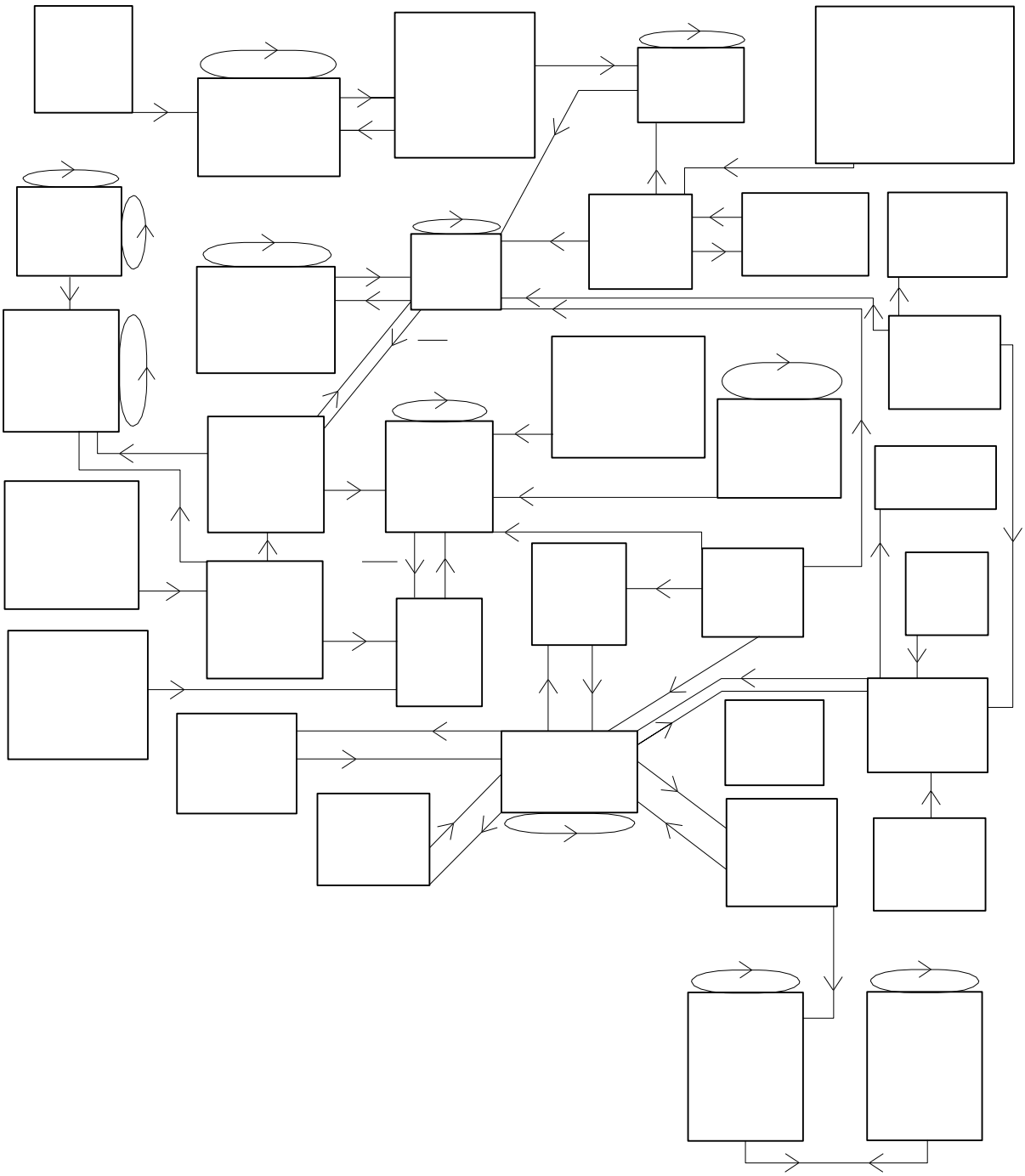
Appendix G
Summary of Random Variables

Relationships Between Families of Random Variables

Families of Random Variables

Probability Program for Small Computers



Families of Random Variables
J. C. Hudson

Name, parameters, f(x), support, E(X) and Var(X) are listed. F(x) is listed below f(x) if there is a useful form. See ks.doc for references.

, sgn(x) = U(x)-U(-x),

is the set of integers, $^{+}$ is the set of positive integers, $\mathbb{R}$ is the set of reals.
Γ(x) is the gamma function.

Name	Parameters	Density/CDF	Support	E(X),Var(X)
Arcsin	a>0		-a<x<a	0, $a^2/2$
Bernoulli	0<p<1		{0,1}	p, p(1-p)
Beta			0<x<1	,
Binomial				np, np(1-p)
Cauchy				,
				,
Chi-Square			x>0	ν, 2ν
		x>0	,	

Erlang			x>0	$n\theta$, $n\theta^2$
Exponential	$\mu>0$		x>0	μ, μ^2
F	x>0			
Noncentral F	x>0	E(X) = , $\nu_2 > 2$ Var(X) = , $\nu_2 > 4$		
Gamma			x>0	$a\theta$, $a\theta^2$
Geometric	$0<p<1$			
			x>0	μ,

