Analysing UML Active Classes and Associated State Machines - A Lightweight Formal Approach

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Abstract. We consider the problem of precisely defining UML active classes with an associated state chart. We are convinced that the first step to make UML precise is to find an underlying formal model for the systems modelled by UML. We argue that labelled transition systems are a sensible choice; indeed they have worked quite successfully for languages as Ada and Java. Moreover, we think that this modelization will help to understand the UML constructs and to improve their use in practice. Here we present the labelled transition system associated with an active class using the algebraic specification language CASL.
The task of making precise this fragment of UML raises many questions about both the "precise" meaning of some constructs and the soundness of some allowed combination of constructs.

1 Introduction

The Unified Modeling Language (UML) [11] is an industry standard language for specifying software systems. This language is unique and important for several reasons:

- UML is an amalgamation of several, in the past competing, notations for object-oriented modelling. For a scientific approach, it is an ideal vehicle to discuss fundamental issues in the context of a language used in industry.
- Compared to other pragmatic modelling notations in Software Engineering, UML is very precisely defined and contains large portions which are similar to a formal specification language, as the OCL language used for the constraints.

It is an important issue in Software Engineering to finally close the gap between pragmatic and formal notations and to apply methods and results from formal specification to the more formal parts of UML. This paper presents an approach contributing to this goal and has been carried out within the European "Common Framework Initiative" (CoFI) for the algebraic specification of software and systems, partially supported by the EU ESPRIT program. Within the CoFI initiative [7], which brings together research institutions from all over Europe, a specification language called "Common Algebraic Specification Language" (CASL) was developed which intends to set a standard unifying the various approaches to algebraic specification and specification of abstract data types. It is a goal of the CoFI group to closely integrate its work into the world of practical engineering. As far as specification languages are concerned, this means an integration with UML, which may form the basis for extensions or experimental alternatives to the use of OCL. That would allow for instance: – to specify user-defined data types that just have values but do not behave like objects; – to use algebraic axioms as a constraints. These new constraints may cover also behavioural aspects, because there exist extensions of algebraic languages that are able to cover with them [4], whereas OCL does not seem to have any support for concurrency. The long-term perspective of this work is to build a bridge between UML specifications and the powerful animation and verification tools that are available for algebraic specifications.

To this end we need to precisely understanding (formally defining) the most relevant UML features. In this paper we present the results of such formalization work, which was guided by the following ideas.

Real UML Our concern is the real UML (i.e., all, or better almost all, its features without simplifications and or idealizations). We are not going to consider a small OO language with a UML syntax, but just what it is presented in the official OMG documentation [12] (shortly UML 1.3 from now on).

Based on an underlying model We are convinced that the first step to make UML precise is to find an underlying formal model for the systems modelled by UML, in the following called *UML-systems*.

Our conviction also comes from a similar experience the two first authors had, many years ago, when tackling the problem of the full formal definition of Ada, within the official EU project (see [1]). There too an underlying model was absolutely needed to clarify the many ambiguities and unanswered questions in the ANSI Manual.

We argue that labelled transition systems could be a sensible model choice; indeed, they were used quite successfully to model concurrent languages as Ada [1], but also a large part of Java [3].

Lightweight formalization By “lightweight” we mean made by using the most simple formal tools and techniques: precisely labelled transition systems algebraically specified using a small subset of the specification language CASL (conditional specification with initial semantics).

Integrated with the formalization of the other fragments of UML In contrast to many other papers on UML semantics, we more or less ignore the issue of class diagram semantics here and concentrate on the state machines[1]. However, the ultimate goal of this work is to have an approach by which it is easily possible to integrate semantically the most relevant diagram types. For this reason, we are using an algebraic approach to the semantics of state machines, since it is well known that class diagrams can be mapped relatively easily onto algebraic specifications. The algebraic semantics described here enables an algebraic access to the semantics also of active, not only of passive, classes. Usually, an active class is statically described in the class diagram and dynamically described in an associated statechart diagram.

The formalization of active classes and state machines has lead to perform a thorough analysis of them uncovering many problematic points. Indeed, the official informal semantics of UML, reported in UML 1.3, is in some points either incomplete, or ambiguous, or inconsistent or dangerous (i.e., the semantics is clearly formulated but the allowed usages seem problematic from a methodological point of view). To stress this aspect and to help the reader we have used the mark pattern **PROBLEM** to highlight them.

In Sect. 2 we define the subset of the active classes with state machines that we consider. Then in Sect. 3 and 4 we introduce the used formal techniques (labelled transition

[1] Following UML terminology *state machine* is the abstract name of the construct, whereas *state chart* is the name of the corresponding diagram; here we always use the former.

systems and algebraic specifications), and present step after step how we have built the lts modelling the objects of an active class with an associated state machine. Due to lack of room part of the definition is reported in Appendix A, while the complete formal model (rather short and simple) is in [9].

2 Introducing UML: Active Classes and State Machines

The UML defines a visual language consisting of several diagram types. These diagrams are strongly interrelated by a common abstract syntax and are also related in their semantics. The semantics of the diagrams is currently defined by informal text only.

The most important diagram types for the direct description of object-oriented software systems are the following:

- Class diagrams, defining the static structure of the software system, i.e., essentially the used classes, their attributes and operations, possible associations (relationships) between them, and the inheritance structure among them. Classes can be passive, in which case the objects are just data containers. For this paper, we are interested in active classes, where each object has its own thread(s) of control.
- Statechart diagrams (state machines), defining the dynamic behaviour of an individual object of a class over its lifetime. This diagram type is very similar to traditional Statecharts. However, UML has modified syntax and semantics according to its overall concepts.
- Interaction diagrams, illustrating the interaction among several objects when carrying out jointly some use case. Interaction diagrams can be drawn either as sequence diagrams or as collaboration diagrams, with almost identical semantics but different graphical representation.

A UML *state machine* is very similar to a classical finite state machine. It depicts states, drawn as rounded boxes carrying a name, and transitions between the states. A transition is decorated by the name of an event, possibly followed by a specification of some action (after a slash symbol). The starting point for the state machine is indicated by a solid black circle, an end point by a solid circle with a surrounding line.

