University of Houston - Clear Lake
PhaseScope energizes the study of mathematics

Higher mathematics has traditionally been a pencil-and-paper discipline employing abstract symbols, sometimes illustrated with rough graphs, that depend on the mathematician's ability to envision correctly the relationships signified by those symbols. For years, students in upper-level math courses learned techniques to solve difficult problemsÐsuch as those in the realm of differential equationsÐbut were unable to visualize the concepts inherent in the solutions. A new computer application called PhaseScope may help change the way students conceptualize complex mathematical problems.

"I wanted a tool for analyzing the solutions and stability of systems of differential equations," says Mike Mezzino, chairman of the department of mathematics at the University of Houston - Clear Lake. "Students can solve the equations, but lack the ability to see the dynamics of the system. PhaseScope illustrates the dynamical aspect of these problems."

Differential equations are fundamental building blocks of a class of mathematical models that attempt to describe real-world situations. Typically, they are a set of related equations, derived from natural laws and hypotheses, that express the rates of change of one or more variablesÐthat is, these equations express change in the dynamics of a model over the course of time. A system of differential equations provides a mathematical framework for understanding dynamical systems, or situations of change, movement, or flux. Differential equations are used to plot the motion of celestial bodies in orbital mechanics, to chart the speed of enzyme reactions in chemistry, to measure the rate of radioactive decay, and to predict demographic changes in competing population groups.

PhaseScope uses the metaphor of an oscilloscope, which measures the fluctuations of an electrical current, to graph the fluctuations of these dynamical systems. With each sweep of PhaseScope's "oscilloscope," the change in a system over time is shown. The innovation of PhaseScope lies in the ease with which it allows students to make changes to the equations, experiment with different parameters, and view the results in simulated real time. This exploration can lead to demonstrations of the new physics of chaos, in which a previously stable system, fluctuating in a consistent manner, can, with a small change in initial values, become seemingly wildly erratic.

The operation of PhaseScope is simple. "Differential equations are entered into PhaseScope, which sends them off to *Mathematica*. PhaseScope then receives the solution from *Mathematica*, parses it, and plots it in either two or three dimensions. Students can then zoom in or out and change the perspective of the resulting graph," explains Mezzino. They can also plot multiple solutions in the same graph and change the display style of individual solutions.

Mathematica and Interface Builder provide a solid foundation

PhaseScope rests securely on the foundations of *Mathematica* and Interface Builder. "*Mathematica* provided the power to solve the equations symbolically, just as students wouldÐthey could type in equations just as they would write them. And the extensibility of *Mathematica* allowed me to integrate it with PhaseScope, writing some of my own routines to handle certain kinds of displays and provide the user with additional numerical integration techniques which can easily be included into the application."

"The key to a product that students will use is in the interface," Mezzino continues. "I've had some experience writing software, and knew that the least appealingÐand most time-consumingÐaspect of programming was creating the user interface. I remember when I saw a demonstration of Interface Builder with sliders and buttons that actually worked, and I was amazed! Here was an application that could be used quickly to design a friendly user interface."

It took Mezzino three months of part-time work to develop the initial version of PhaseScopeÐ"a testament to the rapid development ability of Interface Builder and the NeXTstep environment," says Mezzino. The key was learning Interface Builder, which took about three weeks. He examined the programming examples included with the NeXT bundled software and used some of them in his application. PhaseScope is now a polished software application, with an easy-to-use interface, panel and voice alerts, and the ability to save and print graphs.

Mezzino's students are already using the application, and their response has been very positive. "The students who have used PhaseScope were electrified," he observes. "It's clear that we have been working on the right problem. My students' reactions inspire me to develop other projects like PhaseScope."

Not only have students been impressed; software professionals have also been recognizing the power and originality of PhaseScope. IMPACT Publishing, an software company based in Ithaca, New York, recently sponsored a contest for the most innovative software application developed for the NeXT computers. Mezzino submitted his application, and IMPACT awarded PhaseScope first place. Mezzino received a prize of $1,000 and IMPACT is now pursuing distribution rights.

Building the mathematics lab for the future

The successes of PhaseScope in the NeXTstep development environment have stimulated Mezzino and his colleagues to conceive of other projects using NeXT hardware and software technology to supplement mathematics courses. He and his staff are submitting a proposal to the National Science Foundation to fund a mathematics lab of 15 NeXTstation Color computers and a file server, and a teaching assistant to support students in their mathematical studies.

"We're going to use the lab to fill in the rough spots in students' knowledge of calculus," Dr. Mezzino explained. The University of Houston at Clear Lake enrolls students from the junior year (in college) to masters candidates. Because they spend their first two years at other institutions, the mathematical experience and skills of entering juniors varies. A mathematics lab using *Mathematica*, electronic NotebooksÐstructured documents that can contain *Mathematica* input and output, graphics, and textÐ(including those notebooks developed on other campuses), and custom software will allow students to correct their deficiencies in calculus on an individual basis.

The lab will also provide an opportunity for improved classroom instruction. Mezzino and his colleagues have already modified the syllabuses in four of their courses to include specific laboratory exercises. "The lab work will first encourage the students to explore the software, and then will set specific goals as problems for the week," he adds.

"We're especially excited about Interface Builder and *Mathematica*. These tools will significantly extend and supplement our students' work. The NeXT computers inject a sense of fun into the mathematics curriculum, and that's important," concludes Mezzino.

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