may be the derivative of a one-dimensional map if you want to compute Liapunov exponents (refer to **Computing Liapunov Exponents**).
7. As usually when analyzing maps, set off the **Continuous line** option in the **Format** dialog sheet.

See the examples in CHAOS\BIFURCAT.DS, CHAOS\HENONB.DS, and CHAOS\HENONL.DS.

Computing Liapunov Exponents

Dynamics Solver can be instructed to compute Liapunov exponents (for their definition, see Strogatz [1994]).

When analyzing discrete dynamical systems (iterated maps) of first order, you can draw a bifurcation diagram, for the Liapunov exponent. It is enough to select in the **Vertical axis** of the **Graphics Output** dialog sheet the expression defining the absolute value of the derivative of the map, and set on the **Average value** entry of the same dialog. In this way, you will get a single average value when the index reaches **Last value**, instead of all the values computed from **First value** to **Last value** (in **Iteration Range**).

In more complex systems you can use a bit of ingenuity and the general methods described in Ott [1993]. Do not forget that many problem can be solved by using the full power of *Dynamics Solver*'s expressions and the possibility to increase the order of the problem to introduce new unknowns that will be computed along the problem (see **Hints**). You can analyze the examples CHAOS\CAT.DS (Liapunov exponent for a bidimensional map), HENONL.DS (bifurcation diagram of the Liapunov exponent of a bidimensional map) and LORENZL.DS (bifurcation diagram for the Liapunov exponent of a three-dimensional system of ordinary differential equations).

Computing Histograms

When analyzing discrete dynamical systems, histograms are very useful to compute the natural invariant measure (see Ott [1993]). They can be drawn easily with *Dynamics Solver*. Let us assume that you want to divide the (a,b) interval of a variable or expression in n subintervals ("bins") of length $l = (b-a)/n$ and plot the relative frequency with which each subinterval is visited by the solution; that is, the relative number of times the value of the selected variable or expression is in each subinterval.

1. Select in the **Begin** and **End** entries of the **Iteration Range** dialog sheet the interval end points, a and b respectively.
2. Select in the **Step** value of the **Iteration Range** dialog sheet) the bin length l . The numbers of bins will be **(End-Begin)/Step**. You might want to try first a rather high value (say **(End-Begin)/100**) and once everything else is right, a value small enough to avoid having void vertical lines in the window. It may be convenient to adjust accordingly the dimension of the output window. To help in this task, its dimensions (in pixels) are drawn in the status line when it is resized. You may also prescribe exact values, in pixels, for the width and/or height of the client part of the output window, instead of choosing the window size with the mouse or keyboard. This can be achieved by entering the corresponding value(s) in the **Width** and **Height** entries of the same dialog sheet. Keep in mind that null values for these entries are ignored and that Windows sets minimum values for window dimensions.
3. Set in the **Horizontal axis** entry of the **Graphics Output** dialog sheet the variable or expression to be analyzed. The value in **Vertical axis** will be ignored. You will probably want to enter in **Min. X** and **Max. X** the values in **Begin** and **End**.
4. Select in **Histogram** the number of iterations between consecutive redrawings of the histogram. If this value is 10000, for instance, each time 10000 new values have been computed the window is erased and for each bin in the interval a vertical line of length equal to relative number of times that bin has been visited is drawn. The sum of heights is equal to $1/\text{Step}$, because one normally wants the integral of the invariant measure to be normalized to 1.

See the example in CHAOS\HISTOGRA.DS.

Drawing Cobwebs

When analyzing discrete dynamical systems, cobwebs are very useful to understand many results: pitchfork bifurcations, intermittence, etc. (see Strogatz [1994]). They can be drawn easily with *Dynamics Solver* by using the **Direct graph power** and **Inverse graph power** entries of the **Graphics Output** dialog box.

If **Direct graph power** has a non null value n , the graph of the n -th iterated of the map will be drawn. If **Inverse** is null, the line $y = x$ will be drawn and the iteration will be represented by a line that goes

horizontally from the last point the diagonal and from there vertically to the graph. The points of intersection of the curve with the diagonal line will correspond to (stable or unstable) n -cycles (i.e., fixed points of the n -th iterate of the map).

If **Inverse graph power** is non null, the graph of the n -th iterated of the inverse map will be drawn. If **Direct graph power** is null, the n -th iterate of the map will also be drawn. You can also select different non null values for these entries (for instance 2 and 1 to analyze 3-cycles). The iteration will be represented by a line that goes from one graph to the other, alternatively. The points of intersection of curves corresponding to values n and m will represent (stable or unstable) $(n+m)$ -cycles (i.e., fixed points of the $(n+m)$ -th iterate of the map).

Often it will be helpful to select a non-null value for **Pause** (see **Forcing the Program to Wait**). This will make easier seeing the position of each solution point.

The use of these possibilities is incompatible with the drawing of bifurcation diagrams or histograms or the calculation of **Average** values.

See the examples in CHAOS\GRAPH.DS, INTER.DS and SINGER.DS.

Some Examples

Dynamics Solver includes well over 100 problem files, which go from elementary examples to very sophisticated uses of the program. You might want to study them to learn how to use *Dynamics Solver* in the most effective way. (If the examples files are not installed into your hard disk, you can run INSTALL again.)

Let us mention only the following problem files, which are among the advanced examples:

Illustrated techniques	File
Basins of attraction of a continuous system	CHAOS\BAS.DS
Basins of attraction of a discrete system	CHAOS\BASIN.DS
Bifurcation diagram	CHAOS\BIFURCAT.DS
Bifurcation diagram for the Liapunov exponent	CHAOS\HENONL.DS
Bifurcation diagram for the Liapunov exponent	CHAOS\LIAPUNOV.DS
Boundary conditions: quantum spectrum	QUANTUM\HARMONIC.DS
Boundary conditions: quantum spectrum	QUANTUM\PENK.DS
Boundary conditions: quantum spectrum	QUANTUM\QUARTIC.DS
Cobwebs	CHAOS\GRAPH.DS
Complex quantities and additional variables	DELAY\FP.DS
Constrained initial conditions and Poincaré map	CHAOS\HENONHEI.DS
Current/field lines. Avoiding divergences	DRAWING\DIPOLE.DS
Cursor and nontrivial initial conditions	ART\ELLIPTI1.DS
Devil's staircase. Using additional unknowns	CHAOS\DEVIL.DS
Direction field	MECH\PENDULUM.DS
Equations not solved for the derivative	ODES\FRW2.DS
Histograms	CHAOS\HISTOGRA.DS
Integro-differential equation	NUMERIC\VOLTERRA.DS
Liapunov exponent of a continuous system	CHAOS\LORENZL.DS
Liapunov exponent of a two-dimensional map	CHAOS\CAT.DS
Multiple output in a single window	MECH\BURRAU.DS
Multiple simultaneous initial conditions in a single window	CHAOS\DUFFING4.DS
Numerical quadrature	NUMERIC\ERF.DS
Parameters selected with the cursor	DELAY\DELAY1.DS
Periods	MECH\R3BODYP.DS
Periods	ODES\VDPOLP.DS
Phase space	ODES\PHASE.DS
Poincaré map and projections	ODES\TORUS.DS

Projections	CHAOS\LORENZ.DS
Projections and 3-curves	DRAWING\EMWAVES.DS
Projections and lettering	ODES\AUTOCATA.DS
Pseudo phase space reconstruction	CHAOS\LORENZR.DS
Real time simulation	MECH\R3BODY1.DS
Recurrences	DELAY\FIBONAC.DS
Repeated integration, Power and Pause	CHAOS\DISKS.DS
Repeated integration: chaotic scattering	CHAOS\DISKSCAT.DS
Repeated integration, (very complex)	CHAOS\DISKS.DS
Repeated integration: KAM theorem	CHAOS\HENONMAP.DS
Repeated integration: quantum spectrum	QUANTUM\SPECTRUM.DS
Single precision numbers	CHAOS\HENON1.DS
Stroboscopic Poincaré map	CHAOS\DUFFING1.DS
Stroboscopic Poincaré map and the $i f$ function	CHAOS\DUFFING2.DS
Stroboscopic Poincaré map	ODES\VDPOLF.DS

Using Graphics Elements

You can use graphics elements to add lettering and other informative items (axes, curves, and so on) to the graphics result of your problem files, or simply to draw some figures when preparing graphics for papers and courseware.

If you want to draw a figure with no associated differential or discrete dynamical system, open a graphics window with the Output/**New graph window** entry. You should probably select the in the **Aspect ratio** entry of the **Format** sheet any value different from **Arbitrary**. In this way, circles will look round and other elements can be rotated with no distortion. With graphics elements, the **Axes** entry of the same sheet is obsolete and you may want to clear it.

The graphics elements are created, edited and deleted from the **Draw**, **Action** and **Zoom, step and order** menus or the equivalent toolbar elements, which you can display by using the Configuration/**Toolbar and popup menu** entry.

Most of the actions are also available through the following keyboard shortcuts:

Key	Menu entry
F1	Open the help Index
F2	File/ Save
F3	File/ Open
Shift+F3	Window/ Tile vertical
Shift+F4	Window/ Tile horizontal
F5	Edit/ All settings
Shift+F5	Window/ Cascade
F10	File/ Print
+ *	Draw/ Next element
- *	Draw/ Previous element
* *	Zoom/ Larger step
/ *	Zoom/ Smaller step
Enter	Go/ Start
Esc	Go/ Stop (or stop the redrawing of graphics elements and metafiles)
Del	Window/ Erase window
Ctrl+Del	Window/ Erase all
Ctrl+Ins	Window/ Get screen
Shift+Ins	Window/ Put screen
Ctrl+A	Edit/ Parameters
Ctrl+B	Go/ Backward
Ctrl+C	Window/ Get screen
Ctrl+D	Draw/ Refresh all
Ctrl+E	Edit/ Equations
Ctrl+F	Output/ Graphics format
Ctrl+G	Output/ New graphics window
Ctrl+H	Edit/ Method
Ctrl+I	Edit/ Initial conditions
Ctrl+L	Edit/ Variables
Ctrl+N	Edit/ Notes
Ctrl+O	Output/ Color
Ctrl+P	Output/ Parameters
Ctrl+R	Edit/ Range
Ctrl+S	Output/ Status line
Ctrl+T	Output/ New text window
Ctrl+U	Go/ Continue
Ctrl+V	Window/ Put screen
Ctrl+W	Output/ View point

Ctrl+X	Output/ Text format
Ctrl+Y	Edit/ Type
Ctrl+Z	Edit/ Boundary conditions
C	Draw/ Copy element
D	Draw/ Refresh window
E	Draw/ Edit element
F	Action/ First value
I	Zoom/ Zoom in
L	Action/ Last value
M	Action/ Move around
N	Draw/ New element
O	Action/ Origin
P	Action/ Point
R	Action/ Rotate
S	Action/ Select an element
T	Action/ Third point
X	Action/ Initial condition
W	Zoom/ Zoom out
Z	Zoom/ Actual size
Direction keys	To move the cursor, origin, point, third point, or to rotate, and to select first and last values.

*These keys are those in the numeric keypad

You can also use the **Graphics Element** dialog box to manage graphics elements.

Finally, many useful actions can be accomplished by using the mouse.

To select an element you can use the **Graphics Element** dialog box or the + and - in the numeric keypad; but it is often more convenient to use Action/**Select an element** (or the corresponding toolbar button, or press S) and, then, locate the mouse cursor near the element and double click on the left button (see Preferences/**Other**).

Once the element has been selected, you may use again the entries of **Action** (or the equivalent keys or toolbar buttons) to chose the action that will be performed when using the direction keys (or, in most cases, the mouse) to adjust the element origin, size, direction and so on.

For fine adjustment, you can use the zoom facilities (see **Zooming a Graphics Window**), maybe after selecting the Action/**Move** mode to locate the center of the zoomed window.

Graphics Elements

You can add to any graphics window any number of elements of the following types:

Name	Meaning
Segment	Segment
Circle	(Arc of) circumference
Ellipse	(Arc of) ellipse
Arrow	Arrow head
Text	Text string
2-curve	Parametric plane curve
3-curve	Projection of parametric spatial curve
Special	Special data to be sent directly to the plotter/printer
Data	Read numerical data from an external file
Fractal	Fractal curves

Segment

A straight line starting at the point of coordinates **X** and **Y** and having a **Length** and a **Direction**. A point is obtained, obviously, as a segment of null **Length**.

Attribute	Meaning
X	x coordinate of the origin
Y	y coordinate of the origin
Length	Segment length
Direction	Direction in degrees
Color	Drawing color
Line type	Line width (if negative, a dashed line is used)
Steps	Number of segments (dashes)

Circle

A circle or an arc of circumference defined by its center (the point of coordinates **X** and **Y**) its **Radius**, its **Direction** and its **Begin** and **End** angle values. The last two values are the limit values of the angle a in the usual definition:

$$x = x_0 + r \cdot \cos(a),$$

$$y = y_0 + r \cdot \sin(a).$$

Attribute	Meaning
X	x_0 coordinate of the center
Y	y_0 coordinate of the center
Radius	Circle radius r
Begin	First angle value in degrees
End	Last angle value in degrees
Direction	Direction in degrees
Color	Drawing color
Line type	Line width (if negative, a dashed line is used)
Steps	Number of segments (dashes)

Ellipse

A (arc of) ellipse defined by its center (the point of coordinates **X** and **Y**) its **Horizontal** and **Vertical** semi-axes, its **Direction** and its **Begin** and **End** angle values. The last two values are the limit values of the angle a in the usual definition:

$$x = x_0 + r \cdot \cos(a),$$

$$y = y_0 + s \cdot \sin(a).$$

Attribute	Meaning
X	x coordinate of the center
Y	y coordinate of the center
Vertical	Vertical semi-axis s
Horizontal	Horizontal semi-axis r
Begin	First angle value in degrees
End	Last angle value in degrees
Direction	Direction in degrees
Color	Drawing color
Line type	Line width (if negative, a dashed line is used)
Steps	Number of segments (dashes)

If s and r are equal we have, obviously, a circle.

Arrow

An arrowhead starting at the point of coordinates **X** and **Y**, and having a **Size** and a definite **Direction** and an **Angle**.

Attribute	Meaning
X	x coordinate of the head
Y	y coordinate of the head
Size	Arrow size
Angle	Arrow angle divided by two
Direction	Direction in degrees
Color	Drawing color
Line type	Line width

See the problem files in EXAMPLES\DRAWING.

Text

A character or string of characters defined in **String**. Its left lower corner starts at the point of coordinates **X** and **Y**, and the string is displayed along a **Direction**. Each letter's **Width** and **Height**, can also be controlled by means of **Editing Graphics Elements**. The font used to draw the letters is selected in **Font**.

Attribute	Meaning
String	Text string (including Escape Sequences)
X	x coordinate of the string origin
Y	y coordinate of the string origin
Height	Letter height
Width	Letter width
Direction	Direction in degrees
Color	Drawing color
Line type	Line width
Font	Font number

If a character is not defined in the font, a space is used instead.

Escape Sequences

In an element of type text, a sequence of characters starting by '\' has a special meaning according to the following table:

Symbol	Meaning
\0	Reset all default values
\n	ASCII character $0 < n < 256$
\+n	Advance $-128 < n < 128$ points ⁵
\-n	Move back $-128 < n < 128$ points
\^n	Go up $-128 < n < 128$ points
_n	Go down $-128 < n < 128$ points
*n	Multiply height and width by n

⁵ A character is given by a set of polygonals joining set of points, whose coordinates, x and y , are measured in arbitrary units called "points" ($-128 < x < 128$, $-128 < y < 128$). The usual letter height is about 42 for capital letters and its width ranges from 26 to 54. An undefined character is 0 points width.

<code>\ /n</code>	Divide height and width by n
<code>\ .</code>	Do nothing (useful as separator)
<code>\ :n</code>	Select font $0 \leq n \leq 9$

Any space following a number is skipped. This is a convenient feature to separate the sequence from a contiguous numeric character, but if you actually need a space after a sequence, `\` must be used.

Characters 169-172, 176-197 and 198-223 are accessible as `\A-\D`, `\E-\Z` and `\a-\z`. In some fonts (but not all, see **Show fonts**) they represent Greek characters as follows:

Capital	Normal	Meaning
<code>\A</code>	<code>\a</code>	alpha
<code>\B</code>	<code>\b</code>	beta
<code>\C</code>	<code>\c</code>	chi
<code>\D</code>	<code>\d</code>	delta
<code>\E</code>	<code>\e</code>	epsilon
<code>\F</code>	<code>\f</code>	phi
<code>\G</code>	<code>\g</code>	gamma
<code>\H</code>	<code>\h</code>	eta
<code>\I</code>	<code>\i</code>	iota
<code>\J</code>	<code>\j</code>	theta*, phi*
<code>\K</code>	<code>\k</code>	kappa
<code>\L</code>	<code>\l</code>	lambda
<code>\M</code>	<code>\m</code>	mu
<code>\N</code>	<code>\n</code>	nu
<code>\O</code>	<code>\o</code>	omicron
<code>\P</code>	<code>\p</code>	pi
<code>\Q</code>	<code>\q</code>	theta
<code>\R</code>	<code>\r</code>	rho
<code>\S</code>	<code>\s</code>	sigma
<code>\T</code>	<code>\t</code>	tau
<code>\U</code>	<code>\u</code>	upsilon
<code>\V</code>	<code>\v</code>	sigma*, omega*
<code>\W</code>	<code>\w</code>	omega
<code>\X</code>	<code>\x</code>	xi
<code>\Y</code>	<code>\y</code>	psi
<code>\Z</code>	<code>\z</code>	zeta

*This is an alternative shape for the same letter

The remaining escape sequences are the **User Defined Sequences**, which can be viewed and edited in the **Escape Sequences** dialog box.

User Defined Escape Sequences

In an element of type text, you can use any of the following 24 sequences, which can be viewed and edited in **Escape Sequences** dialog box. They are defined in terms of the built-in **Escape Sequences** and their default values are as follows:

Symbol	Meaning
<code>\!</code>	(undefined by default)
<code>\«</code>	(undefined by default)
<code>\#</code>	(undefined by default)
<code>\\$</code>	(undefined by default)
<code>\%</code>	(undefined by default)
<code>\&</code>	(undefined by default)
<code>\'</code>	(undefined by default)

\(_20\ /1.2 (index for roman font)
\)	\^30\ /1.2 (exponent for roman font)
\,	(undefined by default)
\;	(undefined by default)
\<	(undefined by default)
\=	(undefined by default)
\>	(undefined by default)
\?	(undefined by default)
\@	(undefined by default)
\[\-10_20\ /1.2 (index for italics)
\]	\92 (the backslash character)
\]	\+10\^30\ /1.2 (exponent for italics)
\^	(undefined by default)
\{	(undefined by default)
\	(undefined by default)
\}	(undefined by default)
\~	(undefined by default)

A sequence of this type can freely use other escape sequences or user defined sequences, but direct and indirect recursion does not work. A sequence cannot be longer than 76 characters.

2-curves

An element of this type is defined by its two-dimensional parametric equations:

$$x = f(t, x_0, y_0, s, r, d, t_1, t_2),$$

$$y = g(t, x_0, y_0, s, r, d, t_1, t_2).$$

The parameter, t , goes between t_1 and t_2 , and you can freely use the x_0 , y_0 , s , r and d parameters.

Attribute	Meaning
x	Equation for the x coordinate
y	Equation for the y coordinate
x0	x0 parameter
y0	y0 parameter
s	s parameter
r	r parameter
t1	First value of $t = t_1$
t2	Last value of $t = t_2$
d	d parameter
Color	Drawing color
Line type	Line width (if negative, dashed line)
Steps	Number of segments (dashes)

To avoid premature errors when they are about to be defined, 3-curves with a null x or y entry are not displayed at all.

See the problem files in EXAMPLES\DRAWING (especially SPRINGS.DS).

3-curves

An element of this type is defined by its three-dimensional parametric equations:

$$x = f(t, x_0, y_0, s, r, d, t_1, t_2),$$

$$y = g(t, x_0, y_0, s, r, d, t_1, t_2),$$

$$z = h(t, x_0, y_0, s, r, d, t_1, t_2).$$

The points are automatically projected by using `projx` and `projy` and the settings in **View Point**. An equivalent, but rather messy, way to do the same would be using `projx(f(...),g(...),h(...))` and `projy(f(...),g(...),h(...))` in a 2-curve element.

The parameter, t , goes between t_1 and t_2 , and you can freely use the x_0 , y_0 , s , r and d parameters.

Attribute	Meaning
x	Equation for the x coordinate
y	Equation for the y coordinate
z	Equation for the z coordinate
x_0	x_0 parameter
y_0	y_0 parameter
s	s parameter
r	r parameter
t_1	First value of $t = t_1$
t_2	Last value of $t = t_2$
d	d parameter
Color	Drawing color
Line type	Line width (if negative, dashed line)
Steps	Number of segments (dashes)

To avoid premature errors when they are about to be defined, 3-curves with a null x , y or z entry are not displayed at all.

One useful application of these elements is the drawing of axes when using projections (see `CHAOS\LORENZ.DS` and `ODES\AUTOCAT.DS`, for instance).