The complexity of UML state machines compared to traditional finite state machines comes from several origins:

- The states are interpreted in the context of an object state, so it is possible to make reference, e.g., in action expressions, to object attributes.
- There are constructs for structuring state machines in hierarchies and even concurrently executed regions.
- There are many specialized constructs like entry actions, which are fired whenever a state is entered, or state history indicators.

In order to simplify the semantical consideration in this paper, we assume the following restrictions of different kinds. Please note that none of these assumptions restricts the basic applicability of our semantics to full UML state machines!

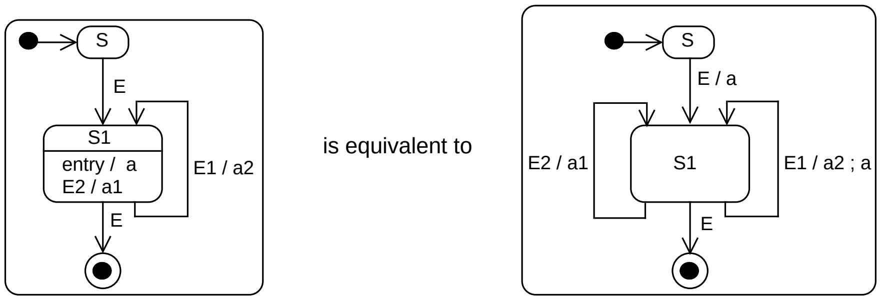
We do not consider the following UML state machine constructs, because they can be replaced by equivalent combinations of other constructs.

Submachines We can eliminate submachines by replacing each stub state with the corresponding submachine as UML 1.3 p. 2.137 states "It is a shorthand that implies a macro-like expansion by another state machine and is semantically equivalent to a composite state".

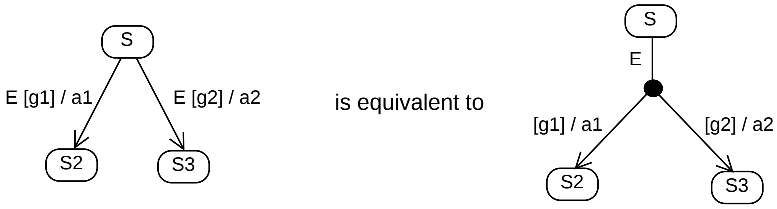
Entry and exit actions Entry and exit actions associated with a state are a kind of shortcut with methodological implication, see, e.g., [11] p. 266, but semantically they are not relevant; indeed we can eliminate them by adding such actions to all transitions entering/leaving such state.

Internal transitions An internal transition differs from a transition whose source and target state coincide only for what concerns entry/exit actions. Because we have dropped entry/exit actions, we can drop also internal transitions.

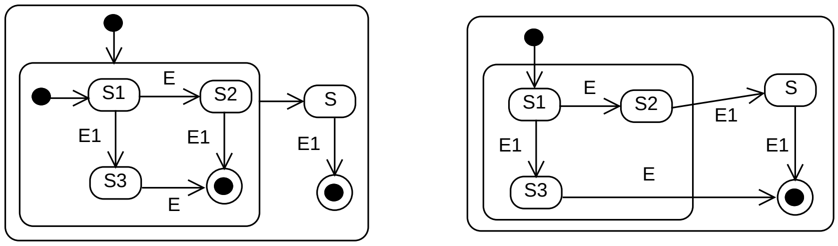
The following picture shows, on an example, how to eliminate entry actions and internal transitions.



Different transitions leaving the same state having the same event trigger We can replace these transitions with compound transitions using junction states. The latter presentation seems better from a methodological point of view, and makes the semantics clearer (what to do when dispatching an event is explained in UML 1.3 by considering the transitions with the same trigger leaving a state all together). The picture below shows on an example of this simplification.



Multiple initial/final states We assume that there is always one unique initial state at the top level (used to determine the initial situation of the class objects) and one unique final state at the top level (when it is active the object will perform a self destruction). The remaining initial/final states can be replaced by using complex transitions. The picture below shows an example of equivalent state machines, differing only in the number of initial/final states.



Compound transitions We assume there are no compound transitions except the complex one and compounds of length two (e.g., as those needed in the two cases above). Indeed compound transitions are used only for presentation reasons and can be replaced by sets of simpler transitions.

Terminate action in the state machine Indeed it can be equivalently replaced by a destroy action addressed to the self.

We do not consider the following features just to save space; indeed we think that they could be modelled without much trouble.

- Operations with return type and return actions
- Synch and history states
- Generalization on the signals (type hierarchy on signals)
- Activities in states (do-activities)

3 Modelling Active Objects with Labelled Transition Systems

3.1 Labelled Transition Systems

A *labeled transition system* (shortly *lts*) is a triple $(ST, LAB, \rightarrow)$, where ST and LAB are two sets, and $\rightarrow \subseteq ST \times LAB \times ST$ is the *transition relation*. A triple $(s, l, s') \in \rightarrow$ is said to be a *transition* and is usually written $s \xrightarrow{l} s'$.

Given an lts we can associate with each $s_0 \in ST$ the tree (*transition tree*) whose root is s_0, where the order of the branches is not considered, two identically decorated subtrees with the same root are considered as a unique subtree, and if it has a node n decorated with s and $s \xrightarrow{l} s'$, then it has a node n' decorated with s' and an arc decorated with l from n to n'.

We model a process P with a transition tree determined by an lts $(ST, LAB, \rightarrow)$ and an initial state $s_0 \in ST$; the nodes in the tree represent the intermediate (interesting) situations of the life of P, and the arcs of the tree the possibilities of P of passing from one situation to another. It is important to note here that an arc (a transition) $s \xrightarrow{l} s'$ has the following meaning: P in the situation s has the *capability* of passing into the situation s' by performing a transition, where the label l represents the interaction with the environment during such a move; thus l contains information on the conditions on the environment for the capability to become effective, and on the transformation of such environment induced by the execution of the transition.

Notice that here by process we do not mean "sequential process", indeed also concurrent processes, which are processes having cooperating components that are in turn other processes (concurrent or not), can be modelled through particular lts, named *structured lts*. A structured lts is obtained by composing other lts describing such components, say *clts*; its states are built by the states of *clts*, and its transitions are determined by composing those of *clts*.

