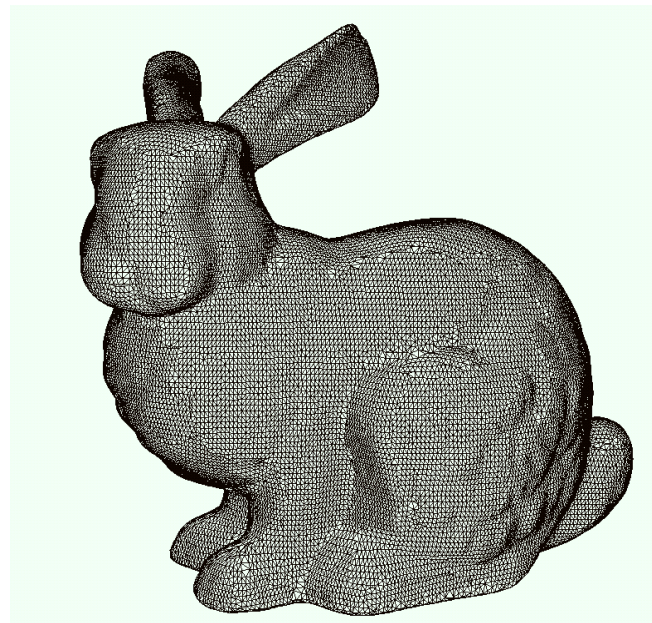


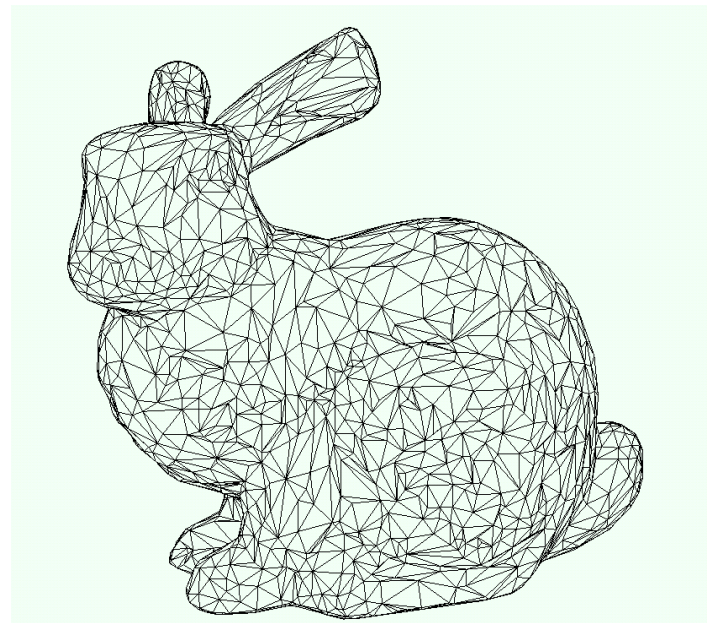
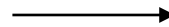
Surface Simplification Algorithms Overview

Surface Simplification

- Problem statement:
given a surface [described with a triangular mesh], find an approximation mesh which minimizes both the **size** and the **approximation error**
- Main issues: **speed, precision, robustness and generality**



70K triangles



2K triangles

Simplification Algorithms

Simplification approaches:

- incremental methods based on local updates
 - ✧ **mesh decimation** [Schroeder et al. '92, ... + others]
 - ▭ **energy function optimization** [Hoppe et al. '93, Hoppe '96, Hoppe '97]
 - ▭ **quadric error metrics** [Garland et al. '97]
- coplanar facets merging [Hinker et al. '93, Kalvin et al. '96]
- re-tiling [Turk '92]
- clustering [Rossignac et al. '93, ... + others]
- wavelet-based [Eck et al. '95, + others]

Incremental methods based on *local updates*

- All of the methods such that :
 - ✧ simplification proceeds as a sequence of *local updates*
 - each update *reduces mesh size* and [monotonically] *decreases* the *approximation precision*

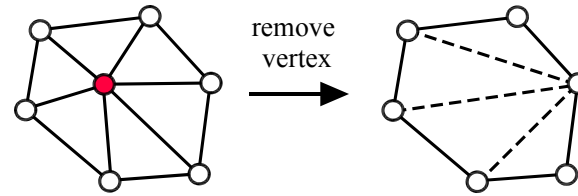
- Different approaches:
 - mesh decimation
 - energy function optimization
 - quadric error metrics

... Incremental methods based on *local updates* ...

Local update actions:

No. Faces

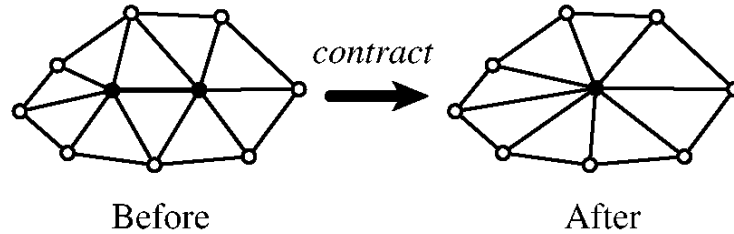
vertex removal



$n-2$

edge collapse

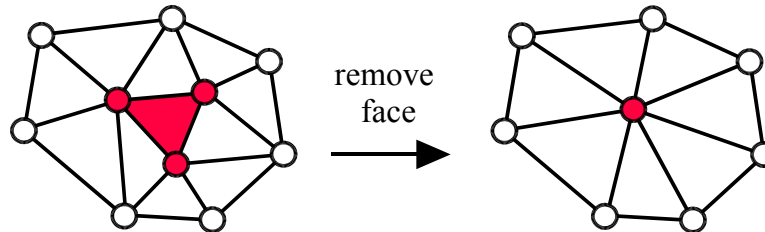
- ☆ preserve location
- new location



$n-2$

triangle collapse

- preserve location
- new location



$n-4$

... Incremental methods based on *local updates* ...

The common framework:

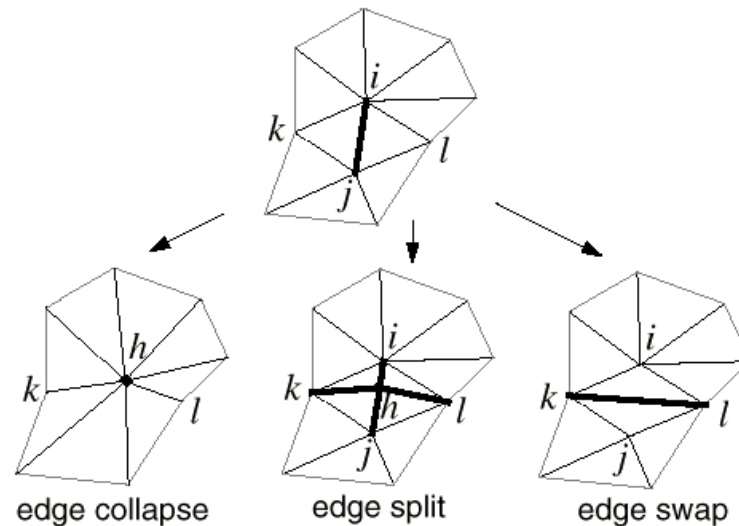
- **loop**
 - ***select*** the element to be deleted/collapsed;
 - ***evaluate approximation*** introduced;
 - ***update*** the mesh after deletion/collapse;
- until** mesh **size/precision** is satisfactory;

Energy function optimization

Mesh Optimization

[Hoppe et al. '93]

- Simplification based on the iterative execution of :
 - edge collapsing
 - edge split
 - edge swap



... Energy function optimization: Mesh Optimization ...

- approximation quality evaluated with an **energy function** :

$$E(M) = E_{\text{dist}}(M) + E_{\text{rep}}(M) + E_{\text{spring}}(M)$$

which evaluates geometric **fitness** and repr. **compactness**

E_{dist} : sum of squared distances of the original points from M

E_{rep} : factor proportional to the no. of vertex in M

E_{spring} : sum of the edge lengths

... Energy function optimization: Mesh Optimization ...

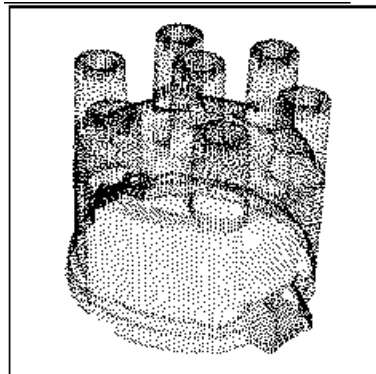
Algorithm structure

- outer minimization cycle (*discrete* optimiz. probl.)
 - choose a legal action (edge collapse, swap, split) which reduces the energy function
 - perform the action and update the mesh ($M_i \rightarrow M_{i+1}$)

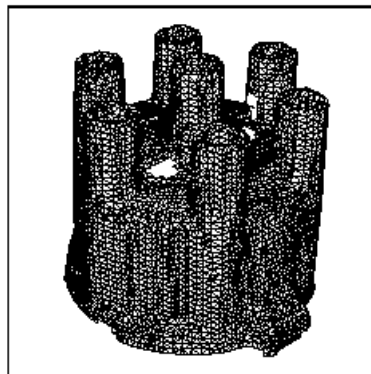
 - inner minimization cycle (*continuous* optimiz. probl.)
 - optimize the vertex positions of M_{i+1} with respect to the initial mesh M_0
- but (to reduce complexity)*
- legal action selection is random
 - inner minimization is solved in a fixed number of iterations

... Energy function optimization: Mesh Optimization ...

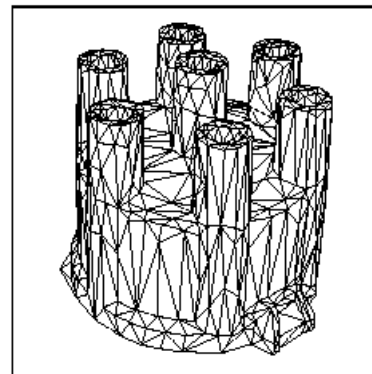
Mesh Optimization - *Examples*



(j) Laser range data ($n = 12,745$)



(k) Output of phase one



(l) Output of phase two

[Image by Hoppe et al.]

... Energy function optimization: Mesh Optimization ...