Special

An element of this type is a string that is directly sent to the plotter device after translating the following sequences:

Sequence	ASCII name	Meaning
<code>\nnn</code>		a decimal code for a character
<code>\a</code>	Bel	character #7
<code>\b</code>	Backspace	character #8
<code>\e</code>	Esc	character #27
<code>\f</code>	FF	character #12
<code>\n</code>	LF	character #10
<code>\onnn</code>		an octal code for a character
<code>\r</code>	CR	character #13
<code>\t</code>	Tab	character #9
<code>\v</code>	VT	character #5
<code>\xnnn</code>		an hexadecimal code for a character
<code>\z</code>	Nul	character #0
<code>\c</code>		the character c in the remaining cases

Data

An element of this type is an external data file whose complete specification is given in **File**. You can add after the file specification four numbers if the form:

```
filename x1 y1 x2 y2
```

to automatically clip the input data (after eventual translation, scaling and rotation, see below) to the rectangle whose diagonal points are (x_1,y_1) and (x_2,y_2) .

Attribute	Meaning
File	filename or filename x1 y1 x2 y2
X	Horizontal translation

Y	Vertical translation
Vertical	Vertical scale factor
Horizontal	Horizontal scale factor
Direction	Direction in degrees
Color	Drawing color
Line type	Line width
Segments	If 0, only points are drawn

The file lines must be composed by two floating points numbers: the coordinates of successive points of a polygonal. A line which has not this structure (if it is void, for instance) starts a new polygonal. Nevertheless, if **Segments** is 0, successive points are never joined by a segment.

Before being drawn, the x and y coordinates are scaled (multiplied) by **Horizontal** and **Vertical**, respectively. These values are ignored if equal to zero (or one). The scaled coordinates are then rotated according to **Direction** and translated by **X** and **Y**.

In many cases it will be better to use relative paths for `filename`. If the data file is located in the problem file directory and you specify only the file name and extension, the drive and path of the latter will be used to open the data file, after trying the current drive and directory.

To avoid repeated errors in some graphics cards, no error is reported if the file has not been defined or the program cannot open it.

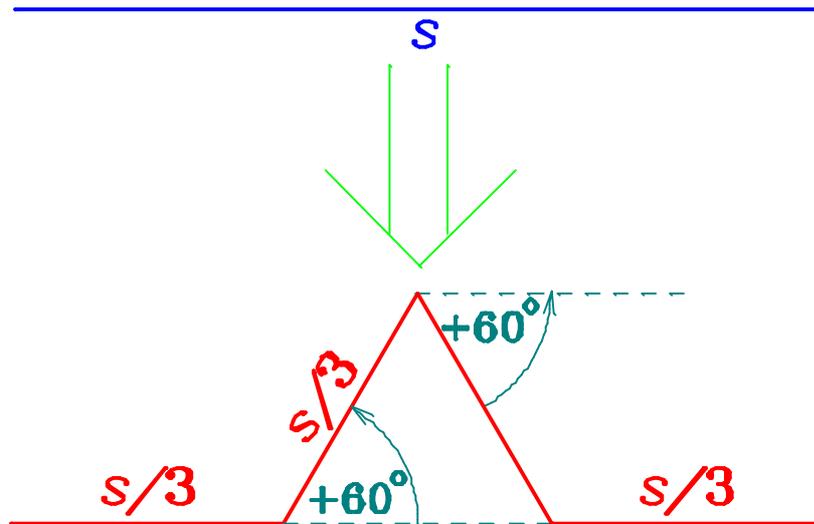
Fractal

An element of this type is an autosimilar fractal based on a straight line starting at the point of coordinates **X** and **Y** and having a **Length** and a **Direction**.

The minimum/maximum order (iteration level) is selected in **Min. order** / **Max. order**. The actual iteration level is obtained adding **Fractal order** (in the **Cursor** dialog sheet) to **Min. order**, as long as the sum is not greater than **Max. order**.

Attribute	Meaning
Divisors	Divisors to obtain the relative lengths of segments in the next generation
Angles	Angles of the relative directions in the next generation. In degrees
X	x coordinate of the origin
Y	y coordinate of the origin
Length	Length of basic segment
Min. order	Minimum fractal order
Max. order	Maximum fractal order
Direction	Direction in degrees of basic segment
Color	Drawing color
Line type	Line width

In the following example (the von Koch curve)



we see the basic (blue) segment of **Length** s . The next generation is formed by four red segments of length $s/3$ and with relative orientations as indicated in the figure. So, four values are needed in **Divisors** (3 3 3 3) to indicate that the length of each new segment is one third of the segment in the previous generation. Another four values (0 60 -60 0) are entered in **Angles** to indicate the relative directions in degrees of each new segment with respect to the one of the previous generation. This construction is repeated again and again up to the desired generation which is indicated by the sum of **Min. order** and **Fractal order** in the **Cursor** dialog sheet, but always less or equal to **Max. order**. See the example VONKOCH.DS.

See also the problem files in EXAMPLES\FRACTAL.

String, x equation, File and Divisors

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Text	String	Text string (including Escape Sequences)
2-curve	x	Equation for the x coordinate
3-curve	x	Equation for the x coordinate
Special	String	Literal string to be included in the output
Data	File	filename or filename x1 y1 x2 y2
Fractal	Divisors	Divisors to obtain the relative length of segments in next generation

If the graphics element is of Data type you can specify only the file name and extension, if it is located in the problem file directory. The drive and path of the latter will be used to open the data file, after trying the current drive and directory.

y equation and Angles

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
2-curve	y	Equation for the y coordinate
3-curve	y	Equation for the y coordinate
Fractal	Angles	Angles to add to the global direction for each segment in the next generation

z equation

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
3-curve	z	Equation for the z coordinate

X coordinate and x0 parameter

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Segment	X	x coordinate of the origin
Circle	X	x coordinate of the center
Ellipse	X	x coordinate of the center
Arrow	X	x coordinate of the head
Text	X	x coordinate of the string start
2-curve	x0	x0 parameter
3-curve	x0	x0 parameter
Data	X	Horizontal translation
Fractal	X	x coordinate of the origin

Y coordinate and y0 parameter

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Segment	Y	y coordinate of the origin
Circle	Y	y coordinate of the center
Ellipse	Y	y coordinate of the center
Arrow	Y	y coordinate of the head
Text	Y	y coordinate of the string start
2-curve	y0	y0 parameter
3-curve	y0	y0 parameter
Data	Y	Horizontal translation
Fractal	Y	y coordinate of the origin

Length, Vertical, Size, Height and s parameter

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Segment	Length	Segment length
Ellipse	Vertical	Vertical semi-axis
Arrow	Size	Size
Text	Height	Letter height
2-curve	s	s parameter
3-curve	s	s parameter
Data	Vertical	Vertical scaling factor
Fractal	Length	Length of basic segment

Radius, Horizontal, Width and r parameter

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Circle	Radius	Radius
Ellipse	Horizontal	Horizontal semi-axis
Text	Width	Letter width
2-curve	r	r parameter
3-curve	r	r parameter
Data	Horizontal	Horizontal scaling factor

Begin, Angle, t1 parameter and Min. order

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Circle	Begin	First angle value in degrees
Ellipse	Begin	First angle value in degrees
Arrow	Angle	Arrow semi-angle
2-curve	t1	t 1 parameter
3-curve	t1	t 1 parameter
Fractal	Min. order	Minimum fractal order

End, t2 parameter and Max. order

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Circle	End	Last angle value in degrees
Ellipse	End	Last angle value in degrees
2-curve	t2	t 2 parameter
3-curve	t2	t 2 parameter
Fractal	Max. order	Maximum fractal order

Direction and d parameter

This entry of **Edit Element** contains the following element information:

Element	Name	Meaning
Segment	Direction	Direction in degrees
Circle	Direction	Direction in degrees
Ellipse	Direction	Direction in degrees
Arrow	Direction	Direction in degrees
Text	Direction	Direction in degrees
2-curve	d	d parameter
3-curve	d	d parameter
Data	Direction	Direction in degrees
Fractal	Direction	Direction in degrees

Line type

This entry of **Edit Element** contains the line type (width) of the current element. For some elements, a negative value indicates that the element will be drawn with as a dashed curve with a **Steps**. The absolute value will be used as line width.

Type	Meaning
Segment	Line width (if negative, dashed line)
Circle	Line width (if negative, dashed line)
Ellipse	Line width (if negative, dashed line)
Arrow	Line width
Text	Line width
2-curve	Line width (if negative, dashed line)
3-curve	Line width (if negative, dashed line)
Data	Line width
Fractal	Line width

Color

This entry of **Edit Element** contains the color to draw the element. You can use the button or double click on the color bar to select a new value.

Steps, Font and Segments

This entry of **Edit Element** contains the number of segments that must be used to draw the current element. In addition, the range of the parameter defining the curve, between the values in **Begin** and **End** is divided by this number.

By convention, a negative value of **Line type** indicates that only alternate segments will be drawn, to have a dashed element. This value indicates the number of segments (alternatively drawn and skipped) to be used. In most cases an odd value looks better.

In an element of type **Text** this value is necessarily between 0 and 9 and it indicates the font used to draw the text. If a value n is used, the corresponding font file is called as defined in the **Font files** dialog box.

In an element of type **Data** if this value is null, successive points are not joined by a segment.

Element	Name	Meaning
Segment	Steps	Number of subsegments
Circle	Steps	Number of segments
Ellipse	Steps	Number of segments
Text	Font	Font number (from 0 to 9)
2-curve	Steps	Number of values of the τ parameter
3-curve	Steps	Number of values of the τ parameter
Data	Segments	If 0, do not join points with segments

Printing

Since *Dynamics Solver* is designed to obtain high quality results, there are different ways to print, which will be discussed in detail in the next sections.

Printing Text

You can print the currently active text window by using File/**Print** (or F10 or the corresponding toolbar button ). The standard **Print** dialog box will appear. You can there select the printer and some printing options.

Fast Graphics Printing

You can print the currently active graphics window by using File/**Print** (or F10 or the corresponding toolbar button ). The **Print** dialog box will appear. You can there select the printing options.

The bitmap formats are rather appropriate for problems in which the graphics results are points, rather than continuous lines (as is often the case in discrete dynamical systems: iterated maps). If the results appear in the form of lines, printing bitmaps will often produce poor results because printers have usually an much higher resolution than screens (and, in consequence, bitmaps). Notice also that you could get only a partial window snapshot (the piece currently visible) if the **Bitmap output** entry in the **Format** sheet was **No bitmap** when they were computed.

Moreover, some graphics card drivers have problems translating bitmaps in some graphics modes. For example, we have found that the drivers of some very popular cards render the white background of windows in 256 and more color modes as a gray texture in PostScript printers (they give the correct white background in injection printers!). Notice that this is a problem (bug) in the graphics driver, not in *Dynamics Solver*. The only thing you can do to avoid it to use another printing mode, as described below.

To obtain good print results you should have prepared the printing before starting solving the problem. See **Preparing Graphics Printing**.

Preparing Printing

Before starting solving the problem (especially if it is a complex one and a large computation time is needed to get the results) you should paid some attention to the kind of printed results you want.

The bitmap formats are rather appropriate for problems in which the graphics results are points, rather than continuous lines (as is often the case in discrete dynamical systems: iterated maps). If the results appear in the form of lines, printing bitmaps will often produce poor results because printers have usually an much higher resolution than screens (and, in consequence, bitmaps). Notice also that you could get only a partial window snapshot (the piece currently visible) if the **Bitmap output** entry in the **Format** sheet was **No bitmap** when they were computed.

The metafile format will produce in most cases better results, but you have to select an appropriate value for the **Basic width** entry in the **Print** dialog box, in order not to have too fine lines. Notice also that this mode will be available only if the **Save output in memory** entry in the **Format** sheet was not null when the integration/iteration results were computed.

Check also the default values in the **Bitmap output** and **Memory output** entries of the Preferences/**Graphics** dialog sheet.

On the other hand, you may want to collect in a single printout the results displayed in different graphics windows. The easier way is to output all the output you want to print to a single window. Though you need a graphics window for each pair of quantities you want displayed on the screen, you may select in the

Window entry of the corresponding **Graphics Output** dialog box the caption of another graphics window, which will then receive all its output (but not its graphics elements). Notice that plotter output, which is described below, will not be redirected (you may get this, at your own risk, by specifying the same plotter for different windows). The **Min. x**, **Max. x**, **Min. Y** and **Max. y** values of the window that will receive the output will be used. If you need different values for each pair of quantities, you have to scale the corresponding expressions.

PostScript Printing

Many formatting programs (such as T_EX, and its dialects L^AT_EX, AMST_EX, and so on) and word processors accept PostScript or, more often, Encapsulated PostScript files. They provide a standard (and, more important, platform-independent) form to include graphics in other documents.

You can easily generate Encapsulated PostScript files from the **Print** dialog box: simply check in the **Encapsulated PostScript** option and select the output file in the **Port/File** entry (maybe by using the **Browse** button).

If the amount of memory necessary to save all the graphics output exceeds your RAM size, you must consider using the **Plotter Output** mode.

Plotter Output

This possibility is reserved to power user who can take advantage of it at least in three instances:

1. When the amount of memory necessary to save all the graphics output exceeds your RAM size and you want to get quality prints from metafiles (maybe to send them to an Encapsulated PostScript file).
2. When the output is to be sent to a plotter, rather than to a printer.
3. When you are a power user that have special output/printing needs. For example, you could be a teacher preparing a collection of exercises that includes many figures to be inserted in the output to a laser printer from a LaTeX source.

In these cases the plotter facilities in *Dynamics Solver* should be used, even if the output will be sent to a printer (probably a laser printer). The process is as follows:

1. Starting from any of the examples provided with the program in the PLOTTER directory, create a plotter driver appropriate for your printer/plotter and printing needs.
2. Install the plotter driver by using the Configuration/**Plotter drivers** menu
3. Select the driver in the **Driver** entry of the **Plotter** dialog sheet.
4. In the same dialog, select the **Output file** and the appropriate values for the remaining parameters (**Frame**, **Pen**, **Width**, **Speed** and **Line type**). Make sure that **On** is selected.
5. Perform some preliminary runs to get a feel of the kind of results you want.
6. For each solution (initial value) that you want sent to the plotter/printer, select the Output/**Plotter output** global setting (or equivalent toolbar button) and start solving the problem. You can set off this setting to continue experimenting with other initial conditions.
7. If you want to include **Axes** or graphics elements, use the Output/**Draw to plotter** menu entry, or equivalent toolbar button.
8. If necessary, use the Output/**Eject page** menu.
9. If necessary, use the Output/**Write epilog** menu.

The plotter drivers are ordinary text files with an extension defaulting to .DRV. They can be adapted to different plotting devices and to different settings of the same plotter/printer (to use different parts of the paper, for instance). Some examples are provided in the distribution disk(s) and the corresponding help

files (which can be accessed from the **Examine** button in the dialog that displays with the Configuration/**Plotter drivers** menu or by using **Driver help** in the **Plotter** dialog box) describe in detail their structure, and how they can be modified. See, for instance, the `EPS.HLP` help file.

Reference Information

Menu commands

Menu	Meaning
File	File and printer management
Edit	View and change problem settings
Output	Control output options
Window	Manage windows
Go	Solve the problem
Draw	Edit drawing elements
Action	Control cursor action
Zoom	Change viewing scale
Configuration	Set preferences
Help	Help system

File

The entries in this menu are used to manage the printer and problem files.

Entry	Key	Meaning
New		Start a new problem
Open	F3	Get a problem file from the disk
Save	F2	Save the current problem to the disk
Save as		Save the current problem to the disk, under a new name
Printer setup		Specify the printer to be used and the corresponding options
Print	F10	Print the current output window
Mail		Send the current problem to the FAX and mail system
Quit	Alt+F4	Exit the program

After the previous menu entries there can appear the file specifications of the last recently used problems. This feature is very convenient to resume previous works. At most four file names are retained. The length of these entries is controlled from the **Filename length in menus** entry of the **Other** dialog sheet.

You will be prompted for a file name when using **Open**, **Save as** and also if **Save** is used when no previous name (i.e., if the problem file was not loaded with **Open** but created with **New**) or if the file was converted from a previous version of the program.

New

Use this entry of the **File** menu (which can be also accessed by using the  toolbar button) to erase all current definitions, close all windows and start analyzing a new problem file.

Open

This entry of the **File** menu (which can be also accessed by pressing F3 or using the  toolbar button) starts an **Open** dialog box that lets you load a problem file from the disk.

Save

This entry of the **File** menu (which can be also accessed by pressing F2 or using the  toolbar button) saves the current problem file to the disk under its current name. (If it has never been saved before, a **Save** dialog box lets you choose it.)

Save as

This entry of the **File** menu (which can be also accessed by using the  toolbar button) starts a **Save** dialog box that lets you save the current problem file to the disk under a new name (or in a different disk or directory).

Printer setup

This entry of the **File** menu (which can be also accessed by using the  toolbar button) opens the **Printer Setup** standard dialog box that lets you choose the printer and its options.

Print

This entry of the **File** menu (which can be also accessed by pressing F10 or using the  toolbar button) opens the **Print** dialog box that lets you print the current window.

Mail

This entry of the **File** menu (which can be also accessed by using the  toolbar button) send the current problem to FAX or mail system (as an attachment).

Quit

This entry of the **File** menu (which can be also accessed by pressing Alt+F4 or using the  toolbar button) ends the current *Dynamics Solver* session. If the current problem file has been modified, you will be prompted and have an opportunity to save the changes.

Most Recently Used Problem Files

The last entries in the **File** menu (from zero to four) are the names (and the corresponding file specifications, if the **Filename length in menus** entry of the **Other** dialog sheet is long enough) of the last recently used problem files. This is a convenient way to resume the analysis of a problem.

Edit

The entries of this menu are used to define the dynamical system under study

Entry	Key	Meaning
Type	Ctrl+Y	Select the type of dynamical system
Variables	Ctrl+L	Select names of variables
Parameters	Ctrl+A	Select names and values of parameters
Equations	Ctrl+E	View and edit the equations defining the problem
Initial conditions	Ctrl+I	Select initial values (and functions)
Range	Ctrl+R	Select the solution range
Method	Ctrl+H	Select the integration method

Notes	Ctrl+N	View and edit problem notes
All settings	F5	View and edit all problem settings in a text window

Type

This entry of the **Edit** menu (which can be also accessed by pressing Ctrl+Y or using the  toolbar button) opens the **Type** dialog sheet where you can select the type of dynamical system to analyze. Using **OK** will open a warning box, because all settings will be lost. To change the type while saving the current settings you may use Edit/**All settings**.

Definitions

This toolbar button, , is equivalent to the Edit/**Variables** menu and is included for convenience, because the corresponding hint⁶ and tip⁷ indicate that from the same dialog box other settings (**Parameters**, **Equations** and **Initial conditions**) can be changed.

Variables

This entry of the **Edit** menu (which can be also accessed by pressing Ctrl+L or using the  toolbar button) opens the **Variables** dialog sheet where you can change the names for the independent and dependent variables and for their initial values.

Parameters

This entry of the **Edit** menu (which can be also accessed by pressing Ctrl+A or using the  toolbar button) opens the **Parameters** dialog sheet where you can change the names and values for parameters, bifurcation variable, Poincaré time and iteration variables.

Equations

This entry of the **Edit** menu (which can be also accessed by pressing Ctrl+E or using the  toolbar button) opens the **Equations** dialog sheet where you can change the equations defining the dynamical system and some other options.

Initial conditions

This entry of the **Edit** menu (which can be also accessed by pressing Ctrl+I or using the  toolbar button) opens the **Initial values** dialog sheet where you can change the initial conditions and functions for the current dynamical system.

⁶ The status line is used, among other things, to display a short string describing the currently selected menu entry or the toolbar button located under the mouse pointer.

⁷You may get a short description inside a small text window when the mouse pointer has paused for about one second over a toolbar button.

Notice that this feature is disabled by default, because some video cards drivers redraw too often the window under the popup tip. You should check if this is the case with your video card driver. You can easily enable them by using the Preferences dialog box.

Boundary conditions

This entry of the **Edit** menu (which can be also accessed by pressing `Ctrl+Z` or using the  toolbar button) opens the **Boundary conditions** dialog sheet where you can change the boundary conditions for the current dynamical system.

Range and Method

This toolbar button () is equivalent to the **Edit/Range** menu and is included for convenience, because the corresponding hint and tip indicate that from the same dialog box other settings (**Method**) can be changed.

Range

This entry of the **Edit** menu (which can be also accessed by pressing `Ctrl+R` or using the  toolbar button) opens the **Range** dialog sheet where you can select the options for the solution range.

Method

This entry of the **Edit** menu (which can be also accessed by pressing `Ctrl+H` or using the  toolbar button) opens the **Method** dialog sheet where you can select the method to solve the differential system (this menu is not available for discrete dynamical systems) and the corresponding options.

Notes

This entry of the **Edit** menu (which can be also accessed by pressing `Ctrl+N` or using the  toolbar button) opens a **text window** where you can view and edit notes related to the current project. If such a window is already opened, it is activated, instead of creating a new one.

See also **Edit Project Notes**.

All settings

This entry of the **Edit** menu (which can be also accessed by pressing `F5` or using the  toolbar button) opens a text window where you can view and edit all the settings of the current problem. If such a window is already opened, it is activated, instead of creating a new one.

See also **Edit all Settings in a Text Window**.

