Multiresolution Representation and Reconstruction of Triangulated Surfaces

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OUTLINE

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2. Level of detail: tradeoff between mesh complexity and accuracy
3. The multi-triangulation (MT): a general multiresolution model
4. Extracting a shape at variable resolution from an MT
5. Building an MT from different data sources
6. Conclusions

Shape representaton with triangle meshes

- an object with a complex shape can be represented with a mesh of triangles
- each triangle approximates a portion of object surface within a given *accuracy*
- triangle meshes are easy to represent, manipulate, and visualize
- different data sources: CAD, sampling (e.g., data from range scanners, medical data), simulation, ...

Remark:

- more accurate representation $\Rightarrow$ more triangles
- more triangles $\Rightarrow$ higher processing times

Ideas:

- adapting the accuracy to the needs of an application can improve efficiency
- the accuracy of representation can be variable over different portions of the object

Level of detail (LOD)

Basic idea: always using the simplest possible mesh that satisfies the accuracy required by a given application.

Alternative approaches:

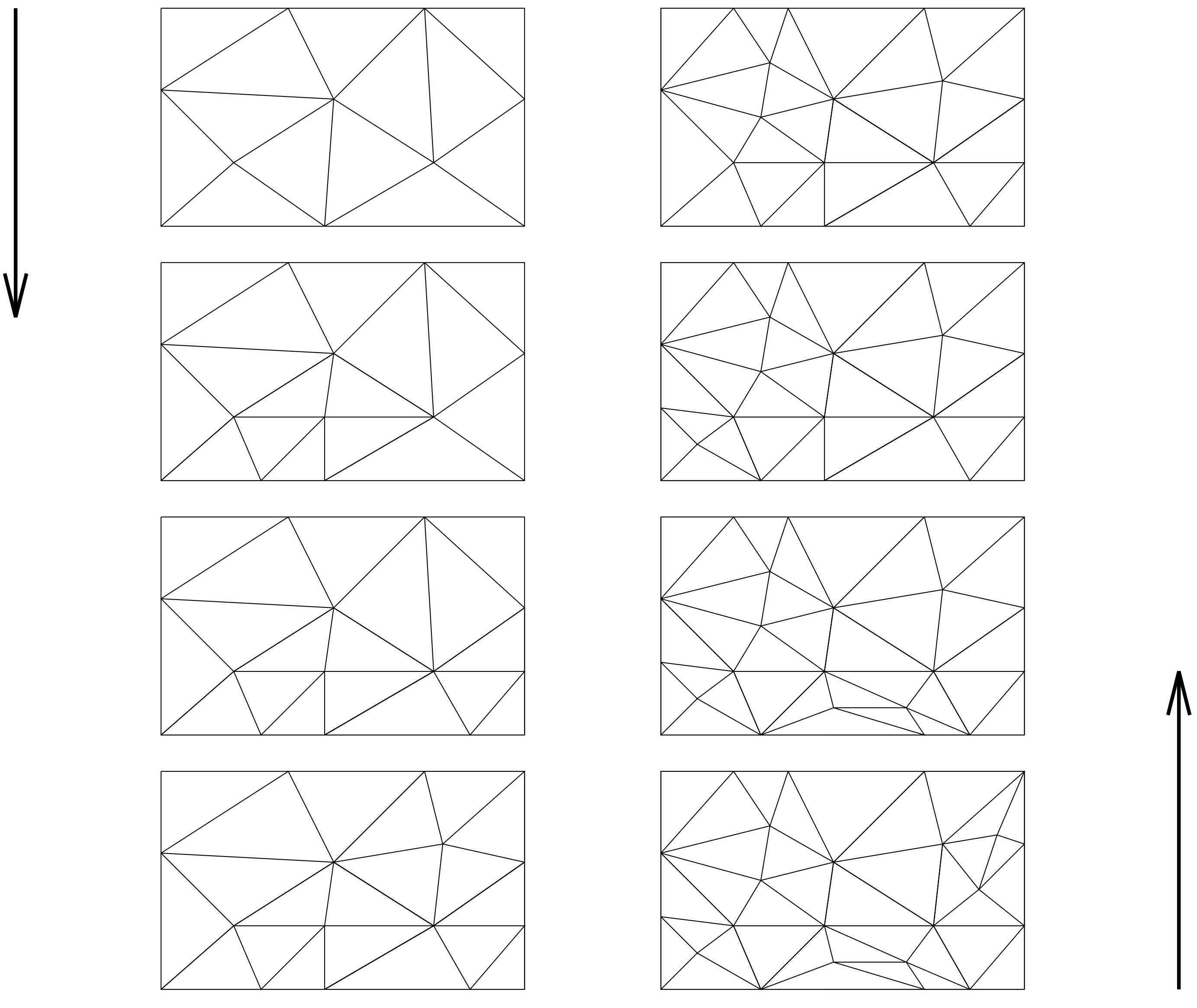
- *on-the-fly simplification*: extract from raw data a mesh of minimal size whose accuracy is sufficient to the application needs.
  - there exist many simplification methods;
  - simplfication is usually an expensive task: no hope to perform it in real time except for some special cases.
- *multiresolution model*: build off-line a model that incorporates many different representations, and can be queried efficiently.
  - more expensive in terms of space
  - much more efficient: support to real time.