Name	Parameters	Density/CDF	Support	E(X),Var(X)
Laplace				
Logistic				μ, σ^2
Lognormal		$x>0$		
				μ, σ^2
				0, 1
Pareto	$b>0$		$x>1$	
Poisson	$\mu>0$			μ, μ
Rayleigh	$b>0$		$x>0$	
Student's t				
		$x\in\mathbb{R}$		

Name	Parameters	Density/CDF	Support	E(X),Var(X)
Triangular		a-b<x<a+b		
	a<b	a<x<b		
			0<x<1	
Weibull			x>δ	

Probability Program for Small Computers

This program is written in Sharp EL-5500III BASIC. Extra spaces are used for clarity. sqr is used instead of Sharp's √ and pi is used instead of π. Expect to make substantial changes in i/o statements if you use the program on another machine.

lines	compute or approximate
100 - 150	binomial pmf and cdf
200 - 290	hypergeometric pmf and cdf
300 - 350	Poisson pmf and cdf
400 - 470	Gaussian right tail probability
500 - 550	Student's t right tail probability
600 - 630	chi - square right tail probability
700 - 750	f distribution right tail probability

```
100 input "x=";x,"n=";n,"p=";p
110 t = (1-p)^n : s = t
120 if x = 0 then 140
130 for i = 1 to x : t = t*(n-i+1)*p/i/(1-p) : s=s+t : next i
140 using : print x;" ";n;" ";p
150 print using "###.####";t+0.00005;s+0.00005 : goto 100

200 input "x=";x,"np=";np,"ns=";ns,"k=";k
210 mi = 0 : if k+ns-np > 0 then let mi=k+ns-np
220 ma = k : if ns < k then let ma = ns
230 if x < mi then let t = 0 : let s = 0 : goto 280
240 if x > ma then let t = 0 : let s = 1 : goto 280
250 j=np-ns : d=k : nt=ns : if mi > 0 then let j=ns : let d=np-k : let nt=np-ns
260 t=1 : for i=1 to nt : j=j+1 : t = t*(j-d)/j : next i : s=t : if x=mi then 280
270 for i = mi+1 to x : t = t*(k-i+1)*(ns-i+1)/i/(np-k-ns+i) : s = s+t : next i
280 using : print x;" ";np;" ";ns;" ";k
290 print using "###.####";t+0.00005;s+0.00005 : goto 200

300 input "x=";x,"m=";m
310 t = exp(-m) : s = t
320 if x = 0 then 340
330 for i = 1 to x : t = t*m/i : s = s+t : next i
340 using : print x;" ";m
350 print using "###.####";t + 0.00005;s + 0.00005 : goto 300

400 input "z=";z : gosub 420
410 using : print z;" qz=; using "##.####";q + 0.00005 : goto 400

420 y = exp(-z*z/2) / sqr(2*pi)
430 w = 1 / (1 + 0.2316419 * abs(z))
440 q=0.31938153+w*(-0.356563782+w*(1.781477937+w*(-1.821255978+w*1.33027449)))
450 q = q*y*w : if z < 0 then let q = 1-q
460 return

500 input "df=";d,"t=";t : if d < 2 then 500
```

```
510 g = sqr(ln(1+t*t/d))
520 s = 0.184 * (8*d+3)/d/g
530 z = sqr(d) * g * (1 - 2*sqr(1 - exp(-s*s)) / (8*d+3))
540 gosub 420 : if t < 0 then let q = 1-q
550 using : print d;t;" qt=";using "###.####";q + 0.00005 : goto 500

600 input "df=";d,"x=";x:if d < 3 then 600
610 z = sqr(x + (d - 1) * (ln((d - 1) / x) - 1))
620 z = z * (x - d + 2/3 - 0.08 / d) / abs(x - d + 1) : gosub 420
630 using : print d;x;"qx=";using "###.####";q + 0.00005 : goto 600

700 input "df1=";d,"df2=";e,"f=";f
710 p = e / (d * f + e) : c = 0.08 * ((1-p) / e - p/d + (.5-p)/(d+e))
720 d = d-1 : e = e-1 : z = e * ln(e/((d+e)*p)) + d * ln(d/((d+e)*(1-p)))
730 z = sqr(3 * (d + e) * z / (3 * (d + e) + 1))
740 z = z * (e+1/3-(d+e+2/3)*p+c) / abs(e - (d+e)*p) : gosub 420
750 using : print d+1;e+1;f;"qf=";using "###.####";q + 0.00005 : goto 700
```

Thisted [1988] gives the Peizer and Pratt [1968] approximation for the chi-square and F distributions and the Wallace [1959] approximation for the t distribution. The Gaussian right tail approximation is given by McConnell [1990].

Errors

The binomial, Poisson and hypergeometric computations are exact to 4 decimal places for all values checked. The program does very well approximating Normal upper tail probabilities. For Student's t, upper tail probabilitiess are acceptable for 4 or more df, and are acceptable for 8 or more df for chi-square. for the F distribution, results are ok if both df are 8 or more, results are not acceptable if both are 4 or less.

The program crashes if t = 0, x^2 = df - 1 or f = 1 with both degrees of freedom equal. The program will not accept df = 1 for Student's t or df < 3 for x^2.