Output

The entries of this menu are used to control the output of the problem solution:

Entry	Key	Meaning
New graph window	<code>Ctrl+G</code>	Open a new graphics window for output
Graphics format	<code>Ctrl+F</code>	Edit format options for a graphics window
Color	<code>Ctrl+O</code>	Select colors
View point	<code>Ctrl+W</code>	Select the view point for projections
New text window	<code>Ctrl+T</code>	Open a new text window for output
Text format	<code>Ctrl+X</code>	Edit format options for a text window
Plotter settings	<code>Ctrl+P</code>	Manage the plotter output
Plotter output		Toggle On/Off the plotter output

Draw to plotter		Send graphics elements to plotter
Eject page		End plotter page
Write epilog		Write plotter epilog
Status line	Ctrl+S	Format options for output in status line

See also **Printing**.

New graph window

This entry of the **Output** menu (which can be also accessed by pressing Ctrl+G or using the  toolbar button) creates a new window for graphics output. A **Graphics Output** dialog box will automatically open to let you select the corresponding options.

Graphics format

This entry of the **Output** menu (which can be also accessed by pressing Ctrl+F or using the  toolbar button) opens a **Graphics Output** dialog box where you can select the format and options for graphics output in the current graphics window.

Color

This entry of the **Output** menu (which can be also accessed by pressing Ctrl+O or using the  toolbar button) opens a **Colors** dialog box where you can select the colors used for different elements in graphics output. You can also use the PenColor, PenWidth, HSV and RGB functions in the **Horizontal axis** and **Vertical axis** entries of the **Graphics Output** dialog box.

View point

This entry of the **Output** menu (which can be also accessed by pressing Ctrl+W or using the  toolbar button) opens a **View Point** box where you can select the view point and other options for projections.

New text window

This entry of the **Output** menu (which can be also accessed by pressing Ctrl+T or using the  toolbar button) creates a new window for text output. A **Text Output** dialog box will automatically open to let you select the corresponding options.

Text format

This entry of the **Output** menu (which can be also accessed by pressing Ctrl+X or using the  toolbar button) opens a **Text Output** dialog box where you can select the format and options for text output in the current text window.

Plotter settings

This entry of the **Output** menu (which can be also accessed by pressing Ctrl+P or using the  toolbar button) opens a **Plotter** dialog box where you can select the options for plotter output.

See also **Printing**.

Plotter output

This entry of the **Output** menu (which can be also accessed by using the  toolbar button) lets you switch On/Off the plotter output.

See also **Printing**.

Draw to plotter

This entry of the **Output** menu (which can be also accessed by using the  toolbar button) send to the high-quality printing device the graphics elements.

See also **Printing**.

Eject page

This entry of the **Output** menu (which can be also accessed by using the  toolbar button) starts a new page in the high-quality printing device.

See also **Printing**.

Write epilog

This entry of the **Output** menu (which can be also accessed by using the  toolbar button) send to the high-quality printing device an epilog. (This is only necessary in very special cases.)

See also **Printing**.

Status line

This entry of the **Output** menu (which can be also accessed by pressing `Ctrl+S` or using the  toolbar button) opens a **Numerical Output in Status Line** dialog box where you can control the format and options for numerical output to the status line.

Window

The entries of this menu are used to manage windows and their contents:

Entry	Key	Meaning
Erase window	Del	Clear the current output window
Erase all	Ctrl+Del	Clear all output windows
Get screen	Ctrl+Ins or Ctrl+C	Copy screen contents to the clipboard
Put screen	Shift+Ins or Ctrl+V	Copy screen contents from the clipboard
Save screen		Copy screen contents to a disk file
Load screen		Copy screen contents from a disk file
Cascade	Shift+F5	Cascade output windows
Tile horizontal	Shift+F4	Tile horizontally output windows
Tile vertical	Shift+F3	Tile vertically output windows
Arrange icons		Arrange icons of minimized output windows
Close	Ctrl+F4	Close the current output window
Close all		Close all output windows

After the previous menu entries there appear the caption of all the output windows in the problem. This is very convenient to locate and access them quickly.

Erase window

This entry of the **Window** menu (which can be also accessed by pressing `Del` or using the  toolbar button) deletes the contents of the currently active (graphics or text) window. In the case of graphics windows, the solution computed so far is lost, but the graphics elements are drawn again.

Erase all

This entry of the **Window** menu (which can be also accessed by pressing `Ctrl+Del` or using the  toolbar button) deletes the contents of every (graphics and text) window. In the case of graphics windows, the solution computed so far is lost, but the graphics elements are drawn again.

Get screen

This entry of the **Window** menu (which can be also accessed by pressing `Ctrl+C` or `Ctrl+Ins` or using the  toolbar button) copies the contents of the current (graphics or text) window to the clipboard, from where it can be pasted in other windows or applications.

See the **Saving and loading graphics and text screens** section.

Put screen

This entry of the **Window** menu (which can be also accessed by pressing `Ctrl+V` or `Shift+Del` or using the  toolbar button) copies the contents of the clipboard to the current graphics window.

Save screen

This entry of the **Window** menu (which can be also accessed by using the  toolbar button) saves the contents of the current (graphics or text) window to an external file, which is selected in a standard **Save** dialog box.

See the **Saving and loading graphics and text screens** section.

Load screen

This entry of the **Window** menu (which can be also accessed by using the  toolbar button) send to the current graphics window the context of an external file, which is selected in a standard **Open** dialog box.

Cascade

This entry of the **Window** menu (which can be also accessed by pressing `Shift+F5` or using the  toolbar button) stacks all open edit windows and overlaps them so that each is the same size as all others and only part of each underlying **client window** is visible.

Tile horizontal

This entry of the **Window** menu (which can be also accessed by pressing `Shift+F4` or using the  toolbar button) arranges your open windows from top to bottom so that they cover the entire width of the client window without overlapping one another, in a manner that allows more width than height for each window.

Tile vertical

This entry of the **Window** menu (which can be also accessed by pressing `Shift+F3` or using the  toolbar button) arranges your open windows from left to right so that they display next to each other, in a manner that allows more height than width for each window.

Arrange icons

This entry of the **Window** menu (which can be also accessed by using the  toolbar button) rearranges any icons on the client window. The rearranged icons are evenly spaced, beginning at the lower left corner of the client window.

Close

This entry of the **Window** menu (which can be also accessed by pressing `Ctrl+F4` or using the  toolbar button) closes the currently active (graphics or text) window.

Close all

This entry of the **Window** menu (which can be also accessed by using the  toolbar button) closes all (graphics and text) windows.

List of Windows

At the end of the **Window** menu there appears a list of all the (graphics and text) output windows. You can switch to any of them by simply selecting the corresponding menu entry.

Go

The entries of this menu are used to solve the mathematical problem:

Entry	Key	Meaning
Start!	Enter	Start solving the problem
Stop!	Esc	Stop solving the problem
Continue	Ctrl+U	Continue solving the problem
Backward	Ctrl+B	Solve for decreasing values of the independent variable

Start!

This entry of the **Go** menu (which can be also accessed by pressing `Enter` or using the  toolbar button) starts solving the continuous or discrete dynamical system.

See also **Solving the system**.

Stop!

This entry of the **Go** menu (which can be also accessed by pressing `Esc` or using the  toolbar button) stops the solution of the continuous or discrete dynamical system. Depending of the complexity of the problem and the selected tolerance, it may be necessary to wait for a while before the solution really stops.

See also **Solving the system**.

Continue

This entry of the **Go** menu (which can be also accessed by pressing `Ctrl+U` or using the  toolbar button) continues, if possible, with a stopped solution of the continuous or discrete dynamical system.

See also **Solving the system**.

Backward

This entry of the **Go** menu (which can be also accessed by pressing `Ctrl+B` or using the  toolbar button) lets you switch the solution direction, but only in the case of continuous (differential) systems.

See also **Solving the system**.

Draw

The entries of this menu are used to manage graphics elements:

Entry	Key	Meaning
Refresh window	D	Redraw graphics elements in current output window
Refresh all	Ctrl+D	Redraw graphics elements in all output windows
Full screen	Ctrl+Home	Use the full screen to display only graph and text windows.
New element	N	Create a new graphics element
Edit element	E	Edit the current graphics element
Next element	+ *	Select the next element
Previous element	- *	Select the previous element
Copy element	C	Duplicate the current element
Remove element		Remove the current element
Remove all		Remove all graphics elements
Action		Select the action of the cursor and the mouse pointer
Zoom, step and order		Open the zoom menu

*These keys are those in the numeric keypad

Refresh window

This entry of the **Draw** menu (which can be also accessed by pressing `D` or using the  toolbar button) redraws the contents (including graphics elements) of the current graphics window.

Refresh all

This entry of the **Draw** menu (which can be also accessed by pressing `Ctrl+D` or using the  toolbar button) redraws the contents (including graphics elements) of every graphics window.

Full screen

This entry of the **Draw** menu (which can be also accessed by pressing `Ctrl+Home` or using the  toolbar button) uses the full screen to display only the graph and text windows. No caption, menu, toolbar or status line is visible. They can still be accessed by using keyboard shortcuts.

The popup menu that opens by clicking on the right mouse button is also available. Use it (or `Ctrl+Home`, or `Alt+D F`) to return to the usual display style.

New element

This entry of the **Draw** menu (which can be also accessed by pressing N or using the  toolbar button) creates a new graphics element. You will select the element type in a **New graphics element** dialog box and the remaining options in a **Graphics Element** dialog.

Edit element

This entry of the **Draw** menu (which can be also accessed by pressing E or using the  toolbar button) opens the **Graphics Element** dialog box where you can edit the graphics elements. The dialog initially displays the options for the current element, but you may easily view and edit any element.

Next element

This entry of the **Draw** menu (which can be also accessed by pressing + in the numeric keypad or using the  toolbar button) selects the next graphics element.

Previous element

This entry of the **Draw** menu (which can be also accessed by pressing - in the numeric keypad or using the  toolbar button) selects the previous graphics element.

Copy element

This entry of the **Draw** menu (which can be also accessed by pressing C or using the  toolbar button) makes a duplicate of the current graphics element.

Remove element

This entry of the **Draw** menu (which can be also accessed by using the  toolbar button) removes the current graphics element.

Remove all

This entry of the **Draw** menu (which can be also accessed by using the  toolbar button) removes all the graphics elements.

Zoom, step and order

The entries of this menu are used to control the viewing scale, the cursor step size and the fractal order:

Entry	Key	Meaning
Zoom in	I	Show finer details around the cursor
Zoom out	W	Show a wider area around the cursor
Actual size	Z	Recover viewing scale before zoom
Larger step	* *	Multiply by 2 the cursor step
Smaller step	/ *	Divide by 2 the cursor step
Next fractal order		Increase the current fractal order and redraw the window
Previous fractal order		Decrease the current fractal order and redraw the window

*These keys are the multiplication and fraction bar in the numeric keypad

Zoom in

This entry of the **Zoom, step and order** menu (which can be also accessed by pressing Γ or using the  toolbar button) will change the current window ranges to show finer details around the cursor. Using the **Graphics Output** dialog box will make permanent these changes. The **Zooming factor** is controlled from the **Cursor** dialog sheet.

Use **Actual size** to recover the original ranges. In particular, you should use it before starting or resuming solving the problem, because if the window does not have its natural scale, the graphics output will not be saved in the bitmap even if the **Bitmap output** entry in the **Format** sheet is different from **No bitmap**. The metafile output, on the other hand, is always saved if the **Save output in memory** entry in the **Format** sheet is not null, even if the window is zoomed.

Zoom out

This entry of the **Zoom, step and order** menu (which can be also accessed by pressing W or using the  toolbar button) will change the current window ranges to show a wider area around the cursor. Using the **Graphics Output** dialog box will make permanent these changes. The **Zooming factor** is controlled from the **Cursor** dialog sheet.

Use **Actual size** to recover the original ranges. In particular, you should use it before starting or resuming solving the problem, because if the window does not have its natural scale, the graphics output will not be saved in the bitmap even if the **Bitmap output** entry in the **Format** sheet is different from **No bitmap**. The metafile output, on the other hand, is always saved if the **Save output in memory** entry in the **Format** sheet is not null, even if the window is zoomed.

Actual size

This entry of the **Zoom, step and order** menu (which can be also accessed by pressing Z or using the  toolbar button) will recover the window ranges active before using **Zoom in** or **Zoom out**, if possible.

You should use it before starting or resuming solving the problem, because if the window does not have its natural scale, the graphics output will not be saved in the bitmap even if the **Bitmap output** entry in the **Format** sheet is different from **No bitmap**. The metafile output, on the other hand, is always saved if the **Save output in memory** entry in the **Format** sheet is not null, even if the window is zoomed.

Larger step

This entry of the **Zoom, step and order** menu (which can be also accessed by pressing $*$ on the numeric keypad or using the  toolbar button) doubles the elementary step that the cursor will move while using the direction keys, or the angle step in **First value**, **Last value**, and **Rotate** modes. These values are also controlled in the **Horizontal step**, **Vertical step** and **Angle** entries of the **Cursor** dialog sheet.

Smaller step

This entry of the **Zoom, step and order** menu (which can be also accessed by pressing $/$ on the numeric keypad or using the  toolbar button) halves the elementary step that the cursor will move while using the direction keys, or the angle step in **First value**, **Last value**, and **Rotate** modes. These values are also controlled in the **Horizontal step**, **Vertical step** and **Angle** entries of the **Cursor** dialog sheet.

Next fractal order

This entry of the **Zoom, step and order** menu (which can be also accessed by using the  toolbar button) increases the fractal order to be added to the **Begin** value of each **fractal** element to get the actual order (always below **End** and above **Begin**). This value is also controlled in the **Fractal order** entry of the **Cursor** dialog sheet.

Previous fractal order

This entry of the **Zoom, step and order** menu (which can be also accessed by using the  toolbar button) decreases the fractal order to be added to the **Begin** value of each **fractal** element to get the actual order (always below **End** and above **Begin**). This value is also controlled in the **Fractal order** entry of the **Cursor** dialog sheet.

Action

The entries of this menu are used to define the action of the cursor and the mouse pointer:

Entry	Key	Meaning
Initial conditions	X	The cursor will select new initial conditions
Select an element	S	The cursor will select a graphics element
Move around	M	The cursor will show its coordinates
Origin	O	The cursor will select the origin
Point	P	The cursor will select the point
Third point	T	The cursor will select the third point
First value	F	The cursor will select the first value
Last value	L	The cursor will select the last value
Rotate	R	The cursor will select the rotation angle

Initial conditions

If this entry of the **Action** menu (which can be also accessed by pressing X or using the  toolbar button) is used the direction keys and the mouse pointer will select new initial conditions.

See also **Solving the system**.

Select an element

If this entry of the **Action** menu (which can be also accessed by pressing S or using the  toolbar button) is used the direction keys and the mouse pointer will select the nearest graphics element when you double click on the left button. (If you have complex graphics elements, you may have to wait while the program computes the nearest element.)

Move around

If this entry of the **Action** menu (which can be also accessed by pressing M or using the  toolbar button) is used the direction keys and the mouse pointer will do nothing but show their coordinates in the status line. Use this to select a point and then zoom around it for fine tuning.

Origin

If this entry of the **Action** menu (which can be also accessed by pressing O or using the  toolbar button) is used the direction keys and the mouse pointer will

	select	change
Segment	the origin	x and y
Circle	the center	x and y
Ellipse	the center	x and y
Arrow	the arrow head	x and y
Text	the string start	x and y

2-curve		x0 and y0 parameters
3-curve		x0 and y0 parameters
Special		(not used)
Data	the translation values	x and y
Fractal	the origin of the basic segment	x and y

Point

if this entry of the **Action** menu (which can be also accessed by pressing P or using the  toolbar button) is used the direction keys and the mouse pointer will

	select	change
Segment	the end	Size and Direction
Circle	a perimeter point	Radius and Direction
Ellipse	a perimeter point	Radius and Direction
Arrow		(not used)
Text	the string end	Radius and Direction
2-curve		(not used)
3-curve		(not used)
Special		(not used)
Data	the horizontal scale and direction	Radius and Direction
Fractal	the end of the basic segment	Size and Direction

Third point

If this entry of the **Action** menu (which can be also accessed by pressing T or using the  toolbar button) is used the direction keys and the mouse pointer will

	select	change
Segment		(not used)
Circle		(not used)
Ellipse	the minor semiaxis	Size
Arrow		(not used)
Text	the string height	Size
2-curve		(not used)
3-curve		(not used)
Special		(not used)
Data	the vertical scale	Size
Fractal		(not used)

First value

If this entry of the **Action** menu (which can be also accessed by pressing F or using the  toolbar button) is used the direction keys and the mouse pointer will

	select	change
Segment		(not used)
Circle	the arc start	First value
Ellipse	the arc start	First value
Arrow	the arrow angle	First value
Text		(not used)
2-curve	the range start	First value
3-curve	the range start	First value
Special		(not used)
Data		(not used)
Fractal		(not used)

Last value

If this entry of the **Action** menu (which can be also accessed by pressing L or using the  toolbar button) is used the direction keys and the mouse pointer will

	select	change
Segment		(not used)
Circle	the arc end	Last value
Ellipse	the arc end	Last value
Arrow		(not used)
Text		(not used)
2-curve	the range end	Last value
3-curve	the range end	Last value
Special		(not used)
Data		(not used)
Fractal		(not used)

Rotate

If this entry of the **Action** menu (which can be also accessed by pressing R or using the  toolbar button) is used the direction keys and the mouse pointer will

	select	change
Segment	the direction	Direction
Circle	the arc direction	Direction
Ellipse	the arc direction	Direction
Arrow	the direction	Direction
Text	the direction	Direction
2-curve		d parameter
3-curve		d parameter
Special		(not used)
Data	the direction	Direction
Fractal	the direction of the basic segment	Direction

Configuration

The entries of this menu are used to define your preferences, and to manage the installed external libraries, plotter drivers, fonts and escape sequences:

Entry	Meaning
Preferences	Miscellaneous options
Toolbar and popup menu	Manage the popup menu and toolbars
External codes	Manage external integration methods
Mathematical extensions	Manage mathematical extensions
Plotter drivers	Manage plotter drivers
Font files	Manage font files
Escape sequences	Manage escape sequences

See also **Customization**

Preferences

This entry of the **Configuration** menu (which can be also accessed by using the  toolbar button) opens the **Preferences** dialog box where you can select different options to customize your copy of *Dynamics Solver*.

See also **Customizing Dynamics Solver**.

Toolbar and popup menu

This entry of the **Configuration** menu (which can be also accessed by using the  toolbar button) opens the **Customize Toolbars and Popup Menu** dialog box where you can control the toolbars and the popup menu that opens when using the right mouse button.

See also **Customizing Dynamics Solver**.

External codes

This entry of the **Configuration** menu (which can be also accessed by using the  toolbar button) opens the **Install External Methods** dialog box where you can control the integration codes contained in external .DLL libraries.

See also **Writing external integration codes**.

Mathematical extensions

This entry of the **Configuration** menu (which can be also accessed by using the  toolbar button) opens the **Install Mathematical DLLs** dialog box where you can control the mathematical extensions contained in external .DLL libraries.

See also **Adding mathematical functions and constants**.

Plotter drivers

This entry of the **Configuration** menu (which can be also accessed by using the  toolbar button) opens the **Install Plotter Drivers** dialog box where you can control the plotter drivers for plotter output.

Font files

This entry of the **Configuration** menu (which can be also accessed by using the  toolbar button) opens the **Install Font Files** dialog box where you can control the font files needed while using graphics elements.

Escape sequences

This entry of the **Configuration** menu (which can be also accessed by using the  toolbar button) opens the **Escape Sequences** dialog box where you can edit the escape sequences (macros) used in **text** elements.

See also **Using graphics elements**.

Help

The entries of this menu may be used to access different points of this and other help files. They also provide information on the installed fonts.

Entry	Key	Meaning
Index	F1	Open the Table of Contents of the help system
Search for help on		Search for keywords in help topics

Help on Help		Show how to use the help system
Show fonts		Display characters in different fonts
About <i>Dynamics Solver</i>		Display the (amusing) information dialog

After the previous menu entries there can appear the names of the help files corresponding to external libraries (integration methods or mathematical extensions) currently in use.

Index

This entry of the **Help** menu (which can be also accessed by pressing F1 or using the  toolbar button) opens the **Table of Contents** of this help file. There is also a **Tutorial**.

Search for help on

This entry of the **Help** menu opens a dialog menu from where you can search for keywords across the help file.

Help on Help

This entry of the **Help** menu opens the **Getting Help** topic where you can learn the different ways of getting help. You should read it.

Show fonts

This entry of the **Help** menu (which can be also accessed by using the  toolbar button) displays characters in different fonts. Useful to know if a character is in a given font, or to know how to get a special symbol.

About *Dynamics Solver*

This entry of the **Help** menu displays the (amusing) information dialog. You should try it at least once. Double click on the left panel!

Help for External DLLs

The last entries in the **Help** menu, if present, will open the table of contents of the help files corresponding to integration codes and mathematical extensions contained in external .DLL libraries.

See also **Writing external integration codes** and **Adding mathematical functions and constants**.