Mesh Optimization - *Evaluation*

- high quality of the results
- preserves topology, re-sample vertices
- high processing times
- not easy to implement
- not easy to use (selection of tuning parameters)
- adopts a global error evaluation, but the resulting approximation is not bounded

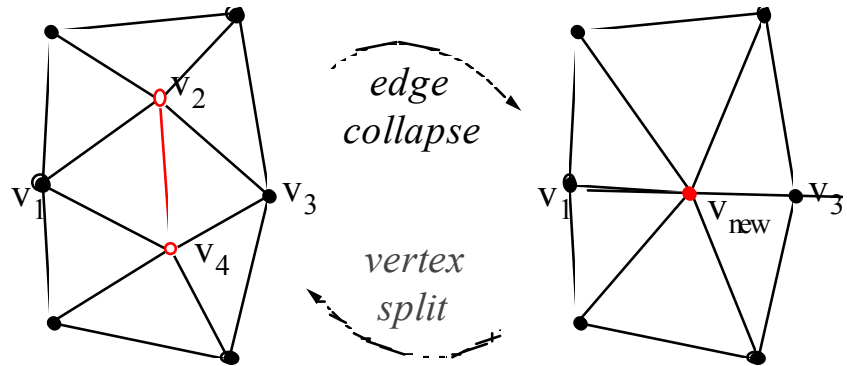
implementation available on the web

... Energy function optimization: **Progressive Meshes** ...

Progressive Meshes

[Hoppe `96]

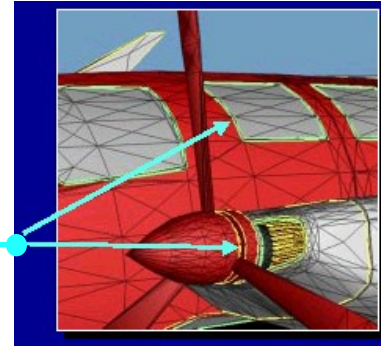
- execute **edge collapsing** *only* to reduce the *energy function*
- *edge collapsing* can be easily inverted ==> store sequence of inverse *vertex split* transformations to support:
 - multiresolution
 - progressive transmission
 - selective refinements
 - geomorphs
- *faster* than MeshOptim.



... Energy function optimization: **Progressive Meshes** ...

Preserving mesh *appearance*

- shape and crease edges
- scalar fields discontinuities (e.g. color, normals)
- discontinuity curves



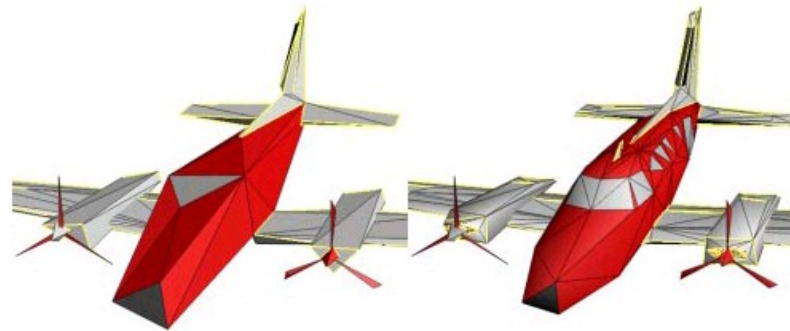
[image by H. Hoppe]

Managed by inserting two new components in the *energy function*:

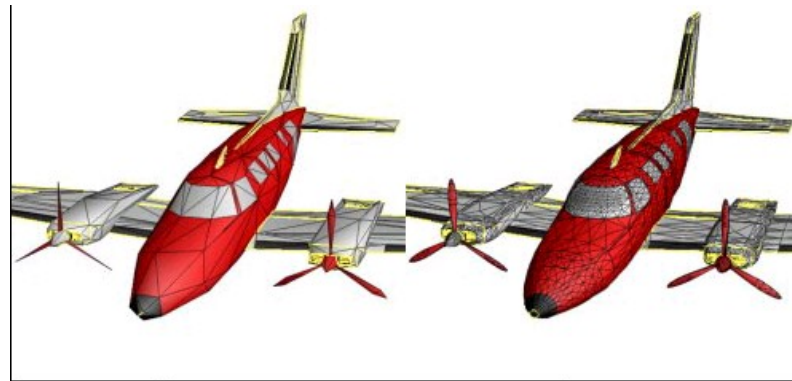
- E_{scalar} : measures the accuracy of scalar attributes
- E_{disc} : measure the geometric accuracy of discontinuity curves

... Energy function optimization: **Progressive Meshes** ...

Progressive Meshes *Examples*



(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces)



(c) Mesh M^{425} (1,000 faces) (d) Original $\hat{M} = M^n$ (13,546 faces)

... Energy function optimization: **Progressive Meshes...**

Progressive Meshes - *Evaluation*

- high quality of the results
- preserves topology, re-sample vertices
- not easy to implement
- not easy to use (selection of tuning parameters)
- adopts a global error evaluation, not-bounded approximation
- preserves vect/scalar attributes (e.g. color) **discontinuities**
- supports **multiresolution** output, geometric morphing, **progressive transmission**, **selective** refinements
- much **faster** than MeshOpt.

will be available in MS DirectX 5.0 graphics interface

Decimation

Mesh Decimation

[Schroeder et al'92]

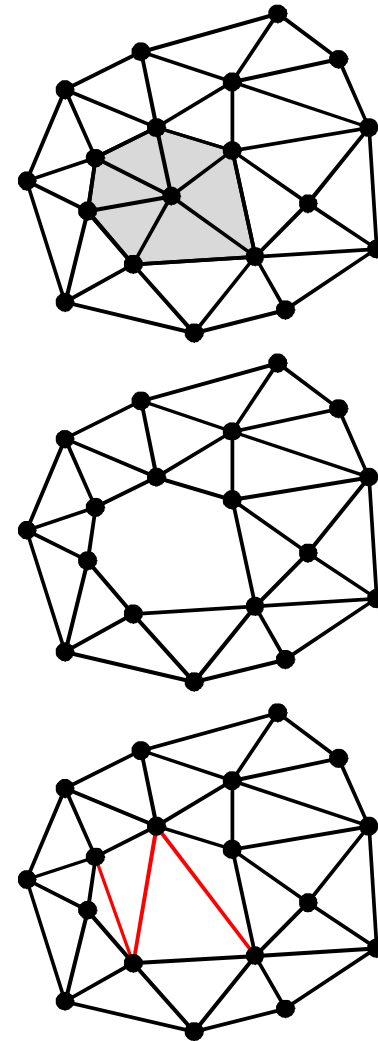
- Based on controlled removal of **vertices**
- Classify vertices as **removable** or **not** (based on local topology / geometry and required precision)

Loop

- choose a *removable* vertex v_i
- delete v_i and the incident faces
- re-triangulate the hole

until

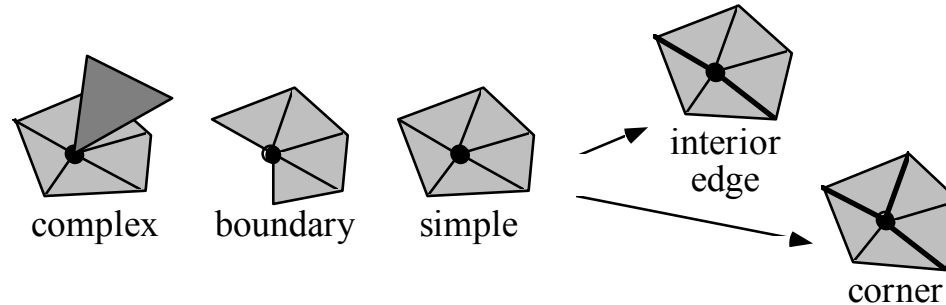
no more removable vertex **or**
reduction rate fulfilled



- General method (manifold/non-manifold *input*)
- Algorithm phases:
 - topologic classification of vertices
 - evaluation of the decimation criterion (error evaluation)
 - re-triangulation of the removed triangles patch

Topologic classification of vertices

- ▶ for each vertex: find and characterize the loop of incident faces



- *interior edge*: if dihedral angle between faces $< k_{\text{angle}}$
(k_{angle} : user driven parameter)
- *not-removable vertices*: complex, [corner]

... Decimation ...

Decimation criterion -- a vertex is *removable* if:

○ **simple** vertex:

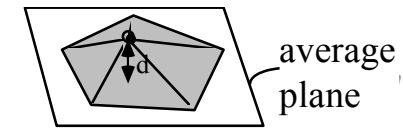
if distance **vertex - face loop average plane**
is lower than ϵ_{\max}

○ **boundary / interior / corner** vertices:

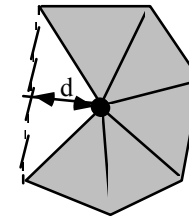
if distance **vertex - new boundary/interior edge**
is lower than ϵ_{\max}

□ adopts *local evaluation* of the approximation!!

□ ϵ_{\max} : value selected by the user



d: distance to plane

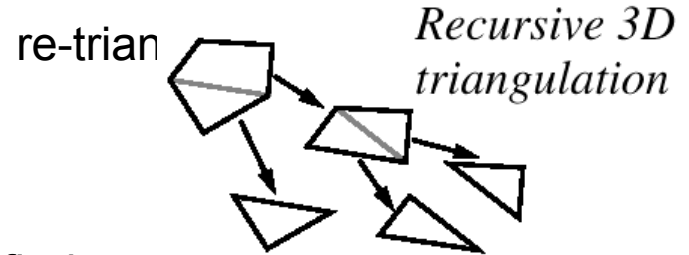


d: distance to edge

Re-triangulation

○ face loops in general non planar ! (but star-shaped)

○ adopts **recursive loop splitting**



○ control *aspect ratio* to ensure simplified mesh quality

○ for each vertex removed:

◇ if simple or boundary vertex ==> 1 loop

▮ if interior edge vertex ==> 2 loops

▮ if boundary vertex ==> - 1 face

▮ otherwise ==> - 2 faces

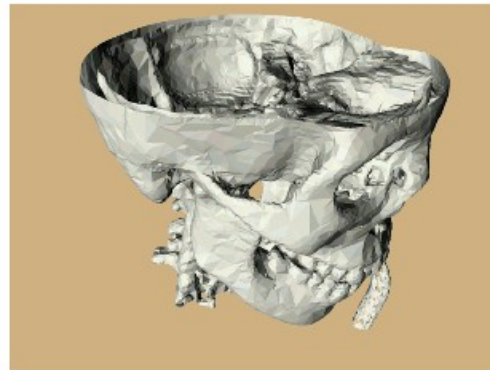
Decimation - *Examples*



Full Resolution
(569K Gouraud shaded triangles)



75% decimated
(142K Gouraud shaded triangles)



75% decimated
(142K flat shaded triangles)



90% decimated
(57K flat shaded triangles)

(images by W. Lorensen)

Original Mesh Decimation - *Evaluation*

- good efficiency (speed & reduction rate)
- simple implementation and use
- good approximation
- works on huge meshes
- preserves topology; vertices are a subset of the original ones
- error is **not** bounded (local evaluation ==> accumulation of error!!)