An lts may be formally specified by using the algebraic specification language CASL (see [8]) with a specification of the following form:

spec LTS =
  STATE **and** LABEL **then**
free { **pred** $\_ \xrightarrow{-} \_ : State \times Label \times State$
  axioms ......
} **end**

whose axioms have the form $\alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \alpha_{n+1}$, where for $i = 1, \ldots, n+1$, α_i is a positive atom (i.e., either a predicate application or an equation). The CASL construct **free** requires that the specification has an initial semantics.

some words on CASL and initial semantics????????

Assume to have a given active class ACL with a given associated state machine SM belonging to the subset of UML introduced in Sect. 2, and assume that ACL and SM are statically correct, as stated in UML 1.3.

We have built the lts L modelling the objects of the class ACL following the steps below, which will be reported in the following sections.

1. check whether L is simple or structured
2. determine the grain of the L-transitions
3. if L is structured, determine its components and the lts modelling them
4. determine the labels of L
5. determine the states of L
6. determine the transitions of L by means of conditional rules (in this case, because we are using CASL, by conditional axioms).

The constraints attached either to ACL or to SM are treated apart in Sect. 4.6, because they do not define a part of L, but just properties on it.

To avoid confusion between the states and the transitions of the state machine SM with those of the lts L, we will write from now on *L-states* and *L-transitions* when referring to those of L.

3.2 Is L simple or structured?

The first question is to decide whether L is simple or structured; this means in terms of UML semantics to answer the following question:

PROBLEM Does an active object correspond to a single thread of control (running concurrently with the others), or to several ones?

Unfortunately, UML 1.3 is rather ambiguous/inconsistent for what concerns this point. Indeed, somewhere it seems to suggest that there is exactly one thread, as in UML 1.3 p. 2-23, p. 2-149, p. 2-150 (**):

> *It is possible to define state machine semantics by allowing the run-to-completion steps to be applied concurrently to the orthogonal regions of a composite state, rather than to the whole state machine. This would allow the event serialization constraint to be relaxed. However, such semantics are quite subtle and difficult to implement. Therefore, the dynamic semantics defined in this document are based on the premise that a single run-to-completion step applies to the entire state machine and includes the concurrent steps taken by concurrent regions in the active state configuration.*

Otherwise, UML 1.3 seems to assume that there are many threads, as in p. 2.133, p. 2-144, p. 3-141:

> *A concurrent transition may have multiple source states and target states. It represents a synchronization and/or a splitting of control into concurrent threads without concurrent substates.*

and in p. 2-150:

> *An event instance can arrive at a state machine that is blocked in the middle of a run-to-completion step from some other object within the same thread, in a circular fashion. This event instance can be treated by orthogonal components of the state machine that are not frozen along transitions at that time.*

In this paper we stick to the interpretation suggested in (**), so each active object corresponds to one thread. However a perhaps better way to solve this point is to introduce two stereotypes: *one-thread* and *many-threads*, to allow the user to decide the amount of parallelism inside an active object.

3.3 Determining the granularity of the L-transitions

Using lts means that we model the behaviour of processes by splitting it into "atomic" pieces (the L-transitions); so, to define L, we must determine the granularity of this splitting, i.e., of the L-transitions.

PROBLEM By looking at UML 1.3 we see that there are two possibilities corresponding to different semantics.

1. each L-transition corresponds to performing a group of transitions of the state machine triggered by the occurrence of the same event starting from a set of active concurrent states. Because L-transitions are mutually exclusive, this choice corresponds to a semantics where L-transitions are implicitly understood as critical regions.
2. each L-transition corresponds to performing a part of a state machine transition; the atomicity of transitions (run-to-completion condition) required by UML 1.3 is guaranteed by the fact that, while executing the various parts of the transition triggered by an event, an object cannot dispatch another event. In this case, the parts of the state machine transitions performed by the same or different objects may be executed concurrently.

The example in Fig. 1, where we assume here that there is another object O2 with an operation OP resulting in a printable value, shows an instance of this problem.
Choice 1 corresponds to say that in any case pairs of identical values will be printed, whereas choice 2 allows for pairs of possibly different values (becuase the value returned by OP can be different in the two occasions due to the activity of other objects).

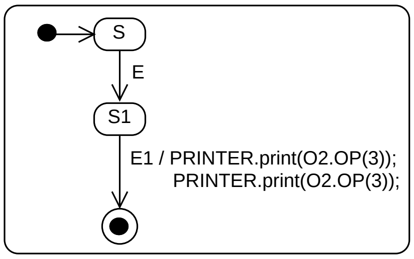


Fig. 1. A simple State Machine

We think that choice 2 is better; however we could similarly model also 1.

3.4 Determining the L-Labels

The L-labels (labels of the lts L) describe the possible interactions/interchanges between the objects of the active class ACL and their external environment (the other objects comprised in the model). By looking at UML 1.3 we find that the basic ways the objects of an active class interact with the other objects are the following, distinguished by us in “input” and “output”:

input:
- to receive a signal from another object
- to receive an operation call from another object
- to read an attribute of another object (+)
- to have an attribute updated by another object (+)
- to be destroyed by another object (+)
- to receive from some clock the actual time (see [11] p. 475)

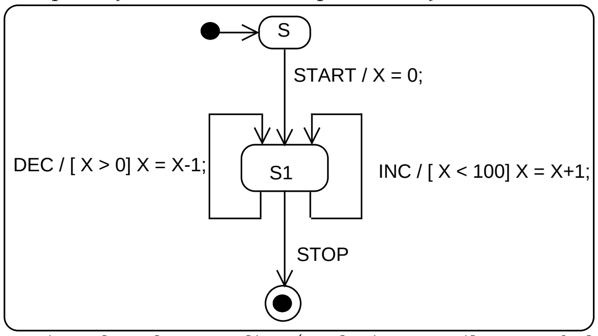
output:
- to send a signal to another object
- to call an operation of another object
- to update an attribute of another object (+)
- to have an attribute read by another object (+)
- to create/destroy another object

UML 1.3 does not consider explicitly the interactions marked by (+), and does not say anything about when they can be performed (e.g., they are not considered by the state machines).