Dialog Boxes

Dialog name	Meaning
Dynamical System Type	Type of dynamical system
Advanced settings	and advanced settings
Definitions	Defining variables,
Parameters	parameters,
Equations	equations and
Initial values	initial values and functions
Boundary conditions	boundary conditions
Iteration Range	Iteration range or
Numerical Method	Numerical integration method and
Range	integration range
Graphics Output	Defining graphics output (ODEs)
Graphics Output for maps	Defining graphics output (maps)
Format	format,
Cursor	cursor definitions and
Plotter	plotter settings
Colors	Select output colors
View Point for Projections	Select view point for projections
Text Output	Defining text output (ODEs)
Text Output for maps	Defining text output (maps)
Numerical Output in Status Line	Numerical values in status line
Preferences	Select several options, including
Boxes	box output and
Graphics	options for graphics output
Other	other options
Print	Print current window
External Libraries	Manage external integration methods/ mathematical extensions/ plotter drivers or font files
Graphics Element	Define graphics element
New Graphics Element	Create a new graphics element
Escape Sequences	Manage escape sequences
Customize Toolbars and Popup Menu	Manage popup menu and toolbars
About Dynamics Solver	Information on <i>Dynamics Solver</i>
Dynamics Solver is already running	To select a previous copy of the program
Mode	How to paste/import a graphics screen
Compiler Error	Error in expression compiler
Input File	Select file and format to read data from
Output File	Select file and format to write data to
Input Value	Input a numeric value

Common Dialog Boxes	Meaning
Color	Customizing colors
Open/Save	Open files for reading/writing
Print	Select printer driver
Printer Setup	Select printer options
Find	Find string expressions
Replace	Replace string expressions

Dynamical System Type

This dialog box is started from the **Edit/Type** menu entry or by pressing **Ctrl+Y**.

Entry	Meaning
Dynamical system	Select one of the following three radio buttons
Single ODE	To solve a single differential equation of arbitrary order
System of ODEs	To solve an arbitrary number of first-order differential equations
Map (iteration)	To solve a discrete dynamical system: an iterated map
Dimension/Order	Enter here the order of the only equation, the number of first-order equations or the dimension of the map
Advanced settings	Open other sheet of the same dialog to select some advanced options
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Before accepting changes to these options, you will be given the opportunity to save the current settings because changing anything in this dialog will reset all definitions. You may also **Cancel** and use the **Edit all settings** menu.

Advanced settings

This is a sheet of the **Type** dialog box, which is started from the **Edit/Type** menu entry or by pressing **Ctrl+Y**.

Entry	Meaning
Compiler options	The following entries should be changed only for very complex problems
Stack length	The number of bytes to be used as stack to evaluate expressions
P-code length	The number of bytes to store compiled expressions
Mathematical option	Select below the value of the Heaviside function at the origin
Heaviside(0)	The most usual values are 1/2, 0 and 1.
Type	Open another sheet in the same dialog to select the type of dynamical system
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Definitions

In the sheets of this dialog box, which opens with **Ctrl+L** or by using the **Edit/Variables** menu entry, you can select names and values for variables, parameters, equations, initial values and functions and boundary conditions.

Entry	Meaning
Independent variable	Name of the independent variable
Index	Name of iteration index (in maps)
Initial value	Name of the initial value of the independent variable or iteration index
Dependent variable(s)	Collective and individual names for the dependent variables
Initial value(s)	and their initial values
Collective name(s)	Collective names to use in the form $x[1]$
Individual name(s)	Individual names
Parameters	Open another sheet of the same dialog to select the names and values of the parameters

Equations	Open another sheet of the same dialog to enter the equations defining the dynamical system
Initial values	Open another sheet of the same dialog to select the initial values
Boundary	Open another sheet of the same dialog to select the boundary conditions
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Parameters

This sheet of the **Definitions** dialog can be directly accessed from the Edit/**Parameters** menu or by using `Ctrl+A`. You can define here the names and values of different parameters.

Entry	Meaning
Parameter # <i>n</i>	Name and value for the problem parameters
Last Poincaré time Bifurcation variable	Name of the last value of the independent variable in a Poincaré section (in differential equations) or name for the bifurcation parameters (in maps)
Repeated solutions and phase portraits	Ranges and names to draw phase portraits or using repeated integration/iteration
Variables	Open another sheet of the same dialog to select the name of variables
Equations	Open another sheet of the same dialog to enter the equations defining the dynamical system
Initial values	Open another sheet of the same dialog to select the initial values
Boundary	Open another sheet of the same dialog to select the boundary conditions
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Equations

This sheet of the **Definitions** dialog can be directly accessed from the Edit/**Equations** menu or by using `Ctrl+E`. You can enter here the dynamical system equations.

Entry	Meaning
Edit definitions	Expression defining the equation or map
Power	(Only for maps) The power of the map whose graphics will be displayed in the screen to have a cobweb diagram. See also <code>CHAOS\DISKS.DS</code> for a tricky example.
Delay	(Not available for maps) The delay in functional-differential equations.
Discontinuities	(Not available for maps) The location of discontinuity points in functional-differential equations.
Variables	Open another sheet of the same dialog to select the names of variables
Parameters	Open another sheet of the same dialog to select the names and values of the parameters
Initial values	Open another sheet of the same dialog to select the initial values
Boundary	Open another sheet of the same dialog to select the boundary conditions
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Initial values

This sheet of the **Definitions** dialog can be directly accessed from the Edit/**Initial values** menu or by using **Ctrl+I**. You can enter here the initial values.

Entry	Meaning
Independent variable	Initial value of the independent variable or index
Dependent variable(s)	Initial value(s) of the dependent variable(s)
Initial functions	Expressions defining the initial values. Useful to study functional-differential equations, subspaces of phase space, constrained systems, phase portraits, etc.
Variables	Open another sheet of the same dialog to select the names of variables
Equations	Open another sheet of the same dialog to enter the equations defining the dynamical system
Parameters	Open another sheet of the same dialog to select the names and values of the parameters
Boundary	Open another sheet of the same dialog to select the boundary conditions
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Boundary conditions

This sheet of the **Definitions** dialog can be directly accessed from the Edit/**Boundary conditions** menu or by using **Ctrl+Z**. You can enter here the boundary conditions.

Entry	Meaning
Point	Value of the independent variable (index) at which the boundary conditions have to be evaluated.
Iterations	Maximum number of Newton iterations to get the initial values that will satisfy the boundary conditions.
Error	Maximum absolute value the conditions below may have in order to consider that the boundary conditions are satisfied.
Boundary conditions	Expressions defining the boundary conditions. Useful to study boundary value problems, subspaces of phase space, constrained systems, phase portraits, etc.
Variables	Open another sheet of the same dialog to select the names of variables
Equations	Open another sheet of the same dialog to enter the equations defining the dynamical system
Parameters	Open another sheet of the same dialog to select the names and values of the parameters
Initial values	Open another sheet of the same dialog to select the initial values
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Iteration Range

This dialog is exclusive to maps and can be directly accessed from the Edit/**Range** menu or by using **Ctrl+R**. You can enter here the solution range.

Entry	Meaning
First value	The output starts when the index equals this value (or the index initial value if this

	entry is 0). It is useful to skip over transients.
Last value	The output ends when the index equals this value
Infinity	The output ends with a warning if the absolute value of any variable reaches this value
Pause	The time (in tenths of second) a blinking cursor will show the last output value. If the value is negative you must press a key to get the next output point. See the examples in CHAOS\DISKS.DS and GRAPH.DS.
Saved points	The number of solution points to be retained in memory
Bifurcation diagram	Select it if you want to draw a bifurcation diagram
Begin	The first value of the parameter in the bifurcation diagram
Step	The step to increase the parameter value in the bifurcation diagram
End	The last value of the parameter in the bifurcation diagram
Real time	If not null, the number of real seconds corresponding to the unity of the independent variable or index. See Forcing the Program to Wait
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Range

This sheet of the **Numerical Method** dialog can be directly accessed from the Edit/**Range** menu or by using **Ctrl+R**. You can enter here the solution range.

Entry	Meaning
First value	The output starts when the independent variable equals this value (or the initial value for the independent variable if this entry is 0). It is useful to skip over transients.
Last value	The output ends when the independent variable equals this value
Infinity	The output ends with a warning if the absolute value of any variable or derivative reaches this value
Pause	The time (in tenths of second) a blinking cursor will show the last output value. If the value is negative you must press a key to get the next output point.
Interpolate	If this entry is not selected, the output happens every time the numerical code computes a new point. If it is selected, the output happens in steps equal to the value selected below.
Step	The independent variable step for interpolated output
Backward	Select the entry to integrate backward
Saved points	The number of solution points to be retained in memory. A non-zero value is necessary to have interpolated output, compute derivatives or Poincaré maps or to solve delay-differential equations
Real time	If not null, the number of real seconds corresponding to the unity of the independent variable or index. See Forcing the Program to Wait
Method	Open another sheet in the same dialog with options for the integration method
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Numerical Method

This sheet of the **Numerical Method** dialog can be directly accessed from the Edit/**Method** menu or by using **Ctrl+H**. You can enter here the parameters of the numerical method. This dialog does not exist for maps.

Entry	Meaning
Tolerance	The tolerance value to be achieved by the integration code
Step h	The starting value of the integration step
Max. h	The maximum value of the integration step
Integration code	The integration code to be used (it may have additional parameters)
Code help	Open the help file corresponding to the selected integration code
Range	Open another sheet to define the integration and output ranges
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Graphics Output

This sheet of the **Graphics Output** dialog can be directly accessed from the Output/**Graphics format** menu or by using **Ctrl+F**. You can enter here the output expressions.

See also the **Graphics Output for Maps**.

Entry	Meaning
Title	The caption for the graphics window
Window	If the default value (<code>this window</code>) is used, the output appears in the current graphics window. Select another window's caption if you want to redirect the output there, in order to have multiple output to a single window.
Min. x	The minimum value of the expression plotter in the horizontal axis
Horizontal axis	The expression represented in the horizontal axis
Max. x	The maximum value of the expression plotter in the horizontal axis
Min. y	The minimum value of the expression plotter in the vertical axis
Vertical axis	The expression represented in the vertical axis
Max. y	The maximum value of the expression plotter in the vertical axis
Poincaré section	Select this to plot a Poincaré section
Increasing	Select this to output a Poincaré map point when the Condition becomes null while increasing from negative to positive
Decreasing	Select this to output a Poincaré map point when the Condition becomes null while decreasing from positive to negative
Error	Select here the tolerance to accept that the Condition is met
Condition	The expression defining the Poincaré map
Colors	Open another dialog to select the output color
View point	Open another dialog to select the projection view point
Format	Open another sheet in the same dialog to define the output format
Cursor	Open another sheet in the same dialog to select the meaning of cursor and mouse action
Plotter	Open another sheet in the same dialog to select driver and options for the plotter
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Graphics Output for Maps

This sheet of the **Graphics Output** dialog can be directly accessed from the Output/**Graphics format** menu or by using **Ctrl+F**. You can enter here the output expressions.

See also **Graphics Output**.

Entry	Meaning
Title	The caption for the graphics window
Min. x	The minimum value of the expression plotter in the horizontal axis
Horizontal axis	The expression represented in the horizontal axis
Max. y	The maximum value of the expression plotter in the horizontal axis
Min. x	The minimum value of the expression plotter in the vertical axis
Vertical axis	The expression represented in the vertical axis
Max. y	The maximum value of the expression plotter in the vertical axis
Direct graph power	The power of the map in a cobweb (ignored if 0)
Inverse graph power	The power of the inverse map in a cobweb (ignored if 0)
Histogram	The number of points between successive drawing of the histogram
Average value	Select this if an averaged value is desired instead of all the values computed from First value to Last value (in Iteration Range). Useful to compute Liapunov Exponents)
Format	Open another sheet in the same dialog to define the output format
Cursor	Open another sheet in the same dialog to select the meaning of cursor and mouse action
Plotter	Open another sheet in the same dialog to select driver and options for the plotter
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Format

This sheet of the **Graphics Output** dialog can be accessed from the Output/**Graphics format** menu or by using `Ctrl+F`. You can enter here the graphics format parameters.

Entry	Meaning
Continuous line	Select this to have the plotted point joined by segments
Frequency	The frequency of graphics output
Axes	Sequence of groups of four coordinates defining the starting and end point of segments. When the graphics screen is created a default value is entered here if the Default axes entry in the Graphics sheet is set.
Direction field	Select this to plot a direction field (this is not available for maps)
Cell	The distance (in pixels) between point in the direction field (this is not available for maps)
Length	The length (in pixels) of the segment of the direction field (this is not available for maps)
Width	Prescribed value for window width (0 is ignored).
Height	Prescribed value for window height (0 is ignored).
Aspect ratio	To have a 1:1 aspect ratio in the graphics window: Arbitrary Ignore Adjust X range Change Min. x and Max. x Adjust Y range Change Min. y and Max. y Square Window is always square Adjust width Change window width Adjust height Change window height When the graphics screen is created the default value is in the Aspect ratio entry of the Graphics sheet.
Bitmap output	Copy output to bitmap No bitmap No bitmap output Copy bitmap Discard bitmap if the window dimensions change Stretch bitmap Stretch bitmap to new window dimensions

	When the graphics screen is created the default value is in the Bitmap output entry of the Graphics sheet.
Save output in memory	Memory (in multiples of 10 kb) to save graphics output
Colors	Open another dialog to select the output color
View point	Open another dialog to select the projection view point
Output	Open another sheet in the same dialog to define the output expressions
Cursor	Open another sheet in the same dialog to select the meaning of cursor and mouse action
Plotter	Open another sheet in the same dialog to select driver and options for the plotter
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Cursor

This sheet of the **Graphics Output** dialog can be accessed from the **Output/Graphics format** menu or by using **Ctrl+F**. You can enter here the action of the mouse/cursor movements.

Entry	Meaning
Variable in axis X	The variable whose initial value is selected with the cursor x coordinate
Horizontal step	The horizontal step when moving the cursor with the direction keys
Variable in axis Y	The variable whose initial value is selected with the cursor y coordinate
Vertical step	The vertical step when moving the cursor with the direction keys
Angle	Step when changing angles
Zooming factor	Scale factor
Fractal order	Fractal order to be added to that of each fractal curve
Output	Open another sheet in the same dialog to define the output expressions
Format	Open another sheet in the same dialog to define the output format
Plotter	Open another sheet in the same dialog to select plotter options
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Colors

This dialog can be directly accessed from the **Output/Color** menu or by using **Ctrl+O**, and also from the **Format** dialog sheet. You can enter here the output colors.

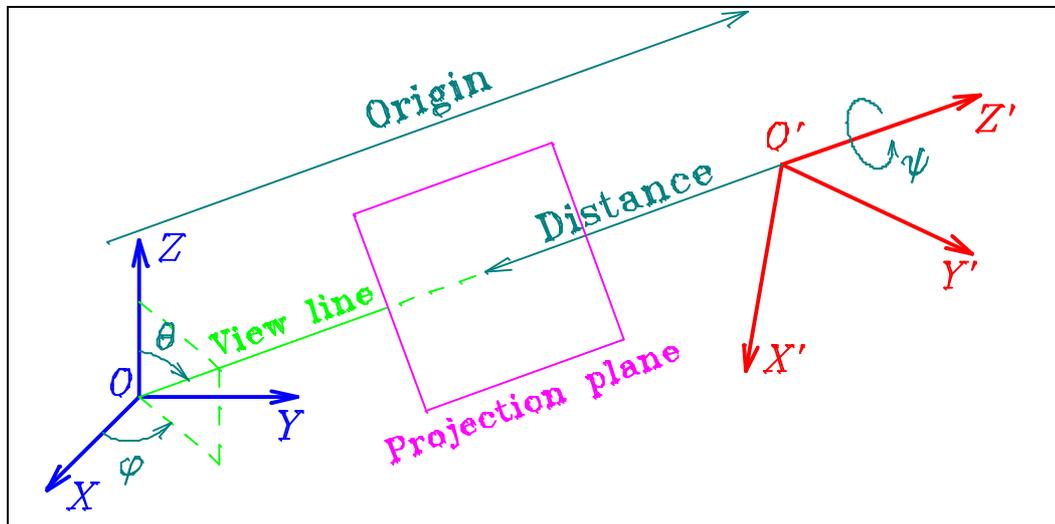
Entry	Meaning
Drawing pen	Select this entry to edit the color for points and lines
Axes	Select this entry to edit the color for axes
Direction field	Select this entry to edit the color for segments in the direction field
Background	Select this entry to edit the color for the window background
Select	Click once to select a color, twice to customize it
Width	Enter here the pen width or select it with the mouse in the list below
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

You can also use the **PenColor**, **PenWidth**, **HSV** and **RGB** functions in the **Horizontal axis** and **Vertical axis** entries of the **Graphics Output** dialog box.

View Point for Projections

This dialog can be directly accessed from the Output/**View point** menu or by using **Ctrl+W**, and also from the **Graphics Output** dialog. You can enter here the parameter for projections.

Most of the entries below can be changed by entering a numerical value, by using a scroll bar or by dragging the cube on the graphics pane.



Entry	Meaning
Theta	First Euler angle (see figure)
Phi	Second Euler angle (see figure)
Psi	Third Euler angle (see figure)
Distance	Distance between observation point and projection plane (see figure)
Origin	Distance between observation point and coordinate origin (see figure)
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Plotter

This sheet of the **Graphics Output** dialog can be directly accessed from the Output/**Plotter settings** menu or by using **Ctrl+P**. You can enter here the plotter driver parameters.

Entry	Meaning
On	Plotter output enabled/disabled for this window. To enable/disable any/all plotter output use the Output/ Plotter output menu or the corresponding button.
Frame	Select this entry to send a frame around the plotter output
Pen	Select here the pen (this entry is highly driver dependent, see Driver help below)
Width	Select here the pen width (this entry is highly driver dependent, see Driver help below)
Speed	Select here the pen speed (this entry is highly driver dependent, see Driver help below)
Line type	Select here the line type (this entry is highly driver dependent, see Driver help below)
Output file	Write here the device (AUX, COM1, COM2, etc.) to collect the plotter output or press the button to open a Open/Save common dialog box to select a file for plotter output
Driver	Select here the plotter driver
Driver help	Pressing this button will provide help on the plotter driver, if available
Output	Open another sheet in the same dialog to define the output expressions
Format	Open another sheet in the same dialog to define the output format

Cursor	Open another sheet in the same dialog to select the meaning of cursor and mouse action
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Text Output

This dialog can be accessed from the Output/**Text format** menu or by using `Ctrl+X`. You can enter here the text format parameters.

Entry	Meaning
Title	The caption for the graphics window
Expression #	The expression(s) to be output
Format	The format string to be used for each Expression
Separator	The format string to be separate different lines/points
Output frequency	The number of solutions points (+1) between to text output lines
Lines in memory	The number of text output line to be retained in memory for backscrolling
Capture	Select this if you want to send the text output to a text file
Output file	Set here the device (PRN, LPT1, LPT2, etc.) to collect the plotter output or
Browse	press this button to open an Open/Save common dialog box to select a file to collect the text output
View point	Open another dialog to select the projection view point
Poincaré section	Select this entry to have a Poincaré section
Condition	The expression defining the Poincaré section
Error	Select here the tolerance to accept that the Condition is met
Increasing	Select this to output a Poincaré map point when the Condition becomes null while increasing from negative to positive
Decreasing	Select this to output a Poincaré map point when the Condition becomes null while decreasing from positive to negative
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Text Output for Maps

This sheet of the **Text Output** dialog can be accessed from the Output/**Text format** menu or by using `Ctrl+X`. You can enter here the text format parameters.

Entry	Meaning
Title	The caption for the graphics window
Expression #	The expression(s) to be output
Format	The format string to be used for each Expression
Separator	The format string to be separate different lines/points
Output frequency	The number of solutions points (+1) between to text output lines
Lines in memory	The number of text output line to be retained in memory for backscrolling
Capture	Select this if you want to send the text output to a text file
Output file	Set here the device (PRN, LPT1, LPT2, etc.) to collect the plotter output or
Browse	Press this button to open an Open/Save common dialog box to select a file to collect the text output
View point	Open another dialog to select the projection view point

Histogram	The number of points between successive writings of the histogram
Average value	Select this if an averaged value is desired (useful to compute Liapunov exponents)
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Numerical Output in Status Line

This dialog can be directly accessed from the Output/**Status line** menu or by using `Ctrl+S`. You can enter here a couple of expressions to be monitored in the status line.

Entry	Meaning
First quantity	The first expression to be output
Second quantity	The second expression to be output
Format string	The format string to be used for each quantity
Frequency	The number of solutions points (+1) between to text output lines
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Compiler Error

This dialog box is opened when the expression compiler finds an error.