*implemented in the **Visualization Toolkit (VTK)**, public domain*

Enhancing Mesh Decimation

- Improve approximation precision, ensure bounded error
 - ◇ **bounded** error [Cohen'96, Gueziec'96]
 - ▮ **global error** evaluation [Soucy'96, Bajaj'96, Klein'96, Ciampalini'97, +...]
 - ▮ **smarter re-triangulation** (edge flipping) [Bajaj'96, Ciampalini'97]

- Multiresolution, dynamic LOD [Ciampalini'97]

- Decimate other entities
 - ▮ **edges** (collapse into vertices) [Gueziec'95-'96, Ronfard'96, Algorri96]
 - ▮ **faces** (collapse into vertices) [Hamann '94]

- Preserve color and attributes info [Soucy'96, Cohen et al 98, Cignoni et al 98, +...]

- Topology simplification [Lorensen 97]

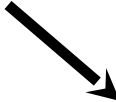



- Extension to 3D meshes (tetrahedral meshes) [Renze'96, Trotts et al 98, Stadt et al 98]

Approximation Error Evaluation

Classification of simplification methods based on **approximation error** evaluation euristics:

User' viewpoint:

- simple to grasp
- simple to drive

- **locally-bounded** error, based on mesh distances
[ex. standard Mesh Decimation] 
- **globally bounded** error, based on mesh distances
[ex. Envelopes + enhanced Decimation + others] 
- control based on **mesh characteristics**
[ex. vertex proximity, mesh curvature] 
- **energy function** evaluation
[ex. Mesh Optim. , Progr. Meshes] 

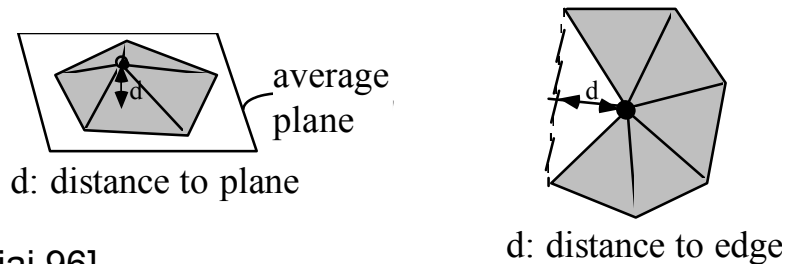
very handy

may be misleading

not easy, many parameters to be selected

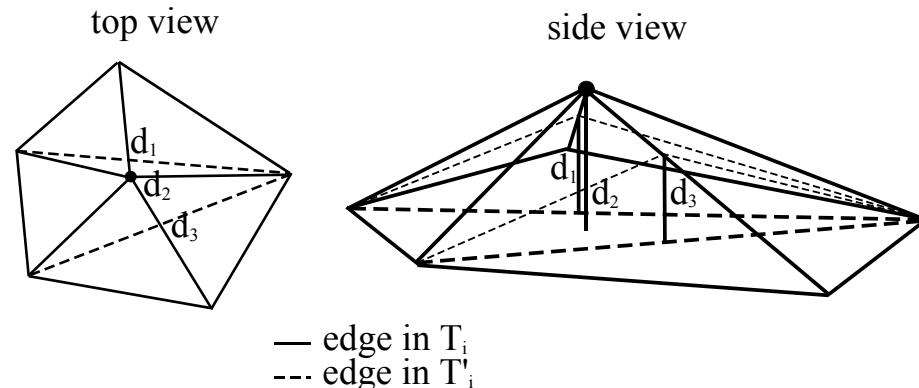
Heuristics proposed for **local error evaluation**:

- **approximate evaluation** [Schroeder 92]



- **correct evaluation** [Bajaj 96]

given two linear patches ==>
the max value of meshes' distance is either on
edges' intersections or on internal vertices



Heuristics proposed for **global error evaluation**:

- **accumulation of local errors**

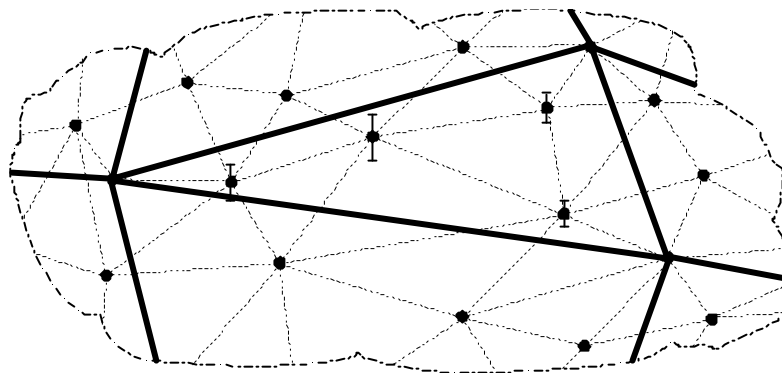
[Ciampalini97]

fast, **but** approximate

- **vertex--to--simplified mesh distance**

[Soucy96]

requires storing which of the original vertices maps to each simplified face;
very near to exact value (but large under-estimation in the first steps)



- edge of initial mesh M_0
- edge of simplified mesh M_i
- error magnitude, $\text{dist}(v, M_i)$

... Heuristics proposed for **global error evaluation**:

- **input mesh -- to -- simplified mesh edges distance** [Ciampalini97]
 - for each internal edge:
 - ◇ select sampling points \mathbf{p}_i (regularly/random)
 - evaluate distance $d(M_0, \mathbf{p}_i)$

sufficiently precise and efficient in time

- **input mesh -- to -- simplified mesh distance** [Klein96]
precise, **but** more complex in time

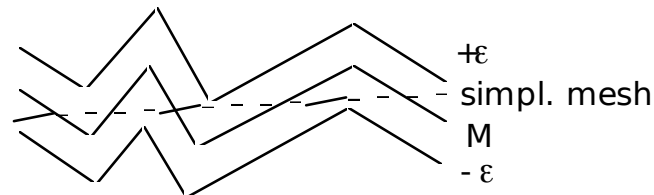
- **use envelopes** [Cohen et al.'96]
precise, no self-intersections **but** complex in time and to be implemented

Enhancing Decimation -- Simplification Envelopes

Simplification Envelopes

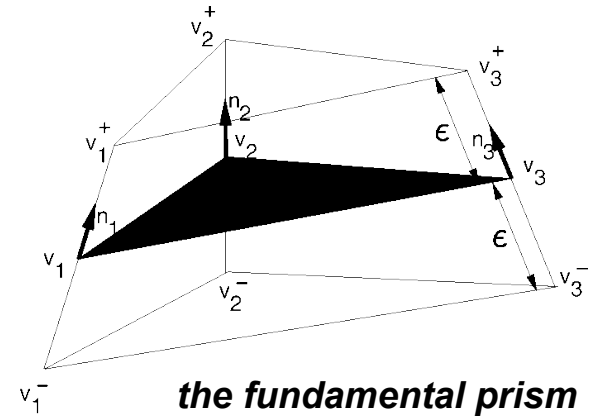
[Cohen et al.'96]

- given the input mesh M
 - build two envelope meshes M_- and M_+ at distance $-\epsilon$ and $+\epsilon$ from M ;
 - simplify M (following a decimation approach) by enforcing the decimation criterion:
a candidate vertex may be removed **only if** the new triangle patch does not intersect neither M_- or M_+



... Enhancing Decimation - Simplification Envelopes ...

- by construction, envelopes do not self-intersect
==> simplified mesh is **not self-intersecting !!**



- distance between envelopes becomes smaller near the bending sections, and simplification harder
- **border tubes** are used to manage open boundaries



(drawing by A. Varshney)

... Enhancing Decimation - **Simplification Envelopes** ...

Simplification Envelopes - *Evaluation*

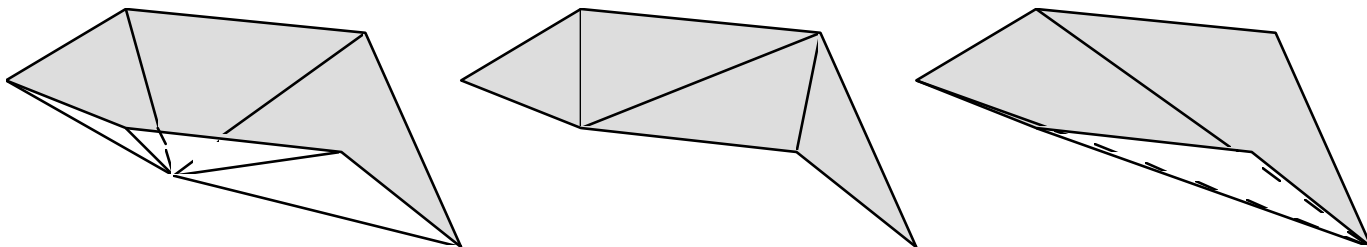
- works on manifold surface **only**
- bounded approximation
- construction of envelopes and intersection tests are not cheap
- > three times more RAM (input mesh + envelopes + border tubes)
- preserve topology, vertices are a subset of the original, prevents self-intersection

available in public domain

Enhancing Decimation -- Smarter re-triangulation

For all methods based on re-triangulation, approximation depends on **new patch quality**

- control new triangles' **aspect ratio**, to avoid slivery faces [equiangularity]
- adopt **edge flipping** to improve mesh quality [Bajaj'96, Ciampalini97]
 - ✦ build a first triangulation and, through a **greedy** optimization process based on edge flipping, adapt it to the original mesh
- **global error** estimate is needed to support flipping



Original

A triangulation

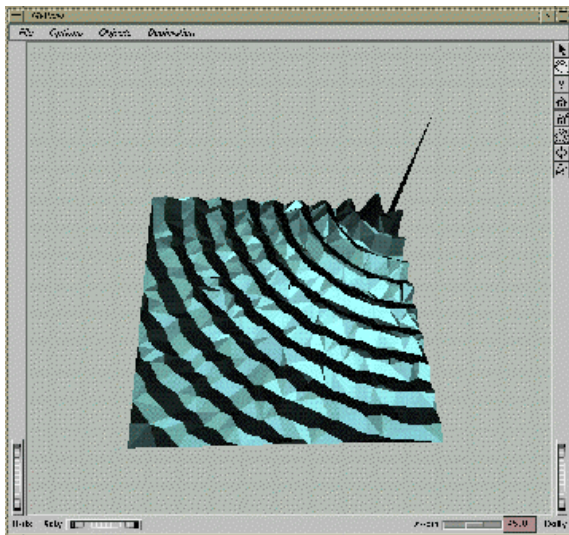
A better triangulation

... Smarter re-triangulation...