A naive LOD model

Basic idea: different accuracies of representation can be obtained through local modifications of a mesh.

A sequence of meshes is obtained by iterative application of local modification steps.

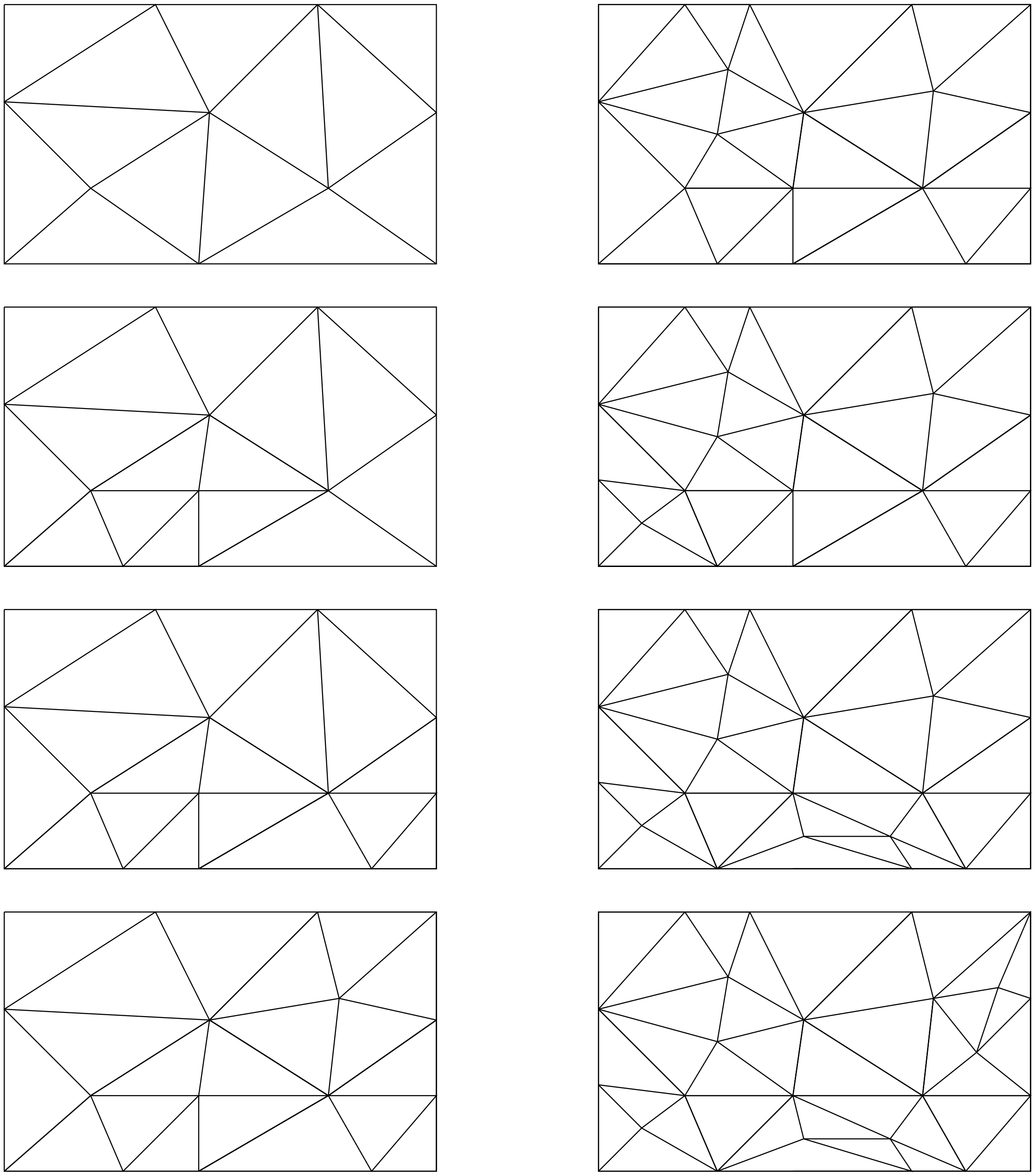
REFINEMENT



SIMPLIFICATION

Remark: the model can be considered as formed by an initial mesh plus a sequence of local modifications (fragments)

Key observation: some fragments are *mutually independent*, i.e., one doesn't need the other to occur