Entry	Meaning
(first line)	The name of the expression in error
(edit box)	The expression in error. You can see and change it
(last line)	The error type (see Messages)
OK	Accept changes and close the dialog
Cancel	Cancel changes, abort compilation and close the dialog
Help	Open this help entry

Mode

This dialog will open when you try to send to the current window the graphics contents of the clipboard (Window/**Put screen** or `Ctrl+V` or `Shift+Ins`) or an external file (Window/**Load screen**):

Entry	Meaning
Put	Sometimes more than one of the following formats is available in the clipboard. Select the one you want:
Device dependent bitmap	The fastest one
Device independent bitmap	Portable among different displays
Windows metafile	Portable and scaleable
Copy colors	(For bitmaps) copy colors:
Copy (only in source)	in bitmap
Or (in source or destination)	in either the source or the destination
And (in source and destination)	in the source as well as in the destination
Not (in the negative of source)	not in the source
Xor (in s. or d.)	in the source or the destination, but not in both
Merge (in dest. or not in source)	in source but not in destination
Erase (in source but not in dest.)	in destination but not in source
Resize	If the source and the destination are not equal:

Figure	resize the source
Window	resize the window
None	copy the source with no resizing
Priority to	When copying give priority to
None	no colors
Black	black color
White	white color
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Print

This dialog will open when you try to print the current window by using File/**Print** or F10:

Entry	Meaning
Encapsulated PostScript	Select this entry to print to a (short) Encapsulated PostScript file (selected in Port/File), instead of using the current Windows printer
Pixels/pt	The number of pixels per point ⁸ in the PostScript file (if Encapsulated PostScript is selected). A value of 10-50 will be right
Printer	The selected printer
Setup	Use this button (or the File/ Printer setup menu) to change the selected printer and to configure its orientation and other options
Port/File	The device/file to collect the printout. Use File/ Printer setup to change, or Browse if Encapsulated PostScript is selected
Browse	This button opens an Open/Save common dialog box to select the file (or device) that will collect the Encapsulated PostScript output
Print	There can be different ways to print a graphic window: Nothing Do not print at all the solution and Axes Metafile Use the points in memory (Output/Save output in memory) Bitmap Print the window bitmap, scaled to printer window Bitmap x 1 Print the window bitmap with no scaling Bitmap x 2 Print the window bitmap with double scale Bitmap x 4 Print the window bitmap with double scale
Invert bitmap	Send to printer the negative of the window
Include elements	Send to printer the graphics elements
Units	Select here the units to be used in the remaining entries of this dialog
Basic width	Pen widths will be multiplied by this value. Use 1-20 depending on the resolution of your printer
Frame width	The width of the frame around the picture (0 to suppress)
Preserve aspect ratio	If this entry is selected, the relative horizontal and vertical dimensions will be the same in the screen and in the printout
Rotate PostScript	If Encapsulated PostScript is allowed and this entry is selected, an explicit rotate instruction is included in the EPS file. Do not use it, unless you want a landscape printout when directly uploading the EPS file to the laser printer
Top margin	The margin above the picture†
Height	The picture height‡
Bottom margin	The margin below the picture†
Left margin	The left margin†

⁸ 1 pt = 0.01389 in = 0.3527 mm

1 in = 72 pt = 25.4 mm

1 mm = 2.8353 pt = 0.03937 in

Width	The picture width‡
Right margin	The right margin†
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

† You can also use the mouse to change this value in the right panel displaying the output size and position.

‡ You can also use the mouse while pressing the `Shift` key to change this value in the right panel displaying the output size and position.

Graphics Element

This dialog can be accessed from the Draw/**Edit** menu entry. It is used to view and change the parameters of the current (and all the remaining) graphics element:

Entry	Meaning
+	Display the next graphics element
-	Display the previous graphics element
<<--	Exchange this element and the previous one
-->>	Exchange this element and the next one
New	Create a new element
Copy	Copy the current element to a new one
Remove	Remove the current element
Erase all	Remove all elements
Type	Select here the element type
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

The number and names of the remaining entries in this dialog depend on the **Type** of the element that is currently displayed. It can be one of the following types:

Segment
Circle
Ellipse
Arrow
Text
2-curve
3-curve
Special
Data
Fractal

New Graphics Element

This dialog is open when a new element is created by using the Draw/**Edit** menu entry. It is used to view and change the parameters of the current (and all the remaining) graphics element:

Entry	Meaning
Type	Select here the type of the new element
OK	Accept changes. The Edit Element dialog will open
Cancel	Cancel changes and close the dialog

Help	Open this help entry
------	----------------------

Preferences

This sheet of the **Preferences** dialog can be accessed from the Configuration/**Preferences** menu. You can enter here your preferences to customize some aspects of *Dynamics Solver*'s behavior:

Entry	Meaning
Toolbar	Options for toolboxes
Change cursor over disabled buttons	The cursor shape will change for disabled buttons
Button size	Size of toolbar buttons:
Small	
Medium	
Big	
Disabled	Disabled buttons will appear:
Hidden	
Normal	
Grayed	
Status hints	Display in status bar hints for menus/toolbars:
Always	always
Enabled	only for enabled buttons/menu entries
Never	never
Popup tips	Display popup tips:
Always	for all toolbar buttons
Enabled	only for enabled buttons
Never	never (to avoid continuous screen redrawing)
Boxes	Open another sheet with options for edit boxes
Graphics	Open another sheet with options for graphics
Other	Open another sheet with miscellaneous options
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

See also **Customization** and the **Configuration** menu.

Boxes

This is a sheet of the **Preferences** dialog, which can be accessed from the Configuration/**Preferences** menu. You can enter here your preferences to customize some aspects of *Dynamics Solver*'s behavior:

Entry	Meaning
Dialog boxes	The font used for dialog boxes will have the following:
Point size	(This entry is not available in the 32-bit edition)
Edit boxes	In edit boxes:
Fixed pitch font	use a fixed pitch font (better for math expressions)
Enter key	use the Enter key to start a new line (instead of OK)
Use bold font in	Use a bold font in:
Status line	
Text windows	
Edit windows	
Startup tips	Control the Tip of the Day displayed at program startup
Next tip	Number of the next tip. 0 to disable this feature.
Toolbox	Open another sheet with options for toolbars and menus
Graphics	Open another sheet with options for graphics

Other	Open another sheet with miscellaneous options
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

See also **Customization** and the **Configuration** menu.

Graphics

This is a sheet of the **Preferences** dialog, which can be accessed from the Configuration/**Preferences** menu. You can enter here your preferences to customize some aspects of *Dynamics Solver*'s behavior:

Entry	Meaning
Copy to clipboard	When copying a graphic window to the clipboard include:
Bitmap	Device dependent bitmap
DIB	Device independent bitmap
Metafile	Windows metafile
Aspect ratio	Default value for the same entry in Format To have a 1:1 aspect ratio in the graphics window: Arbitrary Ignore Adjust X range Change Min. x and Max. x Adjust Y range Change Min. y and Max. y Square Window is always square Adjust width Change window width Adjust height Change window height When the graphics screen is created the default value is in the Aspect ratio entry of the Graphics sheet.
Bitmap output	Default value for the same entry in Format No bitmap No bitmap output Copy bitmap Discard bitmap if the dimensions change Stretch bitmap Stretch bitmap to new window dimensions
Memory output	Default memory size (in multiples of 10 kb) to save graphics output, -1 uses as much memory as necessary
Default axes	Check this entry if you want to use default values for Axes in Format when creating a new graph windows
Redraw while editing elements	Continuous redrawing of graphics elements
Toolbox	Open another sheet with options for toolbars and menus
Boxes	Open another sheet with options for edit boxes
Other	Open another sheet with miscellaneous options
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

See also **Customization** and the **Configuration** menu.

Other

This is a sheet of the **Preferences** dialog, which can be accessed from the Configuration/**Preferences** menu. You can enter here your preferences to customize some aspects of *Dynamics Solver*'s behavior:

Entry	Meaning
Identifier	Enter here the identifier (1 to 8 letters/digits/_) to be able to recover from disk taste-dependent options. (See Special Settings.)
Extensions	By default, <i>Dynamics Solver</i> searches problem files with the .DS extension. This will be fine in most cases, but you might have a lot of problem files with another extension. If you want to search problem

	files with extensions, say, .DS, .MAP and .EF, you should enter here <code>*.ds;*.map;*.ef</code>
Filename length in menus	The maximum length of a filename in the File menu. Select 0 if you want only file names with no path
Show extensions (Windows 3.1+)	Check this entry to show problem file extensions (the default in Windows 3.1).
Show full paths (Windows 95/98/NT4)	In Windows 95/98/NT4 extensions are hidden by default, but if this is checked the full filename is displayed instead of using the current <i>Explorer</i> settings.
Double click to select	If this entry is selected, a double click will be necessary to select new initial conditions or a new graphics element
Check previous instances	If this entry is checked and the program is directly started with no problem file, a Dynamics Solver is already running dialog box will open
Save main window position	Save the main window position and size for the next session
Release button to show popup menu	Check this entry to have to release the right mouse button to see the popup menu (the default in Windows 95/98/NT4). In Windows 3.1 the menu will appear by default before releasing the button.
Do not default to My Documents	A new “feature” of Windows 98 makes your My Documents folder the default directory when trying to open or save a new file. If you find this annoying, check this entry and <i>Dynamics Solver</i> will default to the current directory, as under previous Windows versions.
Toolbox	Open another sheet with options for toolbars and menus
Boxes	Open another sheet with options for edit boxes
Graphics	Open another sheet with options for graphics
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Customize Toolbars and Popup Menu

This dialog can be accessed from the Configuration/**Toolbar and popup menu** entry. It is used to customize toolbars and the popup menu you get by pressing the right mouse button:

Entry	Meaning
Toolbar	Select here the toolbar to customize
Visible	Select this entry to make it visible
Popup menu	Select this entry to assign the popup menu to this toolbar
Available buttons	Select here one of the available buttons and use
->	this to install in the toolbar
Current buttons	Select here one of the installed buttons and use
<-	this to remove it from the toolbar
Up	Exchange this button and the previous one
Down	Exchange this button and the next one
Space	Insert a separator in the toolbar
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

External Libraries

This dialog can be accessed from the Configuration/**External codes, Mathematical extensions, Plotter drivers** and **Font files** menu entries. It is used to add, remove and edit these external complements:

Entry	Meaning
Select	in this list box the element you are customizing
Add	Add a new element
Up	Exchange this element and the previous one
Down	Exchange this element and the next one
Remove	Remove this element from the list
Examine	Display the help file for this element, if available
Restore	Restore the initial values for every element
Name	The element name
DLL file / DRV file / FNT file	Complete file specification for the external element (press the button to open a Open/Save common dialog box to select the file)
Help file	Complete file specification for the external element's help file (press the button to open a Open/Save common dialog box to select the file)
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

Escape Sequences

This dialog can be accessed from the Configuration/**Escape sequences** menu entry. It is used to manage escape sequences:

Entry	Meaning
(first line)	Macro name
(edit box)	Macro definition
OK	Accept changes and close the dialog
Cancel	Cancel changes and close the dialog
Help	Open this help entry

About *Dynamics Solver*

This dialog can be accessed from the Help/**About *Dynamics Solver*** menu and display copyright information on your copy of *Dynamics Solver*:

Entry	Meaning
Copyright and License	Open the Copyright help topic
OK	Close the dialog
Help	Open this help entry

Try double clicking on the graphics panel!

Only in the 32-bit edition: you may send an e-mail message to the author by clicking on his electronic address; you may launch your Internet browser to visit his home page.

Dynamics Solver is already running Dialog Box

If the **Check previous instances** entry of the **Other** dialog sheet is checked and there is another instance of *Dynamics Solver* running when the program is started directly (with no problem file appended after its file specification), this dialog will open.

You can select from a list one of the problem files that are already open or start a new copy of the program. It is also possible to cancel the operation.

Entry	Meaning
Open selected copy	Resume analyzing the problem that is currently selected in the list. It will be brought to the foreground and restored if it was minimized
New copy	Start a new instance of the program
Cancel	Cancel the operation and close the program
Help	Open this help entry

Input File Dialog Box

In this dialog box (which opens the first time the program evaluates `read(i)` or each time it evaluates `openread(i,x)`) you can select the file and format from which the data will be read when using `read(i)`.

Entry	Meaning
File identifier	The integer number identifying the file
File name	The complete file specification. Using the button you can select it in an Open File dialog box
Format string	The format string used to read data
OK	Accept new values
Cancel	Close the dialog and retain old values
Help	Open this help entry

Output File Dialog Box

In this dialog box (which opens the first time the program evaluates `write(i,x)` or each time it evaluates `openwrite(i,x)`) you can select the file and format to which the data will be written when using `write(i,x)`.

Entry	Meaning
File identifier	The integer number identifying the file
File name	The complete file specification. Using the button you can select it in an Open File dialog box
Format string	The format string used to write data
OK	Accept new values
Cancel	Close the dialog and retain old values
Help	Open this help entry

Input Value Dialog Box

In this dialog box (which opens every time the program evaluates `input(i)`) you have to enter a numeric value to be returned by the function `input(i)`.

Entry	Meaning
Input #	The integer number identifying the input value. In the edit box next to this string you have to enter a numeric value

OK	Return the value
Cancel	Close the dialog, return <code>i</code> and end the current iteration (equivalent to returning <code>break(1, i)</code>)
Help	Open this help entry

Tip of the Day Dialog Box

In this dialog box (which opens every time the program starts) you may find some useful hints.

Entry	Meaning
Show tips at startup	Uncheck this entry to disable the Tip of the Day. It may be re-enabled from the Next tip entry of the Boxes dialog box.
Next tip	Display the next tip (its number is stored in the aforementioned entry).
Close	Close the dialog
Help	Open this help entry

Color Common Dialog Box

This common dialog is invoked by *Dynamics Solver* but managed by Windows itself. Refer to your Windows documentation.

Open/Save Common Dialog Box

This common dialog is invoked by *Dynamics Solver* but managed by Windows itself. Refer to your Windows documentation.

Print Common Dialog Box

This common dialog is invoked by *Dynamics Solver* but managed by Windows itself. Refer to your Windows documentation.

Printer Setup Common Dialog Box

This common dialog is invoked by *Dynamics Solver* but managed by Windows itself. Refer to your Windows documentation.

Find Common Dialog Box

This common dialog is invoked by *Dynamics Solver* but managed by Windows itself. Refer to your Windows documentation.

Replace Common Dialog Box

This common dialog is invoked by *Dynamics Solver* but managed by Windows itself. Refer to your Windows documentation.

Keyboard

With a few exceptions, everything can be achieved in *Dynamics Solver* by means of the keyboard. We list below the key actions that are specific to *Dynamics Solver* or standard Windows keys.

Menu Entries

Many entry menus can be directly activated by using one of the following shortcuts:

Key	Menu entry
F1	Open the help Index
F2	File/ Save
F3	File/ Open
Shift+F3	Window/ Tile vertical
Shift+F4	Window/ Tile horizontal
F5	Edit/ All settings
Shift+F5	Window/ Cascade
F10	File/ Print
+ *	Draw/ Next element
- *	Draw/ Previous element
* *	Zoom/ Larger step
/ *	Zoom/ Smaller step
Enter	Go/ Start (or redraw current element)
Esc	Go/ Stop (or stop the redrawing of graphics elements and metafiles)
Del	Window/ Erase window
Ctrl+Del	Window/ Erase all
Ctrl+Ins	Window/ Get screen
Shift+Ins	Window/ Put screen
Ctrl+A	Edit/ Parameters
Ctrl+B	Go/ Backward
Ctrl+C	Window/ Get screen
Ctrl+D	Draw/ Refresh all
Ctrl+E	Edit/ Equations
Ctrl+F	Output/ Graphics format
Ctrl+G	Output/ New graphics window
Ctrl+H	Edit/ Method
Ctrl+I	Edit/ Initial conditions
Ctrl+L	Edit/ Variables
Ctrl+N	Edit/ Notes
Ctrl+O	Output/ Color
Ctrl+P	Output/ Parameters
Ctrl+R	Edit/ Range
Ctrl+S	Output/ Status line
Ctrl+T	Output/ New text window
Ctrl+U	Go/ Continue
Ctrl+V	Window/ Put screen
Ctrl+W	Output/ View point
Ctrl+X	Output/ Text format
Ctrl+Y	Edit/ Type
Ctrl+Z	Edit/ Boundary conditions
C	Draw/ Copy
D	Draw/ Refresh window
E	Draw/ Edit

F	Action/ First value
I	Zoom/ Zoom in
L	Action/ Last value
M	Action/ Move around
N	Draw/ New element
O	Action/ Origin
P	Action/ Point
R	Action/ Rotate
S	Action/ Select an element
T	Action/ Third point
X	Action/ Initial condition
W	Zoom/ Zoom out
Z	Zoom/ Actual size
Direction keys	To move the cursor, origin, point, third point, or to rotate, and to select first and last values.

*These keys are those in the numeric keypad

Cursor Movement Keys

Key(s)	Function
Direction key	Moves the cursor left, right, up, or down in a field or a graphics window.
Shift+direction key	Moves ten times the cursor left, right, up, or down in a graphics window.
End or Ctrl+Right Arrow	Moves to the end of a field.
Home or Ctrl+Left Arrow	Moves to the beginning of a field.
Page Up or Page Down	Moves up or down in a field, one screen at a time.

Dialog Box Keys

Key(s)	Function
Tab	Moves from field to field (left to right and top to bottom).
Shift+Tab	Moves from field to field in reverse order.
Alt+letter	Moves to the option or group whose underlined letter matches the one you type.
Direction key	Moves from option to option within a group of options.
Enter	Executes a command button.
	Or, chooses the selected item in a list box and executes the command.
Esc	Closes a dialog box without completing the command. (Same as Cancel)
Alt+Down Arrow	Opens a drop-down list box.
Alt+Up or Down Arrow	Selects item in a drop-down list box.
Space bar	Cancels a selection in a list box.
	Selects or clears a check box.
Ctrl+/ 	Selects all the items in a list box.

Ctrl+\	Cancels all selections except the current selection.
Shift+direction key	Extends selection in a text box.
Shift+Home	Extends selection to first character in a text box.
Shift+End	Extends selection to last character in a text box

Editing Keys

Key(s)	Function
Backspace	Deletes the character to the left of the cursor. Or, deletes selected text.
Del	Deletes the character to the right of the cursor. Or, deletes selected text.

Help Keys

Key(s)	Function
F1	Displays the help topic of the currently selected menu command, if any, or the Help Index . If the Help window is already open, pressing F1 displays the Using Windows Help topic.
Shift+F1	The mouse pointer will change and you can put it over a toolbar button or screen piece. Click the left button and the corresponding help topic will be shown. Use Esc to abandon the operation.

Menu Keys

Key(s)	Function
Alt	Selects the first menu on the menu bar.
Letter key	Chooses the menu, or menu item, whose underlined letter matches the one you type.
Alt+letter key	Pulls down the menu whose underlined letter matches the one you type.
Left or Right Arrow	Moves among menus.
Up or Down Arrow	Moves among menu items.
Enter	Chooses the selected menu item.

System Keys

The following keys can be used from any window, regardless of the application you are using.

Key(s)	Function
Ctrl+Esc	Switches to the Task List.
Alt+Esc	Switches to the next application window or minimized icon, including full-screen programs.
Alt+Tab	Switches to the next application window, restoring applications that are running as icons.
Ctrl+Tab	Switches to the next window in the same application, restoring minimized windows.
Alt+PrtSc	Copies the entire screen to Clipboard.
Ctrl+F4	Closes the active window.

F1	Gets Help (See Help Keys)
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Text Selection Keys

Key(s)	Function
Shift+Left or Right Arrow	Selects text one character at a time to the left or right.
Shift+Down or Up	Selects one line of text up or down.
Shift+End	Selects text to the end of the line.
Shift+Home	Selects text to the beginning of the line.
Shift+Page Down	Selects text down one window. Or, cancels the selection if the next window is already selected.
Shift+Page Up	Selects text up one window. Or, cancels the selection if the previous window is already selected.
Ctrl+Shift+Left or Right Arrow	Selects text to the next or previous word.
Ctrl+Shift+Up or Down Arrow	Selects text to the beginning (Up Arrow) or end (Down Arrow) of the paragraph.
Ctrl+Shift+End	Selects text to the end of the document.
Ctrl+Shift+Home	Selects text to the beginning of the document.

Window Keys

Key(s)	Function
Alt+Space bar	Opens the Control menu for the main window.
Alt+-	Opens the Control menu for a child (graphics or text) window.
Alt+F4	Closes the main window.
Ctrl+F4	Closes a child (graphics or text) window.
Alt+Esc	Switches to the next application window or minimized icon, including full-screen programs.
Ctrl+Esc	Opens the task list manager.
Alt+Tab	Switches to the next application window, restoring applications that are running as icons.
Ctrl+Tab or Ctrl+F6	Switches to the next child (graphics or text) window, restoring iconized windows.
Alt+Enter	Switches a non-Windows application between running in a window and running full screen.
Direction key	Moves a window when you have chosen Move from the Control menu. Or, changes the size of a window when you have chosen Size from the Control menu.

Edit Windows

Edit windows are used in *Dynamics Solver* to edit all settings in a text window and to edit project notes.

The caret (a vertical bar) indicates the current insertion (or overwriting) position and its coordinates (line and column) permanently displayed in the window last line. To learn how to use editing keys, refer to the **Keyboard** topic.

Notice that Windows limits the maximum line length in an edit window to about 4092 characters (1024 in the 16-bit edition), while *Dynamics Solver* is able to deal with lines five times longer. Problem files with very long lines can be viewed and changed by means of a good editor. In the 16-bit edition Windows limits the total amount of text in an edit window to 64 Kb, while the limit in the 32-bit edition is of 1Mb.

The menu in edit windows is as follows:

Entry	Meaning
File	File and printer management
Edit	Edit commands
Search	Find and replace commands
Help	Help system

File

The entries of this menu of edit windows are used to manage the printer and files:

Entry	Meaning
Accept changes	Save changes, close edit window and go to current output window
Abandon changes	Discard changes, close edit window and go to current output window. The problem could be incomplete!
Switch to main window	Set focus to the current output window. Do not close the text window
New	Clear the edit window
Open	Get a file from the disk
Save	Save the current file to the disk
Save as	Save the current file to the disk under a new name
Print	Print the current file

Edit

The entries of this menu of edit windows are used to manage the clipboard, text changes and text selection:

Entry	Key	Meaning
Undo	Alt+Backspace or Ctrl+Z	Revert the last change
Cut	Shift+Del	Move to clipboard the current selection
Copy	Ctrl+Ins or Ctrl+C	Copy to clipboard the current selection
Paste	Shift+Ins or Ctrl+V	Copy clipboard to the cursor position
Clear	Del	Remove the current selection
Select All		Select all text

Search

The entries of this menu of edit windows are used to find and replace pieces of text:

Entry	Meaning
Search	Open the Find common dialog box
Replace	Open the Replace common dialog box

Help

The entries of this menu of edit windows are used to access some help entry points:

Entry	Meaning
Edit window	Open the edit window entry
Index	Open the index entry
Keyboard	Open the keyboard entry
Help on Help	Open the help on help entry

Edit all Settings in a Text Window

You can use F5 or the File/**All settings** menu (or even the  button) to open a text window where you can view an edit all the settings in the current problem.

