Lab 7: Integration

This lab uses the Integrate command to perform various types of integration of functions of one or more variables.

Do the same that you have done in the past few labs: copy the notebook from the **/math235/notebooks** directory into your own **notebooks** subdirectory; launch the *WriteNow* text processor to set up and save your lab report in your **reports** subdirectory; work through each section of the notebook and write a summary of what you learned in your report; mail your report to **math235**.

As you write up each section of your report, keep these points in mind:

- Use your own words. It is plagiarism to represent someone else's words as your own.
- Examine the "sample" lab reports for previous labs in the **/math235/reports** directory.
- Give at least one example in each section of your report. It's o.k. to use your work from the notebook Exercises here.
- Use the Copy (Command-c) and Paste (Command-v) commands to copy Mathematica input, output, and graphics into your report.
- When you mail in your report for grading, be sure that its file icon is in the Send window before you press the Deliver key.
- In your Mail message, include a comment to tell how much time you spent on this lab.

Mathematica Commands

Here are some of the *Mathematica* commands that you might use in this lab:

`Clear["@*"]`	Deletes all definitions of variables that begin with a lower-case letter.
`Integrate[f[x],x]`	Produces an antiderivative of `f[x]`.
`Integrate[f[x],{x,a,b}]`	Produces the integral of `f` over the interval `[a,b]`.
`Integrate[f[x],{x,a,Infinity}]`	Produces the integral of `f` over the interval `[a,∞)`.
`Integrate[ f[x,y], {x,a,b}, {y,c,d} ]`	Produces the integral of `f` over the region bounded in the plane by: `a` ≤ `x` ≤ `b` and `c` ≤ `y` ≤ `d`.
`Integrate[ Integrate[ f[x,y], {y,g[x],h[x]}], {x,a,b} ]`	Produces the integral of `f` over the region bounded in the plane by: `a` ≤ `x` ≤ `b` and `g[x]` ≤ `y` ≤ `h[x]`.[1]
`Integrate[ f[x,y,z], {x,a,b}, {y,c,d}, {z,p,q} ]`	Produces the integral of `f` over the region bounded in space by: `a` ≤ `x` ≤ `b`, `c` ≤ `y` ≤ `d`, and `p` ≤ `z` ≤ `q`.
`Integrate[ Integrate[ Integrate[ f[x,y,z], {z,u[x,y],v[x,y]}], {y,g[x],h[x]}], {x,a,b} ]`	Produces the integral of `f` over the region bounded in space by: `a` ≤ `x` ≤ `b`, `g[x]` ≤ `y` ≤ `h[x]`, and `u[x,y]` ≤ `z` ≤ `v[x,y]`.
`<<Graphics/ParametricPlot3D.m`	Loads the *Mathematica* package `ParametricPlot3D.m` that is in the `Graphics` subdirectory of the directory `/NextLibrary/Mathematica/Packages`.
`SphericalPlot3D[ f[s,t], {s,0,Pi}, {t,0,2Pi} ]`	Produces the surface that is the graph of the function given in spherical coordinates by ρ `= f[`φ`,`θ`]`, where `0` ≤ φ ≤ `Pi` and `0` ≤ θ ≤ `2Pi`.[2]
`Integrate[ Integrate[ Integrate[ f[r,s,t] r^2 Sin[s], {r,u[s,t],v[s,t]}] ,{s,g[t],h[t]}], {t,a,b} ]`	Produces the integral of `f[`ρ`,`φ`,`θ`]` over the region bounded in spherical coordinates by: `a` ≤ θ ≤ `b`, `g[`θ`]` ≤ φ ≤ `h[`θ`]`, and `u[`φ`,`θ`]` ≤ ρ ≤ `v[`φ`,`θ`]`.
`Integrate[ Integrate[ f[r,s,t] r^2 Sin[s], {r,0,g[s,t]}], {s,0,Pi}, {t,0,2Pi} ]`	Produces the integral of `f[`ρ`,`φ`,`θ`]` over the region bounded in spherical coordinates by: `0` ≤ θ ≤ `2Pi`, `0` ≤ φ ≤ `Pi`, and `0` ≤ ρ ≤ `g[`φ`,`θ`]`.
`// Simplify`	Simplifies the current answer.
`N[%]`	Produces a numerical approximation of the previous answer.

[1] Multiple integrals can be done with a single Integrate command only if all the bounds are constants.

[2] Here, as usual, θ is the polar angle in the XY-plane, and φ is the vertical angle measured down from the Z-axis. This is just the reverse of what the physicists use. (See the last paragraph on page 726 of Stein.)