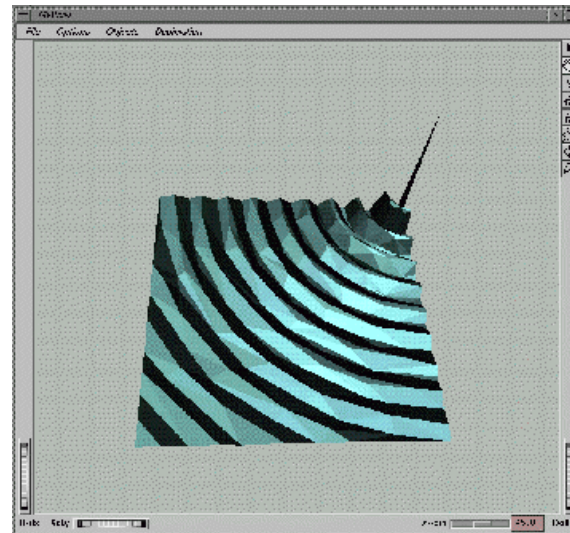
Mesh approximation improvement due to edge flipping (**Jade2.0 code**)

- original mesh: 28,322 triangles
- simplified meshes: same approximation error

no flipping: 1004 faces



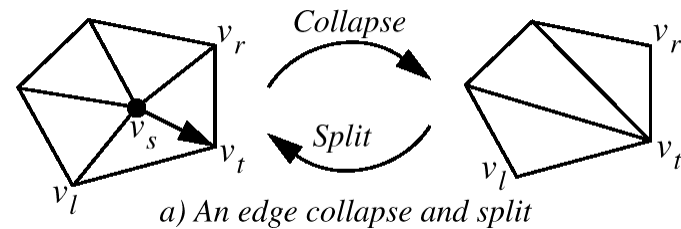
with flipping: 528 faces



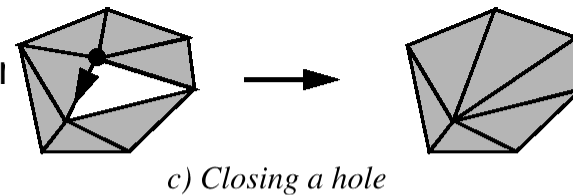
Topology Modifying Progressive Decimation

[Schroeder Vis97]

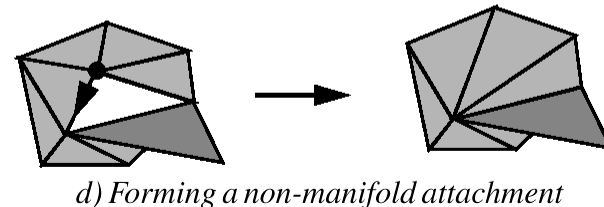
- *topology preservation*: a limiting factor in overall reduction capability
- adopts a **progressive-mesh** approach on top of an **edge-collapse** based mesh decimator
- atomic action: **edge collapse** encoded for progressive storage, transmission, and reconstruction
 ==> holes may close, non-manifold attachments may form



- uses a priority queue to store candidate vertices



- available in the **vtk** system



Candidate vertices selection

- **vertex classification**: same as standard Mesh Decimation
- uses an **heap** to store candidate vertices in order of error
 - *heap initialization*: for all vertices, simulate removal and evaluate approximation introduced
- evaluation of the **error** introduced while removing a vertex:
 - approximated *input_mesh---to---simpl_mesh* distance
 - integrated with *edge flipping* test
- **vertex selection** for removal:
 - in order of **increasing error** (from *heap*)
 - decimating sorted vertices improves mesh quality and is crucial to support multiresolution

Algorithm Jade 2.0 ($M_0, target_err, S$)

Var VH: Heap; \{vertex heap, sorted by increasing error\}

 S := M_0 ;

 \{initialize the heap VH: \}

 FOR EACH vertex v_i in M_0 DO

 compute error e_i associated to the removal of v_i (includes
re-triangulation but not mesh update);

 insert (v_i, e_i) in VH;

 \{main cycle: \}

 REPEAT

 pop first candidate v from VH;

 delete from simplified mesh S;

 retriangulate the hole in S;

 err := current_approximation(S);

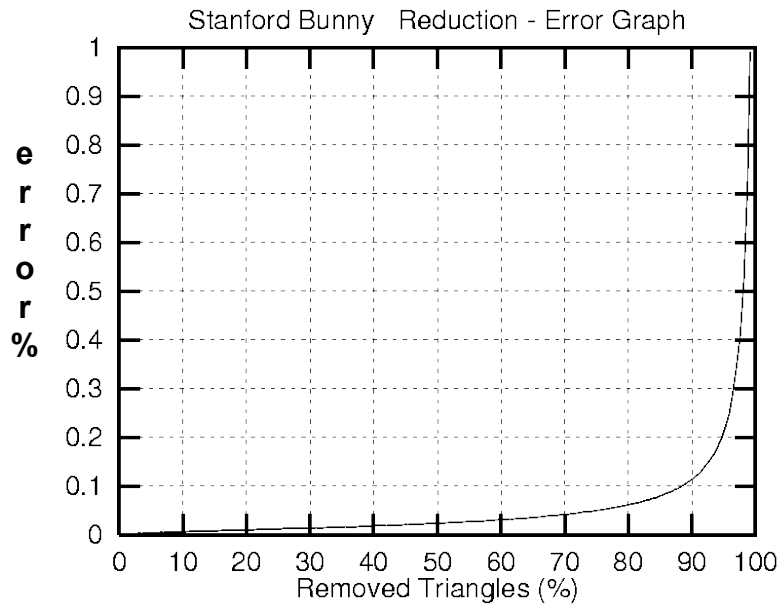
 check error in VH for the vertices on the border of the
re-triangulated hole (and, in case, update heap VH);

 UNTIL err <= target_err;

END;

Results

- Simplification times \approx linear with mesh size

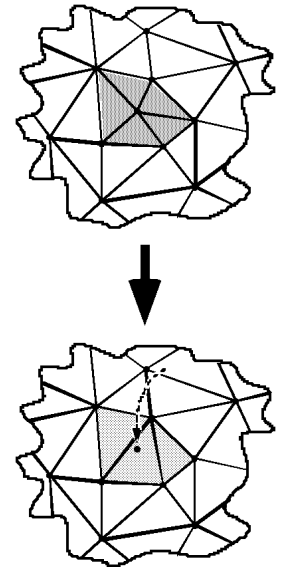


no staircase abrupt
error increase
(fundamental for the quality of
the multiresolution output)

Construction of a multiresolution model

Keep the *history* of the simplification process :

- when we remove a vertex we have **dead** and **newborn** triangles
- assign to each triangle t a **birth error** t_b and a **death error** t_d equal to the error of the simplified mesh just before the removal of the vertex that caused the birth/death of t



By storing the *simplification history* (faces+errors) we can simply extract *any approximation level* in real time

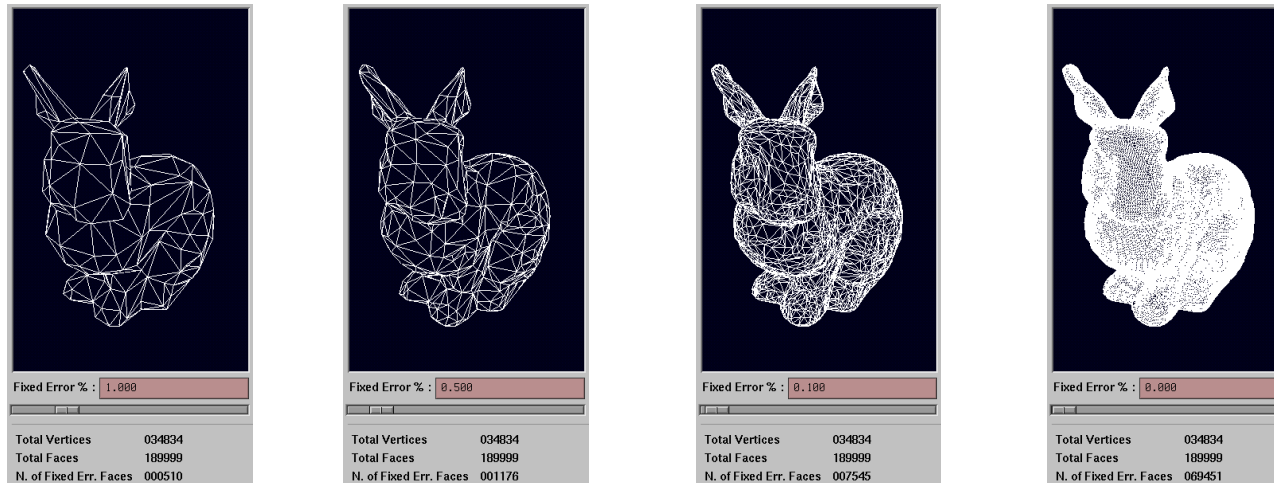
Real-time resolution management

- by extracting from the *history* all the triangles t_i with

$$t_b \leq \epsilon < t_d$$

we obtain a model M_ϵ which satisfies the approximation error ϵ

- maintaining the whole *history* data structure costs approximately 2.5x - 3x the full resolution model



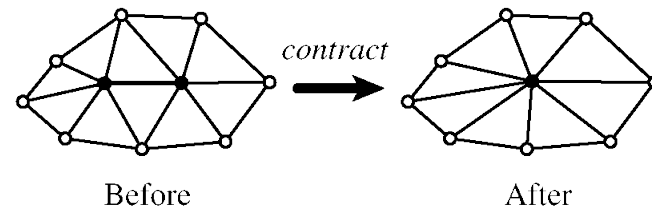
← real-time LOD →

Simplification using Quadric Error Metrics

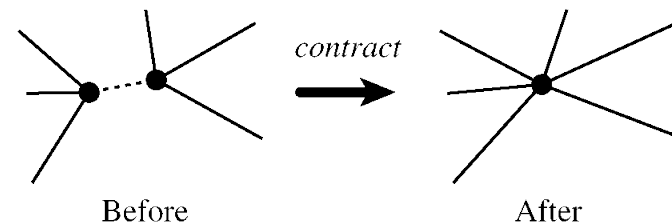
[Garland et al.