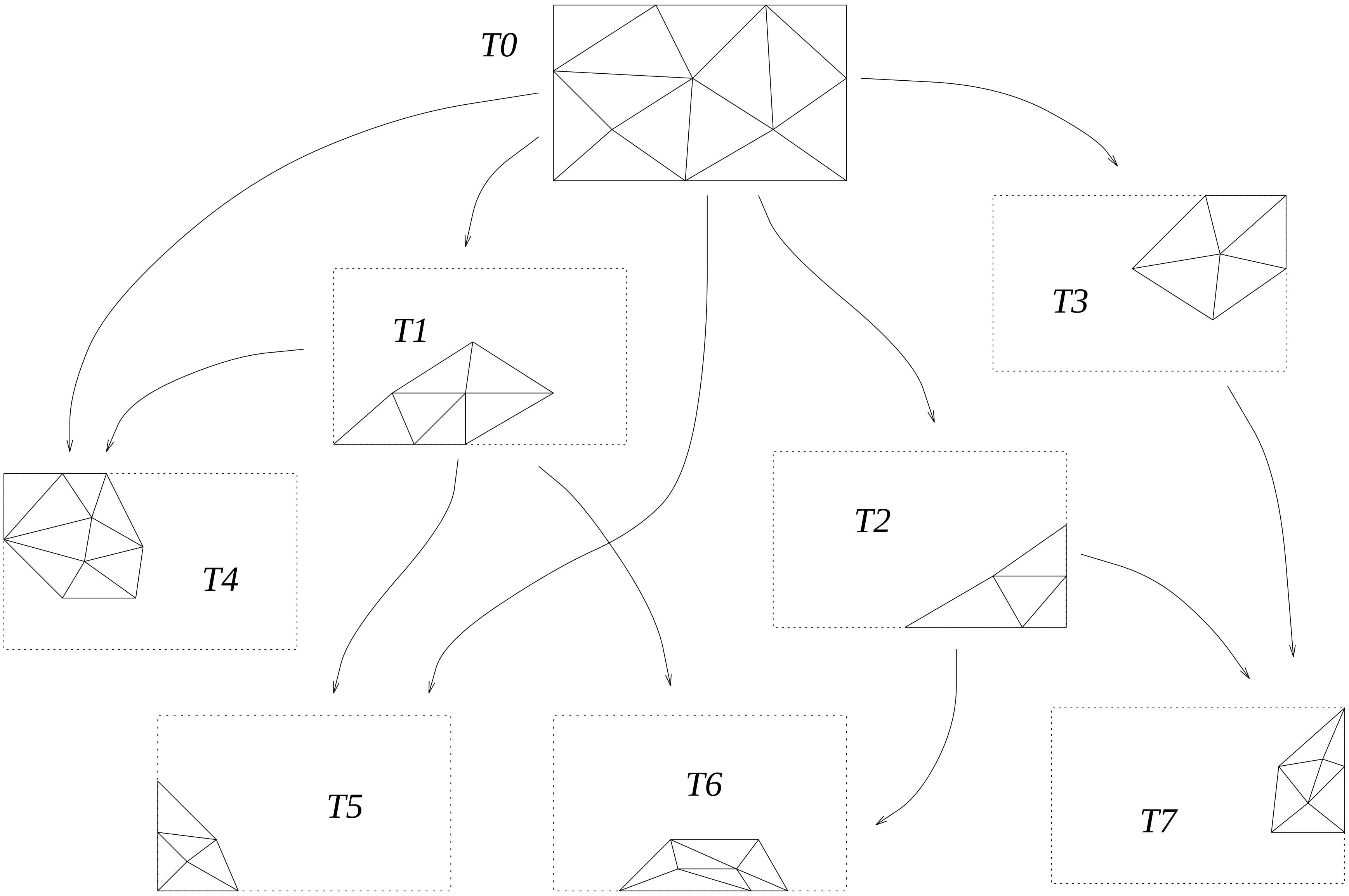


The Multi-Triangulation (MT)

Idea: we don't need a total order!

Fragments are arranged in a partial order:

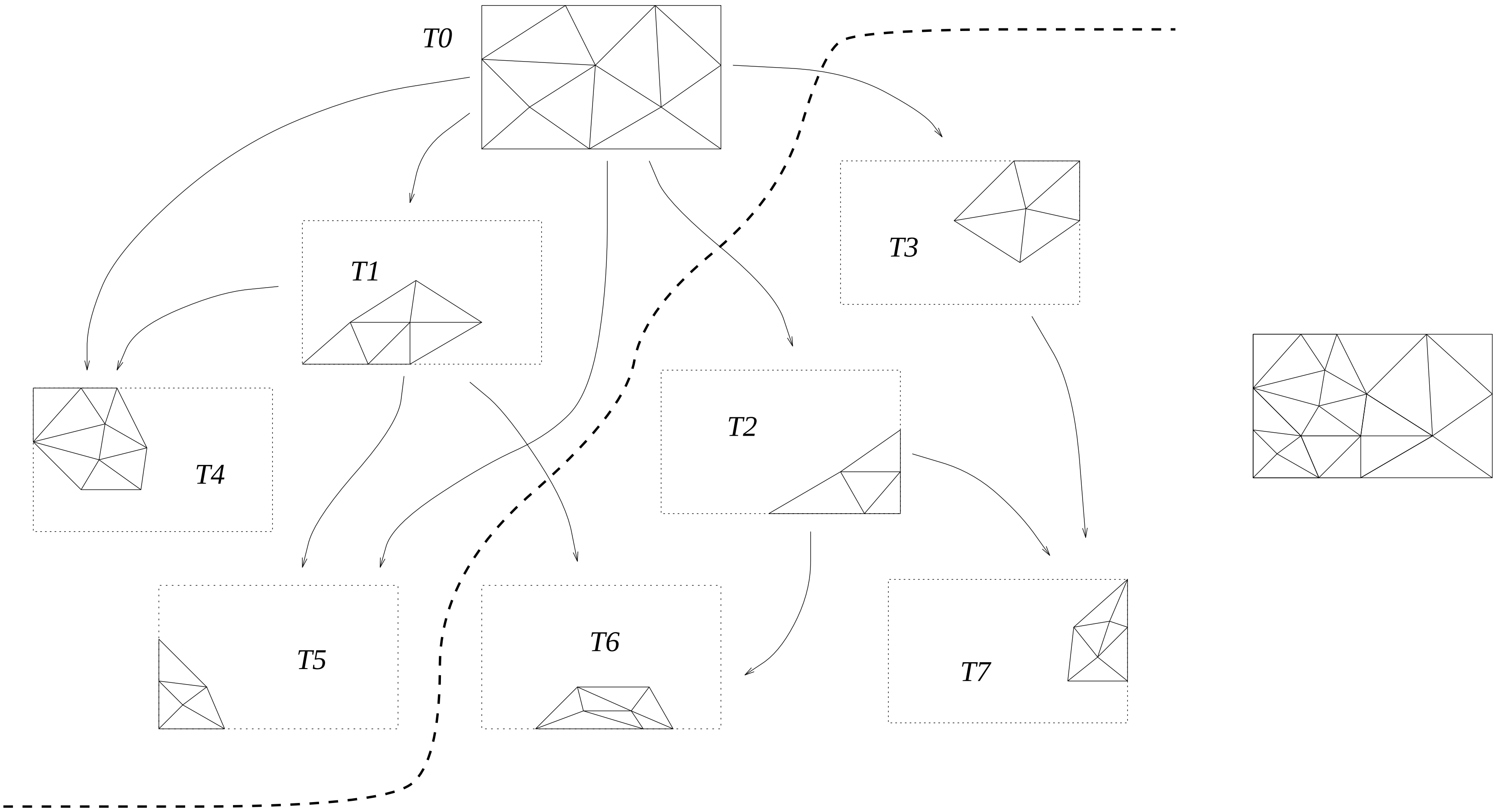
- the coarsest mesh is the minimal element
- fragment T_b follows fragment T_a if T_b refines some portion of T_a