PROBLEM When may an object be destroyed?

A way to settle this point is to make “to be destroyed” an event, which may dispatched when the machine is not in a run-to-completion-step and may appear on the transitions the state machine.

PROBLEM The interactions corresponding to read/update attributes of other objects raise a lot of questions about the UML semantics.
May an object have its attributes updated by some other object?
If the answer is yes, then when such updates may take place? For example, is it allowed during a run-to-completion-step?
Are there any “mutual exclusion” properties on such updates? or may it happen that an object O1 updates an attribute A of O while O is updating it in a different way, or that O1 and O2 updates simultaneously A in two different ways?
May an active object have a behaviour not described by the associated state machine (see UML 1.3 p. 2-136), because another object updates its attributes?
In the following example, another object may perform O.X = O.X + 1000, changing completely the behaviour specified by this state machine.



Notice also that reading/updating attributes of the other active objects is an implicit communication mechanism, thus yielding a dependency between their behaviours that is not explicitly stated.

A way to overcome this point may be to fully encapsulate the attributes with the operations, i.e., an attribute of a class may be read and updated only by the class operations. As consequence, the expressions and the actions appearing in the state machine may use only the object attributes, and not those of other objects.

Then an L-label will be a triple consisting of a set of events received from outside, the received time, and a set of events sent outside; because, due to the above choice, the are no more the interactions marked by (+).

3.5 Determining the *L*-States

The L-states (states of the lts L) describe the intermediate relevant situations in the life of the objects of class ACL.

By looking at UML 1.3 we found that to decide what an object has to do in a given situation we surely need to know:

- the object identity;
- the set of the states (of the state machine SM) that are active in such situation;
- whether the object is in a run-to-completion step, and in such case which are the states that will become active at the end of such step, each one accompanied by the actions to be performed to reach such states;
- the values of object attributes;
- the status of the event queue.

Thus the L-states must contain such informations; successively, when defining the transitions we discovered that, to handle change and time events, we need also to know

- the sequence of the L-states reached by the object during its past life.

The L-states are thus specified by this CASL specification

spec L-STATE =
  IDENT **and** CONFIGURATION **and** ATTRIBUTES **and** EVENT_QUEUE **and** TIME **then**
  free types
    $State ::= Ident : \langle Configuration, Attributes, History, Event\_Queue\rangle \mid$
      $Ident : terminated$
    $History ::= \Lambda \mid \langle State, Time\rangle \,\&\, History$
    ......

where *Ident* : *terminated* are special elements representing terminated objects.

A configuration contains the set of the states that are active in a situation and of those states that will become active at the end of current run-to-completion step (if any), the latter are accompanied by the actions to be performed to reach such states.

spec EVENT_QUEUE =
  SET[EVENT] **then**
  sort *Event_Queue*
  preds $no\_dispatchable\_event : Event\_Queue$
  %% checks whether there is no a dispatchable event
    $\_\_ \in \_\_ : Event \times Event\_Queue$
  %% checks whether a given event in the queue may be selected for dispatching
  ops $put : Set[Event] \times Event\_Queue \to Event\_Queue$
  %% adds some events to the queue
    $remove : Event \times Event\_Queue \to Event\_Queue$
  %% removes an event from the queue
    ......

PROBLEM UML 1.3 explicitly calls the above structure a queue, but it also clearly states that no order must be put on the queued events (UML 1.3 p. 2-144) and so it is really a multiset. This choice of terminology is problematic, because it can induce a user to assume that some order on the received events will be preserved.
The fact that the event queue is just a bag causes other problems: an event may remain for ever in the queue; time and change events may be dispatched disregarding the order in which happened (e.g., "`after` 10" dispatched before "`after` 5"); a change event is dispatched when its condition is false again; two signal or call events generated by the same state machine in some order are dispatched in the reverse order.

To fix this point, we can either change the name of the event queue in the UML documentation in something recalling its true semantics, or define a policy for deciding which event to dispatch first.

In an UML model we cannot assume anything on the order with which some events are received by an object (as the signal and operation calls); we conjecture that this was the motivation for avoiding to order the events in the queue. However, we think that it is better to have a mechanism ensuring that when two events are received in some order they will be dispatched in the same order, also if in many cases we do not know such order.

The specifications of the other components of the L-states are reported in [9].

4 Determining the L-Transitions

An L-transition, i.e., a transition of the lts L, corresponds to

1. either to dispatch an event,
2. or to execute an action,
3. or to receive some events; such events are either received from outside (signals and operation calls) or generated locally (self sent signals and operation calls, change and time events),
4. or to be destroyed by dispatching a special event.

Moreover, (3) may be also performed simultaneously to (1) and (2), because we cannot delay the reception of events.

It is important to notice that the L-transitions of and the transitions of the state machine SM are different in nature and are not in a bijective correspondence. To clarify such relationship we partly report in Fig. 2 the labelled transition tree associated with the simple state machine of Fig. 1, where it is possible to see that one state machine transitions correspond to many L-transitions.

The L-transitions are formally defined by the axioms of this algebraic specification

spec L-Spec =
  L-Label **and** L-State **then**
free { **pred** $\_ \xrightarrow{\_} \_ : State \times Label \times State$
  axioms $\ldots\ldots cond \Rightarrow s \xrightarrow{l} s' \quad \ldots\ldots$

In the following subsections we give the axioms corresponding to the four cases above To master complexity and to improve readability we use several auxiliary functions in such axioms, whose name is written in sans serif font. Some relevant problems come to light when defining some of such functions, and thus we consider them explicitly in Sect. 4.5. The others are reported in the Appendix A.