In the text window you will see and edit the text representation of the problem under study. In fact you will see the problem file text that will be saved to (or loaded from) the disk. This is a convenient way to do extensive changes to the problem, or to start a new one using an old problem as template.

When you close the window (using Alt+F4, the File/**Accept changes** menu or the system menu) the settings will be read. If an error happens, you will be informed and the text window will show you the most probable location of the error. You can correct it, or give up by using the File/**Abandon changes** menu entry, but in the latter case you will probably have an incomplete problem in memory.

Edit Project Notes

You can use Ctrl+N or the File/**Notes** menu (or even the  button) to open a text window where you can view and edit notes related to the current project: information on bibliography, known results, interesting values of parameters or other settings, or whatever information you want to keep with the problem. The contents and format of these notes are completely arbitrary, but its length is limited by Windows.

The notes are part of the problem file and, in consequence, they can be also edited from the **All settings** text window. They will be saved to the disk and recovered from there when you use the **File** menu to save and load problem files in the usual way. You can also save them to (or load from) a separate text file by using the window **File** menu.

Mathematical Expressions

You must enter mathematical expressions to define equations, initial functions, values to plot or display, and so forth. In *Dynamics Solver* an expression is formed, in a very standard way, by the following items optionally separated by spaces:

Numbers	Mathematical Operators
Comparison Operators	Logical Operators
Conditional Operator	Parentheses
Variables for Graphics Elements	Independent Variable
Dependent Variables	Interpolated Solution
Initial Values	User-defined Parameters
Predefined Constants	One-Variable Functions
Two-Variable Functions	Three-Variable Functions
Graphicis Functions	Other Functions
Mathematical Extensions	

Each of these topics is described below in its proper section, but the main differences with the usual mathematical notation and the notation used in *Dynamics Solver* can be summarized as follows:

1. The whole expression must fit on a single line. You must write g/l , e^x , $1.5E5$, $x[1]$ (for derivatives or indexes), x_2 , and $\text{arccot}(x)$.
2. Identifiers cannot be Greek letters, but they can have up to eighty letters or digits after the first character. The underscore character, $_$, is considered a letter in this context. You can use x_2 (or x_2) and theta as identifiers.
3. Many symbols are lacking: $|x|$ must be written as $\text{abs}(x)$, the square root is $\text{sqrt}(x)$ and you write $x \leq y$ for the less or equal operator.
4. Multiplication must be explicitly indicated by means of $*$. So, xy is a two-letter identifier not the product $x*y$. On the other hand $x y$ is not allowed and will raise an error message.
5. Functional arguments must be enclosed between parentheses. For instance, $\cos x$ must be written $\cos(x)$.
6. The precedence of operators is clearly defined: $1/2*pi$ is always interpreted as $(1/2)*pi$ and not as $1/(2*pi)$. If you have any doubt, add some more parentheses.

Numbers

The usual format of computer languages is used by *Dynamics Solver*: -2 , $1E-12$, $-1.02e2$, $.2$, etc. (Refer to the **Compiler Description**.)

Notice that the exponential notation used by most calculators must be used, as in $1.2E7$ in dialog box entries that only have numerical values, but if full expressions can be entered, then the form $1.2*10^7$ is also possible.

Mathematical Operators

Operator	Equivalent function	Meaning
+		Addition
-		Subtraction
*		Multiplication
/		Division
%	mod	Modulus: $x\%y = x \text{ mod } y$

^	pow	Exponentiation
---	-----	----------------

Comparison Operators

They return 1 for True and 0 for False:

Operator	Meaning
<	Less than
<=	Less or equal to
>	Greater than
/	Greater or equal to
==	Equal to
!=	Not equal to

Logical Operators

They return 1 for True and 0 for False:

Operator	Meaning
&&	and
	or
!	not

Only the operands that are necessary to establish the result are evaluated.

Conditional Operator

The expression

$$c ? t : f$$

is equivalent to `if(c,t,f)` and returns `t` if `c != 0` and `f` if `c == 0`.

Operator Precedence

From lowest to highest:

Order	Operator	Type
1	? :	Conditional operator
2		Or
3	&&	And
4	== !=	(In)equality
5	< <= > >=	Comparison
6	+ -	Binary addition/subtraction operators
7	* / %	Multiplication/division
8	^	Exponentiation
9	! + -	Unary operators: not and signs

Parentheses

Use them to change the usual precedence of operators and functions. They are required around function arguments and for any kind of mathematical extensions, even if they are constants.

Identifiers

The user can select the names of variables, parameters, initial values and the interpolated solutions, though most of them have a default value. They are case-sensitive and composed by a letter optionally followed by one to seven letters or digits. In this context, the underscore character, `_`, is considered to be a letter. The identifiers for predefined constants and functions are not case-sensitive.

Variables for Graphics Elements

When using expressions for graphics elements (2-curves and 3-curves) only the following variables are available:

Variable	Meaning
t	The parameter defining the x , y and z equations It runs from t_1 to t_2
x_0	Arbitrary parameter
y_0	Arbitrary parameter
s	Arbitrary parameter
r	Arbitrary parameter
t_1	Minimum value of t
t_2	Maximum value of t
d	Arbitrary parameter

Since the expressions defining curves are entered by the user, the actual meaning of x_0 , y_0 , s , r , d (or even t_1 and t_2 if they are used explicitly) is completely arbitrary.

Independent Variable

This identifier's default name, t for continuous systems and n for maps, can be changed in the **Variables** dialog box.

Dependent Variables

These identifiers default names are $x[1]$, $x[2]$, etc., in the case of a system and for a single equation, $x[0] = x$ ($x[1] = x'$, $x[2] = x''$, etc., are used for derivatives). These default names can be changed in the **Variables** dialog box, where it is also possible to choose individual names for each variable or derivative.

Dependent variable(s) and the solution are different things.

Interpolated Solution

Usually numerical codes used to integrate differential equations compute only isolated solution points at discrete values of the independent variable. The default Dormand-Prince method, as well as other methods, used in *Dynamics Solver* provide much more with the same computational effort: a global

piecewise polynomial approximation to the solution. This powerful characteristic is responsible of the most advanced possibilities in *Dynamics Solver* including fast dense graphics output and the ability to solve a large class of functional-differential equations, compute derivatives of the solution and draw Poincaré sections.

The i -th variable or derivative at a previously computed value τ of the independent variable is $x(i, \tau)$, if the collective name in the **Variables** dialog box is the default x . If you have selected there an individual name for each variable or derivative, the shorter form $x(\tau)$ can be used for a variable called x .

You can even use up to the seventh derivative of a variable or derivative of the solution. So, if the order of the equation is n , the derivatives $d^n x / dt^n = x(n, \tau)$, $d^{n+1} x / dt^{n+1} = x(n+1, \tau)$, ... $d^{n+6} x / dt^{n+6} = x(n+7, \tau)$ are also available. In the case of a system with n equations, $d^k x / dt^k = x(i+k*n, \tau)$, with $1 \leq k \leq 7$, $1 \leq i \leq n$, is the k -th derivative of $x(i, \tau)$. It should be clear that higher derivatives have lower precision, so you should not use them except in very special cases.

Note that the value τ must be a retarded one and that this global solution is stored only if **Saved points** in the **Range** dialog box has not been set to 0, because the value in that menu entry is precisely the number of computed points stored in memory.

The interpolated solution includes the initial function values, if defined in the **Edit initial functions** entry in **Initial values** dialog sheet.

You should not confuse the dependent variables of the previous section and the interpolated solution just discussed. Let us describe their meaning in different contexts.

1. When defining equations and initial values, $x[i]$ refers to the value of x_i or $d^i x / dt^i$ that is being computed currently, while $x(i, \tau)$ corresponds to an already known value. You can use $x(i, \tau')$ for any previous value τ' of the independent variable, provided that it is still stored in memory or defined in the form of an initial function. On the other hand, if you try using $x(i, \tau)$ instead of $x[i]$ for the current value of the independent variable, you will get an interpolation error.
2. When specifying the output quantities in different contexts, both $x[i]$ and $x(i, \tau)$ are already computed and correspond to the last point. In this case, you can use either of them, though the index notation will be slightly faster. You can still use the $x(i, \tau)$ notation for any previous value of the independent variable, or to compute derivatives of the solution.
3. When entering the **Condition** defining a Poincaré section, $x[i]$ is simply unavailable, because in general you are not evaluating the condition at one of the computed points. In this context, *Dynamics Solver* will refuse to compile an expression containing dependent variables. Here you must use the functional notation $x(i, \tau)$.

If individual names have been assigned in the **Variables** dialog box, you can replace $x[i]$ by x and $x(i, \tau)$ by $x(\tau)$ in the previous discussion.

Notice that in the case of discrete dynamical systems, the solution is defined only for integer values of the independent variable (the index) and no interpolation is available (it would be meaningless). So, you should make sure that the n value in $x(i, n)$ and $x(n)$ is an integer.

Initial Values

The default names (τ_0 , x_0 , $x_0[1]$, etc.) can be changed in the **Variables** dialog box. Their values are selected in the **Initial values** dialog sheet or using the mouse or the direction keys.

User-defined Parameters

Their name and values are entered in the **Variables** dialog box.

Predefined Constants

They are searched for after the user-defined parameters and are case-insensitive:

Name	Value	Comment
Catalan	0.915965...	Catalan constant
degree	pi/180	Useful to enter angles in degrees
EulerGamma	0.577215...	Euler-Mascheroni constant
GoldenRatio	1.61803...	Golden ratio $(1+\sqrt{5})/2$
e	2.71828...	Euler constant
pi	3.14159...	Circular constant

Remember that you might also write your own mathematical extensions.

One-Variable Functions

Name	Alias	Comments
abs	fabs	Absolute value
acos	arccos	Inverse cosinus function
asin	arcsin	Inverse sinus function
atan	arctan	Result in $[-\pi/2, \pi/2]$
ceil		$\text{ceil}(x) = -\text{floor}(-x)$
cos		Cosinus
cosh		Hyperbolic cosinus
erf		Error function
erfc		Complementary error function.
exp		Exponential
factorial		Factorial function for $x \geq 0$ (it need not be integer)
floor	int	Integer part $E(x)$: larger integer not higher than x
fracc		$\text{fracc}(x) = x - \text{floor}(x)$
input		$\text{input}(i)$ ask the user to input a value
ln	log	Natural logarithm: $\log(x) = \ln(x)$
lnGamma		Natural logarithm of Euler Gamma function for $x > 0$
log10		Decimal logarithm: $\log_{10}(x) = \ln(x)/\ln(10)$
message		$\text{message}(x)$ displays a message with x and returns x
read		$\text{read}(i)$ reads a number from the i -th input file
round		Nearest integer
sign	sgn	$\text{sign}(x) = x/\text{abs}(x)$, $\text{sign}(0) = 0$
sin		Sinus
single		Returns the number rounded to IEEE single precision (6-7 digits and decimal exponents from -38 to 38)
sinh		Hyperbolic sinus
sqr		$\text{sqr}(x) = x*x$
sqrt		Square root
step		Heaviside function: $\text{step}(x) = (x > 0) ? 1 : 0$ $\text{step}(0)$ defined in Advanced settings
tan		Tangent
tanh		Hyperbolic tangent
time		$\text{time}(x)$ returns the time in milliseconds since: - the current iteration started if $x = 0$, - the current set of iterations started if $x > 0$, - the current <i>Dynamics Solver</i> session started if $x < 0$. (The result accuracy depends heavily on the underlying

		operating system.)
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The following functions are defined in the external library `BESSEL.DLL` (which can be installed by running again `INSTALL` if you did not install it before). You will find specific information on them in the help file that can be accessed by using the **Help/Elliptic** functions menu.

Name	Comment
AiryAi	Airy function Ai
AiryBi	Airy function Bi

Remember that you can also write your own mathematical extensions.

Two-Variable Functions

Name	Alias	Comments
arctan2	atan2	$\text{arctan2}(y, x) = \text{polar angle of } (x, y) \text{ in } (-\pi, \pi]$
break		$\text{break}(c, r)$ returns r and starts a new iteration ⁹ if c is true (non-zero)
continue		$\text{continue}(c, r)$ returns r and starts a new iteration if c is false (zero)
deletewrite		$\text{deletewrite}(i, x)$ deletes the i -th output file and returns x
div		$\text{div}(x, y) = \text{int}(x/y) = (x - x\%y)/y$
hypot		$\text{hypot}(x, y) = \text{sqrt}(x^2 + y^2)$
mod		$\text{mod}(x, y) = x\%y = x \text{ mod } y$
openread		$\text{openread}(i, x)$ allows selecting name and format for the i -th input file and returns x
openwrite		$\text{openwrite}(i, x)$ allows selecting name and format for the i -th output file and returns x
pow		$\text{pow}(x, y) = x^y$
random		$\text{random}(x, y) = \text{random number in } (x, y)$ $\text{floor}(\text{random}(a, b+1))$ to select an integer from a to b
resetread		$\text{resetread}(i, x)$ rewinds the i -th input file and returns x
skip		$\text{skip}(c, r)$ returns r and skips the output of the current graphics or text value if c is true (non-zero). See below Skip function .
write		$\text{write}(i, x)$ writes x to the i -th output file and returns x

The following functions are defined in the external library `ELLIPTIC.DLL` (which can be installed by running again `INSTALL` if you did not install it before). You will find specific information on them in the help file that can be accessed by using the **Help/Elliptic** functions menu.

Name	Comment
EllipticF	Legendre elliptic integral of first kind
EllipticE	Legendre elliptic integral of second kind
JacobiSN	Jacobi elliptic function sn
JacobiCN	Jacobi elliptic function cn

⁹ The next computation in a bifurcation diagram or when using Repeated Integration/Iteration or drawings Phase Portraits. If there is no more iteration the problem solving stops.

JacobiDN	Jacobi elliptic function dn
----------	-----------------------------

Likewise, the following functions are defined in the external library `BESSEL.DLL` (which can be installed by running again `INSTALL` if you did not install it before). You will find specific information on them in the help file that can be accessed by using the **Help**/Elliptic functions menu.

Name	Comment
BesselJ	Bessel function J
BesselY	Bessel function Y
Besselj	Spherical Bessel function j
Bessely	Spherical Bessel function j
BesselI	Modified Bessel function I
BesselK	Modified Bessel function K

Remember that you might also write your own mathematical extensions.

Skip function

To skip the output of the current graphics or text result (maybe because it lies outside the range of interest and will only waste memory, and disk space if the results are being sent to an external file) you may use the `skip` function in the **Horizontal axis** and **Vertical axis** entries of the **Graphics Output** dialog box corresponding to a graphics window or in the **Expressions** entries of a **Text Output** dialog box corresponding to a text window. In fact `skip(c, r)` returns `r` and skips the output of the current graphics or text result if `c` is true (non-zero). If this function is used in other expressions, it simply return `x` and has no side effect.

Notice that in a graphics window with the **Continuous line** selected in the **Format** sheet skipping intermediary points may look awkward. See the example in `CHAOS\DUFFING.DS`.

Input File

You may have global input files that are identified by an integer and used through the complete run of an instance of *Dynamics Solver*. They must be ordinary ASCII files with a list of numbers generated by *Dynamics Solver* or any other program. At least one space, tabulation or end of line should separate consecutive numbers.

The complete specification of the *i*-th input file name has to be entered in a input file dialog box the first time the program evaluates `read(i)` or each time it evaluates `openread(i, x)`. A good place to locate the last function if the input file should change from run to run is one of the **Initial functions** in the **Initial values** dialog sheet.

Each time `read(i)` is evaluated a value is read from the *i*-th input file and the corresponding pointer is advanced to the next value. The pointer can be reset to point to the first value by using `resetread(i, x)`.

For special applications you might change the default format string from its default value, `%lg`, in the **Input file** dialog box.

Input and output files are independent but nothing prevents you from assigning (at your own risk) an input file and an output file to the same physical file.

Output File

You may have global output files that are identified by an integer and used through the complete run of an instance of *Dynamics Solver*. They are ordinary ASCII files with a list of numbers generated by the program.

The complete specification of the *i*-th output file name has to be entered in a **Output file** dialog box the first time the program evaluates `write(i, x)` or each time it evaluates `openwrite(i, x)`. A good place to locate the last function if the output file should change from run to run is one of the **Initial functions** in the **Initial values** dialog sheet.

Each time `write(i, x)` is evaluated the value `x` is appended to the end of the *i*-th output file. The output file can be erased by using `deletewrite(i, x)`.

For special applications you might change the default format string from its default value, `%g\n`, in the **Output file** dialog box.

Output and input files are independent but nothing prevents you from assigning (at your own risk) an input file and an output file to the same physical file.

Three-Variable Functions

The functions `projx` and `projy` take three arguments and return the two coordinates of the projection of the corresponding point. See **Projections**.

The following function is defined in an external library, `ELLIPTIC.DLL` (which can be installed by running again `INSTALL` if you did not install it before). You will find specific information on it in the help file that can be accessed by using the **Help**/Elliptic functions menu.

Name	Comment
<code>EllipticPi</code>	Legendre elliptic integral of third kind

Remember that you might also write your own mathematical extensions.

Graphics Functions

These functions can be used to dynamically change (from time to time, or by using computed expressions) the color and width used to draw graphics results.

`HSV(h, s, v)` sets the drawing color of the current graphics window according to the indicated hue, h , saturation, s , and value, v , components, which value must be in the range $0 \dots 1$. This subjective color model corresponds to the way painters form colors. Pure pigments are given for $s = v = 1$:

Function	Color
<code>HSV(0, 1, 1)</code>	Red
<code>HSV(1/6, 1, 1)</code>	Yellow
<code>HSV(1/3, 1, 1)</code>	Green
<code>HSV(1/2, 1, 1)</code>	Cyan
<code>HSV(2/3, 1, 1)</code>	Blue
<code>HSV(5/6, 1, 1)</code>	Magenta
<code>HSV(1, 1, 1)</code>	Red

On the other hand, tints are formed by decreasing s (i.e., by adding white). Shades are formed by decreasing v (i.e., by adding black) and tones by decreasing both v and s . If $s = 0$, the colors are gray (i.e., achromatic).

`RGB(r, g, b)` sets the drawing color of the current graphics window according to the indicated red, r , green, g , and blue, b , components, which value must be in the range $0 \dots 255$. In this additive scheme, may use:

Function	Color
<code>RGB(255, 0, 0)</code>	Red
<code>RGB(0, 255, 0)</code>	Green
<code>RGB(0, 0, 255)</code>	Blue
<code>RGB(255, 255, 0)</code>	Yellow
<code>RGB(255, 0, 255)</code>	Magenta
<code>RGB(0, 255, 255)</code>	Cyan
<code>RGB(255, 255, 255)</code>	White
<code>RGB(n, n, n)</code>	Gray shades

and different gray shades are obtained by using `RGB(n, n, n)`.

`PenColor(c)` sets the pen color used to draw in the current graphics window to the indicated value $c = 0, 1, 2, \dots, 16777215$. Notice that `PenColor(c) = RGB(r, g, b)` if $c = r + 256 * (g + 256 * b)$. If c is negative, `PenColor(-n)` sets the pen color used to draw in the current graphics window to the n -th color defined in the **Colors** dialog box.

`PenWidth(w)` sets the pen width used to draw in the current graphics window to the indicated value $w = 0, 1, 2, \dots$

These two functions always return 0. In consequence, a convenient way to use them is to add them to an expression. See the examples `EXAMPLES\ART\COS1.DS`, `EXAMPLES\ART\JUMP.DS` and `EXAMPLES\CHAOS\HENONMAP.DS`.

Notice that these functions are available only in the **Horizontal axis** and **Vertical axis** entries of the **Graphics Output** dialog box.

Other Functions

The `max` and `min` functions take an arbitrary number of arguments and return the largest and smaller of them, respectively.

The `if` function has two meanings:

Expression	returns	if
<code>if(cond, true, false)</code>	true	<code>cond != 0</code>
	false	<code>cond == 0</code>
<code>if(cond, pos, nul, neg)</code>	pos	<code>cond > 0</code>
	nul	<code>cond == 0</code>
	neg	<code>cond < 0</code>

Only `cond` and the argument returned are evaluated and they can be full expressions. This function can be seen as a simple control structure and is very useful when dealing with piecewise defined functions. See the example in `ODES\SCROLL.DS`.

Note that `if(c, t, f)` is equivalent to `(c) ? t : f`.

`IterationOrder()` takes no argument and returns the number of times the integration corresponding to the current step has been repeated. It is used only by some advanced methods of solution of singular differential equations.

`Backward()` takes no argument and returns 1 if you are integrating backward and 0 otherwise.