Sig'97]

- Based on incremental **edge-collapsing**



- **but** can also collapse vertex couples which are **not connected** (topology is not preserved)



Geometric error approximation is managed by simplifying an approach based on **plane set distance**

[Ronfard,Rossignac96]

- ✧ INIT_time: store for each vertex the set of incident planes
- Vertex_Collapsing $(v_1, v_2) \Rightarrow v_{new}$
 - ☆ plane_set (v_{new}) = union of the two **plane sets** of v_1, v_2
 - collapse only if v_{new} is not “farther” from its plane set than the selected target error ϵ

criticism:

- storing plane sets and computing distances is not cheap !

Quadric Error Metrics solution:

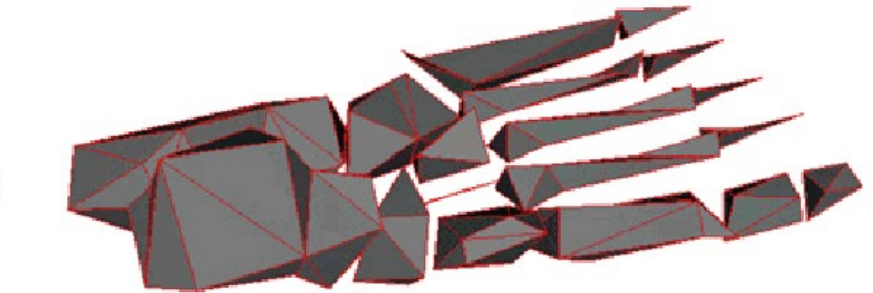
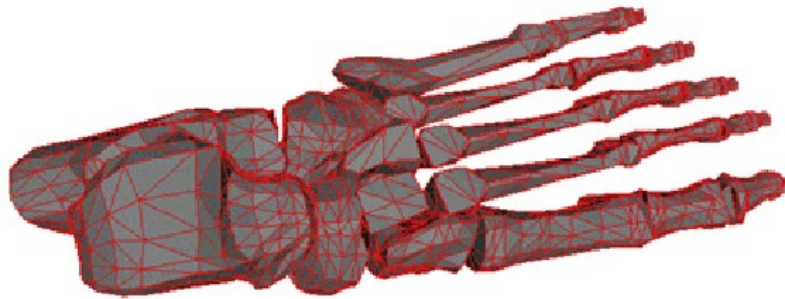
- ✧ quadratic distances to planes represented with **matrices**
 - plane sets merge *via* matrix sums
 - very efficient evaluation of error *via* **matrix operations**
- but**
 - triangle size is taken into account only in an approximate manner (orientation only in Quadrics + weights)

Algorithm structure:

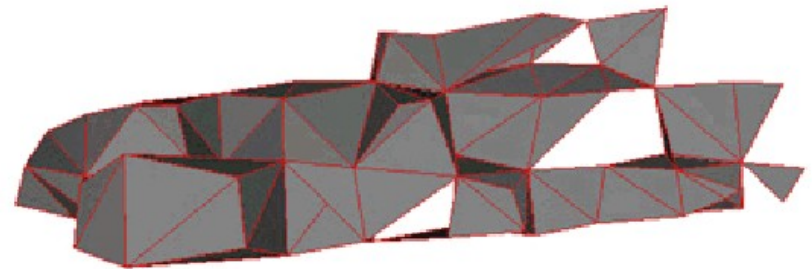
- select valid vertex pairs (upon their distance),
insert them in an heap sorted upon minimum cost;
- **repeat**
 - extract a valid pair V_1, V_2 from heap and contract into V_{new} ;
 - re-compute the cost for all pairs which contain V_1 or V_2 and update the heap;
- until** sufficient reduction/approximation **or** heap empty

An example

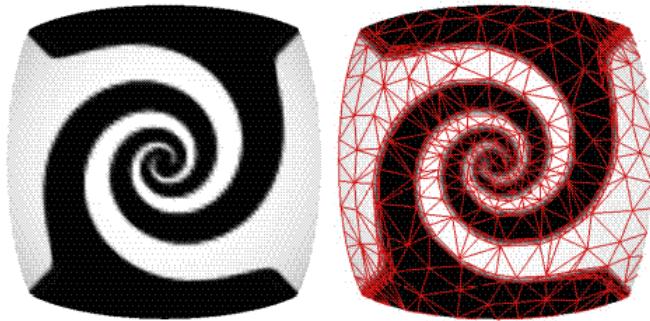
- **Original.** Bones of a human's left foot (4,204 faces).
- Note the many separate bone segments.
- **Edge Contractions.** 250 face approximation.
- Bone segments at the ends of the toes have disappeared; the toes appear to be receding back into the foot.



- **Clustering.** 262 face approximation.

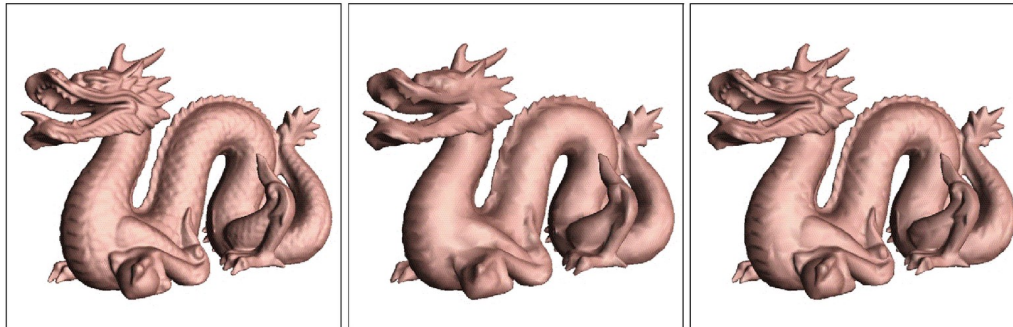
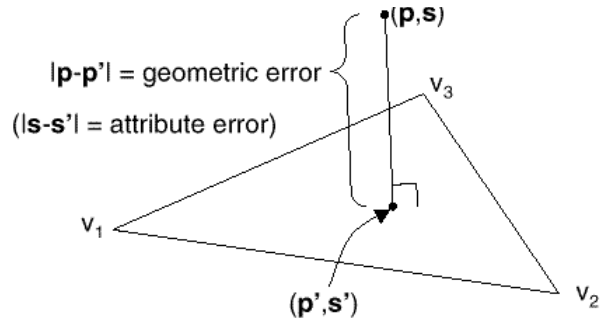


... Quadric Error Metrics Extension ...



Quadric can be extended to take into account:

- color and texture attributes error are computed by projecting them in R^{3+m} [Garland 98]
- by computing attribute error as the squared deviation between original value and the value interpolated [Hoppe 99]



(a) Original mesh

(b) Q is just geometric error

(c) Q also includes normals

Quadric Error Metrics -- *Evaluation*

- iterative, incremental method
- error is bounded
- allows topology simplification (aggregation of disconnected components)
- results are very high quality and ***times incredibly short***
- Various commercial packages use this technique (or variations)

Not-incremental methods:

- coplanar facets merging
- re-tiling
- clustering
+ others]
- wavelet-based

[Hinker et al. '93, Kalvin et al. '96]

[Turk '92]

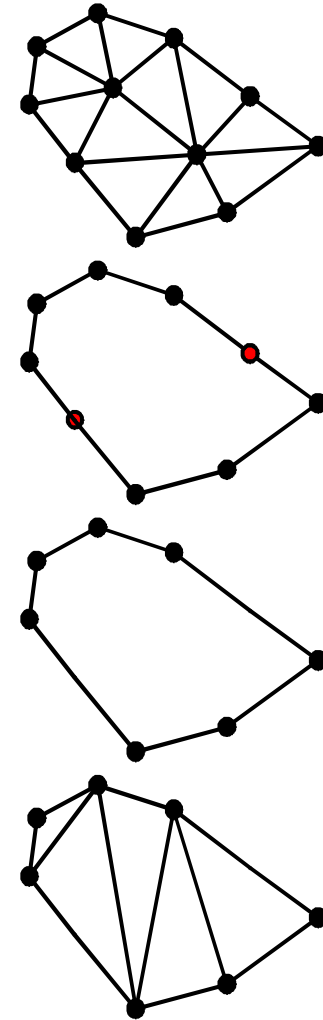
[Rossignac et al. '93, ...]

[Eck et al. '95]

Coplanar Facets Merging

Geometric Optimization [Hinker '93]

- ❑ Construct nearly co-planar sets (comparing normals)
- ❑ Create edge list and remove duplicate edges
- ❑ Remove colinear vertices
- ❑ Triangulate resultant polygons



***Geometric Optimization* - Evaluation**

- simple and efficient heuristic
- evaluation of approximation error is highly inaccurate and not bounded (error depends on relative size of merged faces)
- vertices are a subset of the original
- preserves geometric discontinuities (e.g. sharp edges) and topology

... Coplanar Facets Merging...

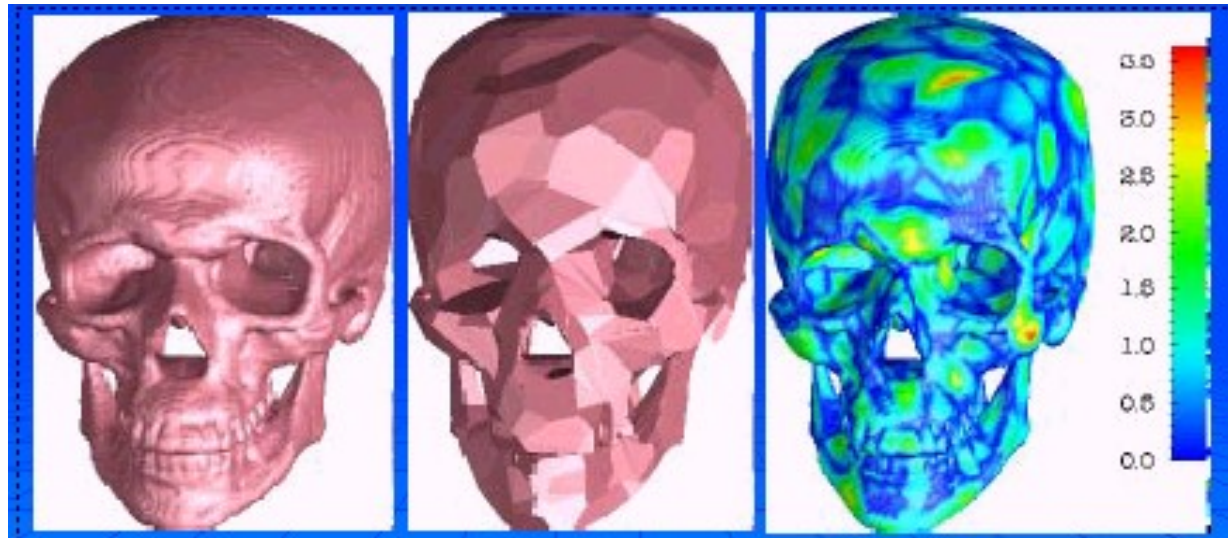
Superfaces

[Kalvin, Taylor '96]