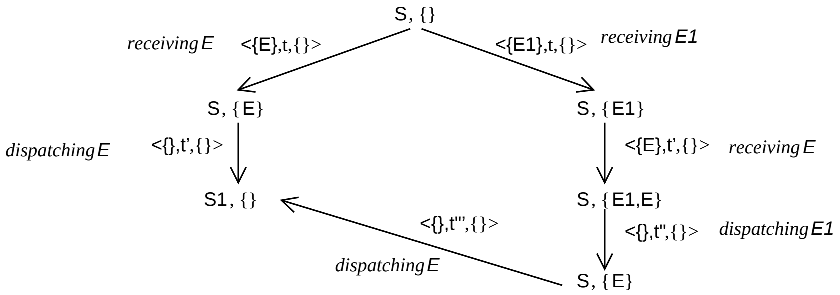


Fig. 2. A fragment of a labelled transition tree

4.1 Dispatching an Event

If the object is not in a run-to-completion step (checked by predicate *not_frozen*), an event ev is in the event queue ready to be dispatched ($ev \in e\_queue$), then there is an L-transition, with the label resulting from the events received from outside *in_evs*, and the time t, where the history has been extended with the current state, and all received events are put in the event queue, as described by Receive_Events.

Recall that $\mathsf{Dispatch}(ev, conf, e\_queue) = conf', e\_queue'$ means that dispatching event ev in the configuration *conf* changes it to $conf'$ and changes *e_queue* to e_queue'.

$$not\_frozen(conf) \ \wedge \ ev \in e\_queue \ \wedge \ \mathsf{Dispatch}(ev, conf, e\_queue) = conf', e\_queue' \ \Rightarrow$$

$$id : \langle conf, attrs, history, e\_queue\rangle \xrightarrow{\langle in\_evs, t, \emptyset\rangle} id : \langle attrs, conf', history', e\_queue''\rangle$$

where

$$e\_queue'' = \mathsf{Receive\_Events}(e\_queue', in\_evs, attrs, history, t)$$

$$history' = \langle id : \langle conf, attrs, history, e\_queue\rangle, t\rangle \ \& \ history$$

4.2 Executing an action

If the object is in a run-to-completion step (checked by predicate *frozen*), and performs an action, then there is an L-transition, with the label resulting from the events received from outside *in_evs*, the time t, and the set of events generated in the action to be propagated outside *out_evs*, where the attributes are updated due to executed action, the history has been extended with the current state, and all received events are put in the event queue, as described by Receive_Events.

$\mathsf{Exec}(id, attrs, conf) = conf', attrs', out\_evs, loc\_evs$ means that the object id with configuration *conf* executes an action changing its configuration to $conf'$, updating its attributes to $attrs'$ and producing the set of output events *out_evs* and the set of local events *loc_evs*.

$$frozen(conf) \ \wedge \ \mathsf{Exec}(id, attrs, conf) = conf', attrs', out\_evs, loc\_evs \ \Rightarrow$$

$$id : \langle conf, attrs, history, e\_queue\rangle \xrightarrow{\langle in\_evs, t, out\_evs\rangle} id : \langle attrs', conf', history', e\_queue'\rangle$$

where

$$history' = \langle id : \langle conf, attrs, history, e\_queue\rangle, t\rangle \ \& \ history$$

$$e\_queue' = \mathsf{Receive\_Events}(e\_queue, in\_evs \cup loc\_evs, attrs, history, t)$$

4.3 Receiving some Events

If the object is not in a run-to-completion step (checked by predicate *not_frozen*), the event queue is empty, then there is an L-transition, with the label resulting from the set of events received from outside in_evs and the time t, where the history has been extended with the current state, and all received events are put in the event queue, as described by Receive_Events.

$$not\_frozen(conf) \ \wedge \ no\_dispatchable\_event(e\_queue) \ \Rightarrow$$
$$id : \langle conf, attrs, history, e\_queue \rangle \xrightarrow{\langle in\_evs, t, \emptyset \rangle} id : \langle attrs, conf, history', e\_queue' \rangle$$

where

$$e\_queue' = \mathsf{Receive\_Events}(e\_queue, in\_evs, attrs, history, t)$$
$$history' = \langle id : \langle conf, attrs, history, e\_queue \rangle, t \rangle \ \& \ history$$

4.4 Being destroyed

We consider a destruction request as an event and assume that an object cannot be destroyed while is in a run-to-completion step.

$$not\_frozen(conf) \ \wedge \ destroy \in e\_queue \ \Rightarrow$$
$$id : \langle conf, attrs, history, e\_queue \rangle \xrightarrow{\langle in\_evs, t, \emptyset \rangle} id : terminated$$

4.5 Auxiliary functions

Receive_Events : $\boldsymbol{Event\_Queue \times Set[Event] \times Attributes \times History \times Time \rightarrow Event\_Queue}$

Receive_Events$(e_queue, evs, attrs, history, t) = e_queue'$ means that e_queue' is e_queue updated by putting in it all received events: the signal and operation call events are given by a function parameter (evs), the time events are detected using t and $history$ (by the function TimeOccur), and the change events are detected by using $attrs$ and $history$ (by the function ChangeOccur).

PROBLEM May operation calls to other objects appear within the expressions of change and time events? If the answer is yes, then we can have more hidden constraints on the mutual behaviour of objects (e.g., a synchronous operation call in the expression of a change event may block an object). We assume no, and this is the reason for the above simple functionality of Receive_Events.

Receive_Events$(e_queue, evs, attrs, history, t) =$
$put(\mathsf{TimeOccur}(t, history) \cup \mathsf{ChangeOccur}(attrs, history) \cup evs, e\_queue)$

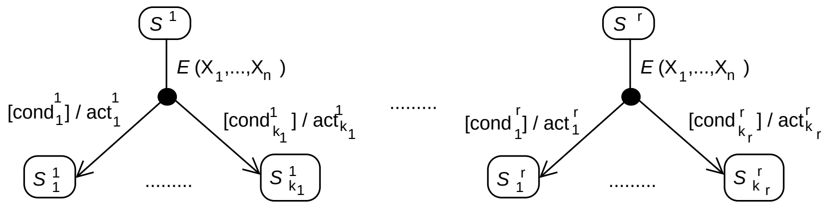
We report the definitions of TimeOccur and of ChangeOccur in Appendix A

Dispatch : $\boldsymbol{Event \times Configuration \times Event\_Queue \rightarrow Configuration \times Event\_Queue}$

Dispatch$(ev, conf, e_queue) = conf', e_queue'$ means that dispatching event ev in the configuration $conf$ changes it to $conf'$ and changes e_queue to e_queue'.

