Lab 7: Series

This lab uses *Mathematica* to investigate geometric series and the p-series. It also describes two famous unsolved mathematical problems: the irrationality of Euler's constant and the location of the zeros of the Riemann zeta function.

Copy the notebook from the **/math212/notebooks** directory into your own **notebooks** subdirectory; launch the *WriteNow* text processor to set up and save your lab report in your **reports** subdirectory; work through each section of the notebook and write a summary of what you learned in your report; mail your report to **math212**.

As you write up each section of your report, keep these points in mind:

- Use your own words. It is plagiarism to represent someone else's words as your own.

- Examine the "sample" lab reports for previous labs in the **/math212/reports** directory.

- Give at least one example in each section of your report. It's o.k. to use your work from the notebook Exercises here.

- Use the Copy (Command-c) and Paste (Command-v) commands to copy *Mathematica* input, output, and graphics into your report.

- Use the Speller (Command-:).

- When you mail in your report for grading, be sure that its file icon is in the Send window before you press the Deliver key.

- In your Mail message, include a comment to tell how much time you spent on this lab.

Reminders and caveats:

- If a *Mathematica* command is taking too long to execute, use the Abort command (Command-.) in the Action submenu.

- When writing the product of two variables in *Mathematica*, the symbols must be separated by either a space or an asterisk. For example, **`xy`** will be interpreted by *Mathematica* as the single variable named "xy"—not the product x times y. The latter has to be written as either **`x y`** or **`x*y`**.

- Save your work (with Command-s) every few minutes.

Here are some ideas to consider when writing your report:

Geometric Series

1. How can *Mathematica* produce the sequence of partial sums of a series?
2. How can you use *Mathematica* to check the <u>rate</u> of convergence?

Harmonic Series and Euler's Constant

1. How can *Mathematica* be used to show that the harmonic series diverges?
2. How is its sequence of partial sums related to the natural logarithm?
3. How does this lead to Euler's constant?
4. What can you say about the rate of convergence to Euler's constant?

p-Series and Riemann's Zeta Function

1. How can *Mathematica* be used to investigate the limits of the p-series?
2. What is the Riemann zeta function?
3. For how many values of t in the range $0 \le t \le 50$ does $\zeta(1/2 + t\ i) = 0$?
4. What is the Riemann Hypothesis?

Mathematica Commands

Here are some of the *Mathematica* commands that you might use in this lab:

`s[n_] := NSum[0.8^i,{i,0,n-1}]`	Defines the sequence **`s[n]`** of partial sums of the geometric series `1 + 0.8^1 + 0.8^2 + ...`, so that `s[n]` is the sum of the first n terms: `s[n] = 1 + 0.8^1 + 0.8^2 + ... + 0.8^(n-1)`.
`Table[{n, s[n]},{n,1,5}]`	Produces the list of pairs: `{{1,s[1]},{2,s[2]},{3,s[3]},{4,s[4]},{5,s[5]}}`.
`// TableForm`	Causes the output to be in tabular (columns) form.
`N[s[100],50]`	Produces the numerical (decimal) value of `s[100]` with 50 significant digits.
`Table[{2^k,s[2^k]},{k,1,16}]`	Produces the list of pairs: `{{2,s[2]},{4,s[4]},{8,s[8]},...,{65536,s[65536]}}`
`EulerGamma`	Euler's constant.
`N[EulerGamma]`	Returns the numerical (decimal) value of Euler's constant.
`Table[f[2^k],{k,1,16}]`	Produces the list: `{f[2],f[4],f[8],f[16],f[32],...,f[65536]}`
`s[p_,n_] := NSum[1/i^p,{i,1,n}]`	Defines the family of sequences **`s[p,n]`** of partial sums of the p-series `1 + 1/2^p + 1/3^p + ...`, so that `s[p,n]` is the sum of the first n terms: `s[p,n] = 1 + 1/2^p + 1/3^p + ... + 1/n^p`.
`Zeta[p]`	The Riemann zeta function, which equals the sum of the p-series: `Zeta[p] = Limit[s[p,n],n->Infinity]`.[1]
`Plot[Abs[Zeta[1/2+t*I]],{t,0,32}]`	Plots the absolute value of the Riemann zeta function at complex numbers `1/2+t*I`, where `t` ranges from

[1] *Mathematica* cannot evaluate this limit directly. Although `Zeta[p]` is equal to the sum of the p-series, it is actually computed by means of an integral.

0 to 32 and `I` is the (imaginary) square root of -1.