T0
T1
T2
T3
T4
T5
T6
T7

Extracting meshes from an MT

Besic ideas:

- given a *cut* in the MT, the combination of all fragments above the cut forms a mesh;
- if a cut is moved below a given fragment, the mesh is refined (i.e., becomes more accurate) over the area covered by that fragment.


T0
T1
T3
T2
T4
T5
T6
T7

Fundamental results [Puppo, 1996]:

Theorem 1 (expressive power): *any possible* mesh made of triangles of the MT corresponds to some cut.

Theorem 2 (efficiency): given a threshold function τ that bounds the maximum approximation error allowed at each point in space, *the smallest* mesh made of triangles of the MT, and satisfying τ can be found *in linear time*.

Possible extraction algorithms:

- **static**: given an MT, extract the minimal mesh satisfying a threshold function τ;
- **dynamic**: given a current cut on the MT, update the cut to obtain the minimal mesh satisfying a threshold function τ.

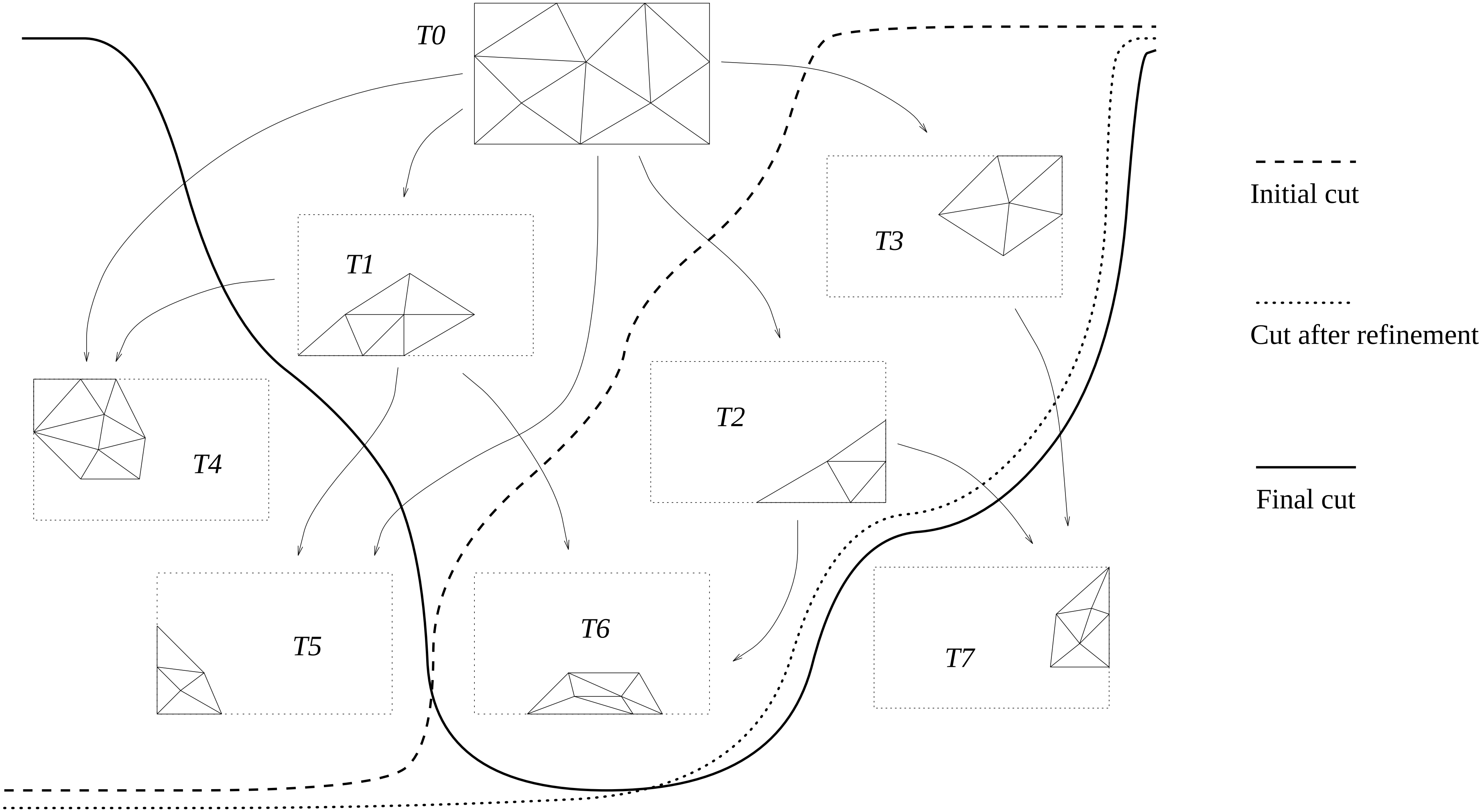
A dynamic extraction algorithm

Idea: given a current cut, and a new threshold:

- move the cut downward where the mesh is not refined enough;
- move the cut upward where the mesh is too refined.

Pseudo-code:

```
1. for every triangle $t$ in the current mesh do
       if $t$ doesn't satisfy $\tau$ then
           move the cut downward until the fragment refining $t$ is above it;
2. while some leaf fragment in the current cut can be eliminated do
       move the cut above it.
```


T0
T1
T2
T3
T4
T5
T6
T7
Initial cut
Cut after refinement
Final cut

Building an MT

From sampled data:

- for height fields (terrains): MT decomposing the domain in the xy plane; triangles are lifted in 3D space by applying the sampled height at each vertex.