It is defined by cases, and here we report the most relevant ones, the others are reported in Appendix A.

Some transitions triggered by an event Assume that in the state machine SM there are the following branched transitions triggered by E starting from the states belonging to $Sset$



Note that inheritance of transitions between nested states and overriding may be handled here by deciding which transitions to consider.

If the active states are $Sset$, and $cond^i_{q_i}$ holds for $i = 1, \ldots, h$ ($h \leq r$), then the object will start a run-to-completion step going to perform $act^i_{q_i}$ and to reach state $S^i_{q_i}$, for $i = 1, \ldots, h$ (the actual parameters of the event are substituted for its formal parameters within the actions to be performed).

$active\_states(conf) = Sset \wedge_{i=1}^{h} \mathsf{Eval}(cond^i_{q_i}[p_j/x_j], attrs) = True \;\Rightarrow$

$\mathsf{Dispatch}(E(p_1, \ldots, p_n), conf, e\_queue) = conf', remove(E(p_1, \ldots, p_n), e\_queue)$

where

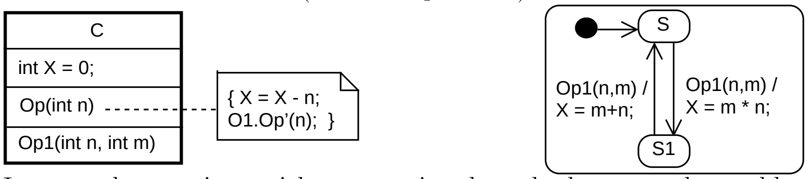
$conf' = run(\ldots run(conf, act^1_{q_1}[p_j/x_j], S^1_{q_1}) \ldots, act^h_{q_h}[p_j/x_j], S^h_{q_h})$

active_states and *run* are operations of the specification CONFIGURATION returning respectively the active states of the state machine and recording the start of a run-to-completion step, going from an active state into another one performing a given action.

PROBLEM What to do when the dispatched event is an operation call for whom also a method has been defined in the class ACL.

The solution in this case is just to prohibit to have a method for the operation appearing in some transition, and it is supported, e.g., by [11] p. 369 and other UML sources ([10]).

PROBLEM A similar problem is posed by the case below: what will happen when someone calls method Op, whose body is described by the note attached to its name in the class diagram? On one hand, assuming that such call is never answered, may lead to produce wrong UML models, because the other classes assume that Op is an available service, since it appears in the class icon. On the other hand, answering to it (perhaps only when the machine is not in a run-to-completion-step) seems in contrast with the role of the state machine (UML 1.3 p. 2-136).



In general operations with an associated method seem to be problematic for the active classes, and so it could be sensible to drop them.

No transitions triggered by a deferred event Assume that in the state machine SM there are no transitions starting from states belonging to $Sset$ triggered by E and that E is deferred in some elements of $Sset$.

PROBLEM UML 1.3 does not say what to do when dispatching E in such case. The possible choices are:
- remove it from the event queue
- put it back in the event queue
- put it back in the event queue, but only for the states in which it was deferred.

Here we assume that it is deferred, and that after it will be available for any state; however, notice, that we could formally handle also the first cases, whereas to consider the last we need to assume that there are many threads in an object with the relative event queues (one for each active state).

If the active states are $Sset$, then the event $E(p_1, \ldots, p_n)$ is left in the event queue for future use.

$active\_states(conf) = Sset \;\Rightarrow$

$\mathsf{Dispatch}(E(p_1, \ldots, p_n), conf, e\_queue) = conf, e\_queue$

4.6 Constraints

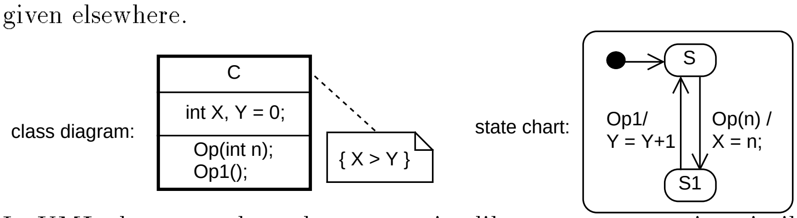
Constraints may be attached to any element of a UML model. For what concerns the fragment of UML considered in this paper we have constraints attached to the class icon (e.g., invariants) and to the operations (e.g., pre-post conditions), in the class diagram, and attached to the state machine (e.g., invariants). The language for expressing the constrains is not fixed in UML (also if OCL is the most used), however the semantics of the constraints is precisely settled, indeed UML 1.3 p. 2-29,2-30 states:

> *A constraint is a semantic condition or restriction expressed in text. In the meta-model, a Constraint is a BooleanExpression on an associated ModelElement(s) which must be true for the model to be well formed. . . . Note that a Constraint is an assertion, not an executable mechanism. It indicates a restriction that must be enforced by correct design of a system.*

Such idea of constraint may be easily formalized in our setting: the semantics of a constraint attached either to ACL or to SM is a property on L. The formulae of CASL allow to express a rich set of relevant properties, recall that the underlying logic of CASL is many sorted first-order logic, and that CASL extensions with temporal logic combinators, based on [4] are under development. Moreover the constraints expressed using OCL may be translated in CASL without too many problems.

Assume we have the constraints $C_1, \ldots, C_n$ attached either to ACL or to SM, then then the UML model containing ACL and SM is well formed iff for $i = 1, \ldots, n$, $L \models \Phi_i$, where Φ_i is the CASL formula corresponding to C_i. Techniques and tools developed for algebraic specification languages may help to verify the well formedness of UML models.

PROBLEM The use of constraints without a proper discipline may be problematic, as in the following case, where the constraint attached to the icon of class C is an invariant that must hold always, and so must be respected also by the transitions triggered by the calls to OP and OP1. These inconsistencies may be hard to detect, because the problematic constraints are in the class diagram, while state machine violating them is given elsewhere.



In UML there are also other constraint-like constructs posing similar problems; as the query qualification for operation (requiring that an operation does not modify the state of the object), or the “specifications” for signal receptions (expressing properties on the effects of receiving such signal). Also in these cases the behaviour described by the state machine may be in contrast with them.

