The use of Continuity 2-1 can be most easily described by using it to investigate the following problem. Is cos(x + y) continuous at
(1, 2)? This computer program will not prove that cos(x + y) is continuous at (1, 2); however, the program will provide us with graphical evidence that it is. Moreover, this program should give us additional insights into the meaning of continuity and a better understanding of epsilon-delta proofs of continuity.

Since cos(1 + 2) is approximately -0.989, center an epsilon neighborhood around that value by entering -0.989 into the center text field (above the button labeled "Set Center" in the box labeled "Neighborhood Controls". Enter an value of 0.5 into the epsilon text field. Since this is a relatively small epsilon, magnify the neighborhood by clicking the button labeled "Zoom In" in the "Range Controls" box. To translate the interval so that it is centered in the view, you can enter -0.989 in the text field next to the button labeled "Set Center" in the "Range Controls" box.

Now, center a delta neighborhood around the point (1, 2) by entering 1 in the text field labeled "x" and 2 in the text field labeled "y" in the "Neighborhood Controls" box. By moving the mouse to different locations in the delta neighborhood, you will notice that not all points in the delta neighborhood are mapped to points in the epsilon neighborhood. It may be easier to see this if you magnify the delta neighborhood by clicking the button labeled "Zoom In" in the "Domain Controls" box. The fact that all points in the delta neighborhood

were not mapped into the epsilon neighborhood does not imply that cos(x+y) is discontinuous at (1, 2). It simply means the value of 1 for delta is not small enough for an epsilon of 0.5. Choose a smaller value for delta by entering a value directly into the delta text field or by moving the delta slider. Try to find a delta such that all points in the delta neighborhood are clearly mapped into the epsilon neighborhood. In the process of searching for a sufficiently small delta, you may wish translate the delta neighborhood so that it is centered in the view. To do this, enter 1 into the text field labeled "x" and enter 2 into the text field labeled "y" in the "Domain Controls" box.

Once you have found a delta neighborhood that maps completely into the 0.5 epsilon neighborhood, repeat the same procedure using a smaller epsilon.

Using the procedure described above, you should be able to convince yourself that no matter how small the epsilon neighborhood around cos(1+2) is, you will always be able to find a corresponding delta neighborhood around (1, 2) with the property that every point in the delta neighborhood maps into the epsilon neighborhood. This evidence indicates (but does not *prove*) that cos(x + y) is continuous at (1, 2).