- group mesh faces in a set of *superfaces*:
 - iteratively choose a seed face f_i as the current *superface* Sf_j
 - find by propagation all faces adjacent to f_i whose vertices are at distance $\epsilon/2$ from the mean plane to Sf_j and insert them in Sf_j
 - moreover, to be merged each face must have orientation similar to those of others in Sf_j
- straighten the *superfaces* border
- re-triangulate each *superface*

Superfaces - an example

- Simplification of a human skull (fitted isosurface), *images courtesy of IBM*



***Superfaces* - Evaluation**

- slightly more complex heuristics
- evaluation of approximation error is more accurate and bounded
- vertices are a subset of the original ones
- preserves geometric discontinuities (e.g. sharp edges) and topology

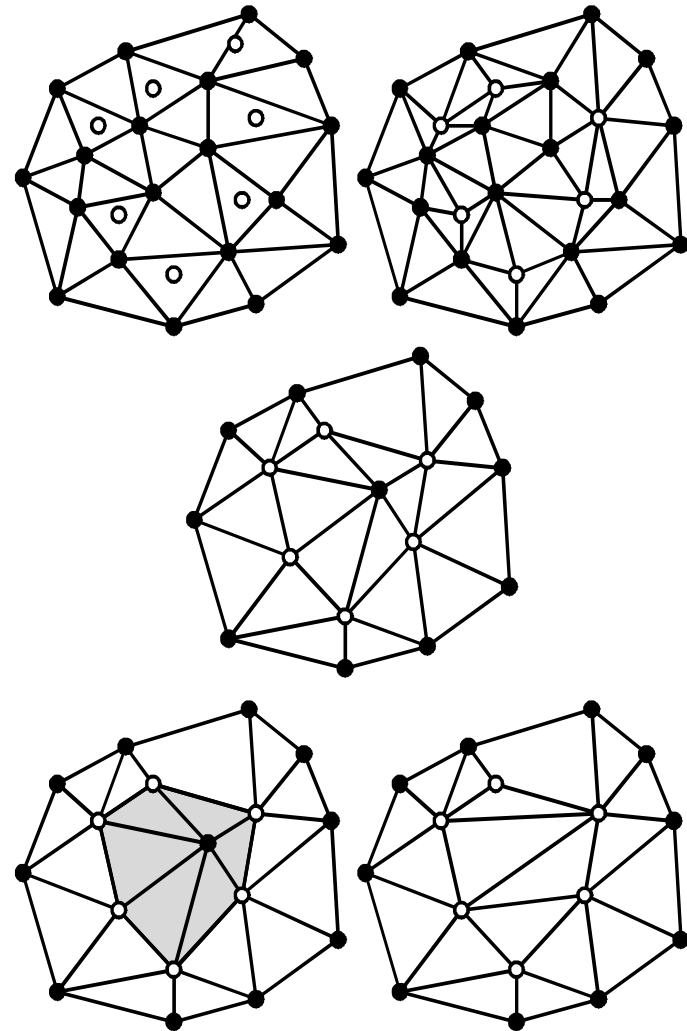
Re-tiling

Re-Tiling

[Turk '92]

- Distribute a new set of vertices into the original triangular mesh (points positioned using repulsion/relaxation to allow optimal surface curvature representation)
- Remove (part of) the original vertices
- Use local re-triangulation

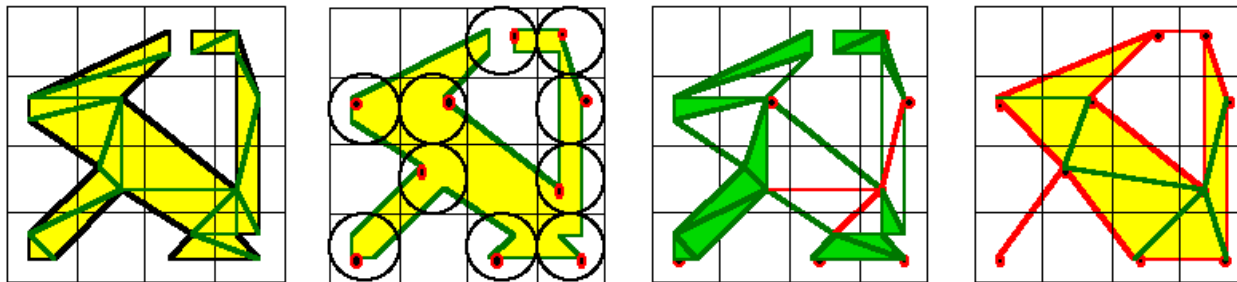
no info in the paper on time complexity!



Vertex Clustering

[Rossignac, Borrel '93]

- detect and unify *clusters* of nearby vertices
(discrete gridding and coordinates truncation)
- all faces with two or three vertices in a cluster are removed
- does not preserve topology (faces may degenerate to edges, genus may change)
- approximation depends on grid resolution



(figure by Rossignac)

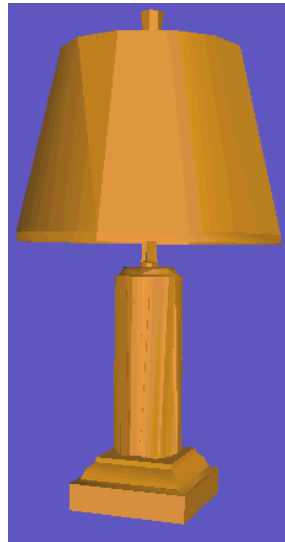
Clustering -- Examples (1)

- Simplification of a table lamp, Interaction Accelerator, courtesy IBM

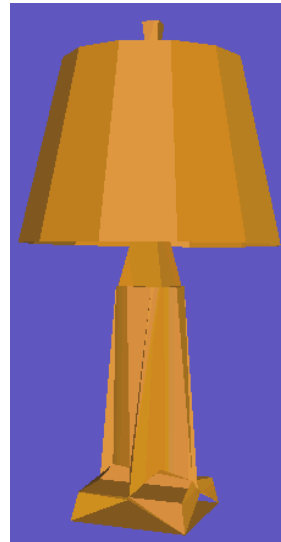
IBM 3D



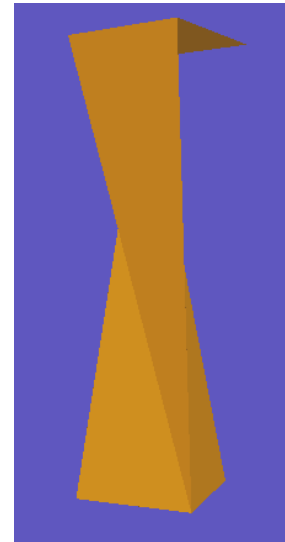
10,108 facets



1,383 facets



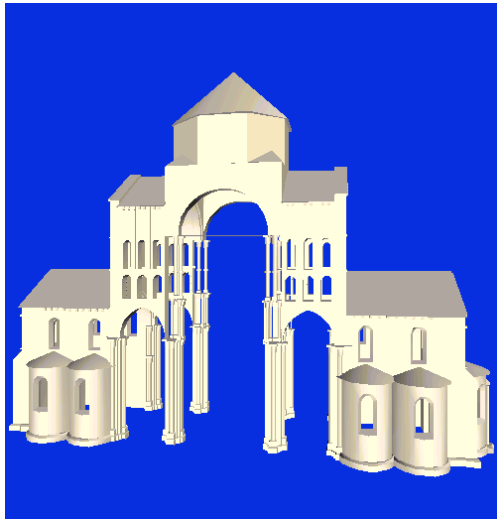
474 facets



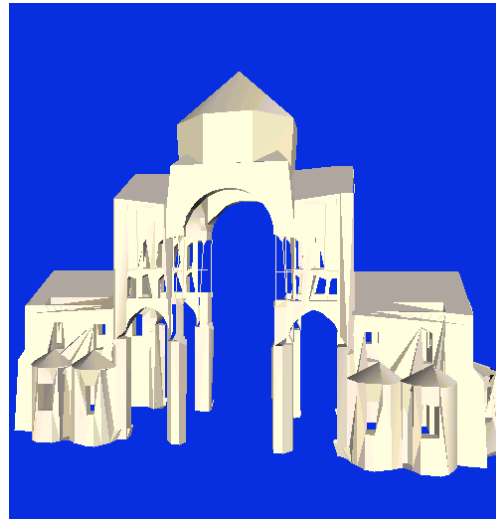
46 facets

Clustering -- Examples (2)

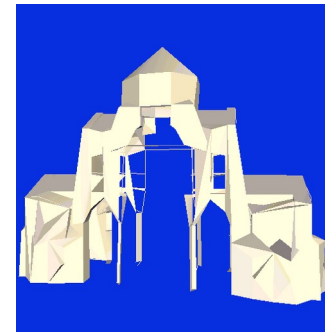
- Simplification of a portion of Cluny Abbey, IBM 3D Interaction Accelerator, courtesy IBM France.



