Lab 6: Sequences

This lab uses *Mathematica* to investigate sequences and their convergence.

Do the same that you have done in the previous labs: copy the notebook from the **/math212/notebooks** directory into your own **notebooks** subdirectory; launch the *WriteNow* text processor to set up and save your lab report in your **reports** subdirectory; work through each section of the notebook and write a summary of what you learned in your report; mail your report to **math212**.

As you write up each section of your report, keep these points in mind:

- Use your own words. It is plagiarism to represent someone else's words as your own.
- Examine the "sample" lab reports for previous labs in the **/math212/reports** directory.
- Give at least one example in each section of your report. Use your work from the notebook Exercises here.
- Use the Copy (Command-c) and Paste (Command-v) commands to copy Mathematica input, output, and graphics into your report.
- Use the Speller (Command-:).
- When you mail in your report for grading, be sure that its file icon is in the Send window before you press the Deliver key.

Reminders and caveats:

- When writing the product of two variables in *Mathematica*, the symbols must be separated by either a space or an asterisk. For example, `xy` will be interpreted by *Mathematica* as the sine of the single variable named "xy"—not the product x times y. The latter has to be written as either `x y` or `x*y`.
- Save your work (with Command-s) every few minutes.
- Don't use the miniaturize button (the little square in the upper left corner of the window). Its purpose is to reduce the window to an icon which is placed at the bottom of the screen. It does *not* close or save the file. Use the Hide command (Command-h) to clear the screen. Then you can restore the windows by double-clicking on the icon in the dock.

Here are some ideas to consider when writing your report:

Simple Sequences

1. How can you use *Mathematica* to check convergence?
2. How can you use *Mathematica* to check the rate of convergence?

Recursive Sequences

1. What is a recursive sequence?
2. Must a divergent sequence be unbounded?

Newton's Method

1. How is Newton's method performed with a recursive sequence?
2. How rapid is the convergence?
3. How does Mathematica carry out a command like **`Solve[f[x]==0,x]`**?

***Mathematica* Commands**

Here are some of the *Mathematica* commands that you might use in this lab:

`Clear[a]`	Deletes the current definition of **`a`**.
`a[n_] := (1+1/n)^n`	Defines the sequence **`a[n]`**.
`Table[ a[n], {n,1,8} ]`	Produces the subsequence: `{a[1],a[2],a[3],a[4],a[5],a[6],a[7],a[8]}`.
`points = Table[ a[n], {n,1,8} ]`	Defines **`points`** to be that subsequence.
`N[points]`	Produces numerical (decimal) values for that subsequence.
`ListPlot[ points ]`	Plots the subsequence.
`PlotStyle -> PointSize[0.02]`	Makes each point 2% as wide as the entire graph.
`Table[ {2^k,a[2^k]}, {k,1,8} ]`	Produces the subsequence: `{a[2],a[4],a[8],a[16],a[32],a[64],a[128],a[256]}`.
`// N`	Returns the current answer in numerical (decimal) form.
`N[%]`	Produces the numerical value of the previous answer.
`TableForm[points]`	Produces the table **`points`** in tabular form.
`Limit[ a[n], n -> Infinity ]`	Produces the limit of the sequence.
`f[n_] := f[n-1] + f[n-2]`	Defines **`f`** recursively.
`f[3]`	Produces the current value of **`f[3]`**.
`?f`	Produces the current definition of **`f`**.
`Table[ a[n], {n,9,12} ] // N`	Produces the numerical values of the subsequence: `{a[9],a[10],a[11],a[12]}`.
`a[n_] := a[n-1] - f[a[n-1]]/f'[a[n-1]];`	Defines the sequence a[n] to be the Newton iterates for the solution to the equation $f(x) = 0$.