Lab 8: Vector Functions in Space

This lab continues our study of vector functions that we began in Lab 2. In Lab 6 we considered general vector-valued functions in two dimensions. Here we study the general vector-valued function in three dimensions. My using *Mathematica*'s animation ability, we can rotate the plots to get a clearer understanding of the graphs of the functions.

Do the same that you have done in the past few labs: copy the notebook from the **/math235/notebooks** directory into your own **notebooks** subdirectory; launch the *WriteNow* text processor to set up and save your lab report in your **reports** subdirectory; work through each section of the notebook and write a summary of what you learned in your report; mail your report to **math235**.

As you write up each section of your report, keep these points in mind:

- Use your own words. It is plagiarism to represent someone else's words as your own.
- Examine the "sample" lab reports for previous labs in the **/math235/reports** directory.
- Give at least one example in each section of your report. It's o.k. to use your work from the notebook Exercises here.
- Use the Copy (Command-c) and Paste (Command-v) commands to copy Mathematica input, output, and graphics into your report.
- Use the Speller (Command-:).
- When you mail in your report for grading, be sure that its file icon is in the Send window before you press the Deliver key.
- In your Mail message, include a comment to tell how much time you spent on this lab.

Reminders and caveats:

- When writing the product of two variables in *Mathematica*, the symbols must be separated by either a space or an asterisk. For example, **`xy`** will be interpreted by *Mathematica* as the sine of the single variable named "xy"—not the product x times y. The latter has to be written as either **`x y`** or **`x*y`**.
- Save your work (with Command-s) every few minutes.
- Check your mail.

Mathematica Commands

Here are some of the *Mathematica* commands that you might use in this lab. Also refer to the listings in the handouts for Lab 6 and Lab 7.

`f[t_] = {3Cos[t],3Sin[t],t}`	Defines the vector-valued function `f[t]` to be `(3Cos[t])i + (3Sin[t])j + (t)k` .
`N[f[t]]`	Produces the numerical (decimal) value of `f[t]`.
`Table[N[f[t]],{t,a,b,0.01}]`	Produces the table of vectors : `{f[a], f[a+0.01], f[a+0.02],...,f[b]}` where each `f[t]` determines a point in space.
`Graphics3D[ Line[ Table[N[f[t]],{t,a,b,0.01}] ] ] ]`	Produces a space curve by connecting the points in the table with line segments.
`Show[ c[f][a,b] ]`	Displays the graph of `f` over the interval `[a,b]`.
`ViewPoint->{7Cos[s],7Sin[s],2}`	Rotates the image by the polar angle θ=s.
`Do[Show[c[f][a,b],ViewPoint->{7Cos[s],7Sin[s],2}],{s,0,Pi/2,Pi/28}]`	Produces a sequence of 15 plots of the same graph. The different view points are chosen so that the graph appear to rotate 90 degrees when the sequence is animated.
`p[f_][t_] := Graphics3D[ Point[f[t]] ]`	Defines `p[f][t]` to be the point at `f[t]`.
`PointSize[.02]`	Sets the point size to be 0.02 as large as the picture.
`v[f_][t_] := Graphics3D[ Line[{f[t],f[t]+f'[t]}] ]`	Defines the velocity vector for the function `f` at time `t`.
`t[f_][a_,b_] := {c[f][a,b],Table[p[f][t],{t,a,b}],Table[v[f][t],{t,a,b}]}}`	Defines the trajectory of `f` on the interval `[a,b]` to be the graph of `f` together with a list of points and its velocity vectors at those points.
`a[f_][t_] := Graphics3D[ Line[{f[t],f[t]+f''[t]}] ]`	Defines the acceleration vector for the function `f` at time `t`.
`Norm[{x_,y_,z_}] := Sqrt[x^2+y^2+z^2]`	Produces the magnitude of the vector `xi+yj+zk`.
`s[f_][t_] := Norm[f'[t]]`	The speed of the particle at time `t`.
`Table[ N[s[f][t]], {t,0,Pi,Pi/8} ]`	A (numerical) list of speeds for the nine times: 0, Pi/8, 2Pi/8, 3Pi/8, 4Pi/8, 5Pi/8, 6Pi/8, 7Pi/8, 8Pi/8.
`l[f_][aa_,b_] := Integrate[ s[f][t], {t,aa,b} ]`	The length of the curve over the interval `[aa,b]`.