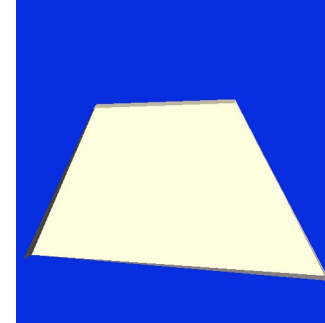
46,918 facets



6,181 facets



1,790 facets



16 facets

Clustering - *Evaluation*

- high efficiency (but timings are not reported in the paper)
- very simple implementation and use
- low quality approximations
- does not preserve topology
- error is bounded by the grid cell size

part of IBM 3D Interaction Accelerator

Multiresolution Analysis

[Eck et al. '95, Lounsbery'97]

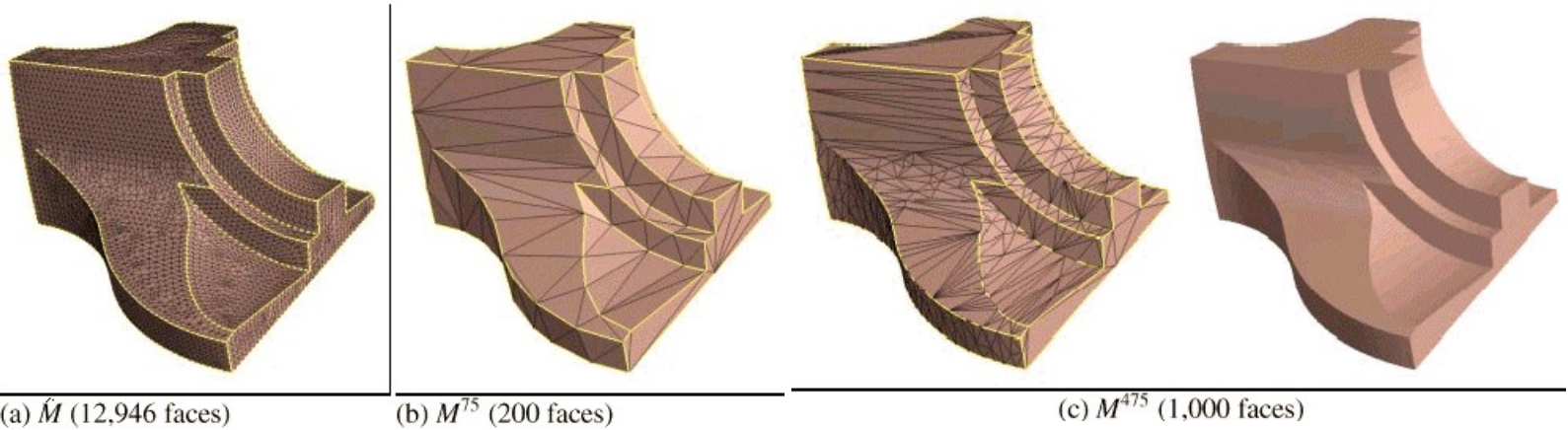
- Based on the **wavelet** approach
 - simple base mesh
 - + local correction terms (wavelet coefficients)

- Given input mesh M:
 - **partition** : build a low resolution base mesh K_0 with tolerance ϵ_1
 - **parametrization** : for each face of K_0 build a parametrization on the corresponding faces of M
 - **resampling** : apply j recursive quaternary subdivision on K_0 to build by parametrization different approximations K_j

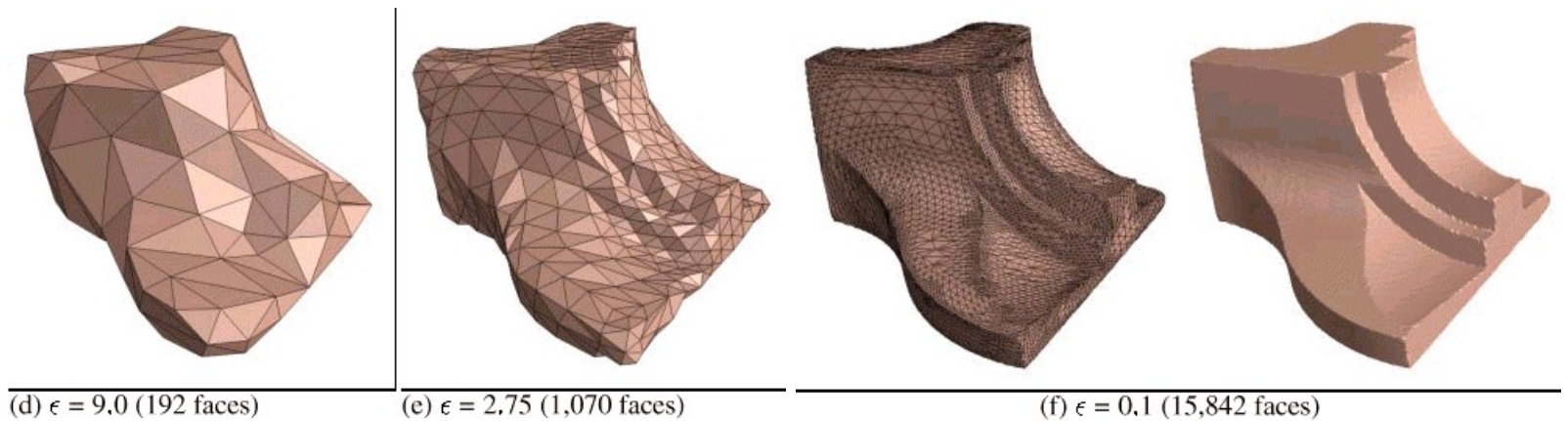
- Supports:
 - bounded error, compact multiresolution repr., mesh editing at multiple scales

Hoppe's experiment: comparative eval. of quality of multiresolution representation

○ Progressive Meshes



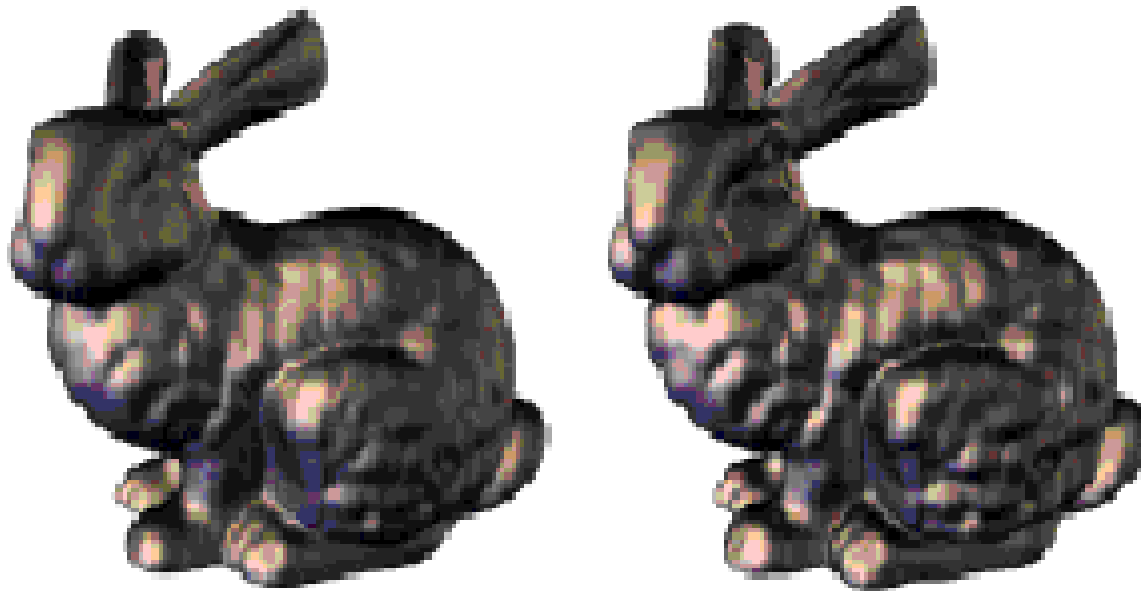
○ Multiresolution Analysis



Multires Signal Processing for Meshes

[Guskov, Swelden, Schroeder 99]

- Still the **Partition, Parametrization and Resampling** approach but the original mesh connectivity is retained:
 - partition is done on the simplified mesh
 - use of a **non-uniform relaxation procedure** (instead of standard triangle quadrisection) that mimics the inverse simplification process
 - Possibility of using signal processing techniques on mesh (eg. Smoothing, detail enhancement ...)



Preserving detail on simplified meshes

- Problem Statement :

how can we preserve in a *simplified* surface
the **detail** (or **attribute value**)
defined on the *original* surface ??

- What one would preserve:

- **color** (per-vertex or texture-based)
- **small variations of shape curvature** (bumps)
- **scalar fields**
- **procedural textures** mapped on the mesh

... Preserving detail on simplified meshes ...

Approaches proposed in literature are:

- **integrated** in the simplification process
(ad hoc solutions **embedded** in the simplification codes)

- **independent** from the simplification process
(post-processing phase to restore attributes detail)

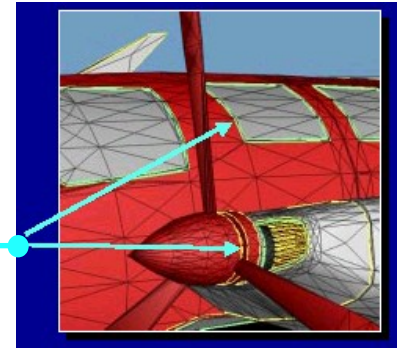
... Preserving detail: Integrated Appr....

Integrated approaches:

- attribute-aware simplification
 - do not simplify an element e **IF** e is on the boundary of two regions with different attribute values
 - or**
 - use an enhanced multi-variate approximation evaluation metrics (shape+color+...)
[Hoppe96, GarHeck98, Frank etal98, Cohen etal98]

- store removed detail in textures
 - *vertex-based* [Maruka95, Soucyetal96]
 - *texture-based* [Krisn.etal96]

- preserve **topology** of the attribute field [Bajaj et al.98]



(image by H. Hoppe)

... Preserving detail: Simplif.-Independent...

