Lab 9: Differential Geometry

This lab continues our study of vector functions in space. Here we see how the differential calculus can is used to describe the geometry of curves.

Copy the notebook from the **/math235/notebooks** directory into your own **notebooks** subdirectory; launch the *WriteNow* text processor to set up and save your lab report in your **reports** subdirectory; work through each section of the notebook and write a summary of what you learned in your report; mail your report to **math235**.

As you write up each section of your report, keep these points in mind:

- Use your own words. It is plagiarism to represent someone else's words as your own.
- Examine the "sample" lab reports for previous labs in the **/math235/reports** directory.
- Give at least one example in each section of your report. It's o.k. to use your work from the notebook Exercises here.
- Use the Copy (Command-c) and Paste (Command-v) commands to copy Mathematica input, output, and graphics into your report.
- Use the Speller (Command-:).
- When you mail in your report for grading, be sure that its file icon is in the Send window before you press the Deliver key.
- In your Mail message, include a comment to tell how much time you spent on this lab.

Reminders and caveats:

- If a *Mathematica* command is taking too long to execute, use the Abort command (Command-.) in the Action submenu.
- When writing the product of two variables in *Mathematica*, the symbols must be separated by either a space or an asterisk. For example, `xy` will be interpreted by *Mathematica* as the sine of the single variable named "xy"—not the product x times y. The latter has to be written as either `x y` or `x*y`.
- Save your work (with Command-s) every few minutes.
- Check your mail.

Mathematica Commands

Here are some of the *Mathematica* commands that you might use in this lab.

mag[v_] := Sqrt[v.v]	Defines the magnitude of the vector **v**.
uT[r_][t_] := r'[t]/mag[r'[t]]	Defines the unit tangent function **uT[r][t]** .
uN[r_][t_] := uT[r]'[t]/mag[uT[r]'[t]]	Defines the unit normal function **uN[r][t]** .
g[r_][a_,b_] := Graphics3D[Line[Table[N[r[t]], {t,a,b,0.02}]]]	Defines the graph of **r** on the interval **[a,b]**.
GrayLevel[0.6]	Makes the graph light gray.[1]
Table[N[f[t]], {t,a,b,0.01}]	Produces the table of vectors : `{f[a], f[a+0.01], f[a+0.02],...,f[b]}` where each `f[t]` determines a point in space.
vector[p_,q_] := Graphics3D[Line[{p,p+q}]]	Defines the vector **q** emanating from point **p** to be the line seqment from p to p+q.
Thickness[0.01]	Sets the line's thickness to be 0.01 the width of the picture.as thick .
trajectory[r_][a_,b_] := Do[Show[{ g[r][a,b], frame[r][u] }], {u,a,b,(b-a)/15}]	Produces a sequence of 16 pictures, each with the same curve **g[r][a,b]** but with the frame of vectors emanating from different points on the curve. The resulting sequence shows the moving frame when animated.
<<Algebra/Trigonometry.m	Loads the external package **Trigonometry.m** .[2]
// TrigReduce	Applied the **TrigReduce** function to the result, thereby simplifying the expression by means of certain trigonometric identities.
uT[r][t].uN[r][t]	The dot product of **uT[r][t]** and **uN[r][t]**.
a[r_][t_] := r''[t]	Defines the acceleration vector of **r[t]**.
<<LinearAlgebra/Cross.m	Loads the external package **Cross.m** .
?Cross	Displays a summary of the **Cross** command.
uB[r_][t_] := Cross[uT[r][t],uN[r][t]]	Defines **uB[r][t]** to be the cross product of the vectors **uT[r][t]** and **uN[r][t]**..
kappa[r][t]	The curvature of the function **r[t]**.
tau[r][t]	The torsion of the function **r[t]**.
s[r_][t_] := Integrate[mag[r'[u]], {u,0,t}]	Defines the arc-length function of **r[t]**.

[1] The scale runs from 0.0 to 1.0, with 0.0 being black and 1.0 being white.

[2] Look at the **Trigonometry.m** file in the **NextLibrary/Mathematica/Packages/Algebra** directory.