We think that a way to settle those problems is to develop a precise methodology for using these constraints, making precise their role and when to use them in the development process.

5 Conclusions and related work

The task of formalizing UML has been addressed using various available formal techniques, as we will discuss below. Most of these attempts are complementary, because they approach the task from different viewpoints and aims.

In this respect our work has several distinguishing features. First of all we are using a very elementary and well-known machinery, namely lts (which are at the basis of formalisms like CCS), and conditional algebraic specifications. As happened for Ada, [1], this simple model provides a powerful setting for revealing ambiguities, incompleteness and perhaps questionable features, both from the purely technical and the methodological point of view. For example, we have discussed several naturally arising questions concerning UML, such as “what is the behaviour of an object with several active subthreads?”, and “does the run to completion execution of a state machine transition imply that a transition is a kind of critical region?”

Furthermore we have used a modular framework where the various possible interpretations of the aspects of UML (and of its many variants) may be made precise and the various problems exemplified and discussed also with users lacking a formal background (a lightweight formal method approach).

Within the “Precise UML” group and also outside, several proposals for formalizing UML or parts of them have been presented; we briefly report on some paradigmatic directions.

Some papers are addressing specific questions; for example [5] shows how to use graph rewriting techniques to transform UML state machines into another simplified machine (a kind of normal form); but, e.g., the execution of actions is not considered. That paper could be seen as providing a method for eliminating what we called shortcuts in Sect. 2.

Some other papers try to formalize UML by using a particular specification language; for example, as in [6], using Real-Time Action Logic, a form of real time temporal logic. With respect to these approaches we put less emphasis on the specification language and more on the underlying model, because we think that in this setting it is easier to explain UML features (also the possible problems) and to derive a revised informal presentation.

The relevance of the underlying model for making precise UML has been considered in [2], where a different model, a kind of stream processing function, is used. But the main aim there is methodological: how a software engineering method can benefit from an integrative mathematical foundation. While one of the main results of our work could be a basis for a revised reference manual for UML.

Finally, we have not considered the large number of papers on the semantics of classical state machines (as those supported by the Statemate tool), because the semantics of the UML state machines is rather different; e.g., in the former many events may occur simultaneously, whereas that cannot happen in UML (see [11] page 440).

We plan to go on analysing UML, to give a sound basis for our work in the CoFI project, trying to give an underlying formal (lightweight) model to the other constituents of a UML-system, i.e., to instances of passive classes and the system itself, and to consider the other kinds of diagrams, as class, sequence and collaboration.

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A Auxiliary Functions

TimeOccur : $\boldsymbol{Time} \times \boldsymbol{History} \ \rightarrow \boldsymbol{Set}[\boldsymbol{Event}]$

Assume that the time events appearing in the state machine SM are $TE_1, \ldots, TE_n$.

$\mathsf{TimeOccur}(t, history) = tev_1(t, history) \cup \ldots \cup tev_n(t, history)$

where for $i = 1, \ldots, n$ tev_i is an operation of sort $Set[Event]$ corresponding to TE_i defined by cases on the form of TE_i

TE_i = `after` exp (absolute time event) This event occurs when the difference between the actual time and the time the object entered the source state of the transition in which appear the event (S) is equal to the value of exp.

$\mathsf{Eval}(exp, attrs) = t' \ \wedge \ (t - entered\_state(history, S) = t') \ \Rightarrow \ tev_i(t, history) = \{TE_i\}$

$\mathsf{Eval}(exp, attrs) = t' \ \wedge \ (t - entered\_state(history, S)) \neq t' \ \Rightarrow \ tev_i(t, history) = \emptyset$

TE_i = `after` exp `since` S (relative time event) This event occurs when the difference between the actual time and the time the object entered state S is equal to the value of exp.

$\mathsf{Eval}(exp, attrs) = t' \ \wedge \ (t - entered\_state(history, S)) = t' \ \Rightarrow \ tev_i(t, history) = \{TE_i\}$

$\mathsf{Eval}(exp, attrs) = t' \ \wedge \ (t - entered\_state(history, S)) \neq t' \ \Rightarrow \ tev_i(t, history) = \emptyset$

ChangeOccur : $\boldsymbol{Attributes} \times \boldsymbol{History} \ \rightarrow \boldsymbol{Set}[\boldsymbol{Event}]$

Assume that the change events appearing in the state machine SM are $CE_1, \ldots, CE_n$.

$\mathsf{ChangeOccur}(attrs, history) = cev_1(attrs, history) \cup \ldots \cup cev_n(attrs, history)$

where for $i = 1, \ldots, n$ cev_i is an operation of sort $Set[Event]$ corresponding to CE_i defined as follows.

Assume CE_i = `change` $bexp$. This event occurs when the value of $bexp$ is true in the current state and was false in the previous state.

$\mathsf{Eval}(bexp, attrs) = True \ \wedge \ \mathsf{Eval}(bexp, attrs(previous\_state(history))) = False \ \Rightarrow$
$\quad cev_i(bexp, attrs) = \{CE_i\}$

$\mathsf{Eval}(bexp, attrs) = False \ \Rightarrow \ cev_i(bexp, attrs) = \emptyset$

$\mathsf{Eval}(bexp, attrs) = True \ \wedge \ \mathsf{Eval}(bexp, attrs(previous\_state(history))) = True \ \Rightarrow$
$\quad cev_i(bexp, attrs) = \emptyset$

Dispatch

Here we report the lacking cases of the definition of Dispatch.

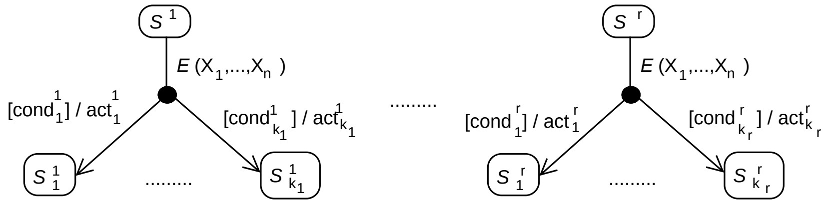
No transitions triggered by a nondeferred event Assume that in the state machine SM there are no transitions starting from the states belonging to $Sset$ triggered by E and that E is not deferred in any element of $Sset$.