Simplification-Independent approach:

our Vis'98 paper

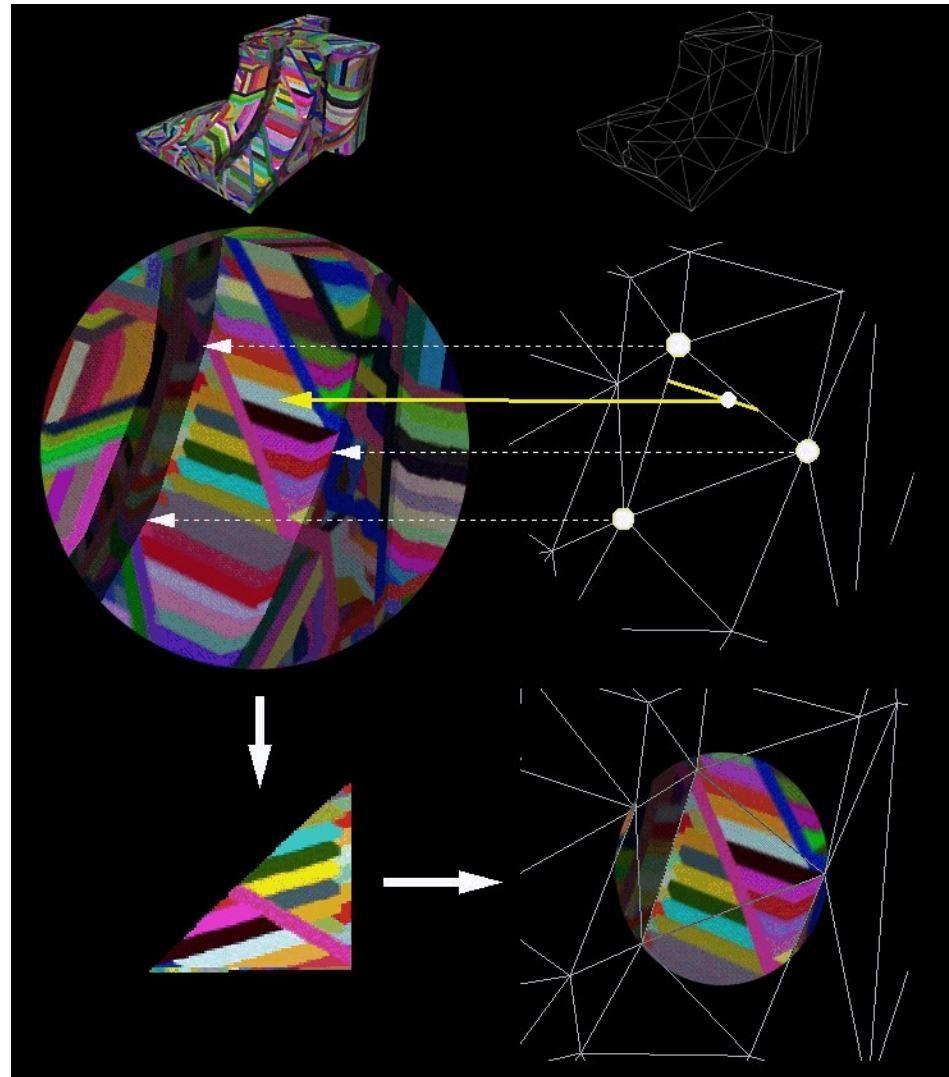
[Cignoni etal 98]

- **higher generality:** attribute/detail preservation is not part of the simplification process
- performed as a **post-processing** phase (after simplification)
- any attribute can be preserved, by constructing an ad-hoc **texture map**

... Preserving detail: Simplif.-Independent...

A simple idea:

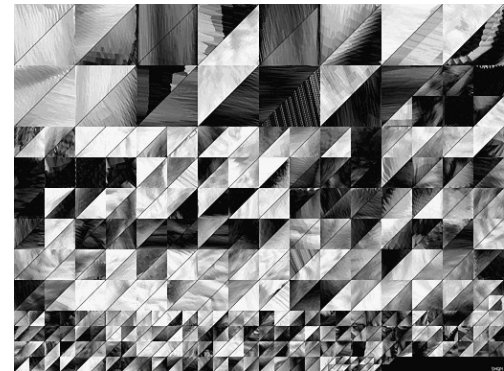
- for each simplified face:
 - detect the original detail
 - code it into a triangular texture map
- pack all textures patches in a std. rectangular texture



... Preserving detail: Simplif.-Independent...

More in detail:

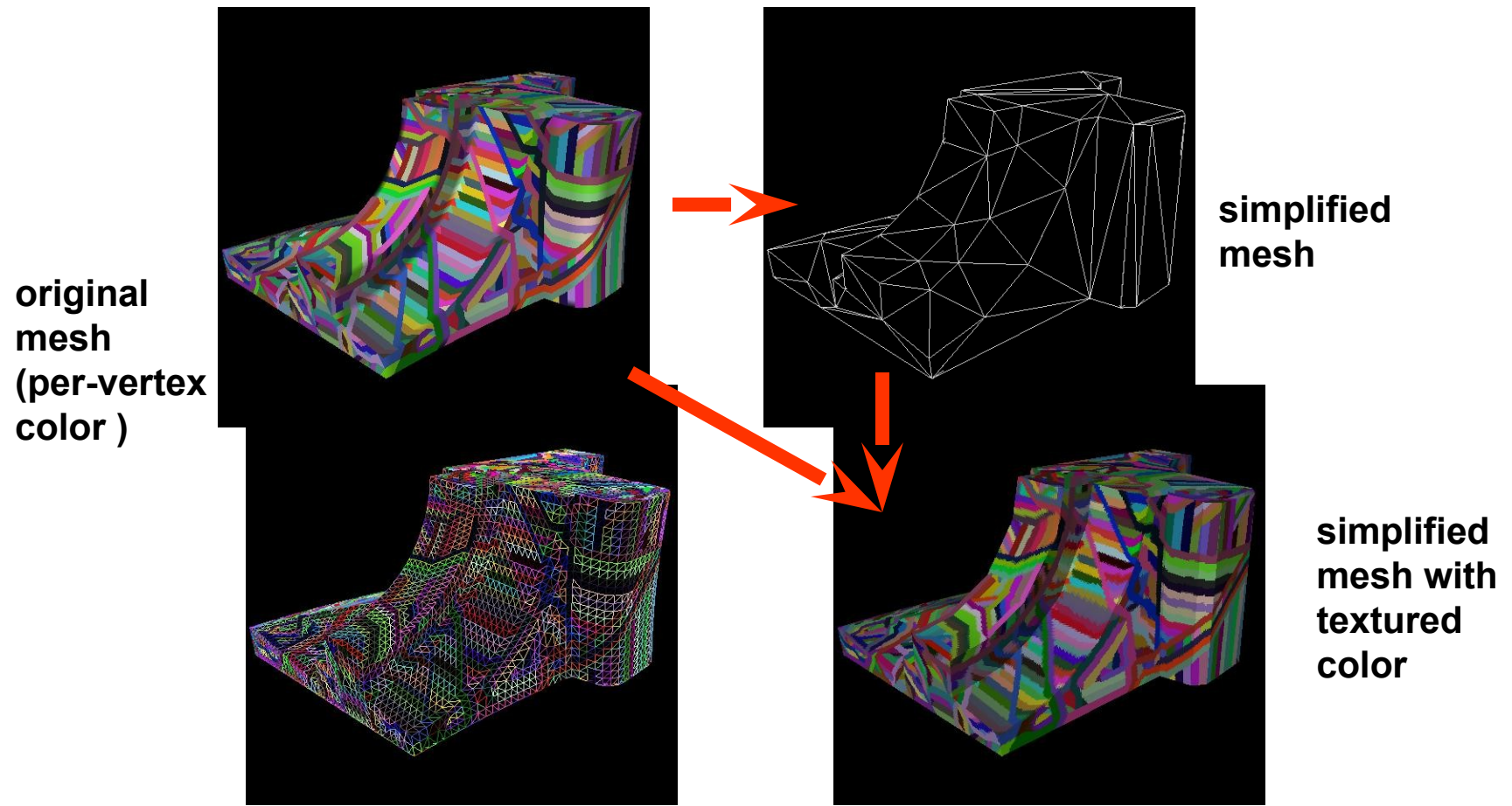
- For each triangular face produce a texture patch, which encodes the “detail” of \mathbf{S} lost in \mathbf{S}_1
 - scan-convert each face of simplified mesh \mathbf{S}_1
 - ✧ for each sample point \mathbf{p} :
 - ☆ find the corresponding point \mathbf{p}' on original \mathbf{S}
 - ▣ **compute** the attribute value in \mathbf{S} on \mathbf{p}'
 - ▣ **store** this value in a **triangular texture patch**
- Texture patches are stored in an efficient manner into a single, rectangular texture
- Use std. texture mapping (sw/hw) to render in real time
Times: tens of seconds



... Preserving detail: Simplif.-Independent...

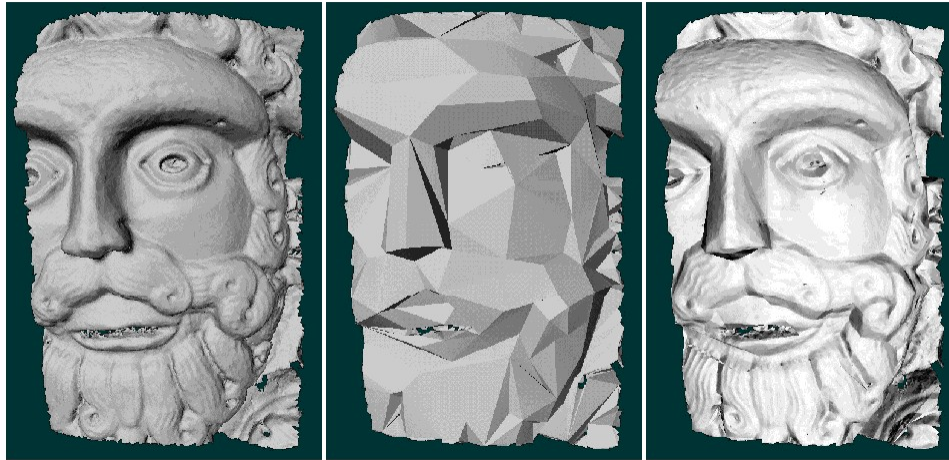
S
I
M
P
L
I
F
I
C
A
T
I
O
N
|
A
L
G
O
R
I
T
H
M
S

- an example of **color** preservation

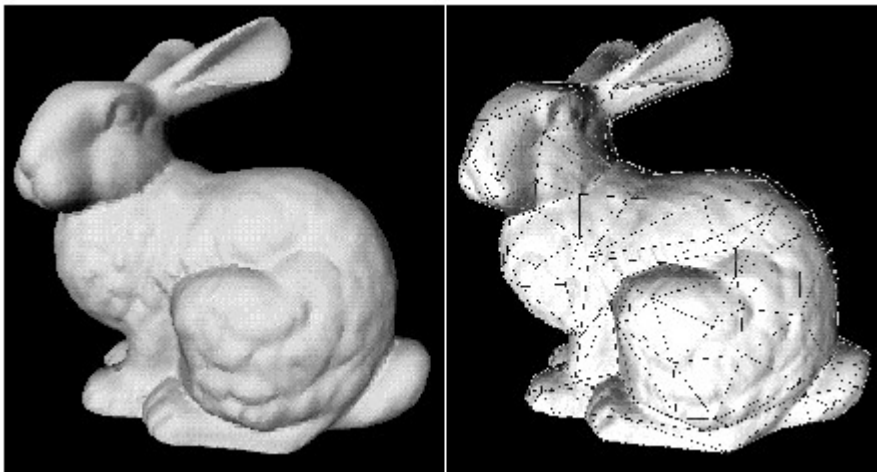


... Preserving detail: Simplif.-Independent...

- example of **geometric detail** preservation by **displacement mapping**



Original 20k face
simplified 500 face



Original 60k faces
simplified 250 faces