  - construction by refinement: start with a simple mesh covering the domain, and incrementally add data points; each point insertion makes a new fragment.
  - construction by simplification: start with a triangulation of all data points, and progressively simplify by decimation; each point deletion makes a new fragment.
- for manifold surfaces: build an initial mesh on all data points, then build the MT by simplification.

  Methods for building the initial mesh:
  - from scattered data: a reconstruction method is needed [Boissonnat 1984, Veltkamp 1993, Bajaj et al. 1995-1996]
  - from contours: the mesh is obtained by connecting contours lying on consecutive planes [Keppel 1985, Fuch et al. 1977, Boissonnat 1988, Geiger 1993]

- from CAD surfaces: data are a collection of adjacent parametric patches (trimmed surfaces)
  - build a 1D multiresolution model for each trimming curve (boundary of a patch), through refinement;
  - build an MT for each patch through refinement, in accordance with the 1D models of its boundaries;
  - weld models of different patches along their common boundaries;
  - the result is a unique MT of the whole surface.

Conclusions

- The MT is a powerful multiresolution structure that generalizes over previously known models
- The MT is suitable to any sort of shapes that can be modeled with triangle meshes
- A minimal mesh satisfying a given accuracy can be extracted in real time

- Prototype systems for real-time visualization of terrains and free-form surfaces (implemented)
- Prototype system for CAD surfaces (under implementation)
- Extension to volume data: Multi-Tetrahedization (under implementation)
- Possible extension to other techniques of shape representation that admit a multiresolution approach based on decomposition, and local updates (e.g.: skeletons)