Mathematical Extensions

One can use the **External Libraries** dialog box which open with the Configuration/**Mathematical extensions** menu entry to add constants (which must be followed by `()`) and functions. They will usually be described in a help file that can be accessed from the **Help** menu. Some of them (implementing elliptic functions, for example) are provided with the package.

Compiler Description

To describe formally the expression compiler, we will use a simplified version of the metalanguage known as “Backus-Naur-Form.” The metasymbol `::=` denotes equality and a vertical bar `|` indicates an option. In consequence, a line in the form

$$a ::= b c \mid d$$

means that `a` is either `bc` (the concatenation of `b` and `c`) or `d`. Elements between curly braces are optional and may appear repeatedly. So,

$$d ::= e \{ f \}$$

indicates that `d` is equal to `e`, or to `ef`, or to `eff`, etc.

The “Backus-Naur-Form” description of the expression compiler/interpreter is as follows:

```

expression ::= or-expression | or-expression ? expression :
              expression
or-expression ::= and-expression {|| and-expression}
and-expression ::= equality {&& equality}
equality ::= relation | relation equality-operator relation
equality-operator ::= == | !=
relation ::= addend | addend comparison-operator addend
comparison-operator ::= < | > | <= | >=
addend ::= term {adding-operator term}
term ::= signed-power-factor {multiplying-operator signed-power-
              factor}
adding-operator ::= + | -
signed-power-factor ::= power-factor | sign power-factor
power-factor ::= factor | factor ^ signed-factor
signed-factor ::= factor | sign factor
multiplying-operator ::= * | / | div | %
factor ::= constant | variable | number | function | ( expression )
sign ::= + | -
variable ::= identifier | identifier [ integer ] | identifier ' {'}
constant ::= identifier
number ::= digit {digit . {digit scale |
              digit {digit . {digit
              digit {digit          scale |
              digit {digit
              . digit {digit          scale |
              . digit {digit
function ::= identifier ( expression {, expression } )
identifier ::= letter {letter | digit}
integer ::= digit {digit}
letter ::= A | B | ... | Z | a | b | ... | z | _
digit ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
scale ::= E scale-factor | e scale-factor
scale-factor ::= digit {digit} | sign digit {digit}

```

Examples

In the following table you may find some examples:

Mathematical expression	Dynamics Solver expression
$\frac{d\theta}{dt} + \frac{g}{l} \sin\theta$	theta[1]+g/l*sin(theta)
$\sigma(y-x)$	sigma*(y-x)
$-\frac{GMx}{(x^2+y^2)^{3/2}}$	-G*M*x/(x^2+y^2)^(3/2)

$\sqrt{\frac{1}{R}-1}$	$\text{sqrt}(1/R-1)$
$-x(t) + A^2(1 + 2B\cos x(t-r))$	$-x+A^2*(1+2*B*\cos(x(t-r)))$
$\frac{dx}{dt}(t-x(t))$	$x(1,t-x(0,t))$
$(c-au'-cu)u'$	$(c-a*u[1]-c*u)*u[1]$ or, in the case of a single equation, $(c-a*u'-c*u)*u'$
$A \sin^2 \theta (g^2 + h^2) - \left(\frac{v}{Q} + x + x_0 \right)$	$A*\sin(\text{theta})^2*(g^2+h^2) - (v/Q+x+x0)$
$\frac{1-v(t)/c}{1+v(t-r)/c}$	$(1-v/c)/(1+v(t-r)/c)$

Appendices

Customization

The program can be easily customized by using the entries in the **Configuration** menu and the **Preferences** dialog box.

There exists a couple of advanced settings that have not been included in that menu, because it will be needed only by power users in very special cases.

The installation program has copied the help file, DSOLVER.HLP, to the directory containing the executable program, DSOLVER.EXE. If, for some reason, you prefer the help file in another directory, *Dynamics Solver* will find it if you enter in the Options section of DSOLVER.INI configuration file (the registry entry HKEY_LOCAL_MACHINE\SOFTWARE\Juan M. Aguirregabiria\Dynamics Solver\Current version for the 32-bit edition) a line with the full file specification in the form:

```
Help file=c:\newdir\dsolver.hlp
```

By default, *Dynamics Solver* assigns to each graphics window its own graphics context. This speeds up the output and should not be a problem except, perhaps, if the memory is very low. You could try setting in the Options section of DSOLVER.INI configuration file (the registry entry HKEY_CURRENT_USER\SOFTWARE\Juan M. Aguirregabiria\Dynamics Solver\Current version for the 32-bit edition) an entry in the form:

```
Own DC=0
```

If the Mask 87 option is set to 1, the floating point exceptions raised by the numeric data processor are masked off. The reason for this is that there seems to be an important amount of flawed or bad installed coprocessors. I have personally had problems with coprocessors in different systems. My old 8087 treated underflows in a way contrary to the documented one (this error happened in all the 8087 I tried). I also know an 80387 that hangs systematically the system if an invalid argument exception occurs, and only in this case. Several years ago I tested a 486 system with a bad installed error line: the system failed completely after the second floating point exception. Finally, in my 486 the difficulties arise sometimes when an invalid argument exception occurs. (I never detected problems in an 80287 I used during three years and my new Pentiums seem to work correctly.)

When using *Dynamics Solver* to explore problems, floating point errors occur rather often. This is normal and the program handles them well, in most cases. If you are experiencing problems with floating point errors, you could set to 1 the Mask 87 option: floating point exceptions will be masked off. Instead of using them to detect errors when they happen, the program will test periodically the coprocessor status to see if any error happened so far. This seems to work in all systems. The main drawback is that the error report can be delayed and that, in some cases, instead of the first error it is reported an error that happened as consequence of the first one. Then, it can be more difficult to identify the source of trouble.

On the other hand, by default *Dynamics Solver* will mask off the denormal, underflow and inexact floating point exceptions. But you can change this by setting the control word you prefer in the 'hidden' option called Control 87. You have only to specify its decimal value by summing up the values corresponding to each exception:

Exception	value
invalid operation (0/0, ...)	1
denormal value (precision loss)	2
divide by zero (1/0, ...)	4
overflow (too big)	8
underflow (too small)	16
inexact operation (sin 10 ²⁰⁰)	32

See also **Writing External Integration Codes, Adding Mathematical Functions, Constants and Editing Fonts**.

Writing External Integration Codes

Dynamics Solver includes 15 integration methods to solve ordinary differential equations (see **Selecting the Integration Method**).

Moreover *Dynamics Solver* is extensible, you can easily add new integration methods. You have to create a Windows DLL library in your language of choice and follow some simple rules. The subdirectory DEMOMET (which can be created by running again INSTALL if you did not install it before) contains complete examples in Borland C++, Turbo Pascal for Windows and Microsoft FORTRAN. The files include complete documentation and can be used as templates to write your own libraries. If you are using a different programming language, you can still use the information in those files to write the extension. Once the library has been written you can install it in *Dynamics Solver* by using the Configuration/**External codes** menu and the **Install External Methods** dialog box that will open.

Adding Mathematical Functions and Constants

Dynamics Solver includes a large class of mathematical constants and functions that can be used in any expression. Some of them are included in external libraries (CONSTANT.DLL for some less usual constants and ELLIPTIC.DLL for some elliptic functions).

Moreover *Dynamics Solver* is extensible, you can easily add new constants and functions. You have to create a Windows DLL library in your language of choice and follow some simple rules. The subdirectory DEMOMATH (which can be created by running again INSTALL if you did not install it before) contains complete examples in Borland C++, Turbo Pascal for Windows and Microsoft FORTRAN. The files include complete documentation and can be used as templates to write your own libraries. If you are using a different programming language, you can still use the information in those files to write the extension. Once the library has been written you can install it in *Dynamics Solver* by using the Configuration/**Mathematical extensions** menu and the **Install Mathematical DLLs** dialog box that will open.

Editing Fonts

Dynamics Solver does not use Windows fonts in graphics elements, but some special fonts that look worse on the screen but are more portable among platforms and printing devices. The printed result is reasonably good and the files in portable format (PostScript and HPGL, for instance) are small. Some of these fonts are provided with the program, and the user can easily modify them (or create new fonts) by using the Font Manager program, FM.EXE. If you have not installed it, run again INSTALL. The new fonts will be used by *Dynamics Solver* once you they are installed in the **Install Font Files** dialog box .

If you want to use Windows fonts (including TrueType fonts), you can copy the screen to the clipboard and from there to any drawing program, like Paintbrush. You can then add lettering with Windows fonts.

Problem Files

The mathematical problem definition is saved to the disk (and retrieved from there) by using the **File** menu in the form of a text file that you can view and edit in any text editor (or word processor, but remember to save it as a text file) or by using the **All settings** command.

This feature allows the knowledgeable user preparing difficult problems with other programs. For instance, you could use a computer algebra system to generate the equations to be solved.

Since the structure of the problem file is rather strict, you should always start from a problem file generated by the program.

Any line whose first nonblank character is a semicolon is considered a comment that is ignored. On the other hand, completely blank lines are not ignored. They represent undefined values and cannot be added or erased at will.

Some special settings are configuration- or taste-dependent and will be taken into account only if you are reading the problem file from a **All settings** text window or (when reading from the disk) if **Identifier** is set and has the same value in the problem file and in the Preferences/**Other** dialog sheet.

See also **Example Files**.

Special Settings

The following options are configuration- or taste-dependent and will be taken into account only if you are reading the problem file from a **All settings** text window or (when reading from the disk) if **Identifier** is set and has the same value in the problem file and in the Preferences/**Other** dialog sheet.

Setting	Dialog box entry
Stack length	Advanced settings/Stack length
P-code length	Advanced settings/P-code length
Capture	Format/Save output in memory
Plotter	Plotter/On
Device	Plotter/Output file
No bitmap	Format/Bitmap output
Copy bitmap	Format/Bitmap output
Stretch bitmap	Format/Bitmap output
Capture	Text Output/Capture
Output file	Text Output/Output file

Example Files

The current *Dynamics Solver* release includes a lot of problem files. They can be used as examples when teaching ordinary differential equations, mechanics, nonlinear dynamics and chaos. They are also very useful to learn how to use the program for complex tasks (see also **Some Examples**).

They collected in different directories (but take into account that many files could fit in more than one directory).

Decorative Examples

The EXAMPLES\ART directory contains the following example files. Apart from their decorative interest, some of them are important examples in some scientific fields.

Name	Comment
ARCS	Decorative arcs
COS	Decorative map
COS1	COS with automatic parameter selection

ELLIPTI1	ELLIPTIC with rotated initial functions and variables
ELLIPTIC	Gingerman map
GSIN	Decorative map
GUMOWSKI	Decorative map
JUMP	Decorative map
MOIRE	Parametric resonance used to get a decorative motif
NESSIE	Decorative map
SIN	Decorative iteration
TORMAP	Decorative map
TWIST	Another decorative chaotic map
VOLUTE1	Decorative map
VOLUTE2	Decorative map

Deterministic Chaos

The EXAMPLES\CHAOS directory contains the following example files. They show some uses of *Dynamics Solver* when analyzing chaotic dynamics.

Name	Comment
ANOSOV	Ergodic toral automorphism of Anosov/Arnold (cat map)
BAKER	Baker map
BALL	Bouncing ball
BAS	Basins of attraction of the Duffing equation
BASIN	Basins of attraction of a map
BIFURCAT	Bifurcation diagram of the logistic equation
CAT	Arnold's cat map and its Liapunov exponent
CRUTCHF1	Bifurcation diagram for the Crutchfield map
CURRY	The map of Curry and Yorke
DEVIL	Devil's staircase. Using more unknowns
DISKS	Chaotic scattering around hard disks. Tricky!
DISKSCAT	Chaotic scattering around hard disks. Very tricky!
DUFFING	Duffing equation and stroboscopic Poincaré maps
DUFFING1	Duffing equation and multiple stroboscopic Poincaré maps
DUFFING2	A trickier version of the previous problem file
DUFFING3	Another version of the previous problem file
DUFFING4	Duffing equation and sensitive dependence on initial conditions
GEISEL	A diffusive chaotic system
GRAPH	Cobweb for the logistic map
HENON	Hénon map
HENON1	Generates HENON2 . DAT with double and single precision
HENON2	HENON2 . DAT shows sensible dependence on working precision
HENONB	Hénon map: bifurcation diagram
HENONHEI	Hénon-Heiles system: Poincaré application
HENONL	Bifurcation diagram for the Liapunov exponent of Hénon map
HENONMAP	Hénon symplectic map
HISTOGRA	Histogram for the logistic map
INTER	Intermittence and cobweb diagrams
JULIA	Julia set
LIAPUNOV	Logistic map: Bifurcation diagram and Liapunov exponent
LOGISTIC	Logistic map
LORENZ	Lorenz attractor: projections
LORENZL	Liapunov exponent of the Lorenz attractor
LORENZR	Pseudo phase space reconstruction of the Lorenz attractor

LOZI	Lozi's map: a chaotic attractor
MANDEL	Mandelbrot set
NEWTON	Newton method to find the square roots of 1
NOCHAOT	Strange nonchaotic attractor
PREYPRED	Discrete prey-predator model
ROSSLER	Rosler system
SCAT	Chaotic scattering function
SCATTER	Chaotic scattering orbit. Uses SCATTER.DAT
SCROLL	Double scroll attractor of Chua's circuit
SHIFTED	A shifted map
SINGER	A one-hump map with two attractors
STANDARD	The standard map
SYMMETRY	A symmetric map
TAN	Intermittence in a map and cobweb
TENT	Tent map: bifurcation diagram
YAMAGUCH	A map analyzed by Yamaguchi and Isima

Functional-Differential Equations

The EXAMPLES\DELAY directory contains the following example files of functional-differential equations:

Name	Comment
DELAY1	Functional-differential equation with constant delay
DELAY2	Functional-differential equation with variable delay
EPIDEMIC	Model of Kermack-McKendrick for epidemics propagation
FIBONAC	Recurrence to compute the Fibonacci numbers
FP	Very complex example: a Fabri-Pérot cavity
IKEDA	Ikeda's functional-differential equation

Drawing Examples

The EXAMPLES\DRAWING directory contains the following example files. They show how one can use *Dynamics Solver* as a drawing tool to prepare figures for courseware and scientific papers.

Name	Comment
2-CURVES	Drawing complex elements with 2-curves
2-PEND	Drawing of a double pendulum for a physics exercise
BESSEL	Drawing mathematical functions
CURVE1	Using <i>Dynamics Solver</i> to draw curves
CURVE2	A better way to draw curves in <i>Dynamics Solver</i>
DIPOLE	Current lines of the electrostatic dipolar field
EMWAVES	Electromagnetic waves. Using 3-curves
GAMMA	Using <i>Dynamics Solver</i> to draw Euler Γ function
LISSAJ	A table of Lissajous figures
LISSAJO3	Lissajous three-dimensional curves
LISSAJOU	Lissajous figures
SPHER	True spherical coordinates
SPRING	Typical drawing for an exercise in mechanics
SPRINGS	How to draw a circular spring
WAVES	Wave propagation in a three-dimensional drawing

Fractal Examples

The EXAMPLES\FRACTAL directory contains the following example files with beautiful autosimilar fractal curves.

Name	Comment
C-CURVE	C-like fractal curve
FILL2	Peano curve filling von Koch island
FILLSNOW	Peano curve filling von Koch island
ISLAND	von Koch island
PEANO	Peano curve
QUADRAT	A finite area inside an infinitely long perimeter
SNOWFLAK	The von Koch fractal
TREE	Fractal trees
VONKOCH	von Koch fractal

Classical Mechanics

The EXAMPLES\MECH directory contains the following example files. They show how to use *Dynamics Solver* in courses on Classical Mechanics.

Name	Comment
BURREAU	Chaotic planar 3-body problem
BINARY	Moving around two Newtonian poles
CENTRAL	Bertrand's theorem
DISCRETE	Discrete analogue of Kepler's problem
DOUBLE	Double pendulum
DRY	Spring-mass motion with dry friction
ELASPEN1	Elastic pendulum: Complex evolution
ELASPEN2	Elastic pendulum: Regular motion
ELASPEN3	Elastic pendulum: Nonlinear Lissajous curves
INVERTED	Stability of the inverted driven pendulum
KEPLER	Kepler's problem
LAGRANGE	Lagrangian points in the Earth-Moon system
PENDULUM	Pendulum phase space
R3BODY	Restricted three-body problem: Arenstorf's periodic orbit
R3BODY1	Restricted three-body problem: aperiodic orbit
R3BODYP	Computing periods in the restricted three-body problem

Numerical Calculus

The EXAMPLES\NUMERIC directory contains the following example files.

Name	Comment
ELLIPTIC	Numerical quadrature: Legendre elliptic integral of first kind
ERF	Numerical quadrature: erf function
EULER	Break down of integration methods
NEWTON	Square roots by the Newton-Raphson method
OREGO	Oregonator: famous stiff system

RANDOM	Testing congruential generators for random numbers
STIFF	A stiff system
UNSTABLE	Getting numerical approximations to unstable solutions
VDPSTIFF	Stiff case with van der Pol's oscillator
VOLTERRA	Integro-differential equation

Ordinary Differential Equations

The EXAMPLES\ODES directory contains the following example files. They are (only) some examples to be used in course on differential equations.

Name	Comment
AUTOCATA	Autocatalator model
CYCLE	Saddle point, sinks and a limit cycle
CYCLES	Fixed point and two limit cycles
FRW1	Dealing with equations not solved for the derivative
FRW2	Dealing with equations not solved for the derivative
NOHYPER	Non hyperbolic equilibrium point
PHASE	Drawing phase spaces
SYSTEM2	A general second-order differential system
TORUS	Toroidal attractor
TRANSIEN	Very long transient due to Volta
VDPOL	Limit cycle in the van der Pol oscillator
VDPOLF	Forced van der Pol oscillator and additional variables
VDPOLP	Period of the van der Pol oscillator

Quantum Mechanics

The EXAMPLES\QUANTUM directory contains the following example files. Only some simple examples are contained in this directory.

Name	Comment
HARMONIC	Spectrum and wave-functions of the harmonic oscillator
PENK	Ground-level for a quartic potential with high accuracy
QOSCILL	Quantum oscillator. Guessing energy levels
HARMONIC	Spectrum and wave-functions of the harmonic oscillator
SPECTRUM	Shooting the spectrum of the quantum harmonic oscillator

Technical Information

To save memory, the numbers stored to redraw graphics windows are single precision numbers (7 decimal digits and values ranging approximately from 3.4E-38 and 3.4E38).

IEEE double precision (15-16 decimal digits and values ranging approximately from 1.7E-308 to 1.7E308) is used everywhere else in *Dynamics Solver*,

You may explicitly use `single` to convert any quantity to single precision (see the example in `CHAOS\HENON1.DS`).

The default integration method is the embedded Runge-Kutta method of order 8(5,3) with automatic step size control developed by Prince and Dormand in 1981. It is a continuous method that provides a global piecewise polynomial approximation of seventh order to the solution. The latter is thus available for dense graphical output, to solve a large class of functional-differential equations, to calculate derivatives of the solution and to compute Poincaré sections. It is described in Hairer [1993]. See also **Selecting the Integration Method**.

We have developed the algorithm to compute derivatives by using the classical central-point formulae along with Richardson's h^2 extrapolation (*deferred approach to the limit*).

Some other numerical algorithms are based on ideas from the excellent `netlib` (<http://www.netlib.org>), as well as from the books listed in the bibliography.

Dynamics Solver files

The INSTALL program has copied the following files to the *Dynamics Solver* directory of your hard disk. (Of course, if you do not have selected the **Complete Install** option, some of the files will be missing.)

Directory	File	Meaning
		<i>Dynamics Solver</i> directory
	README.TXT	Last minute notes
	DSOLVER.EXE	Executable program
	...	Data files for the uninstaller
DEMOMATH	*.*	How to add mathematical functions in Borland C++, Microsoft Fortran and Borland Turbo Pascal
DEMOMET	*.*	How to add integration methods in Borland C++, Microsoft FORTRAN and Borland Turbo Pascal
EXAMPLES		Problem files
	ART*.DS	Decorative examples
	CHAOS*.DS	Nonlinear dynamics and chaos (see also DELAY and MECH)
	DELAY*.DS	Functional-differential equations
	DRAWING*.DS	<i>Dynamics Solver</i> as a drawing tool
	FRACTAL*.DS	Fractal examples
	MECH*.DS	Examples in Mechanics
	NUMERIC*.DS	Examples in Numerical Calculus
	ODES*.DS	Ordinary differential equations
	QUANTUM*.DS	Examples in Quantum Mechanics
FONTS		Fonts
	*.F'T	Font files
	FONTS.HLP	Help file describing the font files
	FM.EXE	Font Manager program for DOS
	FM.HLP	Help file for Font Manager
	FM.ICO	Icon for Font Manager
HELP	*.HLP	Help and tutorial files
	*.CNT	Contents file (only in the 32-bit edition)
MATH	*.DLL	Additional mathematical functions
METHODS	*.DLL	Additional integration methods
PLOTTER	*.DRV	Plotter drivers
WINDOWS	LLATSNI.EXE or UNINST.EXE	Uninstaller

It should be noted that, as a consequence of the installation process, the font files, mathematical function libraries, additional integration methods and plotter drivers of the last four directories are already integrated in the program. The same happens with the corresponding help files, which can be directly accessed from the **Help** menu and the **Plotter** dialog box. The settings for all these files can be checked and changed from the **Configuration** menu.