If the active states are $Sset$, then event $E(p_1, \ldots, p_n)$ is just removed from the event queue.

$active\_states(conf) = Sset \ \Rightarrow$

$\mathsf{Dispatch}(E(p_1, \ldots, p_n), conf, e\_queue) = conf, remove(E(p_1, \ldots, p_n), e\_queue)$

Some transitions triggered by the event Assume that in the state machine SM there are the following branched transitions triggered by E starting from the states belonging to $Sset$



(inheritance of transitions and overriding may be handled here by deciding which transition to consider).

- If the active states are $Sset$, E is not deferred in any of the states in $Sset$, and for $i = 1, \ldots, r$, $j = 1, \ldots, k_i$ $cond^i_j$ does not hold, then such event is removed from the queue.
  $active\_states(conf) = S \land {}^{r}_{i=1} \land {}^{k_i}_{j=1} \mathsf{Eval}(cond^i_j[p_j/x_j], attrs) = False \Rightarrow$
  $\mathsf{Dispatch}(EV(p_1, \ldots, p_n), conf, e\_queue) = conf, remove(E(p_1, \ldots, p_n), e\_queue)$
- If the active states are $Sset$, E is deferred in some of the states in $Sset$, and for $i = 1, \ldots, r$, $j = 1, \ldots, k_i$ $cond^i_j$ does not hold, then such event is left in the queue.
  $active\_states(conf) = S \land {}^{r}_{i=1} \land {}^{k_j}_{j=1} \mathsf{Eval}(cond_i[p_j/x_j], attrs) = False \Rightarrow$
  $\mathsf{Dispatch}(EV(p_1, \ldots, p_n), conf, e\_queue) = conf, e\_queue$

Exec : $\boldsymbol{Ident} \times \boldsymbol{Attributes} \times \boldsymbol{Configuration} \rightarrow$
$\boldsymbol{Configuration} \times \boldsymbol{Attributes} \times \boldsymbol{Set}[\boldsymbol{Event}] \times \boldsymbol{Set}[\boldsymbol{Event}]$

$\mathsf{Exec}(id, attrs, conf) = conf', attrs', out\_evs, loc\_evs$ means that the object id with configuration $conf$ may execute an action changing its configuration to $conf'$, updating its attributes to $attrs'$ and producing the set of output events out_evs and the set of local events loc_evs.

Exec is defined by cases on the form of the action to be executed.

calling an operation

- Self calling
  $to\_execute(conf, id.Op(exp_1, \ldots, exp_n), S) \land {}^{n}_{i=1} \mathsf{Eval}(exp_i, attrs) = v_i \Rightarrow$
  $\mathsf{Exec}(id, attrs, conf) =$
  $execute(conf, id.Op(exp_1, \ldots, exp_n), S), attrs, \{Op(v_1, \ldots, v_n)\}, \emptyset$
- Calling another object
  $to\_execute(conf, id'.Op(exp_1, \ldots, exp_n), S) \land$
  $\land {}^{n}_{i=1} \mathsf{Eval}(exp_i, attrs) = v_i \land id' \neq id \Rightarrow$
  $\mathsf{Exec}(id, attrs, conf) =$
  $execute(conf, id.Op(exp_1, \ldots, exp_n), S), attrs, \emptyset, \{id.Op(v_1, \ldots, v_n)\}$

sending a signal

- Sig is a signal for whom ACL has a reception (self sending). Notice that Sig is sent also outside because it must be received also by the other objects of the same class and by the objects of other classes having a reception for it.
  $to\_execute(conf, Sig(exp_1, \ldots, exp_n), S) \land {}^{n}_{i=1} \mathsf{Eval}(exp_i, attrs) = v_i \Rightarrow$
  $\mathsf{Exec}(id, attrs, conf) =$
  $execute(conf, Sig(exp_1, \ldots, exp_n), S), attrs, \{Sig(v_1, \ldots, v_n)\}, \{Sig(v_1, \ldots, v_n)\}$

- *Sig* is a signal for whom ACL has not a reception (sending outside)
  $to\_execute(conf, Sig(exp_1, \ldots, exp_n), S) \;\wedge\; {}_{i=1}^{n}\mathsf{Eval}(exp_i, attrs) = v_i \;\Rightarrow$
  $\mathsf{Exec}(id, attrs, conf) = execute(conf, Sig(exp_1, \ldots, exp_n), S), attrs, \emptyset, \{Sig(v_1, \ldots, v_n)\}$

destroy

- Self destruction
  $to\_execute(conf, id.destroy, S) \;\Rightarrow$
  $\qquad \mathsf{Exec}(id, attrs, conf) = execute(conf, id.destroy, S), attrs, \{destroy\}, \emptyset$
- Destroying another object
  $to\_execute(conf, id'.destroy, S) \;\wedge\; id' \neq id \;\Rightarrow$
  $\qquad \mathsf{Exec}(id, attrs, conf) = execute(conf, id'.destroy, S), attrs, \emptyset, \{id'.destroy\}$

assignment to one of its attributes as an example of uninterpreted action

$to\_execute(conf, x = exp, S) \;\wedge\; \mathsf{Eval}(exp, attrs) = v \;\Rightarrow$
$\qquad \mathsf{Exec}(id, attrs, conf) = execute(conf, x = exp, S), attrs[v/x], \emptyset, \emptyset$

Eval : $\boldsymbol{Expression} \times \boldsymbol{Attributes} \times \boldsymbol{Set}[\boldsymbol{Inputs}] \rightarrow \boldsymbol{Value}$

$\mathsf{Eval}(exp, attrs) = v$ means that the value of exp calculated using $attrs$ is v.

- attribute of the object
  $\mathsf{Eval}(x, attrs) = value\_of(attrs, x)$
- data operator
  $\wedge_{i=1}^{n}\mathsf{Eval}(exp_i, attrs) = v_i \;\Rightarrow\; \mathsf{Eval}(OP(exp_1, \ldots, exp_n), attrs) = op(v_1, \ldots, v_n)$