The installation program has also created (unless you have chosen not to do it) a *Start/Programs* menu (or, in the 16-bit edition a *Program Manager* group), called *Dynamics Solver*, with some entries.

The 16-bit program will also create the file `DSOLVER.INI` in the Windows directory. Do not delete it, because it contains important information. The 32-bit edition saves its options in the registry.

In the same way, the 16-bit edition installer will put the file `RDSEXAM1.DLL` in the `WINDOWS\SYSTEM` directory. This file is used by the help and tutorial system to start example files when you click in their names. The 32-bit edition has an equivalent file, `RDSEXAM2.DLL`, but it is located in the same directory as the help and tutorial files.

Finally, the 16-bit installer will put, if necessary, the `CTL3DV2.DLL` file in your `WINDOWS\SYSTEM` directory. This file is necessary and provides three-dimensional dialog boxes. In the 32-bit edition there is no equivalent file, because Windows 95/98/NT4 automatically provide the 3D effect.

Warning and Error Messages

1. Every time *Dynamics Solver* evaluates the function `message(x)` it will issue a message containing the numeric value `x`.

Dynamics Solver Message

Every time *Dynamics Solver* evaluates the function `message(x)` it will issue a message containing the numeric value of expression `x`. This allows power users dealing with some complex problems.

See also the **Input File** and **Output File** sections.

2. **Warning messages.** You should consider carefully your answer (**Yes/No/Cancel** or **OK/Cancel**), because some irreversible changes might happen. (Click on the button to obtain information on the corresponding message.)

File already exists, OK to overwrite?

You have selected a file specification corresponding to an already existing file. If you press `Enter` or use **OK**, the previous file contents will be lost. Consider pressing `Esc` (or using **Cancel**) and choosing a different file name or path.

EPS files do not usually contain rotation statements. Do you want to include an explicit rotation?

The **Print/Rotate PostScript** option should be used only to get a landscape printout when uploading the EPS file directly to the printer. When inserting the file in a document, the page orientation will be under the control of the program that is currently managing the whole document. Most programs will ignore the rotation instruction, but a less careful program might get confused.

Not enough memory OK to exit the program?

This very rare warning will be issued if there is not enough memory to analyze the current problem (it must be really big!).

Press `Esc` (or use **Cancel**), to continue trying. You should lower the **Dimension/Order** entry in the **Dynamical System Type** dialog box that will open.

Using `Enter` or **OK** will close the program.

The problem has changed, do you want to save it?

You have changed some settings in the current problem file and are trying to load a new file (or change something in the **Dynamical System Type** dialog). Answer:

- **Yes** to save the current settings to the disk and load the new file (or accept the type change).
- **No** to abandon the current settings and load the new file (or accept the type change).
- **Cancel** (or press `Esc`) to cancel the loading of the new file (or the type change). The current problem and settings will be retained.

If you have changed some settings in the **Dynamical System Type** dialog box (such as the type of dynamical system or its dimension), *Dynamics Solver* cannot retain your definitions for equations, variables etc. If you press `Enter` or use **Yes**, those definitions will be lost. You could instead try modifying directly the problem file by using the **Edit/All settings** menu.

3. **Error messages.** (Click on the button to obtain information on the corresponding message.)

[expected

In an expression a collective name needs a square bracket to select one of its components.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

) expected

There is an unbalanced parenthesis in an expression.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

] expected

There is an unbalanced square bracket in an expression.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

: expected

There is an incomplete `? :` operator in an expression.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

, expected

There is a function that requires more arguments in an expression.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Argument domain error

While evaluating an expression, a function error happened because a function was given an invalid argument such as $\log(-1)$, $\arcsin(2)$, etc. The function and argument are included in the first line of the error message.

Argument out of range

The value of the independent variable in the interpolated solution lies out of the stored range. See the discussion in '**argument out of range**' **Interpolation Error**.

This error can be annoying and difficult to interpret in some cases. First, if you forget a `*` sign and write, for instance, $x(a+b)$ instead of $x*(a+b)$, *Dynamics Solver* may try to find an interpolated value for x (or the value of a function called x) and fail because it is unknown.

In functional-differential equations there can be other reasons for this error. It can be issued if you are trying to integrate backward the problem. This is not possible in most cases, because at each step the previous values must be known. It may also be due to a delay that is too long for the number of points stored in memory. Try increasing the **Tolerance** or the **Saved points** entries in the **Range** dialog sheet (which can be activated from the **Edit/Range** menu entry or by pressing **Ctrl+R**). Another more subtle cause for this error may be that in an integration step, from t to $t+h$, the functions defining the equations must be evaluated at an intermediate value lying between t and $t+h$ when the solution is not yet known in this interval. So, if the solution, $x(i, t)$, appears in these functions and it is evaluated for such an intermediate value of t , the aforementioned error will stop the integration. $x(i, t)$ must always be used with already known values of t . For instance, you cannot use $x(i, t)$ instead of $x[i]$ (refer to solution). It is also impossible to start from a null value of t the integration of equations with variable delays in the form $x(i, t-a*t)$, for example, because the initial delay will be null. Instead try a small but not null value for the initial value of the independent variable. Even in a truly retarded equation the error could happen if the integration step size is (or becomes later) larger than the delay. This can be

avoided with smaller values for the **Step h** and **Max. h** entries in the **Method** dialog sheet (which can be activated from the Edit/**Method** menu entry or by pressing `Ctrl+H`).

Argument singularity

While evaluating an expression, a function error happened because a function was given an invalid argument such as 0^{-2} , etc. The function and argument are included in the first line of the error message.

Bad name or file

You have an incomplete name or file specification of an external library, font or plotter driver. Press `Enter` or use **OK** and you will return to the **External Libraries** dialog box where the incomplete definition happened.

Bad plotter driver Line: #n

Line n in the current plotter driver is incorrect. Refer to the **Plotter Drivers** section.

Cannot create child window

This very rare error indicates that an output window could not be open. Maybe there are too many windows, or the memory is very low.

Cannot get the screen

The program was not able to copy the contents of the current output window to the clipboard. Maybe there is not enough memory (or your video card driver is too old).

Cannot load the screen

The program was not able to load a graphics screen from the disk. Maybe there is not enough memory (or your video card driver is too old).

Cannot print

The program was not able to print the current window from the disk. Maybe there is not enough memory.

Cannot put the screen

The program was not able to paste from the clipboard to the current graphics window. Maybe there is not enough memory (or your video card driver is too old).

Cannot save the screen

The program was not able to save to the disk the contents of the current output window. Maybe there is not enough memory (or your video card driver is too old).

Code DLL already in use

Some integration codes contained in external libraries cannot be used by more than one instance of the program at the same time. You must make sure that no other copy of the program is using that external integration code (check the **Integration code** entry in the **Method** dialog box).

Code DLL error #n

A poorly designed external library (containing an integration method or additional mathematical function) could issue this message instead of giving a more explicit error. Refer to the library author. The libraries provided with the program do not use this message.

Code for erf failed

This very rare error indicates that the numerical code used to evaluate the error functions `erf` and `erfc` is not converging. Probably the argument is out of range.

Could not print

The *Print Manager* reported this obscure error while trying to send to the printer the *Dynamics Solver* output. Check the printer setup.

Could not send the mail

An error happened while trying to send the current problem to the FAX and mail system.

Could not solve in real time\nNeed at least # s

The value in **Range/Real time** is too low.

Could not start Dynamics Solver

This very rare error indicates that the program could not complete the initialization process. Probably the Windows session is in an unstable state or the resources are very low.

Denormal operand

This floating point error should not occur. See **Bugs, Suggestions and Fixed Versions**.

Division by 0

While evaluating an expression, a floating point error (like trying to compute $1/0$) happened.

Error in problem line #n. Do you want to correct it?

Line *n* in the problem file you are trying to load from the disk or from an **All settings** edit window is incorrect.

If you press **Enter** (or use **Yes**), an **All settings** edit window will open and you can correct there the error, or give up by using the **File/Abandon changes** menu. Notice that in the latter case you will most probably have an incomplete problem file in memory and you will have to change it before it could be run.

If you answer **No**, the error will not be corrected and you will have an incomplete problem in memory.

Extra character(s)

Some characters appear after a complete expression. Check for unbalanced parentheses, operators and so on.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Gauss method failed

Probably the boundary value problem has no solution. However, you may try changing the initial values, decreasing the Error and increasing the Iterations.

General error

The *Print Manager* reported this obscure error while trying to send to the printer the *Dynamics Solver* output. Check the printer setup.

Index out of bounds

In an expression an index is negative or larger than the highest available derivative or component.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Inexact operation

This floating point error should not occur. See **Bugs, Suggestions and Fixed Versions**.

Infinity reached

The problem solution was automatically stopped because the value of a variable or derivative has reached the **Infinity** value you chose in the **Range** dialog box.

Insufficient disk space

The *Print Manager* reports that there is not enough disk space to store the metafile.

Insufficient memory

While trying to send to the printer the *Dynamics Solver* output, the *Print Manager* reported that there is not enough memory to create the metafile necessary to create the printout.

Integer division by 0

This error should not happen. See **Bugs, Suggestions and Fixed Versions**.

Integer overflow

This error should not happen. See **Bugs, Suggestions and Fixed Versions**.

Invalid character

In an expression there appears an invalid character.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Invalid code

This internal error of the expression interpreter should not happen. See **Bugs, Suggestions and Fixed Versions**.

Invalid dimension

This internal error of the expression interpreter should not happen. See **Bugs, Suggestions and Fixed Versions**.

Invalid font file

You are trying to use an incompatible font file. See **Editing Fonts**.

Invalid index

In an expression an index is not a constant integer.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Invalid operation

While evaluating an expression, a floating point error (like trying to compute 0/0) happened.

Invalid printer

No printer has been specified. Use the File/**Printer setup** menu entry.

Invalid stack

This internal error of the expression interpreter should not happen. See **Bugs, Suggestions and Fixed Versions**.

name library not found

An external library (with additional mathematical functions or integration methods) has not been found. Check the installation in the **Configuration** menu.

Newton's method failed

Probably the boundary value problem has no solution. However, you may try changing the initial values, decreasing the Error and increasing the Iterations.

No available timer

Windows has no available timer because too many programs are using these scarce resources (or, maybe, some impolite program forgot to free unused timers).

Dynamics Solver will not be able to do timed pauses (according to the **Pause** setting in the **Range** dialog box). If you need this ability, try closing some other applications or, as the last resort; restarting Windows.

No font file

Dynamics Solver has not found a font file. Check the installed fonts in the Configuration/**Font files** menu (or run `INSTALL` again).

No help file was specified for this driver

Dynamics Solver has not found the help file for a potter driver. Check the installation of the driver in the Configuration/**Plotter drivers** menu.

No input file

No file name has been provided for a graphics element of type data. You must provide a file specification for the data file.

No line in memory

It is not possible to print a text window because no line is stored in memory. Check the **Lines in Memory** entry in the **Text Output** dialog box.

No line to print

Nothing to print because the **Edit Window** is empty.

No print mode

No format is available to print a graphics screen. This error should only happen if you are trying to print to a device that does not support bitmaps (as some plotters) and the solution has not been stored in memory by setting a non-zero value for **Save output in memory** in the **Format** dialog box.

No segment length in element #*n*

Incorrect definition of a graphics element of type **Fractal**: you have provided no **Divisor**.

Not enough memory

This very rare error indicates that your system has not enough memory to solve a problem. Try closing other applications or simplifying the problem..

Only #*n* solution point(s) will be retained!

You are trying to solve a differential equation by means of an external integration method that will not store in memory more than *n* solution points. This will be only a real problem if you need to evaluate the interpolated solution for very retarded values of the independent variable.

Order too high

In an expression a derivative order is too high. The exact number of available derivatives depend on the integration method you are using.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Output to file failed

When trying to send the text output to an external file an error happened. The most probable cause is that an incorrect file specification has been entered in the **Output file** box of the **Text Output** dialog box.

The error could also happen, for the same reasons, when using the `w r i t e` function to send values to an output file.

Overflow

While evaluating an expression, a floating point error happened because the result was too large. For example, the program might be trying to evaluate $1E200*1E200$, but the result is higher than the maximum IEEE double precision value. See **Technical Information**.

Overflow range error

While evaluating an expression, a function error happened because the result was too large. For example, the program might be trying to evaluate E^{2000} , but the result is higher than the maximum IEEE double precision value. See **Technical Information**.

P-code too long

This rare error could happen when trying to analyze a very complex problem if an expression happens to be too complex. You must increase the value in the **P-code length** entry in the **Advanced settings** dialog box.

Partial loss of significance

This function error should not occur. See **Bugs, Suggestions and Fixed Versions**.

Root finding failed

This very rare error could happen when trying to compute a **Poincaré section**. Try increasing the **Error** entry in the corresponding **Graphics Output** or **Text Output** dialog box.

Stack overflow

Dynamics Solver tried to evaluate a too complex expression. This should not happen: see **Bugs, Suggestions and Fixed Versions**.

Stack overflow

Dynamics Solver tried to evaluate a too complex expression You must change the **Stack length** in the **Advanced settings** dialog box.

Step too small

The integration step became too small because it is not possible to meet the current value of **Tolerance** in the **Numerical Method** dialog box. Try increasing that value, but take into account that very often this error indicates that the differential system is stiff or that some kind of singularity arises (the right side value might be approaching zero, for instance).

To read correctly this problem file, version *n* or newer is required. Continue anyway?

You should upgrade your copy of *Dynamics Solver* to avoid errors reading this problem file, which has been created with a newer version.

Too few angles in element #*n*

Incorrect definition of a graphics element of type **Fractal**: the number of **Angles** must be equal to that of **Divisors**.

Too many steps

The integration code failed to meet the current value of **Tolerance** in the **Numerical Method** dialog box. Try increasing that value, but take into account that very often this error indicates that the differential system is stiff or that some kind of singularity arises (the right side value might be approaching zero, for instance).

Too much text

When trying to load a problem file or to edit it (or another kind of external file) in an **Edit Window** *Dynamics Solver* found that the file exceeds the maximum length the program can handle. This maximum length is about 64Kb for the 16-bit edition of *Dynamics Solver* due to the limitations of edit boxes in 16-bit versions of Windows. Sorry, you can do nothing about that, except to use the 32-bit version of this program under a 32-bit environment.

Total loss of significance

While evaluating an expression, a function error happened because a function was given a legal but very unusual (probably too large) argument, such as $\sin(1E70)$ etc.

Underflow

This floating point error should not occur. See **Bugs, Suggestions and Fixed Versions**.

Underflow range error

This floating point error should not occur. See **Bugs, Suggestions and Fixed Versions**.

Unexpected end of expression

In an expression there is an incomplete expression. Check for unbalanced parentheses and operators. The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Unknown Windows error

This very rare message indicates that an undocumented Windows error happened.

Unknown function

Dynamics Solver has found some unknown function in an expression.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

Unknown identifier

Dynamics Solver has found an unknown variable or constant in an expression.

The error location is indicated by a flashing cursor in the edit box of a **Compiler Error** dialog box.

User canceled printing

You're always right.

Variable with no name

Check the entries in **Variables**: a variable has no name.

Windows error

This error message is issued by the underlying operating system and is probably due to the fact that you are trying to read from or write to a non-existing file or directory. Refer to your Windows reference manual for more information about Windows error messages.

Hints

1. Use this documentation and the help file! Lots of tricks are described here. Try using the **Search** function of the help engine. Read the **Getting help** topic.
2. Each error message and dialog box has its own help button, which can be used to obtain information on how to deal with it.
3. Read the **Customization** and the **Frequently Asked Questions** sections and all their subsections.
4. The right mouse button opens a useful (and customizable) menu. The contents of the popup menu is controlled from Configuration/**Toolbar and popup menu**.
5. You can use toolbar buttons for every menu entry. The displayed toolbars and buttons are controlled from Configuration/**Toolbar and popup menu**.
6. To improve the visibility of toolbar buttons you can change their size from the **Preferences** dialog box.
7. Many menu entries can be directly activated from the keyboard. See **Keyboard**.
8. Useful information is displayed in the status line when the mouse pointer is over a toolbar button or a menu entry. If you find that the character font used to display these hints is too small, try setting the **Use bold font / in status line** entry of the **Boxes** dialog box.
9. Depending on the settings in the **Preferences** dialog box, you may also get a short description in the form of a popup window when the mouse pointer is over a toolbar button. But, notice that this feature is disabled by default, because some video cards drivers redraw too often the window under the popup tip.
10. Read the **Zooming a Graphics Window** section.
11. Use **Move around** and **Zoom in** to see the details around some point and fine tune graphics elements.
12. If you want to see on the screen the finest detail, try using the **Full screen** mode.
13. By default, *Dynamics Solver* searches problem files with the `.DS` extension. This will be fine in most cases, but you might have a lot of problem files with another extension. If you want to search problem files with extensions, say, `.DS`, `.MAP` and `.EF`, you should enter in the **Extensions** entry of the Preferences/**Other** dialog sheet the string `*.ds;*.map;*.ef`.
14. Spend some time understanding all the **Printing and Plotting Features**.
15. Many complex problems can be solved by increasing the order of the problem and introducing new unknowns. In this way a finite equation can be differentiated to obtain a more complex differential equation (but *Dynamics Solver* is expert in differential equation). See **Equations Not Written in Normal Form, Integro-Differential Equations, Definite Integrals, Complex quantities** and the examples `BAS.DS`, `BASIN.DS`, `HENONL.DS`, `CAT.DS`, `FP.DS`, `FRW2.DS` and `DEVIL.DS`, for instance. A tricky example is shown in `DISKSCAT.DS`.
16. A wise combination of the full power of *Dynamics Solver*'s Expressions, **Repeated Integration/Iteration** and **Constrained Initial Conditions** gives unlimited possibilities. Try them!
17. In highly complex problems, you could take advantage of the ability to write external mathematical functions and constants. Let us assume that you have a very complex equation (say ten pages of output from a computer algebra system, like Mathematica). Even if you write it in *Dynamics Solver* the repeated evaluation of such a function could be exceedingly slow. You could instead write a procedure in your language of choice (FORTRAN, C, C++, Pascal, etc.) and call it from *Dynamics Solver*.
18. In many cases, errors that are issued by the program when evaluating expressions can be avoided by using a mathematically equivalent form, which is evaluated in a different way by *Dynamics Solver*. Suppose you need the expression $x \cdot \log(x)$ evaluated at $x = 0$. We know that this limit is finite but *Dynamics Solver* stops with an error when trying to compute $\log(0)$. The

problem is readily solved by using the equivalent, but messy, $x \cdot \log(x + \text{step}(-x^2))$. This kind of tricks often involves the Heaviside step function and need $\text{step}(0) \neq 0$ in

Heaviside(0) of **Advanced settings**. You may also use the `if` function: `if(x, x*log(x), 0)` or the equivalent $x \cdot \log(x) : 0$.

19. Use graphics elements to add letters, axes and other informative elements to your graphics results.
20. When using *Dynamics Solver* to draw figures (with graphics elements), you should probably select the in the **Aspect ratio** entry of the **Format** sheet any value different from **Arbitrary**. In this way, circles will look round and other elements can be rotated with no distortion. With graphics elements, the **Axes** entry of the same sheet is obsolete and you may want to clear it.
21. Very artistic results can be achieved by dynamically changing the color and pen width used to draw the results by means of the `PenColor`, `PenWidth`, `HSV` and `RGB` functions. See the examples in `ART\COS1.DS` and `CHAOS\HENONMAP.DS`.
22. Explore the examples included with the program. You may find there a lot of inspiring advanced techniques.

Frequently Asked Questions

How may I stop a long window redrawing?

Press `ESC`

How may I print the results?

This is a complex issue. Please read carefully **Printing** and the following sections.

How may I add lettering to a figure?

Use graphics elements.

How may I uninstall *Dynamics Solver*?

For the 16-bit edition, use the uninstaller program from the *Dynamics Solver* group of *Program Manager*. It will uninstall the program, if the `FILOG.INI` file has not been deleted. To complete the uninstallation you could delete the `DSOLVER.INI` file from the Windows directory.

For the 32-bit edition use the *Control Panel* as usual. You may have to uninstall by hand some files you have added.

May I use TrueType fonts (and other Windows fonts) in *Dynamics Solver*?

Dynamics Solver does not use Windows fonts in graphics elements, but some special fonts that look worse on the screen but are more portable among platforms and printing devices. The printed result is reasonably good and the files in portable format (PostScript and HPGL, for instance) are small. Some of these fonts are provided with the program, and the user can easily modify them (or create new fonts) by using the Font Manager program, `FM.EXE`. If you have not installed it, run again `INSTALL`. The new fonts will be used by *Dynamics Solver* once you they are installed in the **Install Font Files** dialog box .

If you want to use Windows fonts (including TrueType fonts), you can copy the screen to the clipboard and from there to any drawing program, like Paintbrush. You can then add lettering with Windows fonts.

Bugs, Suggestions and Fixed Versions

I would greatly appreciate receiving information on bugs, as well as suggestions to improve *Dynamics Solver*. Bug fixes will be available as fast as possible, and the last version can be downloaded from the program page:

<http://tp.lc.ehu.es/jma/ds/ds.html>

Help with the documentation and the help files would be especially useful: I have not had the time to revise it and corrections to my English will be welcome.

Clever solutions to complex or unusual problems will be acknowledged and included in the documentation with full credit to their author.

Contact the author preferably through e-mail: <wtpagagj@lg.ehu.es>.

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